PROBLEM LIST 2

(1) (10 points) **Stigler diet**. The diet planning problem is considered. There are 77 foods reached of 9 nutrients. Recommended daily consumption of nutrient i is defined by number k_i. For each food j and nutrient i the amount of nutrient per 1$ spent on this food is defined by $s_{j,i}$. The goal is to find amounts of money spent for each type of food in 365 days and consume all nutrients in numbers which ratio to required consumption lays in $[1-\delta, 1+\delta]$. Objective – minimize amount of total spent money. Create Linear programming model and find the solutions for given data set and $\delta = 0.1$.

(2) (10 points) **Sudoku puzzle**. Sudoku puzzle objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 boxes contain all of the digits from 1 to 9. Create Integer Programming model and find the solution of the following instance.

			8		1			
							4	3
5								
				7		8		
						1		
6							7	5
		3	4					
			2			6		

(3) (10 points) **Bin Packing**. There are n items with different weights and m bins with equal capacity. These items should be packed into bins such that total weight of items packed into one bin doesn't exceed capacity of this bin. Find minimal number of required bins.

(4) (20 points) **No 4-cycles**. How many edges can be in the graph with n vertices without cycles of the length 4? Create Integer Programming model and find solution for $n = 7, 8$.

(5) (15 points) **Single Machine Scheduling**. There are n tasks which should be processed by a single machine for the time interval [0,..,T]. We consider discrete time moments. Each task has release time, processing time and deadline. Only one

task can be processed by the machine at each time moment. Makespan of schedule is the completion time of the latest task. Find a minimal makespan schedule.

(6) (15 points) **Shortest path.** There is a direct graph $G(V, E)$. For each edge $(i, j) \in V$ its length is defined by l_{ij}. If there is no edge from i to j, then $l_{ij} = -1$ and $l_{ii} = 0$ for each $i \in V$. The objective is to find shortest path from a to b. Create Integer Programming model and find solutions for three different pairs of vertices defined in the attached materials.

(7) (20 points) **Assignment with minimal cost.** There is a set of workers W and a set of tasks N, such that $|W| = |N|$ and assignment cost function c_{ij} defined for each $i \in W, j \in N$. The objective is to assign workers on tasks with minimal cost. The correct Integer Programming model is described below.

- Binary variables: $\forall i \in W, j \in N x_{ij}$ equals to 1 iff worker i assigned on task i.
- Objective: $\min \sum_{i \in W} \sum_{j \in N} c_{ij} x_{ij}$.
- Constraints:
  - each worker should be assigned on one task: $\forall i \in W : \sum_{j \in N} x_{ij} = 1$;
  - each task should be assigned to one worker: $\forall j \in N : \sum_{i \in W} x_{ij} = 1$.

Prove that for this model fractional solution of linear relaxation is optimal solution of Integer Programming model. There is no need to implement model, only theoretical proof is required.