Two grain types in sandpiles

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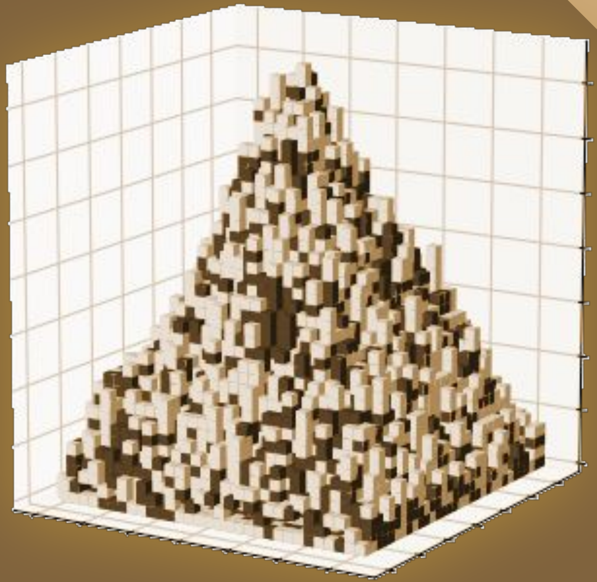


Introduction

What is self organized criticality (SOC)?

- Convergence
- Scale invariant

Why study SOC?

- Understanding complex systems

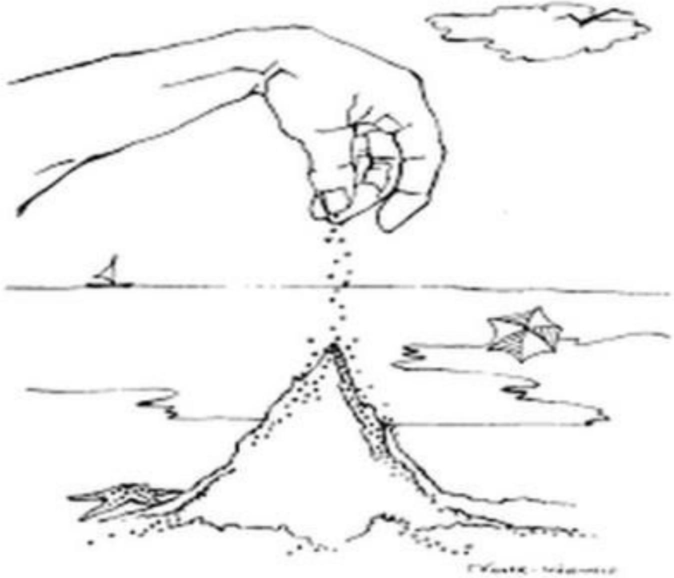


Figure 1: Example of SOC (J. Verdon, 2016)

The sandpile model

Implementation

- Adding grains to a pile of sand
- Observing the avalanches in the pile

SOC in the sandpile model

- Power law distribution

Previous Research

Basic sandpile model (Bak & Chen, 1991)

- 1D model
- Single grain
- Fixed position drop

Multiple grain types (Van de walle, 1999)

- 1D model
- Grain dynamics

2D sandpile models (Kalinin et al., 2018)

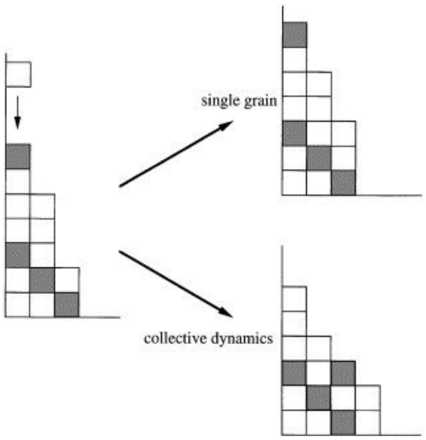


Figure 2: 1D multigrain sandpile model (Van de walle, 1999)

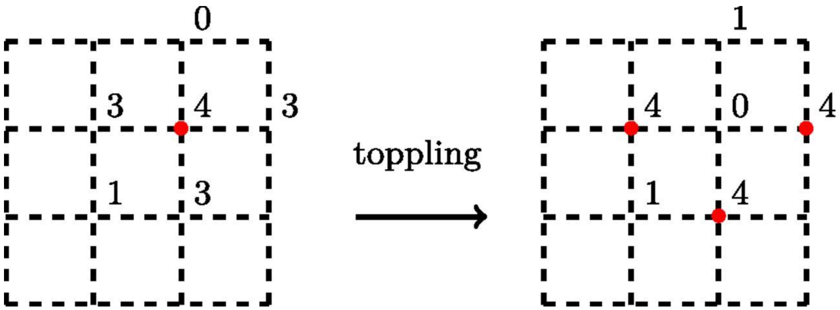


Figure 3: Basic 2D sandpile model(Van de walle, 1999)

Research Question

Goal: Increasing the realism of the previous multigrain model

Methods: Increasing the dimension of the model and using additional grain dropping methods.

Research Question: What are the effects of multiple grain types and the effects of different drop methods on the avalanche sizes, avalanche frequencies and the distribution of the grain types in the 2D sandpile model.

