From Waveforms to Bits

Diogo Fonseca
Professor: João Dias

UALG
MESTRADO EM ENGENHARIA INFORMÁTICA

8 de Novembro de 2025

Table of contents

Table of contents

UAlg
UNIVERSIDADE DO ALGARVE

Fourier Analysis

Fourier Series

$$g(t) = \frac{1}{2}\boldsymbol{c} + \sum_{n=1}^{\infty} \boldsymbol{a_n} sin(2\pi nft) + \sum_{n=1}^{\infty} \boldsymbol{b_n} cos(2\pi nft)$$

Fourier Series

$$g(t) = \frac{1}{2}\boldsymbol{c} + \sum_{n=1}^{\infty} \boldsymbol{a_n} sin(2\pi n f t) + \sum_{n=1}^{\infty} \boldsymbol{b_n} cos(2\pi n f t)$$

$\boldsymbol{a_n} = \frac{2}{T} \int_0^T g(t) sin(2\pi n f t) dt$

Fourier Analysis

Fourier Series

$$g(t) = \frac{1}{2}\boldsymbol{c} + \sum_{n=1}^{\infty} \boldsymbol{a_n} sin(2\pi nft) + \sum_{n=1}^{\infty} \boldsymbol{b_n} cos(2\pi nft)$$

$\boldsymbol{a_n} = \frac{2}{T}\int_0^T g(t)sin(2\pi nft)dt$

$\boldsymbol{b_n} = \frac{2}{T}\int_0^T g(t)cos(2\pi nft)dt$

Fourier Series

$$g(t) = \frac{1}{2}\boldsymbol{c} + \sum_{n=1}^{\infty} \boldsymbol{a_n} sin(2\pi n f t) + \sum_{n=1}^{\infty} \boldsymbol{b_n} cos(2\pi n f t)$$

$\boldsymbol{a_n} = \frac{2}{T}\int_0^T g(t) sin(2\pi n f t) dt$

$\boldsymbol{b_n} = \frac{2}{T}\int_0^T g(t) cos(2\pi n f t) dt$

$\boldsymbol{c} = \frac{2}{T}\int_0^T g(t) dt$

Bandwidth-Limited Signals

Some definitions

- Bandwidth (analog)
- Bandwidth (digital)
- Cutoff
- Baseband
- Passband

UAlg UNIVERSIDADE DO ALGARVE

Bandwidth-Limited Signals

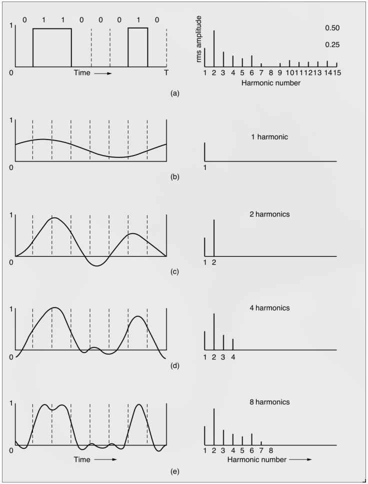


Figure 1: (a) A binary signal and its root-mean-square Fourier amplitudes. (b)-(e) Successive approximations to the original signal.

Bandwidth-Limited Signals

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Table 1: Relation between data rate and harmonics.

UAlg UNIVERSIDADE DO ALGARVE

Maximum Data Rate of a Channel

Maximum Data Rate of a Channel

Assume a perfect channel for communication with **no noise or interferance**. Furthermore, assume an arbitrary signal that has been run through a low-pass filter of bandwidth $\boldsymbol{B}$.

Maximum Data Rate of a Channel

Assume a perfect channel for communication with **no noise or interferance**. Furthermore, assume an arbitrary signal that has been run through a low-pass filter of bandwidth $\boldsymbol{B}$.

Nyquist's sample rate proof

The filtered signal can be completely reconstructed by making exactly $\boldsymbol{2B}$ samples per second.

Maximum Data Rate of a Channel

Assume a perfect channel for communication with **no noise or interferance**. Furthermore, assume an arbitrary signal that has been run through a low-pass filter of bandwidth $\boldsymbol{B}$.

