From Waveforms to Bits

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MESTRADO EM ENGENHARIA INFORMÁTICA

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Fourier Analysis

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Fourier Series

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Bandwidth-Limited Signals

Some definitions

- Bandwidth (analog)
- Bandwidth (digital)
- Cutoff
- Baseband
- Passband

Bandwidth-Limited Signals

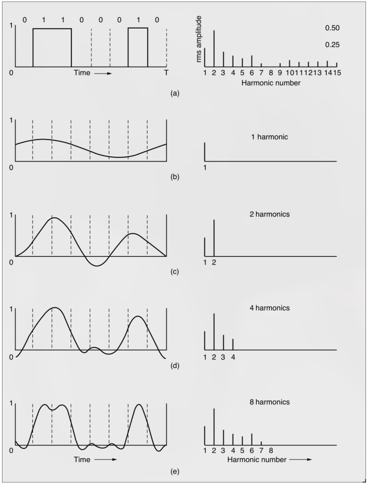

Figure 1: (a) A binary signal and its root-mean-square Fourier amplitudes. (b)-(e) Successive approximations to the original signal.

Bandwidth-Limited Signals

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Table 1: Relation between data rate and harmonics.

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Nyquist's theorem

Maximum data rate of channel $= \mathbf{2}\boldsymbol{B}\,\mathbf{log_2}(\boldsymbol{V})$ bits/sec

What about interferance?

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Assume a channel analogous to the last, with a signal power $\boldsymbol{S}$ and a noise power $\boldsymbol{N}$.

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Capacity of a real channel

Maximum data rate of channel $= \boldsymbol{B}\,\mathbf{log_2}(\mathbf{1}+\frac{\boldsymbol{S}}{\boldsymbol{N}})$ bits/sec

Digital Modulation

Baseband transmission

NRZ scheme (Non-Return-to-Zero)

Baseband Transmission

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- Signal follows the data. (i.e. Positive voltage for one, negative voltage for zero)
- Sender sends the signal.
- Receiver samples the signal at regular intervals of time.
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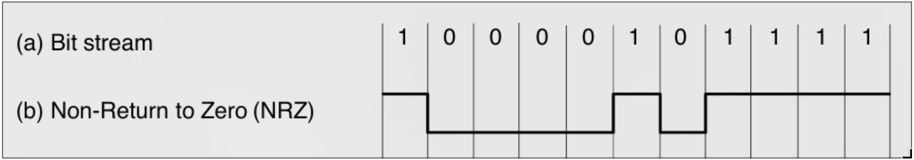

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- Bitrate can be interpreted as **(symbol rate × bits per symbol)**.

Clock Recovery

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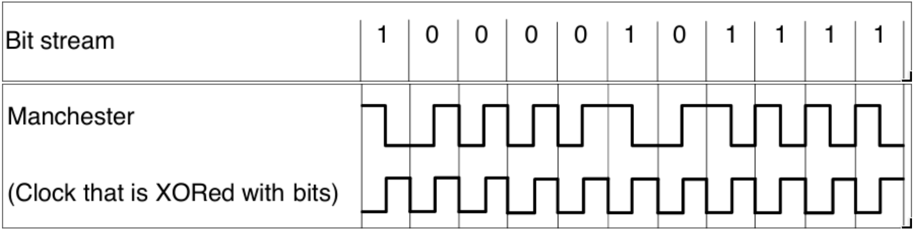
- After a while it's hard to tell the bits apart, 15 zeroes look much like 16 zeroes unless you have a very accurate clock.
- Very accurate clocks are expensive.