Model

2D sandpile model with multiple grains

Initial condition

- ❑ Random dependent on critical values
- ❑ Limited by critical values of the grains

Boundary condition

- ❑ Open
- ❑ Removes grains from the system

Model (assumptions)

Critical condition instead of height

Multigrain falls instead of single grain falls

Fall independent of parameters in the model

Model (Pseudocode)

Step 1: initialize grid

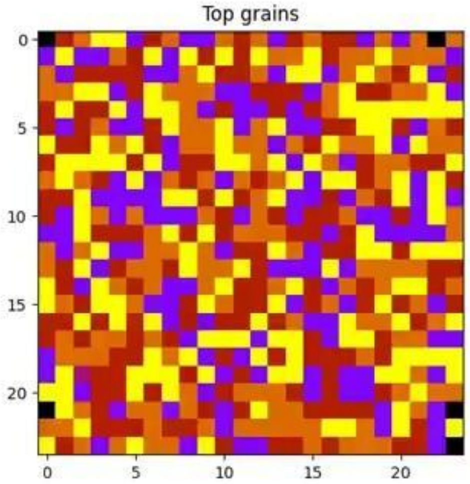
Step 2: Add grain to grid (of given type)

Step 3: If new grain threshold value exceeds stack size

Step 4: Move top 4 grains to neighbours

Step 5: Repeat steps 3 and 4 for neighbours

Step 6: Repeat steps 2 to 5 for given amount of steps



Model (Adding grain types)

Different type of grains with different critical thresholds

Methods

- ❏ Position drop
- ❏ Random drop
- ❏ Normal distributed drop

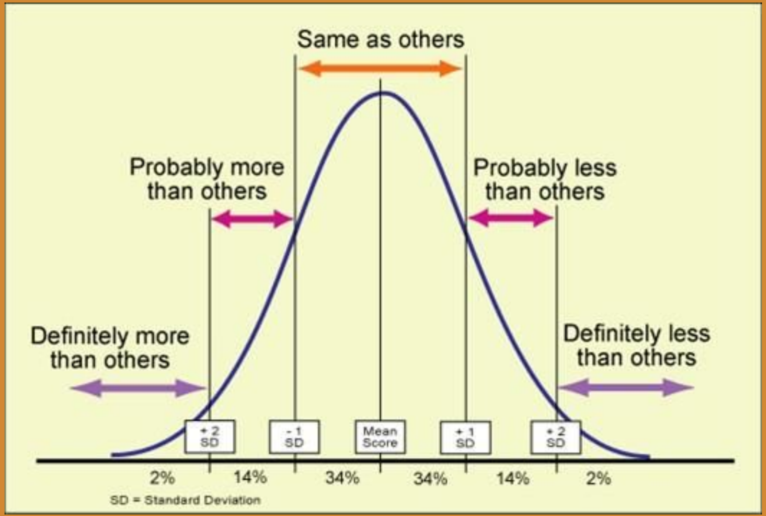


Figure 4: Normal distribution (J. Verdon, 2016)

Model (General parameters)

