

From Waveforms to Bits

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Fourier Analysis

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Fourier Series

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Bandwidth-Limited Signals

Some definitions

- Bandwidth (analog)
- Bandwidth (digital)
- Cutoff
- Baseband
- Passband

Bandwidth-Limited Signals

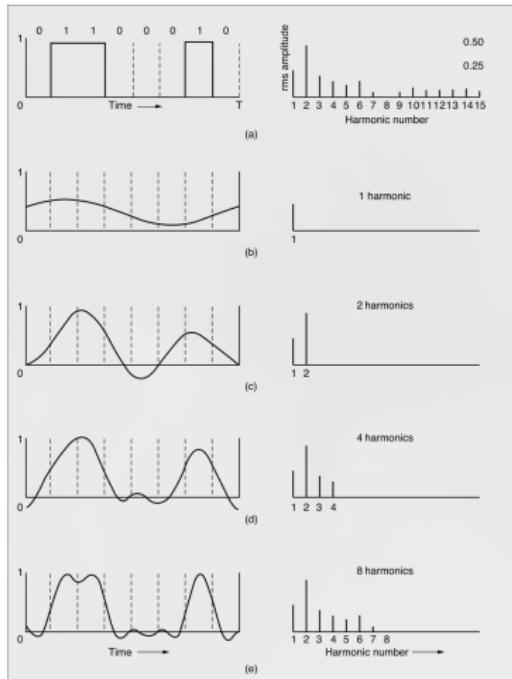


Figure 1: (a) A binary signal and its root-mean-square Fourier amplitudes. (b)-(e) Successive approximations to the original signal.

Bandwidth-Limited Signals

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Table 1: Relation between data rate and harmonics.

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Nyquist's theorem

Maximum data rate of channel = $2B \log_2(V)$ bits/sec

What about interference?

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Capacity of a real channel

Maximum data rate of channel = $B \log_2(1 + \frac{S}{N})$ bits/sec

Digital Modulation

Baseband transmission

Baseband Transmission

NRZ scheme (Non-Return-to-Zero)

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NRZ scheme (Non-Return-to-Zero)

- Signal follows the data. (i.e. Positive voltage for one, negative voltage for zero)
- Sender sends the signal.
- Receiver samples the signal at regular intervals of time.
- Receiver maps signal samples to the closes symbols (i.e. {0, 1})

Baseband Transmission

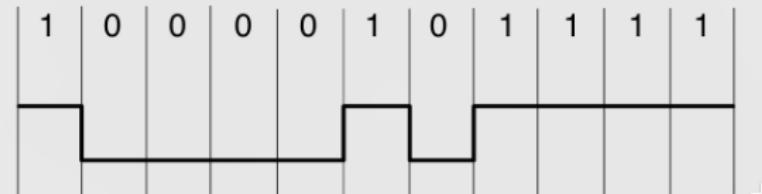
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(a) Bit stream

1	0	0	0	0	1	0	1	1	1	1
---	---	---	---	---	---	---	---	---	---	---

(b) Non-Return to Zero (NRZ)



Bandwidth Efficiency

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- Bitrate can be interpreted as (**symbol rate** \times **bits per symbol**).

Clock Recovery

The problem

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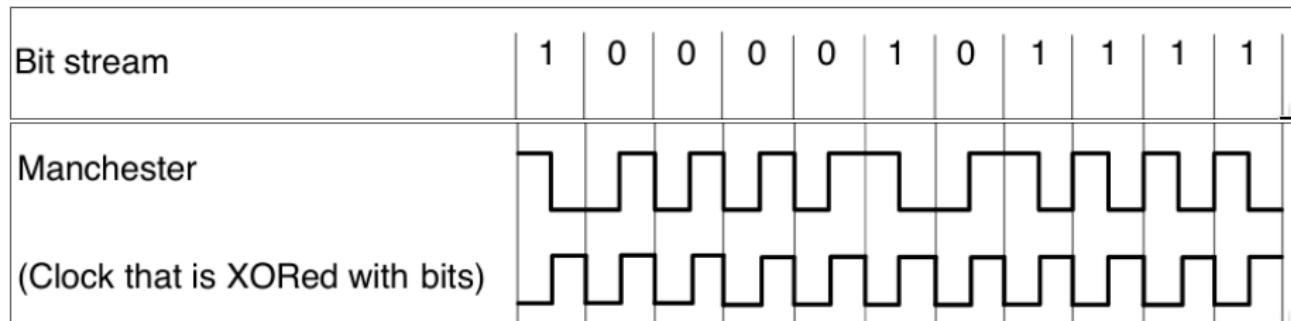
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- After a while it's hard to tell the bits apart, 15 zeroes look much like 16 zeroes unless you have a very accurate clock.
- Very accurate clocks are expensive.