Manchester encoding



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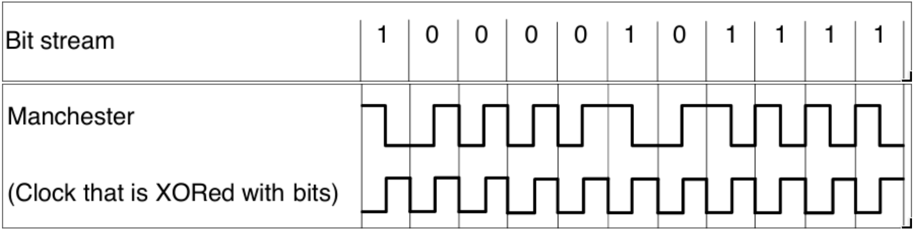
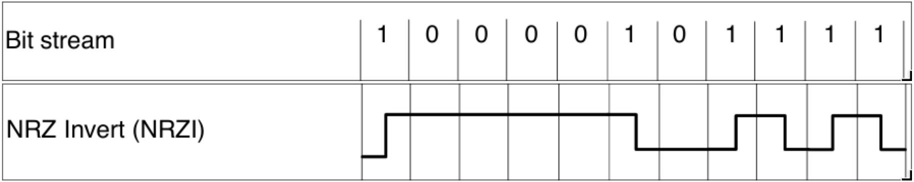


- Requires twice the bandwidth.
- Used in classic Ethernet.

Clock Recovery

NRZI (Non-Return-to-Zero Inverse)



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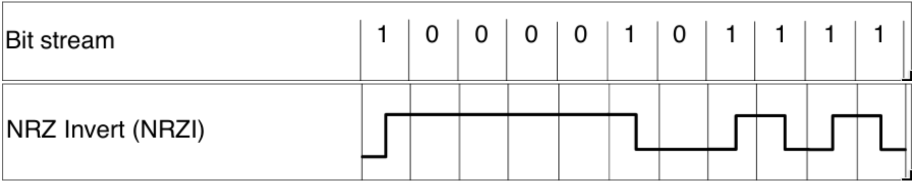

- Long streaks of 0 still have the same problem.
- Used in USB.

4B/5B

Data (4B)	Codeword (5B)	Data (4B)	Codeword (5B)
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

Table 2: 4B/5B mapping.

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Table 2: 4B/5B mapping.

- 25% overhead.

Scrambling

Scrambles the data by XORing it with a pseudorandom sequence.
Receiver XORs the data with the same sequence.

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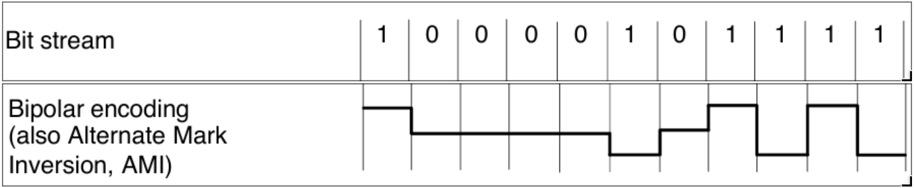
Receiver XORs the data with the same sequence.

- Used in early versions of IP over SONET.
- Still possible to get long streaks of identical symbols.
- Possible malicious usage, "killer packets".

Balanced Signals

Balanced Signals

Bipolar Encoding



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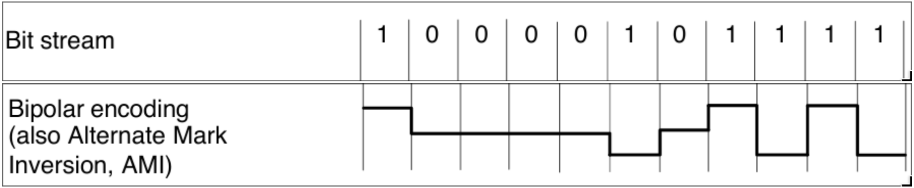

- Adds a voltage level.
- Average of 0.
- Indirectly helps with clock recovery.

Balanced Signals

4B/5B (again) or 8B/10B

Balanced Signals

4B/5B (again) or 8B/10B

- 8B/10B has at most a disparity of 2 bits.
- 8B/10B is 80% efficient.
- Helps with clock recovery.