Initial grid

16 to 1024 (square area) in steps of power 2

Everly distributed random values

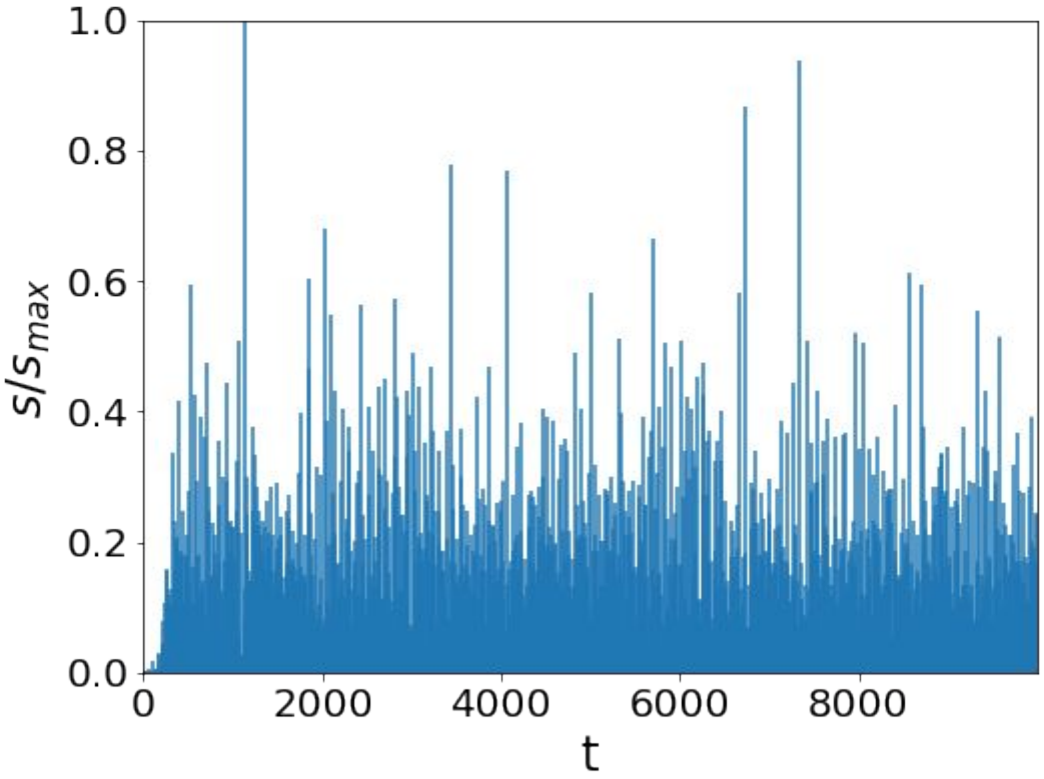
Experiments

Trails = 1 million

Center position with a SD of 1/16 of grid length

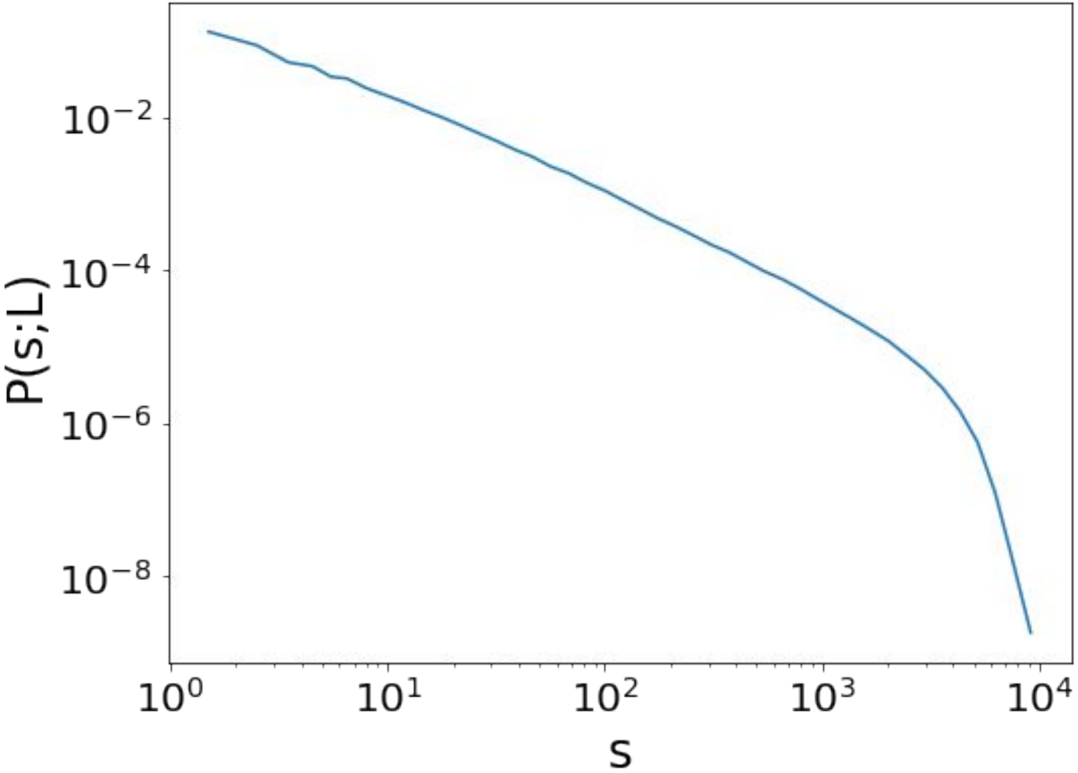
Grain odds = 50/50

Results - Time series



- Normalized avalanche sizes