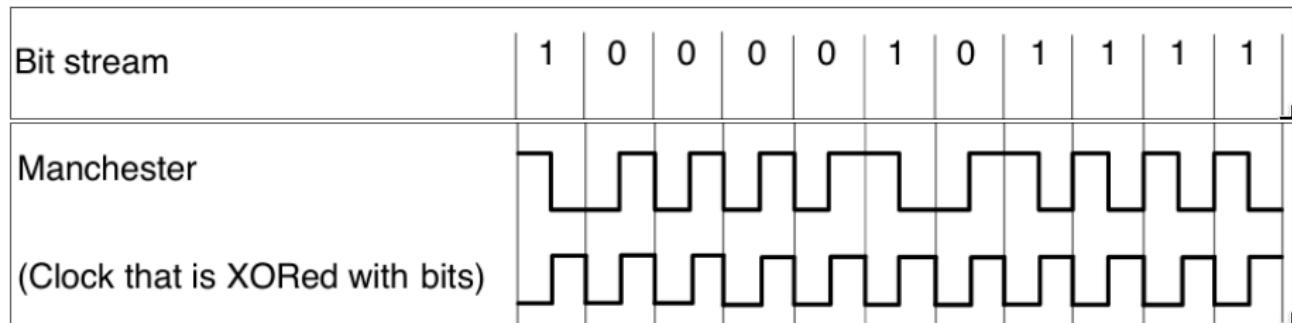
Clock Recovery

Manchester encoding



Clock Recovery

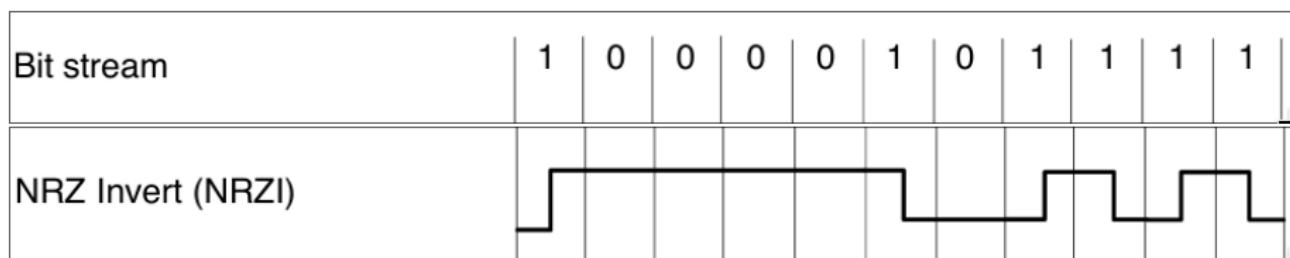
Manchester encoding



- Requires twice the bandwidth.
- Used in classic Ethernet.

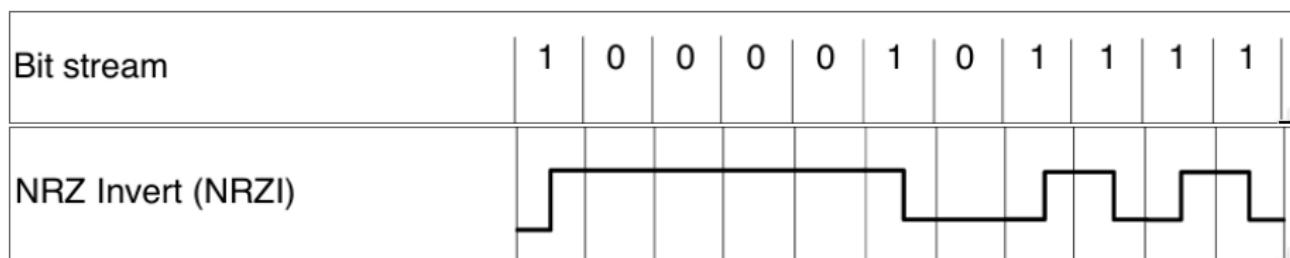
Clock Recovery

NRZI (Non-Return-to-Zero Inverse)



Clock Recovery

NRZI (Non-Return-to-Zero Inverse)



- Long streaks of 0 still have the same problem.
- Used in USB.

Clock Recovery

4B/5B

Data (4B)	Codeword (5B)	Data (4B)	Codeword (5B)
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

Table 2: 4B/5B mapping.