Nyquist's sample rate proof

The filtered signal can be completely reconstructed by making exactly $\boldsymbol{2B}$ samples per second.

And given $\boldsymbol{V}$, the number of discrete states a signal can transmit at each sampling instant:

Maximum Data Rate of a Channel

Assume a perfect channel for communication with **no noise or interferance**. Furthermore, assume an arbitrary signal that has been run through a low-pass filter of bandwidth $\boldsymbol{B}$.

Nyquist's sample rate proof

The filtered signal can be completely reconstructed by making exactly $\mathbf{2}\boldsymbol{B}$ samples per second.

And given $\boldsymbol{V}$, the number of discrete states a signal can transmit at each sampling instant:

Nyquist's theorem

Maximum data rate of channel $= \mathbf{2}\boldsymbol{B}\,\mathbf{log_2(}\boldsymbol{V}\mathbf{)}$ bits/sec

UAlg UNIVERSIDADE DO ALGARVE

What about interferance?

Maximum Data Rate of a Channel

Assume a channel analogous to the last, with a signal power $\boldsymbol{S}$ and a noise power $\boldsymbol{N}$.

Maximum Data Rate of a Channel

Assume a channel analogous to the last, with a signal power $\boldsymbol{S}$ and a noise power $\boldsymbol{N}$.

SNR (Signal-to-Noise Ratio)

$$\frac{\boldsymbol{S}}{\boldsymbol{N}}$$

Maximum Data Rate of a Channel

Assume a channel analogous to the last, with a signal power $\boldsymbol{S}$ and a noise power $\boldsymbol{N}$.

SNR (Signal-to-Noise Ratio)

$$\frac{\boldsymbol{S}}{\boldsymbol{N}}$$

Capacity of a real channel

Maximum data rate of channel $= \boldsymbol{B \log_2 (1 + \frac{S}{N})}$ bits/sec

Digital Modulation

Baseband transmission

NRZ scheme (Non-Return-to-Zero)

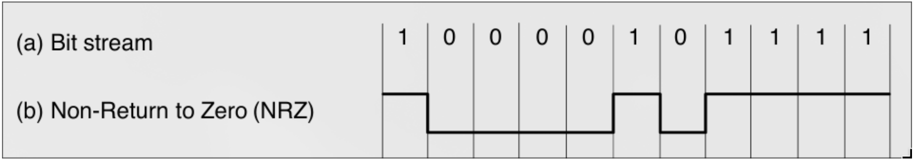
Baseband Transmission

NRZ scheme (Non-Return-to-Zero)

- Signal follows the data. (i.e. Positive voltage for one, negative voltage for zero)
- Sender sends the signal.
- Receiver samples the signal at regular intervals of time.
- Receiver maps signal samples to the closes symbols (i.e. $\{0, 1\}$)

NRZ scheme (Non-Return-to-Zero)

- Signal follows the data. (i.e. Positive voltage for one, negative voltage for zero)
- Sender sends the signal.
- Receiver samples the signal at regular intervals of time.
- Receiver maps signal samples to the closes symbols (i.e. $\{0, 1\}$)



Bandwidth Efficiency

Bandwidth Efficiency

NRZ scheme

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.
- Given this is a fundamental limit, we cannot run NRZ faster without using additional bandwidth.

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.
- Given this is a fundamental limit, we cannot run NRZ faster without using additional bandwidth.
- One strategy to increase efficiency is to use more than two signaling levels.

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.
- Given this is a fundamental limit, we cannot run NRZ faster without using additional bandwidth.
- One strategy to increase efficiency is to use more than two signaling levels.
- By using 4 different voltages, for instance, we can send 2 bits at once as a single **symbol**.

UAlg
UNIVERSIDADE DO ALGARVE

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.
- Given this is a fundamental limit, we cannot run NRZ faster without using additional bandwidth.
- One strategy to increase efficiency is to use more than two signaling levels.
- By using 4 different voltages, for instance, we can send 2 bits at once as a single **symbol**.
- This works as long as the signal at the receiver is sufficiently strong to distinguish all the levels.

