From Waveforms to Bits

Diogo Fonseca
Professor: João Dias

UALG
MESTRADO EM ENGENHARIA INFORMÁTICA

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Fourier Series

$$g(t) = \frac{1}{2}\boldsymbol{c} + \sum_{n=1}^{\infty} \boldsymbol{a_n} sin(2\pi nft) + \sum_{n=1}^{\infty} \boldsymbol{b_n} cos(2\pi nft)$$

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Bandwidth-Limited Signals

Some definitions

- Bandwidth (analog)
- Bandwidth (digital)
- Cutoff
- Baseband
- Passband

Bandwidth-Limited Signals

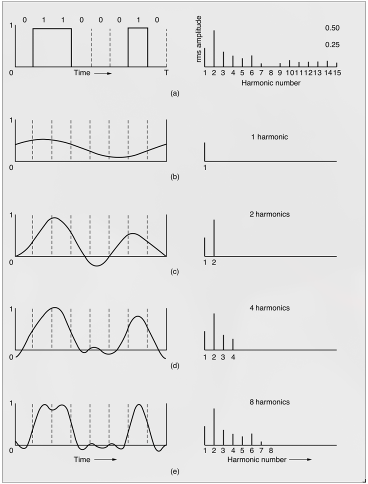

Figure 1: (a) A binary signal and its root-mean-square Fourier amplitudes. (b)-(e) Successive approximations to the original signal.

Bandwidth-Limited Signals

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Table 1: Relation between data rate and harmonics.

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Nyquist's theorem

Maximum data rate of channel $= \boldsymbol{2B\log_2(V)}$ bits/sec

What about interferance?

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Capacity of a real channel

Maximum data rate of channel $= \boldsymbol{B} \log_2(1 + \frac{S}{N})$ bits/sec

Digital Modulation

Baseband transmission

NRZ scheme (Non-Return-to-Zero)

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- Signal follows the data. (i.e. Positive voltage for one, negative voltage for zero)
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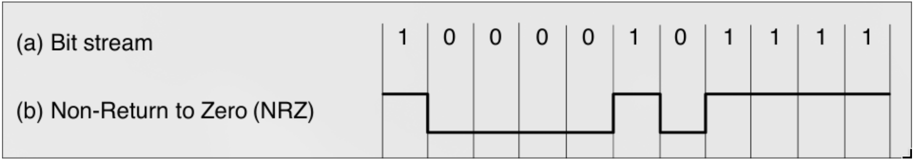

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- Bitrate can be interpreted as **(symbol rate $\times$ bits per symbol)**.

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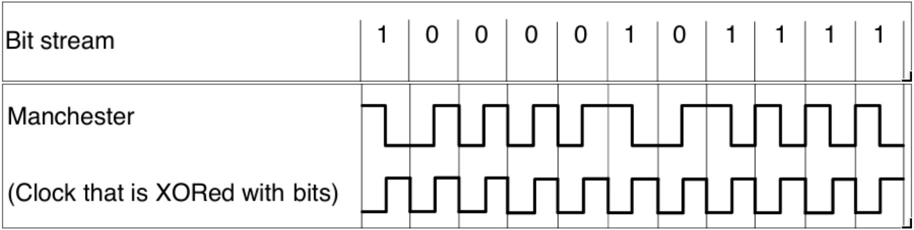
- After a while it's hard to tell the bits apart, 15 zeroes look much like 16 zeroes unless you have a very accurate clock.
- Very accurate clocks are expensive.

Manchester encoding



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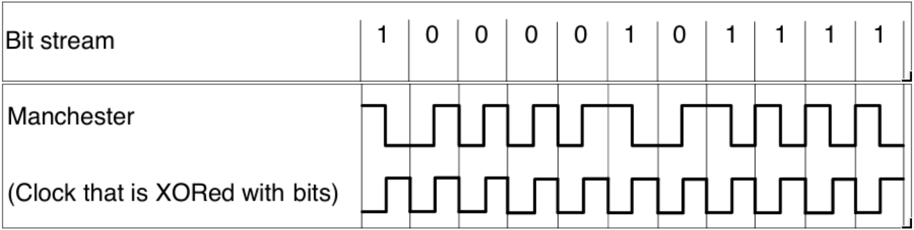
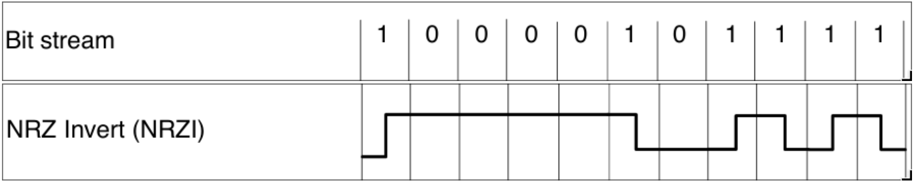


- Requires twice the bandwidth.
- Used in classic Ethernet.

NRZI (Non-Return-to-Zero Inverse)



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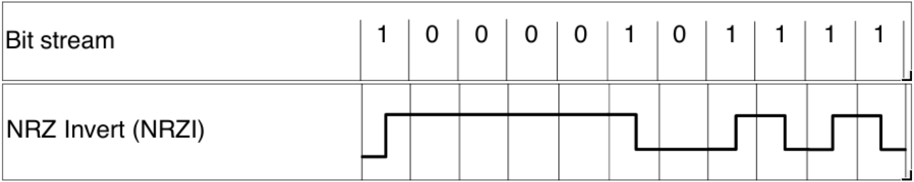

- Long streaks of 0 still have the same problem.
- Used in USB.

4B/5B

Data (4B)	Codeword (5B)	Data (4B)	Codeword (5B)
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

Table 2: 4B/5B mapping.

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- 25% overhead.

Scrambling

Scrambles the data by XORing it with a pseudorandom sequence.
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- Used in early versions of IP over SONET.
- Still possible to get long streaks of identical symbols.
- Possible malicious usage, "killer packets".

Balanced Signals