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Table 2: 4B/5B mapping.

- 25% overhead.

Scrambling

Scrambles the data by XORing it with a pseudorandom sequence.
Receiver XORs the data with the same sequence.

Scrambling

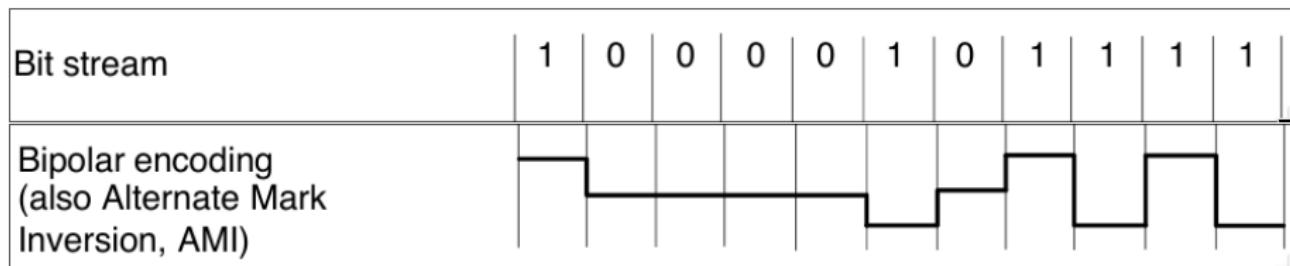
Scrambles the data by XORing it with a pseudorandom sequence.
Receiver XORs the data with the same sequence.

- Used in early versions of IP over SONET.
- Still possible to get long streaks of identical symbols.
- Possible malicious usage, "killer packets".

Balanced Signals

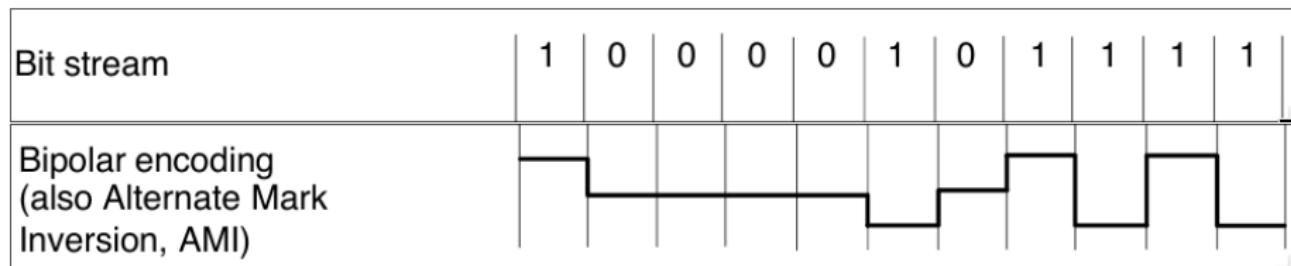
Balanced Signals

Bipolar Encoding



Balanced Signals

Bipolar Encoding



- Adds a voltage level.
- Average of 0.
- Indirectly helps with clock recovery.

Balanced Signals

4B/5B (again) or 8B/10B

Balanced Signals

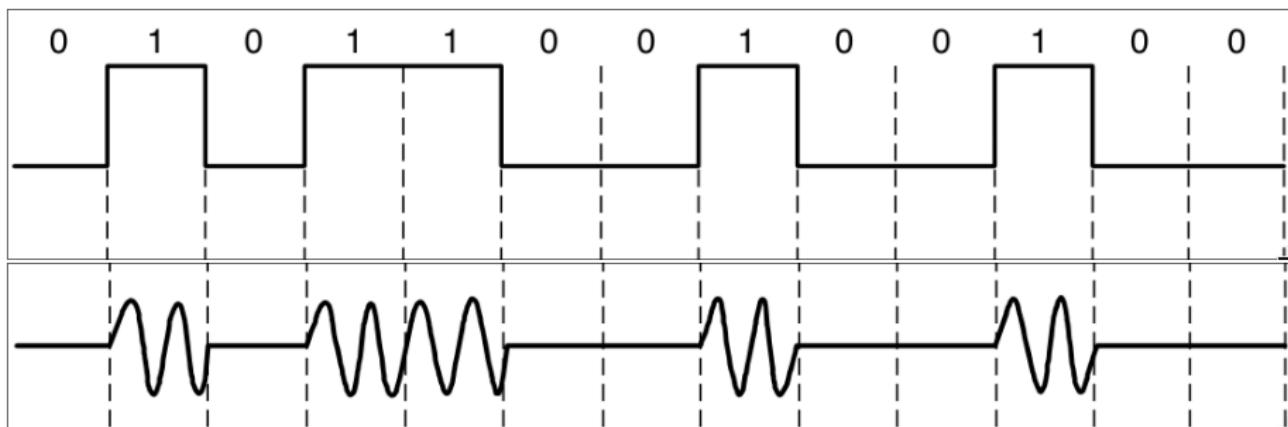
4B/5B (again) or 8B/10B

- 8B/10B has at most a disparity of 2 bits.
- 8B/10B is 80% efficient.
- Helps with clock recovery.