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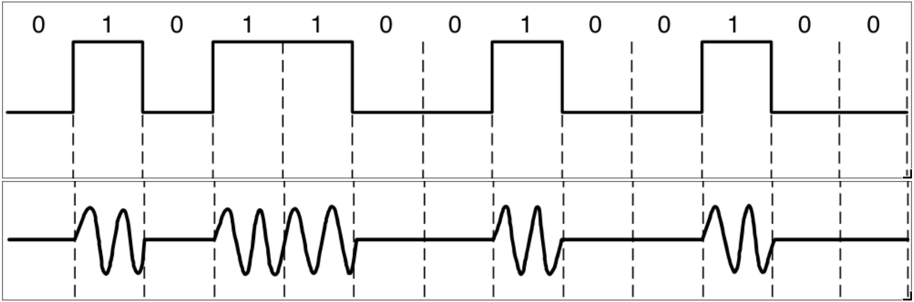
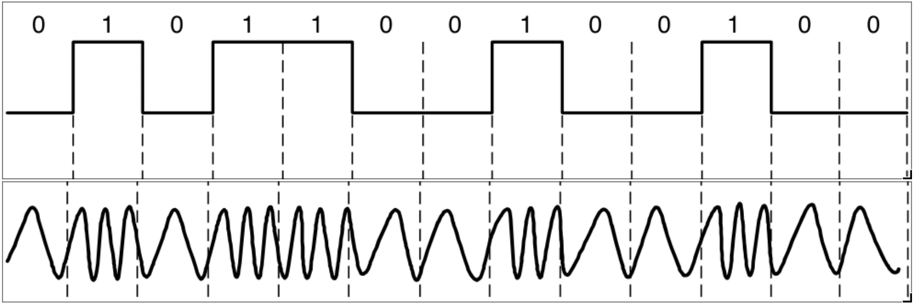
Passband Transmission

ASK (Amplitude Shift Keying)

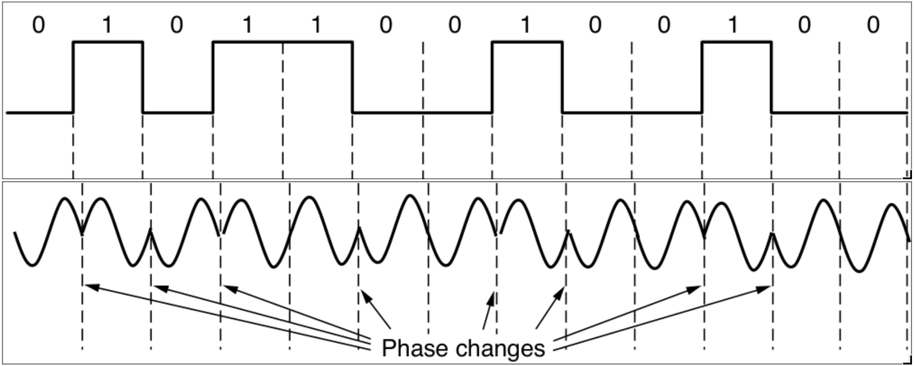


Passband Transmission

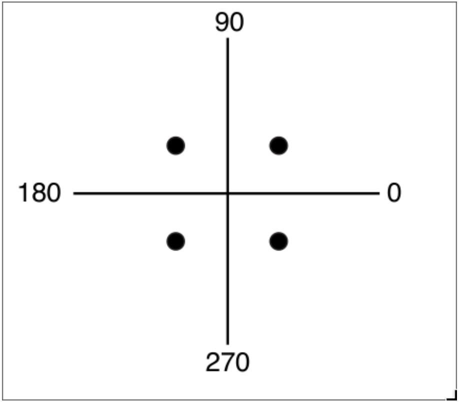
FSK (Frequency Shift Keying)