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.
- Given this is a fundamental limit, we cannot run NRZ faster without using additional bandwidth.
- One strategy to increase efficiency is to use more than two signaling levels.
- By using 4 different voltages, for instance, we can send 2 bits at once as a single **symbol**.
- This works as long as the signal at the receiver is sufficiently strong to distinguish all the levels.
- The rate at which the signal changes (**symbol rate**) is then half the bit rate, so bandwidth has been reduced.

UAlg
UNIVERSIDADE DO ALGARVE

Bandwidth Efficiency

NRZ scheme

- For a data rate of B bits/sec we need a bandwidth of at least $\boldsymbol{\frac{B}{2}}$ Hz.
- Given this is a fundamental limit, we cannot run NRZ faster without using additional bandwidth.
- One strategy to increase efficiency is to use more than two signaling levels.
- By using 4 different voltages, for instance, we can send 2 bits at once as a single **symbol**.
- This works as long as the signal at the receiver is sufficiently strong to distinguish all the levels.
- The rate at which the signal changes (**symbol rate**) is then half the bit rate, so bandwidth has been reduced.
- Bitrate can be interpreted as **(symbol rate $\times$ bits per symbol)**.

UAlg UNIVERSIDADE DO ALGARVE

Clock Recovery

The problem

The problem

- The receiver must know when one symbol ends and the next symbol appears.

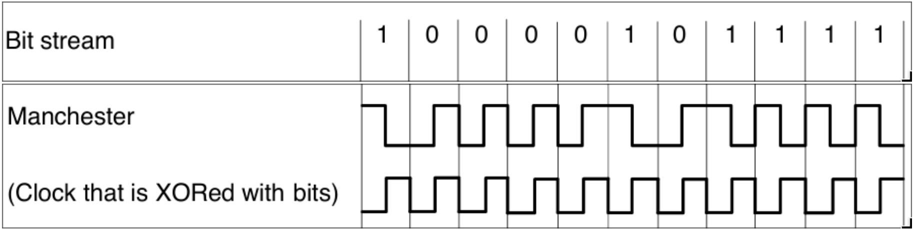
The problem

- The receiver must know when one symbol ends and the next symbol appears.
- After a while it's hard to tell the bits apart, 15 zeroes look much like 16 zeroes unless you have a very accurate clock.

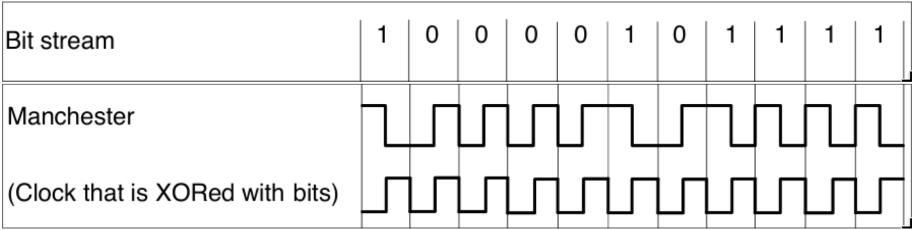
The problem

- The receiver must know when one symbol ends and the next symbol appears.
- After a while it's hard to tell the bits apart, 15 zeroes look much like 16 zeroes unless you have a very accurate clock.
- Very accurate clocks are expensive.

Manchester encoding



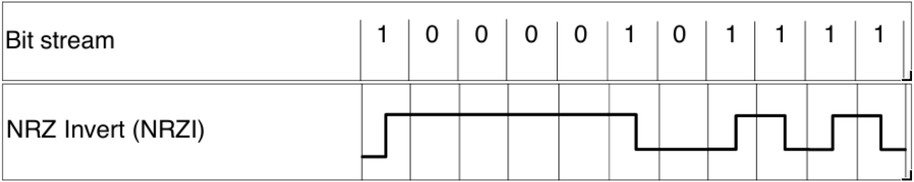
Manchester encoding



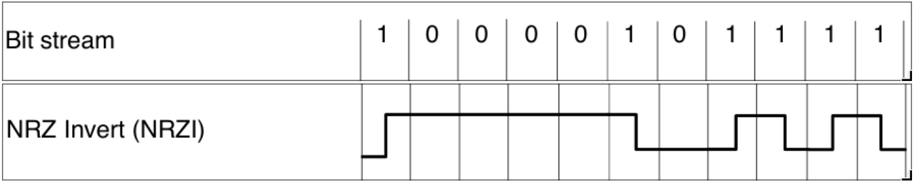
- Requires twice the bandwidth.
- Used in classic Ethernet.