Passband Transmission

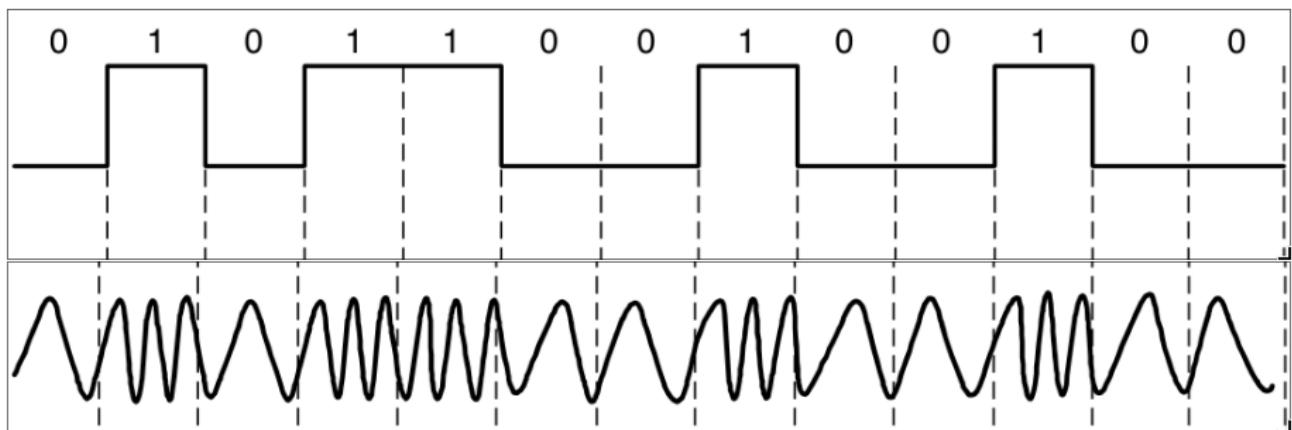
Passband Transmission

ASK (Amplitude Shift Keying)



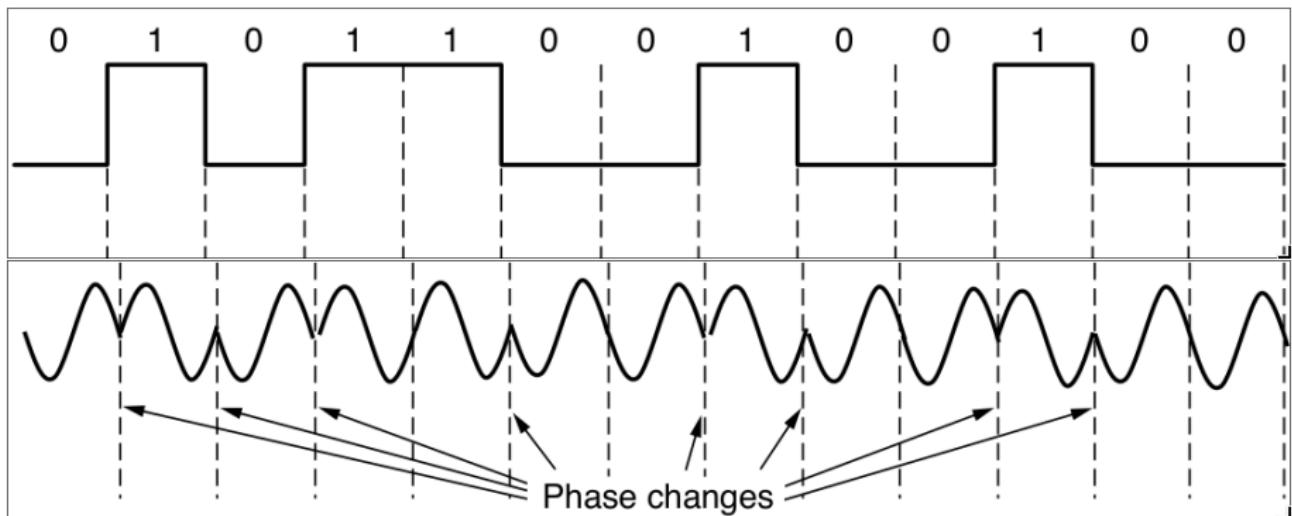
Passband Transmission

FSK (Frequency Shift Keying)