- Initial settle phase

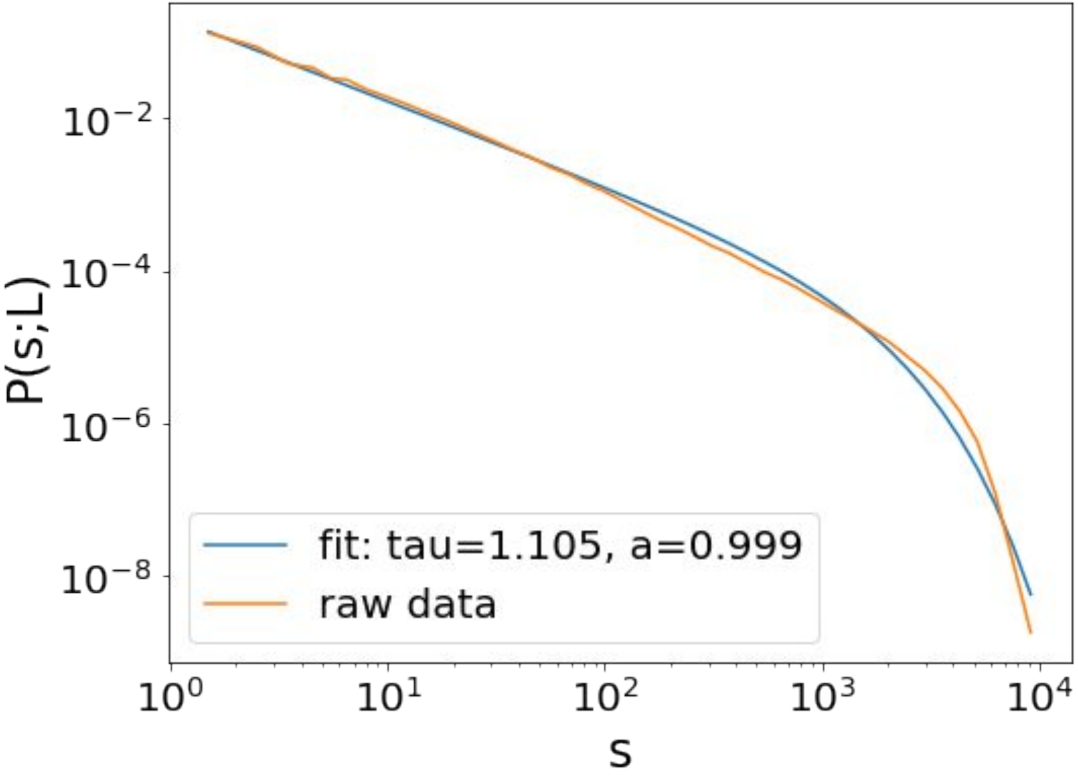
Results - Probability density example



- Powerlaw distribution with exponential falloff

$$P(s) \propto s^{-\tau} a^{s}$$

Results - Probability density fit

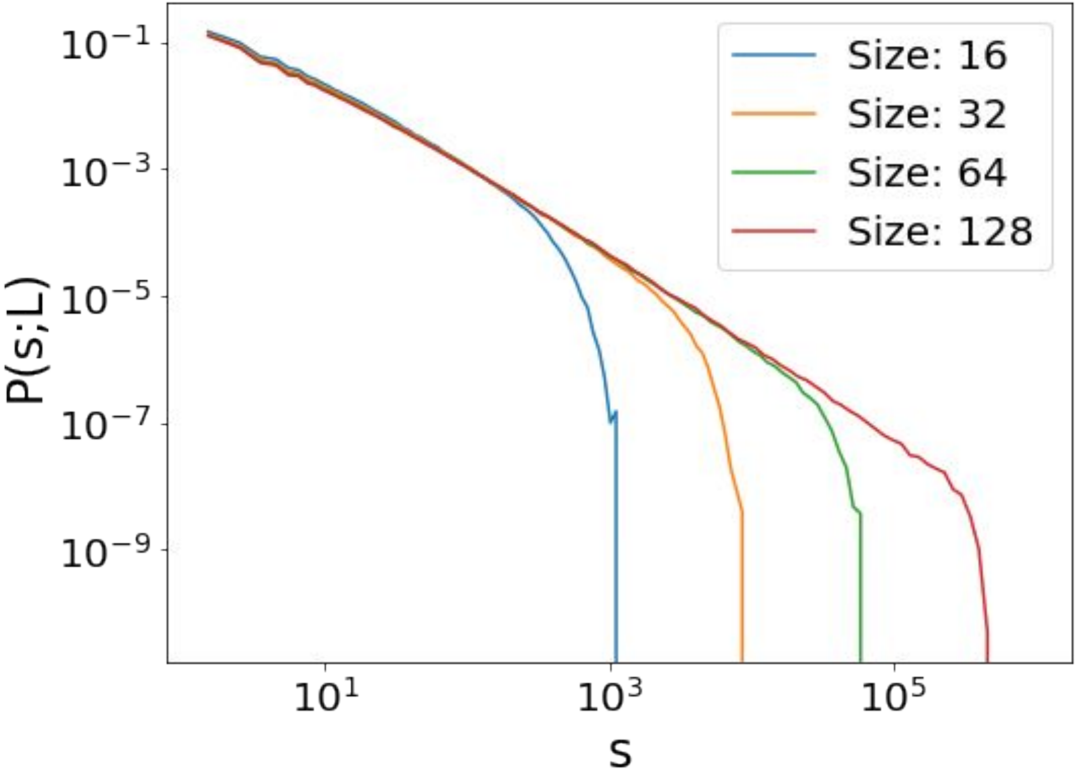


- ❑ Fit to reduce relative error

$$P(s) = C\, s^{-\tau} a^{s}$$

