2. Counting Problems

① unusual → 7 letters

U - 3 times
n - 1 time
s - 1 time
a - 1 time
L - 1 time

$U_1 n U_2 s U_3 a l$

3 groups

$$\frac{7!}{3!\,1!\,1!\,1!\,1!} = \frac{7!}{3!}$$

(U) (n) (s) (a) (L)

---

strings: $5! + \frac{4 \cdot 5!}{\quad} \quad \frac{6 \cdot 5!}{\quad}$

$\frac{}{1!}$ $\frac{}{2!}$ $\frac{}{3!}$

4 letters left after U choose 3 spots

UU _ _ _ _

$$\frac{4!}{3!} = 4$$

$$\frac{4!}{2!2!} = 6$$

Total subsets: $1 + \binom{4}{3} + \binom{4}{2}$

$*|*|*|*|*$

2. $**|**|*$

13 unique values

$$\binom{13}{2}\times\binom{4}{2}\times\binom{4}{2}\times\binom{44}{1}$$

3. FC = Fighting couple

$_|***|**|***|**|*|*$

$*|***|*|***|**|*|*$

Case 1: FC

Case 1: FC gets no song

16 songs - 1 + 6 couples

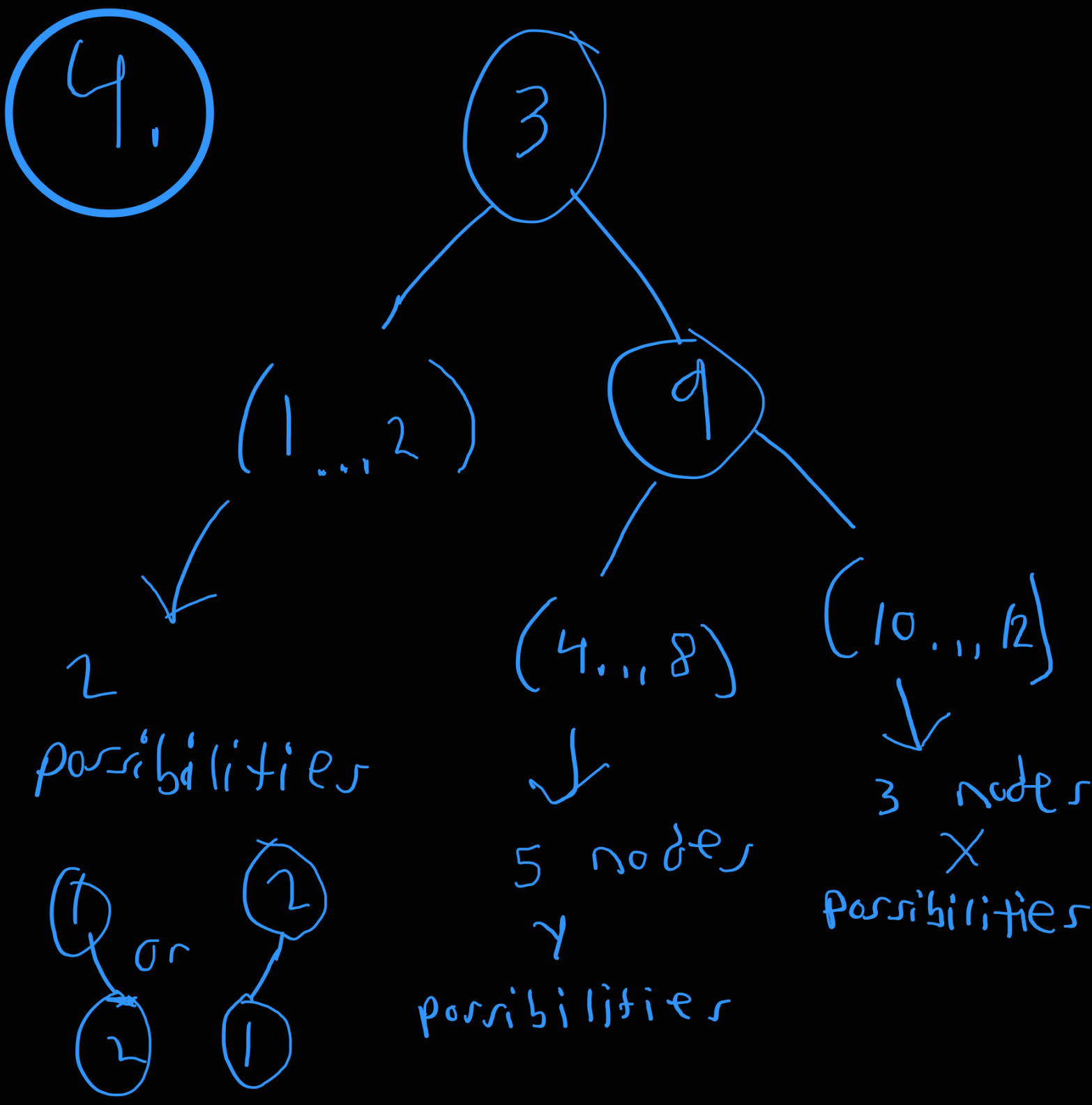
$= 21 \rightarrow \binom{21}{6-1}$

Case 2: FC gets a song

