Probability

1. $$1\left(\frac{14}{15}\right)\left(\frac{13}{15}\right)\left(\frac{12}{15}\right)\left(\frac{11}{15}\right)\left(\frac{10}{15}\right)\left(\frac{9}{15}\right)\left(\frac{8}{15}\right)$$

$$= \boxed{\frac{17297280}{15^7}}$$

2. 100,000 total

numbers.

5 possibilities for 1st digit.
4 possibilities for 2nd digit
3rd: 7 possibilities
4th: 6
5th: 5

$4 \cdot 5 \cdot 7 \cdot 6 \cdot 5$

Probability of success:

4,200

$100,000$

$Q = 1 -$ probability of success

Bernouli Trial:

8 trials, want 5 successes, $n=8$, $k=5$

$$\binom{8}{5}(0.042)^5(1-0.042)^{8-5}$$

$$= \boxed{56(0.042)^5(0.958)^3}$$

must do Bernouli trials

3. Probability for a 4 or above on dice roll: $3/6 = \frac{1}{2}$

$$P(A) = \underbrace{P(x=2)}_{\substack{2 \text{ succ} \\ 1 \text{ fail} \\ \text{so } n=3 \\ k=2}} + \underbrace{P(x=3)}_{\substack{3 \text{ succ} \\ 0 \text{ fail} \\ \text{so } n=3 \\ k=3}}$$

$$P(x=2) = \binom{3}{2}\left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^1$$

$$= 3\left(\frac{1}{8}\right) = \frac{3}{8}$$

$$P(x=3) = \binom{3}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$$

$$= 1\left(\frac{1}{8}\right)(1) = \frac{1}{8}$$

$P(A) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

Alt method: $\frac{6}{6}\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

$$P(B) = \frac{6}{(6)^3} = \frac{1}{36}$$

111
222
333
444
555
666

6^3 total possibilities

$P(A \cap B) = P(444)$
$+ P(555) + P(666)$
$= \frac{1}{6^3} + \frac{1}{6^3} + \frac{1}{6^3} = \frac{3}{216}$

$= \frac{1}{72}$

$P(A) \cdot P(B) = \left(\frac{1}{2}\right)\left(\frac{1}{36}\right) = \frac{1}{72}$

$P(A \cap B) = P(A) \cdot P(B)$
so A and B are

$\boxed{\text{independent}}$.

(P

1. (Poker)

$$\binom{13}{5} \cdot 4 \text{ suits}$$

$= 1287 \cdot 4 = 5148$ total flushes

$5148 - 4 - 36 = 5108$

(royal flushes) (straight flushes)

Total Poker hands $= \binom{52}{5}$

$= 2{,}598{,}960$

$$\frac{5108}{2{,}598{,}960} \approx \frac{1}{508.8}$$

$\boxed{509}$

Takes about $\boxed{509}$ games for a flush in Poker).

2. (Basketball)

F = Team wins 1 game.

E = Superstar plays.

$$p(F|E) = 70\% = \frac{7}{10}$$

$$p(F|\bar{E}) = 50\% = \frac{5}{10} = \frac{1}{2}$$

$$p(4 \text{ wins} | E) = \binom{5}{4} \cdot \left(\frac{7}{10}\right)^4$$

$$= \frac{5}{3}\left(\frac{7}{10}\right)^4$$

$$p(4\text{ wins} \mid \overline{E}) = \binom{5}{4} \cdot \left(\frac{1}{2}\right)^4$$

plug into total probability formula after calculating Baye's from previous step.

$$p\left(\begin{array}{c}\text{superstar}\\\text{plays}\end{array} \middle| \begin{array}{c}4/5\\\text{wins}\end{array}\right)$$

$$= \frac{p\left(\begin{array}{c}4/5\\\text{games}\end{array} \middle| \begin{array}{c}\text{superstar}\\\text{plays}\end{array}\right) \cdot p\left(\begin{array}{c}\text{superstar}\\\text{plays}\end{array}\right)}{p(E \mid F) \cdot p(F) + p(E \mid \overline{F}) \cdot p(\overline{F})}$$

$$(0.7)^4 \cdot \binom{5}{4} 0.75$$

$$= (0{,}7)^4 \cdot \binom{5}{4} \cdot 0{,}75 + (0{,}5)^4 \binom{5}{4} \cdot (0{,}25)$$

$$\approx 0{,}92 \qquad \boxed{92\%}$$