

Probability Theory and Statistics 2, Assignment 4

1. [4] Suppose X_1, \dots, X_m is a random sample drawn from $N(0, \sigma_X^2)$ and Y_1, \dots, Y_n is a random sample drawn from $N(0, \sigma_Y^2)$. The two samples are (mutually) independent. Let

$$P\left[\sum_{i=1}^m (X_i)^2 \leq \sum_{j=1}^n (Y_j)^2\right] = \beta,$$

where $\beta \in [0, 1]$. Derive an expression (a formula) for σ_Y in terms of the other parameters in the problem and a critical value of a well-known distribution.

2. [4] Suppose X_1, \dots, X_n is a random sample drawn from $\text{EXP}(1/\theta)$. Let M be the sample median and assume n is odd. Express $P[M > \theta]$ in terms of an integral. Note: you do not need to solve the integral!
3. The file “PTS2_assignment4_BMI_data.csv” on Canvas consists of three columns of data. The first column gives the body length (in meters) of 100 patients. They were randomly selected from the Dutch population. The second column gives the weight (in kg) of these patients before treatment. The third column gives the weight (in kg) of the same patients after treatment.

The so-called Body Mass Index (BMI) is defined as the weight (in kg) divided by the body length (in meters) squared and can be considered as a measure for a healthy weight. A BMI between 18.5 and 25 is generally considered as “normal” (see Wikipedia).

For this exercise, you must use computer software (for example R, or Python). Please take a screenshot of your code and include this image in the pdf-file that you submit on Canvas.¹ If you fail to do this, you will loose points.

- [4] Compute a point estimate and a 95%-confidence interval of the mean BMI of the Dutch population.
- [4] Compute a point estimate and a 95%-confidence interval of the percentage of people in the Dutch population with a “normal weight”.
- [4] Suppose you would give the whole Dutch population the same treatment as was given to the sample of 100 patients. Construct a 95%-confidence interval for the difference in mean BMI before and after treatment. [Hint: see exercise 7.25 of the reader.]

¹The screenshot should be visible inside the pdf-document that you submit on Canvas. For this, you can use the `graphicx` package and the command `\includegraphics[scale=]{}`.