

Physics 426 - Fluid Mechanics

Reading: Week 1

Q1: What does density (tend) to depend on?

The density of a fluid is dependent on temperature and pressure. This is because of the equation of state of a fluid.

Q2: What is pressure?

Pressure is the force exerted by molecules on one another and on the boundaries of whatever is containing the fluid. If a surface is put in a fluid the pressure exerts a force normal to the contour of the surface. Pressure does not have a direction, as it is uniform in all directions.

Q3: How does pressure support water columns?

Since the U pipe system is initially at equilibrium and a weight of 10kg is added to one side (lets call it the left for simplicity) we know that to restore equilibrium the 10kg of water would need to transfer to the other side (the right). So, we can now figure out what the how high 10kg of water in the pipe would be, since we know the density of water:

$$\begin{aligned}\rho_{water} &= 1000\text{kg}/\text{m}^3 = \frac{m_{water}}{V_{water}} \\ V_{water} &= \frac{m_{water}}{\rho_{water}} = \frac{10\text{kg}}{1000\text{kg}/\text{m}^3} \\ V_{water} &= 0.01\text{m}^3\end{aligned}$$

Knowing the area of the cross section is $A = 0.1\text{m}^2$ we can calculate the height that the right side will be above the left side:

$$\begin{aligned}V_{water} &= A_{CrossSection} \times h_{water} \\ h_{water} &= \frac{V_{water}}{A_{CrossSection}} \\ h_{water} &= \frac{0.01\text{m}^3}{0.1\text{m}^2} = 0.1\text{m}\end{aligned}$$

Therefore, the difference in height between the two sides of the pipe would be $h = 0.1\text{m}$. Since this U pipe is a closed system (i.e. this is a two variable system) we know that the amount one side lowers has to be the same as how much the other side rises. Therefore we can say:

$$\begin{aligned}\Delta h_{left} &= \Delta h_{right} \\ \text{and} \\ \Delta h_{total} &= \Delta h_{left} + \Delta h_{right} = h_{water} = 0.1\text{m}\end{aligned}$$

with these two equations it's evident that the side with the weight has lowered by $0.05m$ to $0.95m$ and the side without the weight has risen by $0.05m$ to $1.05m$. In hind sight using a pressure equilibrium argument would have been more robust and efficient.

Q4: Kinematics: How does advection change the Eulerian perception of a quantity?

First find the rate of change in the concentration:

Time spent traveling 100km at 1m/s is 100000 seconds

$$\Delta\rho = \rho_{Upstream} - \rho_{Downstream} = 150g/m^3 - 100g/m^3 = 50g/m^3$$

$$\frac{\Delta\rho}{\Delta t} = \frac{50g/m^3}{100000seconds} = 5.0 \times 10^{-4}g/(m^3 \cdot seconds)$$

After three hours ($\Delta t = 10800$ seconds) at this rate of change the decrease in concentration at the downstream station would be:

$$\Delta\rho_{Downstream} = \frac{\Delta\rho}{\Delta t} \times \Delta t = 5.0 \times 10^{-4}g/(m^3 \cdot seconds) \times 10800seconds = 5.4g/m^3$$

Therefore, after three hours the concentration of Caffeine in the river at the downstream station would be:

$$\rho_{Downstream_{final}} = \rho_{Downstream_{initial}} - \Delta\rho_{Downstream} = 100g/m^3 - 5.4g/m^3 = 94.6g/m^3$$