Clock Recovery

NRZI (Non-Return-to-Zero Inverse)



NRZI (Non-Return-to-Zero Inverse)



- Long streaks of 0 still have the same problem.
- Used in USB.

4B/5B

Data (4B)	Codeword (5B)	Data (4B)	Codeword (5B)
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

Table 2: 4B/5B mapping.

4B/5B

Data (4B)	Codeword (5B)	Data (4B)	Codeword (5B)
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

Table 2: 4B/5B mapping.

- 25% overhead.

Scrambling

Scrambles the data by XORing it with a pseudorandom sequence.
Receiver XORs the data with the same sequence.

Scrambling

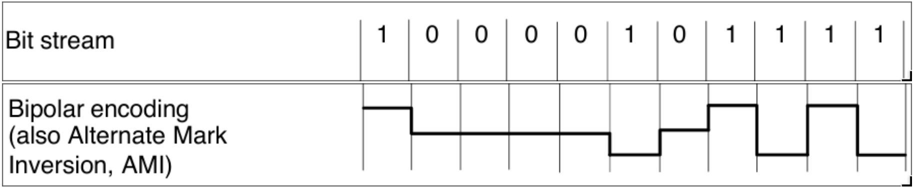
Scrambles the data by XORing it with a pseudorandom sequence.
Receiver XORs the data with the same sequence.

- Used in early versions of IP over SONET.
- Still possible to get long streaks of identical symbols.
- Possible malicious usage, "killer packets".

Balanced Signals

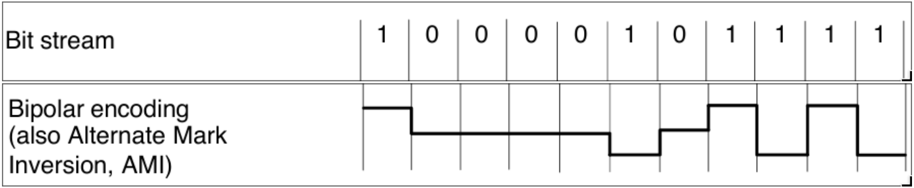
Balanced Signals

Bipolar Encoding



Balanced Signals

Bipolar Encoding



- Adds a voltage level.
- Average of 0.
- Indirectly helps with clock recovery.