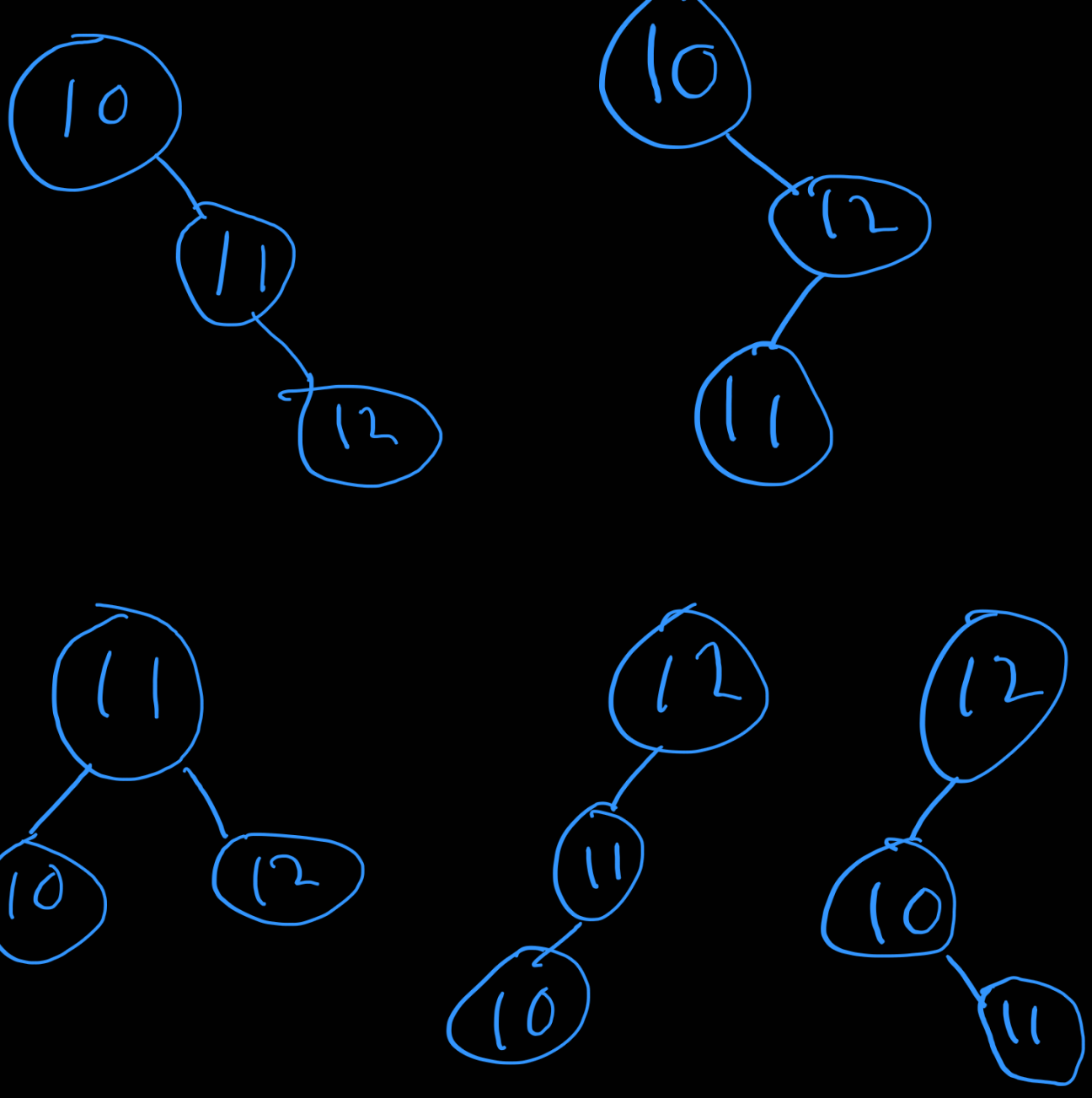
15 songs - 1 + 6 couples

$= 20 \rightarrow \binom{20}{6-1}$

$$\binom{21}{5} + \binom{20}{5}$$



$$Total = 2 \cdot x \cdot y$$



5 possible subtrees

so $X = 5$.

$$\frac{(2n)!}{n!(n+1)!}$$

$n=2$: $\dfrac{(2\cdot 2)!}{2!\,(2+1)!} = \dfrac{24}{2(6)} = 2$ ✓

$n=3$: $\dfrac{(2\cdot 3)!}{3!\,(3+1)!} = \dfrac{6!}{6(24)} = \dfrac{720}{144} = 5$ ✓

$n=5$: $\dfrac{(2\cdot 5)!}{5!\,(5+1)!} = 42$

(4)

$= 42 \cdot$ **420**

Total = 2 · 5 · 12 = 120

5.

(Need to count manually)

Indistinguishable objects over indistinguishable boxes.

Case 1: ☐ ☐ ☐ ☐

7 1 1 1

6 2 1 1

5 3 1 1

5 3 (1)

4 4 1 1

5 2 2 1

4 3 2 1

3 3 3 1

4 2 2 2

3 3 2 2

9 sub-cases

Don't increase from left to right (i.e. (2 3 4))

Case 2:

8 1 1

7 2 1

6 3 1

5 4 1

6 2 2

5 3 2

4 4 2

4 3 3

8 rub-carers

9+8 = **17 carers**

total