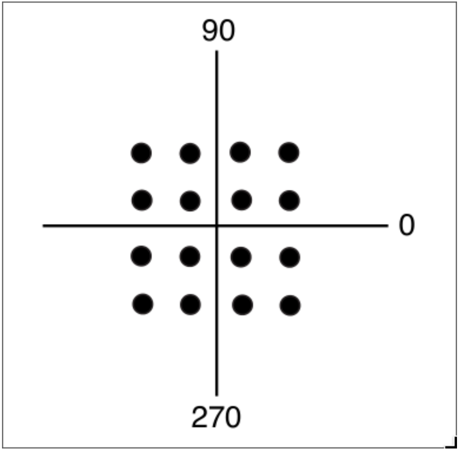
PSK (Phase Shift Keying)



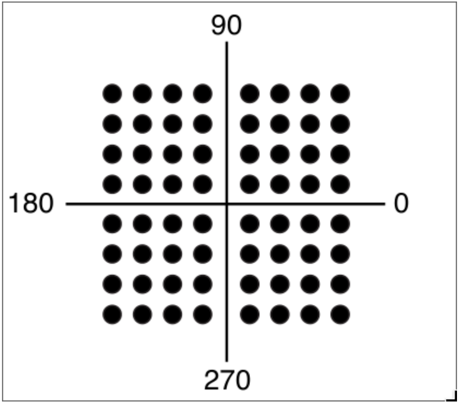
QPSK (Quadrature Phase Shift Keying)



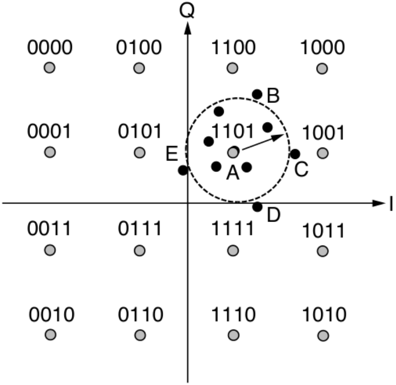
QAM-16 (Quadrature Amplitude Modulation - 16)



QAM-64 (Quadrature Amplitude Modulation - 64)



Gray-coded QAM-16



When 1101 is sent:

Point	Decodes as	Bit errors
A	1101	0
B	1100	1
C	1001	1
D	1111	1
E	0101	1

Multiplexing

FDM (Frequency Division Multiplexing)

- Divides the spectrum into frequency bands.
- Requires guard bands.
- Used by AM radio, telephone networks, cellular, terrestrial wireless and satelite networks.

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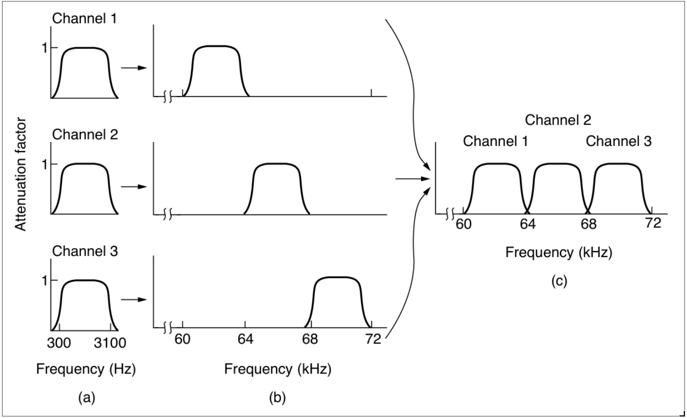
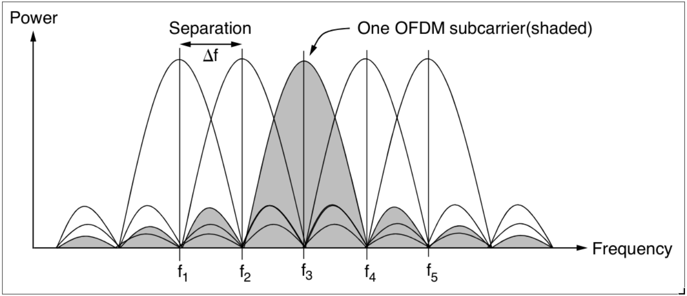


OFDM (Orthogonal Frequency Division Multiplexing)

- Channel bandwidth is divided into many independent subcarriers (i.e. QAM).
- Subcarriers are packed tightly in the frequency domain.
- Frequency response of each subcarrier is designed to be zero at the center of adjacent subcarriers.
- Guard time is needed to repeat a portion of the signals.
- Used in 802.11, cable networks, power-line networking, and 4G cellular systems.