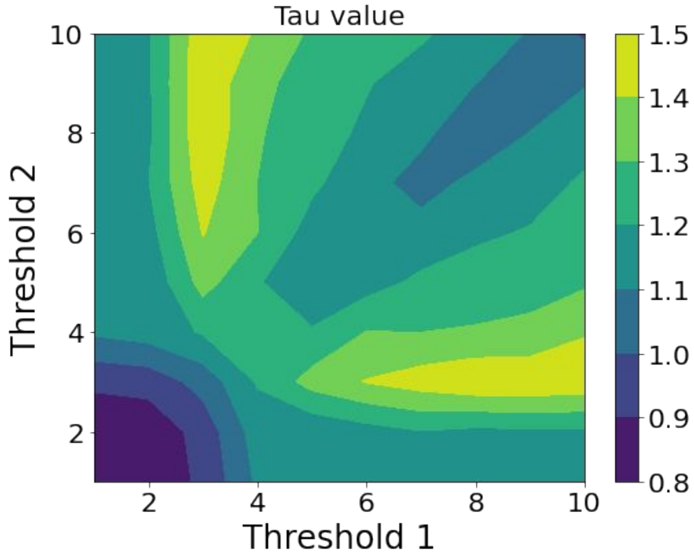
- ❑ C is a normalization constant

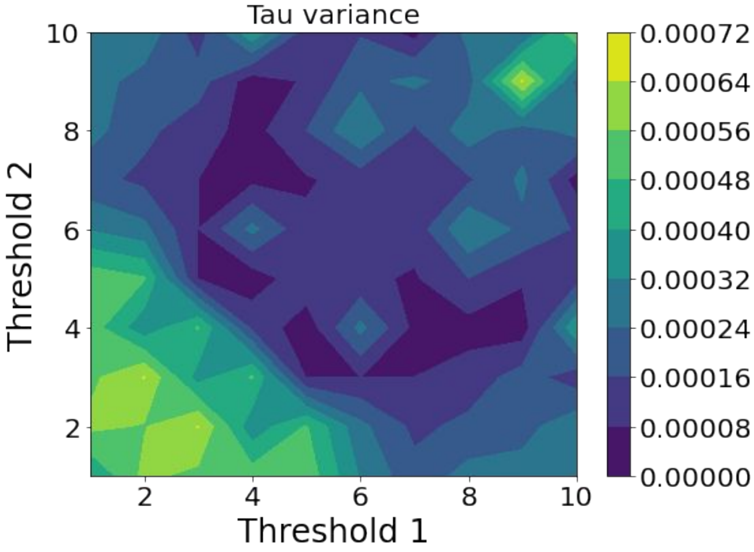
Results - Probability density for different grid sizes



- τ independent on grid size
- a depends on grid size, as expected
- Grid size of 32 used

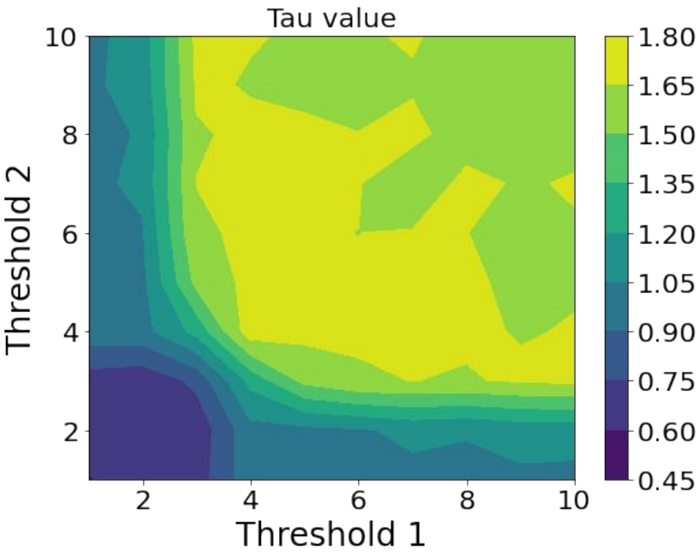
Results - Threshold comparison (Random)