Balanced Signals

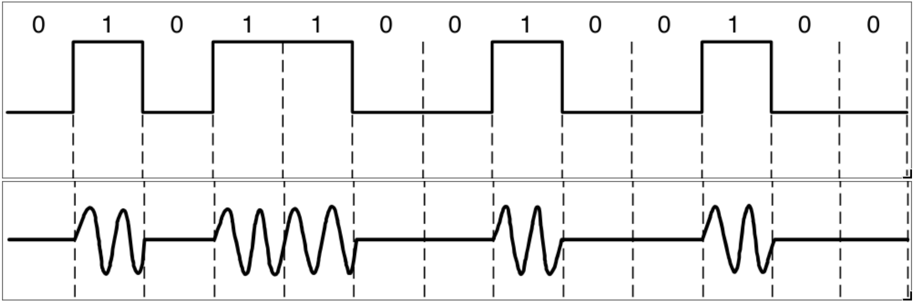
4B/5B (again) or 8B/10B

4B/5B (again) or 8B/10B

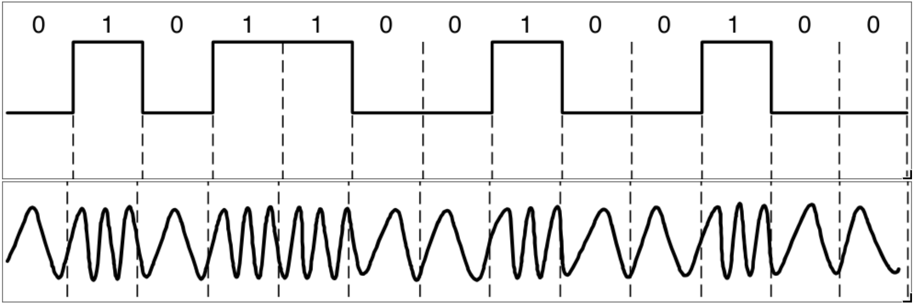
- 8B/10B has at most a disparity of 2 bits.
- 8B/10B is 80% efficient.
- Helps with clock recovery.

Passband Transmission

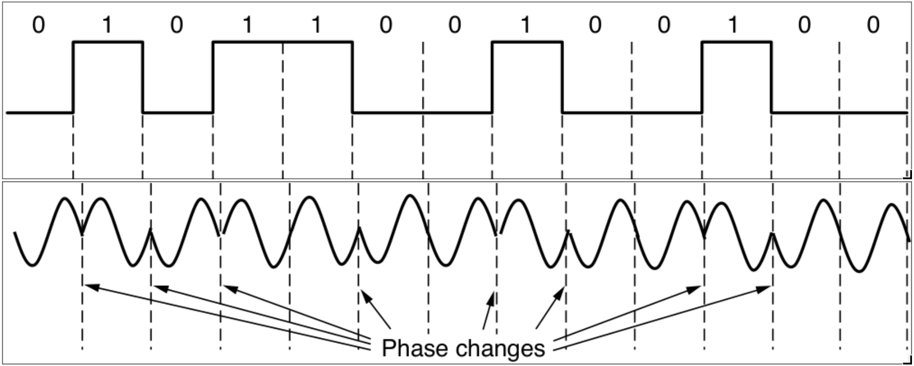
ASK (Amplitude Shift Keying)



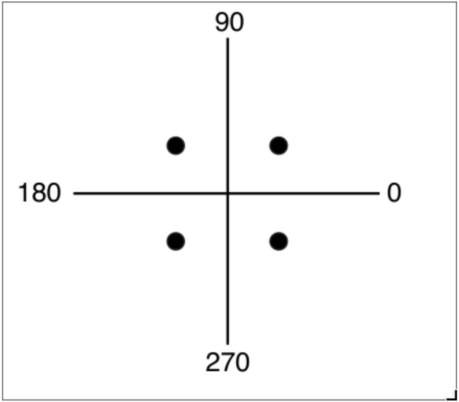
FSK (Frequency Shift Keying)



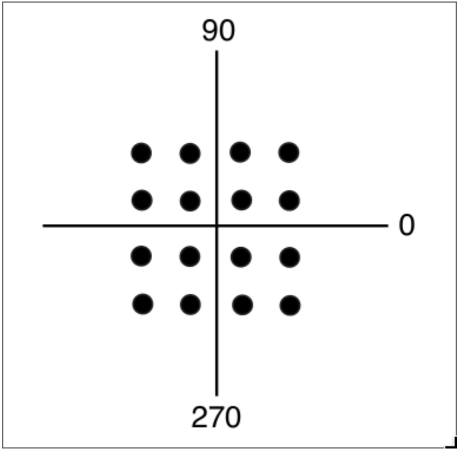
PSK (Phase Shift Keying)



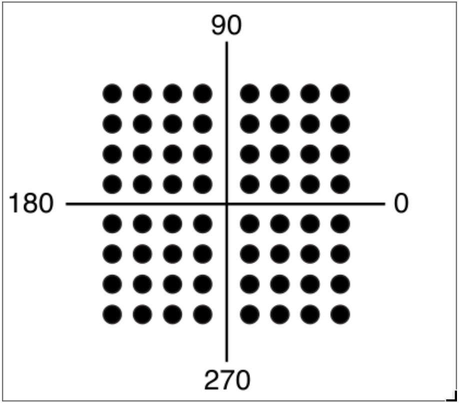
QPSK (Quadrature Phase Shift Keying)