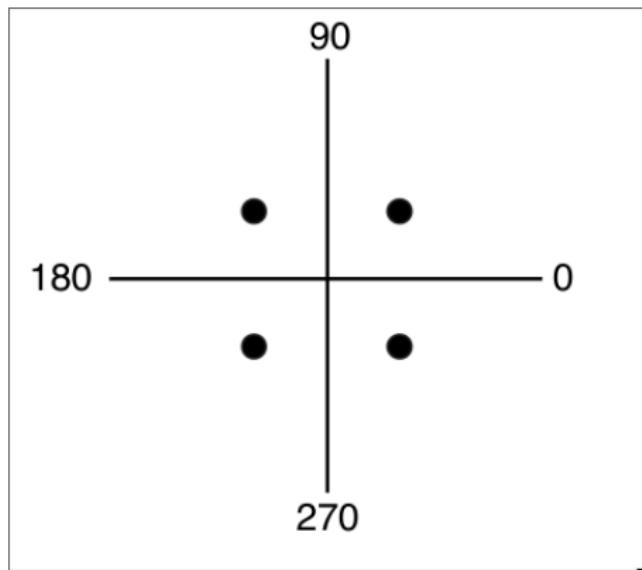
Passband Transmission

PSK (Phase Shift Keying)



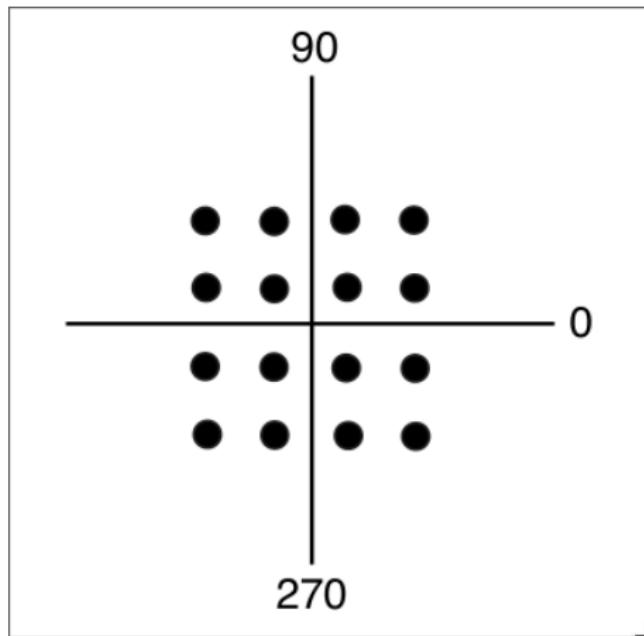
Passband Transmission

QPSK (Quadrature Phase Shift Keying)



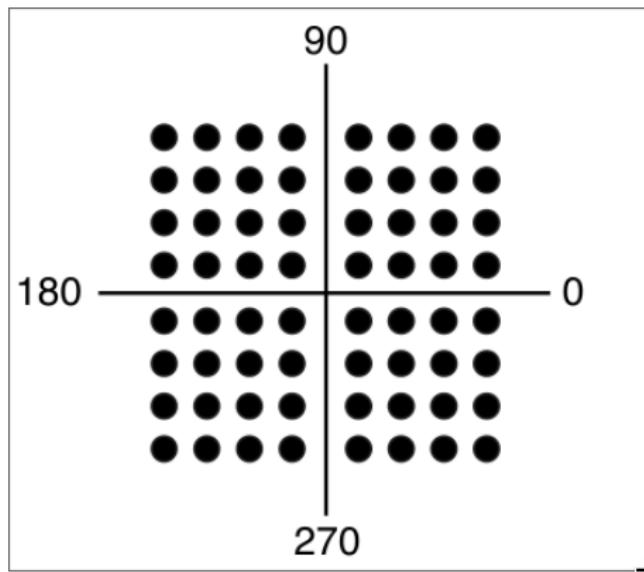
Passband Transmission

QAM-16 (Quadrature Amplitude Modulation - 16)



Passband Transmission

QAM-64 (Quadrature Amplitude Modulation - 64)



Passband Transmission

Gray-coded QAM-16