TDM (Time Division Multiplexing)

- Users take turns to share the whole bandwidth (round-robin).
- Bits from each input stream are taken in a fixed time slot.
- Input stream is output to an aggregate stream.
- Used in telephone and celullar networks.

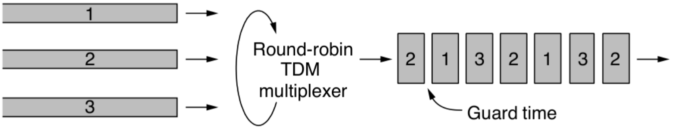


Figure 2-21. Time Division Multiplexing (TDM).

STDM (Statistical Time Division Multiplexing)

- Almost analogous to TDM.
- No fixed schedule.
- Schedule is decided based on usage information.

CDM/CDMA (Code Division Multiplexing / Code Division Multiple Access)

- Very wide frequency band
- Users share the whole frequency spectrum at all time.
- $\boldsymbol{m}$ unique chip vectors $\boldsymbol{S}$ are chosen for each station (generated with Walsh codes).
- each chip vector $\boldsymbol{S}$ represents a 1, and it's complement $\bar{\boldsymbol{S}}$ represents a 0.

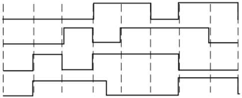
A = (−1 −1 −1 +1 +1 −1 +1 +1)

B = (−1 −1 +1 −1 +1 +1 +1 −1)

C = (−1 +1 −1 +1 +1 +1 −1 −1)

D = (−1 +1 −1 −1 −1 −1 +1 −1)

(a)



(b)

S_1 = C = (−1 +1 −1 +1 +1 +1 −1 −1)
S_2 = B+C = (−2 0 0 0 +2 +2 0 −2)
S_3 = A+$\bar{B}$ = (0 0 −2 +2 0 −2 0 +2)
S_4 = A+$\bar{B}$+C = (−1 +1 −3 +3 +1 −1 −1 +1)
S_5 = A+B+C+D = (−4 0 −2 0 +2 0 +2 −2)
S_6 = A+B+$\bar{C}$+D = (−2 −2 0 −2 0 −2 +4 0)

(c)

S_1•C = [1+1+1+1+1+1+1+1]/8 = 1
S_2•C = [2+0+0+0+2+2+0+2]/8 = 1
S_3•C = [0+0+2+2+0−2+0−2]/8 = 0
S_4•C = [1+1+3+3+1−1+1−1]/8 = 1
S_5•C = [4+0+2+0+2+0−2+2]/8 = 1
S_6•C = [2−2+0−2+0−2−4+0]/8 = −1

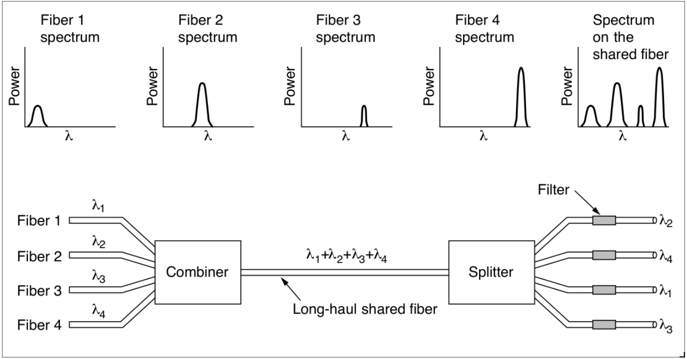
(d)

OFDMA (Orthogonal Frequency Division Multiple Access)

-

WDM (Wavelength Division Multiplexing)

- Basically just FDM but for very high frequencies (optical fiber)



DWDM (Dense Wavelength Division Multiplexing)

Analogous to DWDM but higher number of channels and little space between each channel.