Introduction

The following report details the creation of the linear optimization model used to calculate the optimal combination of ingredients in the animal feed and its subsequent adjustments.

1.1 The Linear Model

- I - set of available ingredients
- N - set of nutrients
- R_p - p sets of available ingredient combinations with rules with $p \in \{1, 2, 3, 4, 5, 6, 7\}$

Parameters
- c_i - cost of ingredient $i \in I$
- v_{ij} - amount of nutrient $j \in N$ per kilogram of ingredient $i \in I$
- b_i^{lb}, b_i^{ub} - lower and upper bound on each ingredient $i \in I$
- d_j^{lb}, d_j^{ub} - lower and upper bound on each nutrient $j \in N$
- r_p - upper bound for ingredient combination $p \in \{1, 2, 3, 4, 5, 6, 7\}$

Variables
- x_i: fraction of the feed mix that consists of ingredient $i \in I$

Model's Constraints

$d_j^{lb} \leq \sum_{i\in I} v_{ij}x_i \leq d_j^{ub}$ $\forall j \in N$ -> constraint for nutrient bounds

$b_j^{lb} \leq x_i \leq b_j^{ub}$ $\forall i \in I$ -> constraint for ingredient bounds

$\sum_{i\in I} x_i = 1$ -> all proportion must sum up to 1

$x_i \geq 0$ -> all proportions must not be negative

$\sum_{l\in C_p} x_{pl} \leq r_p$ $\forall p \in \{1, 2, 3, 4, 5, 6, 7\}$ -> all ingredients with rules must obey rules

Model's Objective

$min \sum_{i\in I} c_i x_i$ - minimizing the sum of costs of each ingredient in its proportion

1.2 Binary variable
- I - set of available ingredients
- N - set of nutrients
- R_p - j sets of available ingredient combinations with rules with $p \in \{1, 2, 3, 4, 5, 6, 7\}$

Parameters
- c_i - cost of ingredient $i \in I$
- v_{ij} - amount of nutrient $j \in N$ per kilogram of ingredient $i \in I$
- b_i^{lb}, b_i^{ub} - lower and upper bound on each ingredient $i \in I$
- d_j^{lb}, d_j^{ub} - lower and upper bound on each nutrient $j \in N$

- r_p - upper bound for ingredient combinations $p \in \{1, 2, 3, 4, 5, 6, 7\}$

Variables

- x_i: fraction of the feed mix that consists of ingredient $i \in I$
- y_i - a binary variable.
    - $y_i = 1$ if ingredient i is selected;
    - $y_i = 0$ if the ingredient i is not selected.

Model's Constraints

$$d_j^{lb} \leq \sum_{i \in I} v_{ij} x_i \leq d_j^{ub} \quad \forall j \in N$$

$$b_j^{lb} \leq x_i \leq b_j^{ub} \qquad \forall i \in I$$

$$\sum_{i \in I} x_i = 1$$

$$x_i \geq 0$$

$$\sum_{l \in C_p} x_{pl} \leq r_p \ \forall p \in \{1, 2, 3, 4, 5, 6, 7\}$$

$y_i \in \{0, 1\}$ $\quad \forall i \in I$ -> y is a binary variable

$x_i \leq y_i$ $\quad \forall i \in I$ -> x not equal to 0 only if corresponding y = 1

Model's Objective

$min \sum_{i \in I} y_i$ - minimizing the number of ingredients

1.3 Minimum cost for the minimum number of ingredients

- I - set of available ingredients
- N - set of nutrients
- R_p - j sets of available ingredient combinations with rules with $p \in \{1, 2, 3, 4, 5, 6, 7\}$

Parameters

- c_i - cost of ingredient $i \in I$
- v_{ij} - amount of nutrient $j \in N$ per kilogram of ingredient $i \in I$
- b_i^{lb}, b_i^{ub} - lower and upper bound on each ingredient $i \in I$
- d_j^{lb}, d_j^{ub} - lower and upper bound on each nutrient $j \in N$
- r_p - upper bound for ingredient combinations $p \in \{1, 2, 3, 4, 5, 6, 7\}$
- a - number of ingredients needed

Variables

- x_i: fraction of the feed mix that consists of ingredient $i \in I$
- y_i - a binary variable.
    - $y_i = 1$ if ingredient i is selected;
    - $y_i = 0$ if the ingredient i is not selected.

