Probability Notes

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Chapter 1

Introduction to Probability

1.1 Interpretation of Probability

1.1.1 The Frequency Interpretation of Probability

Definition

The probablity that some specific outcome of a process can be intrepreted to mean the relative frequency with which the outcome can be obtained if the process is repeated for a large number of times under similar conditions.

Example

Toss coin for 1,000,000 times, number of heads is nearly 500,000, but may not exactly 500,000.

Shortcoming

- number of tests: how large is enough
- similar conditions: conditions cannot be completely the same, otherwise always same outcome
- frequency of outcomes: should approximate theoritical probablity, but no permissible variation
- repetition: many important problems have no repetition. For instance, probablity of a aquaintance

1.1.2 Classical Interpretation & Subjective Interpretation

Classical

Based on equally likely outcome. Paradox: this concept is based on the probablity we are trying to define. Example: six-sided dice, equally 1/6.

Subjective

Based on personal belief and information.

1.2 Experiments and Events

1.2.1 Definition

Experiment

any process in which the possible outcomes can be identified.

Event

a well-define set of possible outcomes of the experiment.

1.2.2 Explanation

Not every set of possible outcomes will be called an event. The probability of an event will be how likely it is that the outcome is in the event.

1.3 Set Theory

1.3.1 Definiton

Set

Collection of process outcomes of an experiment.

Empty Set

Some events are impossible.

Infinite set

Infinitely many outcomes. If countable, there is one-to-one correspondence. If either finite or countable, a set has at most coutably many items.

1.3.2 Operations on Sets

Union of Sets

If $A_1,\ A_2,\ \ldots$ are countable collection of events,then $\cup_{i=1}^{\infty}$ is also an event. But $\cup_{i=1}^{\infty}$ does not necessarily have to be an event.

Overlaps of Evnets

Look at the pictures at page 7, if events are not disjoint, their union can be deduced from intersections.

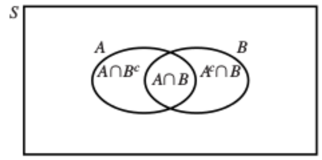

Figure 1.1: intersection of two events

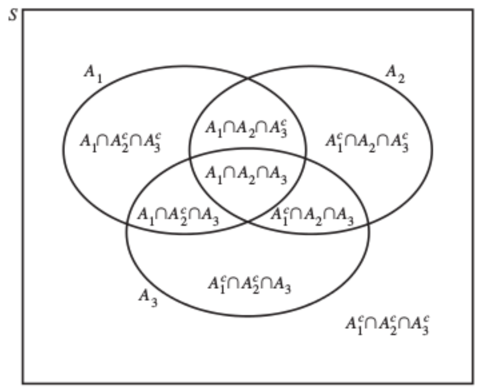

Figure 1.2: intersection of three events

Useful Theorem

$$A = (A \cap B) \cup (A \cap B^c)$$
$$A \cup B = B \cup (A \cap B^c)$$

1.4 Probability

1.4.1 Axioms

- For every event A , $Pr(A) \geq 0$
- $Pr(S) = 1$
- For every infinite sequence of disjoint events

$$Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} Pr(A_i)$$

For finite sequence of disjoint events, equation above still holds

1.4.2 Theorems

- For every two events A and B, $Pr(A \cap B^c) = Pr(A) - Pr(A \cap B)$.

  The reason is

  $$A = (A \cap B) \cup (A \cap B^c)$$

  According to Axiom 3, we have

  $$Pr(A) = Pr(A \cap B) + Pr(A \cap B^c)$$

  Therefore

  $$Pr(A \cap B) = Pr(A) - Pr(A \cap B^c)$$

- For every two events A and B, $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$.

  $$A \cup B = B \cup (A \cap B^c)$$
  $$Pr(A \cup B) = Pr(B) + Pr(A \cap B^c)$$

  From theorem above

  $$A = (A \cap B) \cup (A \cap B^c)$$
  $$Pr(A) = Pr(A \cap B) + Pr(A \cap B^c)$$

  Switch Pr(A) to the right

  $$Pr(A \cap B^c) = Pr(A) - Pr(A \cap B)$$

  Finally

  $$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap$$

- Bonferroni Inequality. For all events $A_1, \ldots, A_n$,

  $$Pr(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} Pr(A_i) \text{ and } Pr(\bigcap_{i=1^n A_i}) \leq 1 - \sum_{i=1}^{n} Pr(A_i^c)$$

1.5 Counting Methods

1.5.1 Multiplication Rule

- An experiment has $k(k \leq 2)$ parts, that the ith part has n_i outcomes
- The outcomes of each part is independent of each other
- The total number of possible outcomes is $n_1 n_2 \cdots n_k$.

1.5.2 Permutation

Selecting k elements from a set of n at a time without replacement will be a permuation of n elements taken k at a time. Denoted by $P_{n,k}$, be awared of $0! = 1$

$$P_{n,k} = n(n-1)(n-2)\cdots(n-k+1)$$

Another form

$$P_{n,k} = n(n-1)(n-2)\cdots(n-k+1)\frac{(n-k)(n-k-1)\cdots 1}{(n-k)(n-k-1)\cdots 1} = \frac{n!}{(n-k)!}$$

When $k = n$,

$$P_{n,n} = n!$$

Bibliography

[1] Ronald L. Granham, Donald E. Knuth, and Oren Patashnik, *Concrete Mathematics*, Addison-Wesley, Reading, MA, 1995.