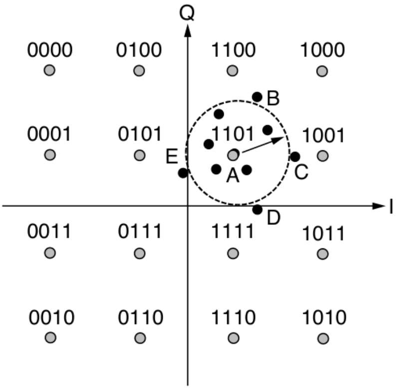
QAM-16 (Quadrature Amplitude Modulation - 16)



QAM-64 (Quadrature Amplitude Modulation - 64)



Gray-coded QAM-16



When 1101 is sent:

Point	Decodes as	Bit errors
A	1101	0
B	110<u>0</u>	1
C	1<u>0</u>01	1
D	11<u>1</u>1	1
E	<u>0</u>101	1

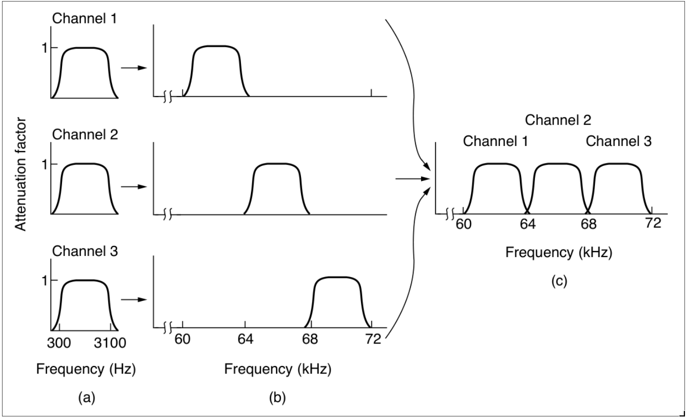
Multiplexing

FDM (Frequency Division Multiplexing)

- Divides the spectrum into frequency bands.
- Requires guard bands.
- Used by AM radio, telephone networks, cellular, terrestrial wireless and satelite networks.

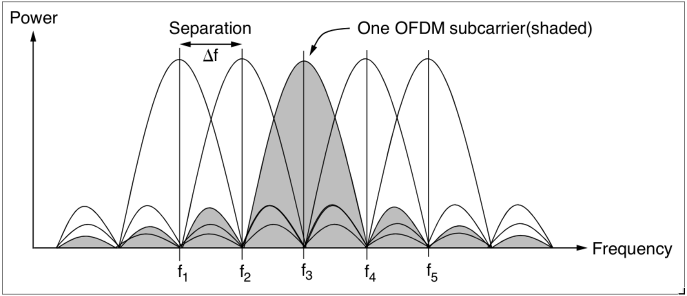
FDM (Frequency Division Multiplexing)

- Divides the spectrum into frequency bands.
- Requires guard bands.
- Used by AM radio, telephone networks, cellular, terrestrial wireless and satelite networks.



OFDM (Orthogonal Frequency Division Multiplexing)

- Channel bandwidth is divided into many independent subcarriers (i.e. QAM).
- Subcarriers are packed tightly in the frequency domain.
- Frequency response of each subcarrier is designed to be zero at the center of adjacent subcarriers.
- Guard time is needed to repeat a portion of the signals.
- Used in 802.11, cable networks, power-line networking, and 4G cellular systems.



OFDMA (Orthogonal Frequency Division Multiple Access)

Analogous to OFDM except one user may take up several subcarriers (called Resource Units, RU), depending on the amount of usage.

TDM (Time Division Multiplexing)

- Users take turns to share the whole bandwidth (round-robin).
- Bits from each input stream are taken in a fixed time slot.
- Input stream is output to an aggregate stream.
- Used in telephone and celullar networks.