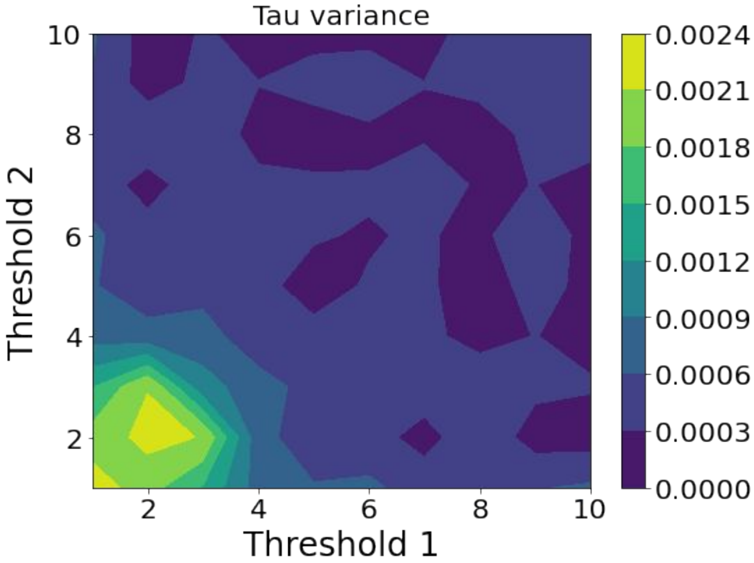




- ❏ Low variance in τ implies good fit with the powerlaw distribution

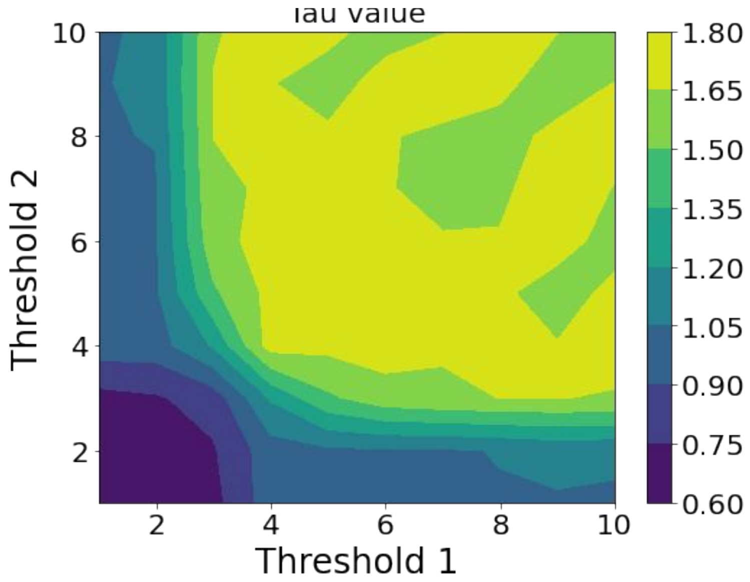
Results - Threshold comparison (Position)

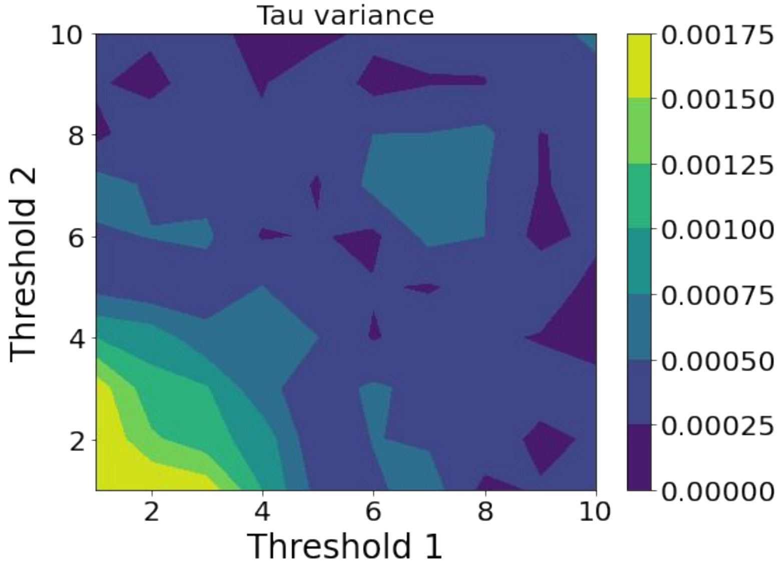




- Increased thresholds cause larger τ values
- Difference in τ appears larger than it actually is
- Low variance in τ implies good fit with the powerlaw distribution

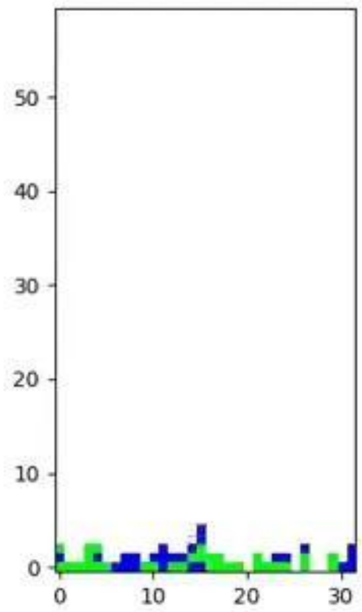
Results - Threshold comparison (Normal)



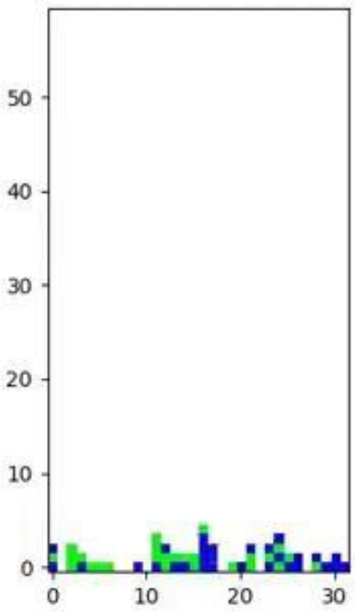


- ❏ Practically the same as (position)
- ❏ Low variance in τ implies good fit with the powerlaw distribution

