Physics 426 - Fluid Mechanics

Reading: Week 3 - Chapter 5

Q1: Meaning of vorticity

ω is defined as the angular frequency (or angular speed) and $\frac{d\Theta}{dt}$ is defined as the rate of angular rotation. Therefore, they are equivalent, $\omega = \frac{d\Theta}{dt}$. The units are in radians per second.

Q2: Conservation of vorticity

$$\frac{D\omega}{Dt} = (\omega \cdot \nabla)\boldsymbol{u} + v\nabla^2\omega$$

Let ν be very small and $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$ and ω is only in the vertical direction (i.e. $\omega = (0, 0, \omega_z)$):

$$\frac{D\omega}{Dt} = \cancelto{0}{\omega_x \frac{\partial}{\partial x}(u + v + w)} + \cancelto{0}{\omega_y \frac{\partial}{\partial y}(u + v + w)} + \omega_z \frac{\partial}{\partial z}(\cancel{u} + \cancel{v} + w) + \cancelto{0}{\nu\nabla^2\omega}$$

$$\frac{D\omega}{Dt} = \omega_z \frac{\partial w}{\partial z}$$

From the initial conditions we know $\frac{\partial w}{\partial z} > 0$, therefore it's evident that if $\omega > 0$ then $\frac{D\omega}{Dt} > 0$.