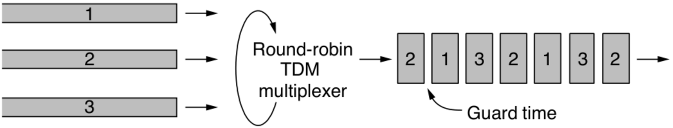


Figure 2-21. Time Division Multiplexing (TDM).

STDM (Statistical Time Division Multiplexing)

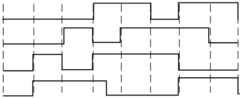
- Almost analogous to TDM.
- No fixed schedule.
- Schedule is decided based on usage information.

CDM/CDMA (Code Division Multiplexing / Code Division Multiple Access)

- Very wide frequency band
- Users share the whole frequency spectrum at all time.
- $\boldsymbol{m}$ unique chip vectors $\boldsymbol{S}$ are chosen for each station (generated with Walsh codes).
- each chip vector $\boldsymbol{S}$ represents a 1, and it's complement $\bar{\boldsymbol{S}}$ represents a 0.

$$
\begin{aligned}
A &= (-1\ -1\ -1\ +1\ +1\ -1\ +1\ +1)\\
B &= (-1\ -1\ +1\ -1\ +1\ +1\ +1\ -1)\\
C &= (-1\ +1\ -1\ +1\ +1\ +1\ -1\ -1)\\
D &= (-1\ +1\ -1\ -1\ -1\ -1\ +1\ -1)
\end{aligned}
$$

(a)



(b)

$$
\begin{aligned}
S_1 &= C &&= (-1\ +1\ -1\ +1\ +1\ +1\ -1\ -1)\\
S_2 &= B+C &&= (-2\ \ 0\ \ 0\ \ 0\ +2\ +2\ \ 0\ -2)\\
S_3 &= A+\bar{B} &&= (\ 0\ \ 0\ -2\ +2\ \ 0\ -2\ \ 0\ +2)\\
S_4 &= A+\bar{B}+C &&= (-1\ +1\ -3\ +3\ +1\ -1\ -1\ +1)\\
S_5 &= A+B+C+D &&= (-4\ \ 0\ -2\ \ 0\ +2\ \ 0\ +2\ -2)\\
S_6 &= A+B+\bar{C}+D &&= (-2\ -2\ \ 0\ -2\ \ 0\ -2\ +4\ \ 0)
\end{aligned}
$$

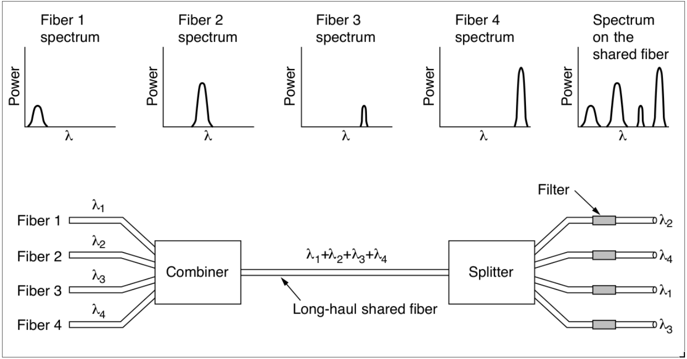
(c)

$$
\begin{aligned}
S_1 \bullet C &= [1+1+1+1+1+1+1+1]/8 = 1\\
S_2 \bullet C &= [2+0+0+0+2+2+0+2]/8 = 1\\
S_3 \bullet C &= [0+0+2+2+0-2+0-2]/8 = 0\\
S_4 \bullet C &= [1+1+3+3+1-1+1-1]/8 = 1\\
S_5 \bullet C &= [4+0+2+0+2+0-2+2]/8 = 1\\
S_6 \bullet C &= [2-2+0-2+0-2-4+0]/8 = -1
\end{aligned}
$$

(d)

WDM (Wavelength Division Multiplexing)

- Basically just FDM but for very high frequencies (optical fiber)



DWDM (Dense Wavelength Division Multiplexing)

Analogous to DWDM but higher number of channels and little space between each channel.