Results

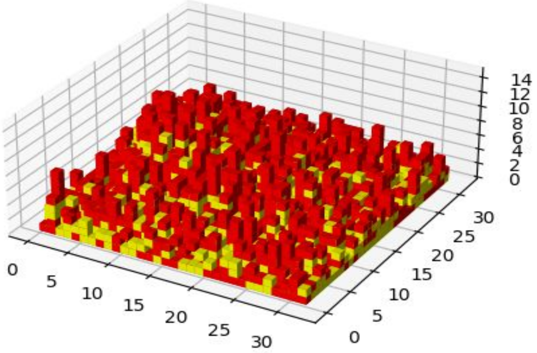


Thresholds 6 and 3

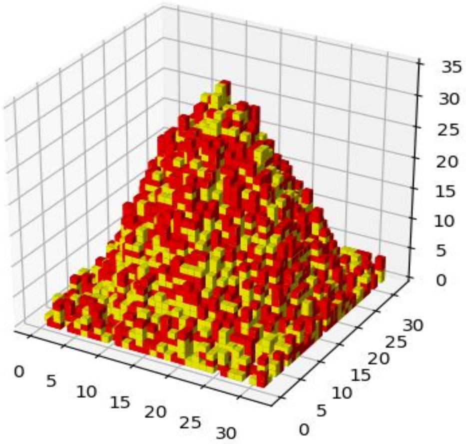


Thresholds 6 and 4

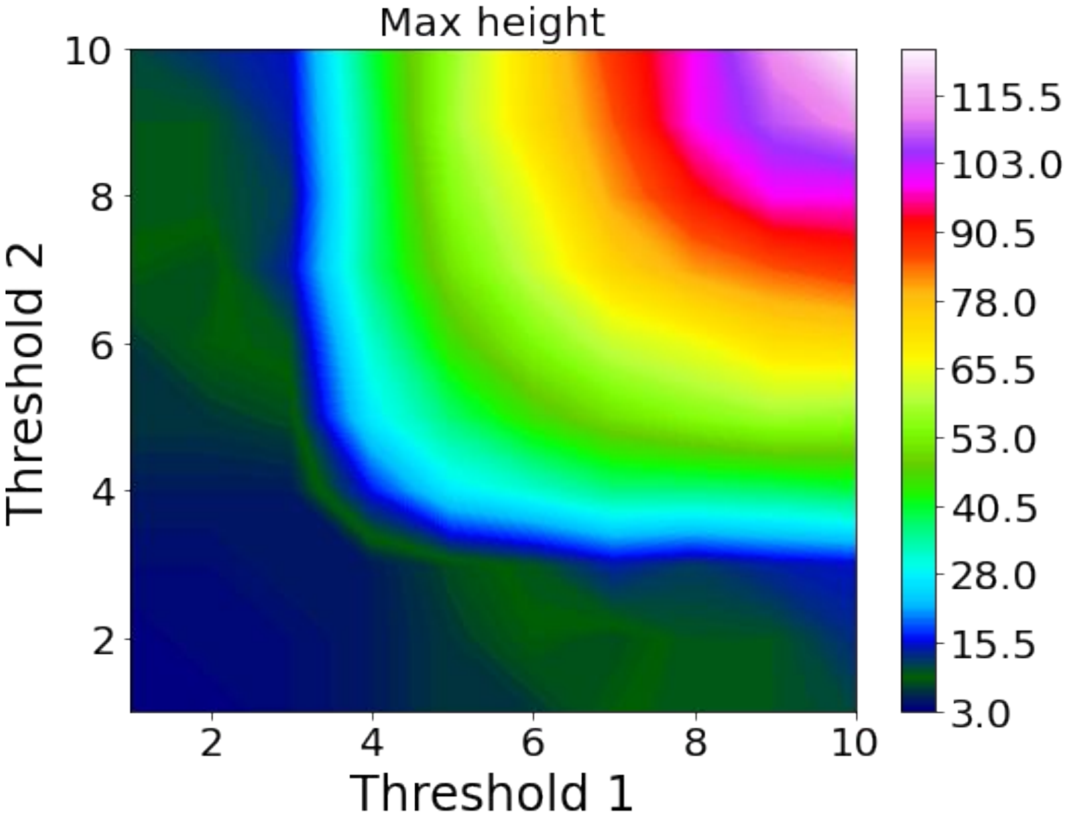
Random stacks



Pyramid shape

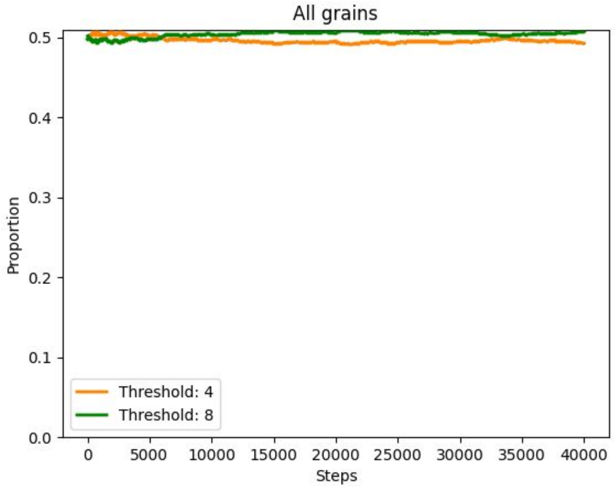


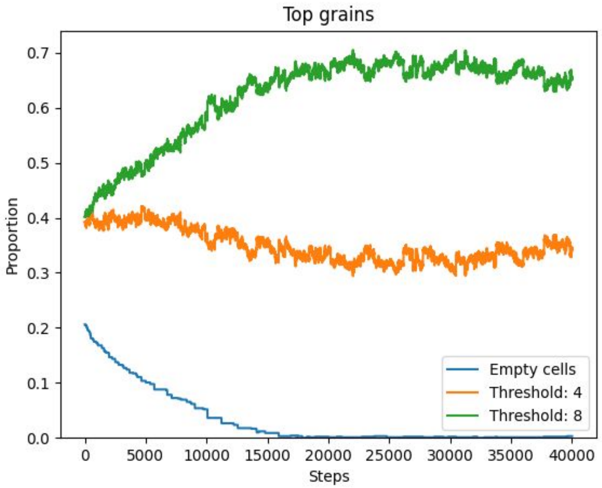
Results - Emergent pyramids

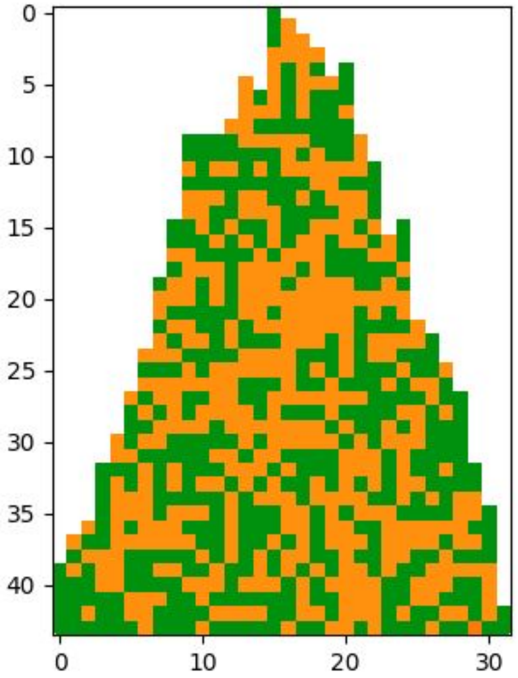


- Randomly dropped
- Pyramids appear when both thresholds are larger than 4
- Otherwise smaller random piles appear that fail to grow

Results

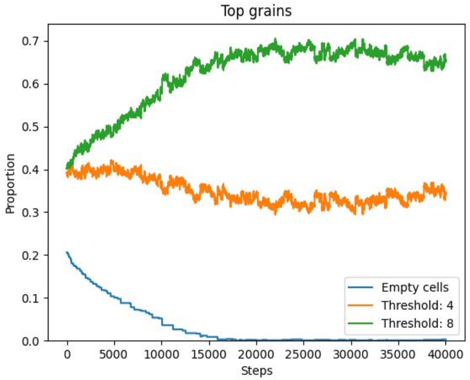

All grains
Proportion
Threshold: 4
Threshold: 8
Steps