Model's Constraints

$$d_j^{lb} \leq \sum_{i \in I} v_{ij} x_i \leq d_j^{ub} \quad \forall j \in N$$

$$b_j^{lb} \leq x_i \leq b_j^{ub} \qquad \forall i \in I$$

$$\sum_{i \in I} x_i = 1$$

$$x_i \geq 0$$

$$\sum_{l \in C_p} x_{pl} \leq r_p \ \forall p \in \{1, 2, 3, 4, 5, 6, 7\}$$

$$y_i \in \{0, 1\} \qquad \forall\, i \in I$$

$$x_i \leq y_i \qquad \forall\, i \in I$$

$\sum_{i \in I} y_i = a$ -> number of all elements must be equal to a

Model's Objective

$$min \sum_{i \in I} c_i x_i$$

Resulting variables

```
1 Var Declarations
    available_ingredients_var : Size=26, Index=a
        Key           : Lower : Value
               barley :     0 :            0.0
                blood :     0 :            0.0
              boneash :     0 :            0.0
              casfine :     0 :            0.0
             caswhole :     0 :     0.18676756
               cotton :     0 :           0.05
               dicaph :     0 : 0.00052158349
                   dl :     0 : 0.00073393208
                 fish :     0 :    0.014126323
               fishlq :     0 :            0.0
              gnseeds :     0 :    0.055750927
                ltryp :     0 :            0.0
               lysine :     0 :  0.0015550651
                maize :     0 :            0.2
        maizebranhighq :     0 :           0.25
         maizebranlowq :     0 :            0.0
               mbmeal :     0 :           0.05
                 salt :     0 :  0.0021850656
               shells :     0 :            0.0
            soybeanexp :     0 :    0.068359545
           soybeanmeal :     0 :            0.0
                sugars :     0 :            0.0
             sunflower :     0 :           0.12
        sunflowerseeds :     0 :            0.0
               tapbran :     0 :            0.0
             wheatbran :     0 :            0.0
```

```
2 Var Declarations
    available_ingredients_var : Size=26, Index=a
        Key           : Lower : Value        :
               barley :     0 :          0.0 :
                blood :     0 :          0.0 :
              boneash :     0 :   0.02662492 :
              casfine :     0 :          0.0 :
             caswhole :     0 :          0.0 :
               cotton :     0 :          0.0 :
               dicaph :     0 :          0.0 :
                   dl :     0 : 0.0012161349 :
                 fish :     0 :          0.0 :
               fishlq :     0 :          0.0 :
              gnseeds :     0 :          0.0 :
                ltryp :     0 :          0.0 :
               lysine :     0 :          0.0 :
                maize :     0 :   0.66196751 :
        maizebranhighq :     0 :          0.0 :
         maizebranlowq :     0 :          0.0 :
               mbmeal :     0 :          0.0 :
                 salt :     0 :        0.003 :
               shells :     0 :          0.0 :
            soybeanexp :     0 :          0.0 :
           soybeanmeal :     0 :   0.30719143 :
                sugars :     0 :          0.0 :
             sunflower :     0 :          0.0 :
        sunflowerseeds :     0 :          0.0 :
               tapbran :     0 :          0.0 :
             wheatbran :     0 :          0.0 :
```

```
    available_ingredients_var : Size=26, Index=ava
        Key           : Lower : Value         : U
               barley :     0 :           0.0 :
                blood :     0 :           0.0 :
              boneash :     0 :   0.028172935 :
              casfine :     0 :           0.0 :
             caswhole :     0 :           0.0 :
               cotton :     0 :           0.0 :
               dicaph :     0 :           0.0 :
                   dl :     0 : 0.00063225114 :
                 fish :     0 :           0.0 :
               fishlq :     0 :           0.0 :
              gnseeds :     0 :           0.0 :
                ltryp :     0 :           0.0 :
               lysine :     0 :           0.0 :
                maize :     0 :    0.71727804 :
        maizebranhighq :     0 :           0.0 :
         maizebranlowq :     0 :           0.0 :
               mbmeal :     0 :           0.0 :
                 salt :     0 :         0.003 :
               shells :     0 :           0.0 :
            soybeanexp :     0 :           0.0 :
           soybeanmeal :     0 :    0.25091677 :
                sugars :     0 :           0.0 :
             sunflower :     0 :           0.0 :
        sunflowerseeds :     0 :           0.0 :
               tapbran :     0 :           0.0 :
             wheatbran :     0 :           0.0 :
    existence_of_product : Size=26, Index=availabl
```

From left to right. Figure 1. The resulting variable values to solve the model 1.1. Figure 2. The resulting variable values required to solve the model 1.2. Figure 3. The resulting variable values required to solve the model 1.3.

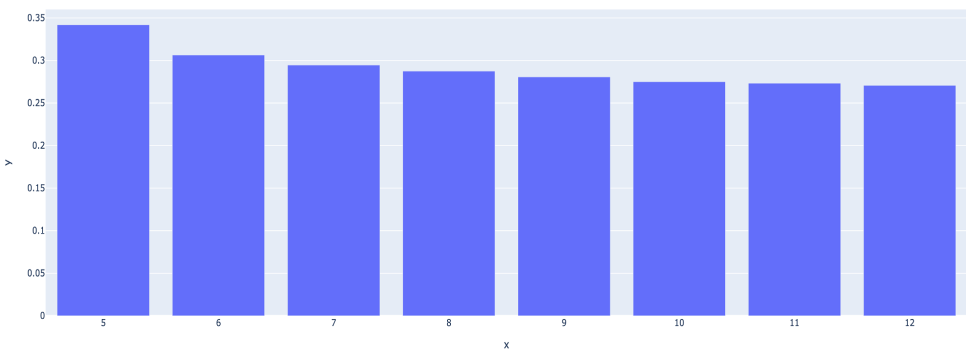

Figure 4. Graph showing the relationship between the number of ingredients used and the total cost of the feed mix.

Our team suggests choosing 7 as a number of ingredients to be used in a mix. Rationale - the cost of the feed mix does not decrease much after 7 ingredients have been used.

Solutions for 1.1. to 1.3.: 0.2697643896412; 5; 0.341822823725.