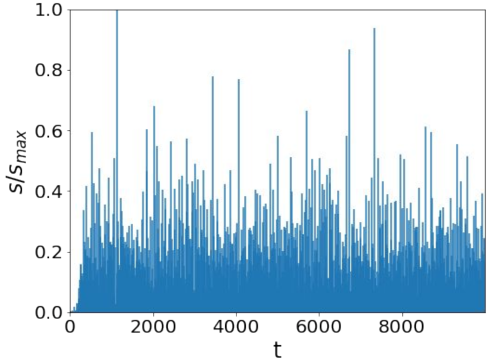

Top grains
Proportion
Empty cells
Threshold: 4
Threshold: 8
Steps



Discussion

- Stochasticity
- Top grain distribution dependence on parameters
- More than 2 types
- Initial startup phase (4%)





Conclusions

SOC holds for two thresholds

τ depends highly on specific thresholds and adding method

Formation of pyramids depends mostly on lowest threshold

Top grain in stacks dominated by highest threshold

Questions | Reference List

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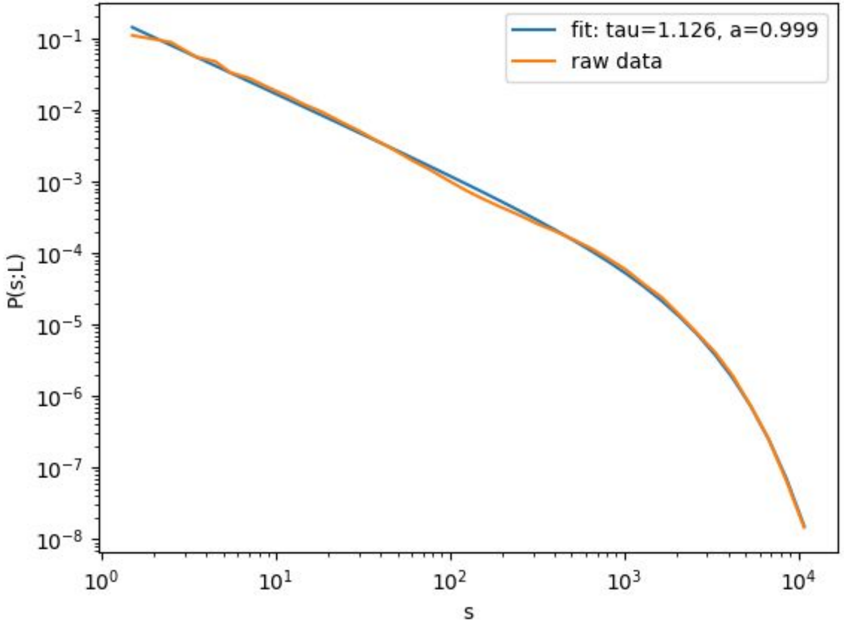
Extra - Best fit



Extra - Worst fit