Binary logistic regression: Conceptual overview and example using jamovi

In this video, I provide a general introduction to binary logistic regression and discuss how to interpret regression output based on an analysis using the jamovi program. Jamovi is an open-source program that can be freely downloaded at https://www.jamovi.org/download.html.

I will begin my discussion with a general overview of binary logistic regression (henceforth, BLR) and then proceed with an example in which I analyze a set of data.

This Powerpoint will be made available for download under the video description and incorporates more detail (in terms of review of logistic regression concepts) than I provide in the video. The dataset is available for download as well at https://drive.google.com/open?id=1Etmudy8b6SZRykSPxCyFzG8ANQZwv966, with this link included in the video description. The data are contained in an SPSS file, which can be imported directly into jamovi.

Finally, the demonstration incorporates some discussion of the use of dummy variables when factor variables are input as predictors. For another video that provides an overview of dummy coding (albeit in SPSS), you can go here: https://www.youtube.com/watch?v=XGlbGaOsV9U

Binary logistic regression

Overview:

Binary logistic regression (BLR) is utilized when a researcher desires to model the relationship between one or more predictor variables and a binary dependent variable. Fundamentally, the researcher is addressing the question, "What is the probability that a given case falls into one of two categories on the dependent variable, given the predictors in the model?"

One might be inclined to ask why we don't use standard ordinary least squares regression (OLS) instead of BLR. OLS regression assumes (a)there is a linear relationship between the independent variables and the dependent variable and (b) the residuals are normally distributed and (c) exhibit constant variance (Pitcuh & Stevens, 2016). All three assumptions are violated if the outcome variable in an OLS model is binary. And pivoting off (a), the relationship between one's predictor variables and the probability of a target outcome is inherently non-linear (and takes on an S-shaped curve). This is because probabilities are bounded at 0 and 1. When modeling a binary outcome using OLS regression, the estimation of model parameters ignores this boundedness, which has the notable effect of producing predicted probabilities that fall outside the 0-1 range. BLR estimates regression parameters by taking into account the fact that probabilities are bounded and 0 and 1. It also does not assume that residuals are normally distributed and exhibit constant variance. Because of this, Osborne (2015) classifies BLR as a "non-parametric technique" (p. 10).

Binary logistic regression

Overview (cont'd):

Although the relationship between predictors and the probability of (target) group membership is inherently non-linear, BLR still relies on a linear function to model that relationship. It does so by using a link function (log link) that converts the non-linear relationship (in this case, reflected in an S-shaped curve) into one that is linear. [One might say that BLR 'linearizes' the previous non-linear relationship.] Unfortunately, this process transforms the fitted values on the dependent variable from a metric that is fairly intuitive (i.e., the probability of target group membership) into one that is rather non-intuitive (i.e., log odds of target group membership; also referred to as the logit). As such, the regression coefficients in the model are interpreted as the predicted increase in log odds associated with target group membership per unit increase on the predictor.

$$logit = \ln\left(odds\right) = \ln\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

Here, we see that a person's predicted logit is a function of the predictors in the model. The logit is equal to the natural log of the person's odds of being in the target group. The person's predicted odds of target group membership is the ratio of the probability of target group membership to the probability of non-target group membership.

Binary logistic regression

Overview (cont'd):

To make things a bit more concrete, the equation below shows the relationship between the probability of an event (A) and the odds for that event. As you see, the odds for an event is simply a ratio of the probability of the event to the probability the event does not occur.

$$odds(A) = \frac{P(A)}{1 - P(A)}$$

You may be thinking this is more intuitive than log-odds and wondering, "Why don't we model the relationship between the predictors and the odds of target group membership?" The reason is that although there is no upper bound to the odds, it is still bounded at 0 on the lower end. As such, the relationship between predictors and the odds would still be non-linear. By taking the natural log of the odds (resulting in logits), we now have a dependent variable that is unbounded, as potential values for the logit can range from negative infinity to positive infinity. As a result, we can model the relationship between the predictors and the outcome using a linear function (see previous slide).

A few other details (in case you are wondering)

If you wish to transform a predicted logit back into odds, you simply exponentiate the logit. That is, raise the base of the natural logarithms (e= 2.7182818...) to the power of the logit (which is a linear function based on the set of predictors).

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To transform odds back into probability, you simply take the ratio of the odds to 1+odds.

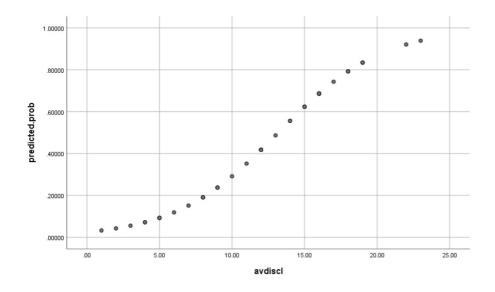
$$P robability = \frac{odds}{1 + odds}$$

Stitching these equations together, we can get a more direct feel of the relationship between logits, odds, and probabilities.

$$Probability = \frac{odds}{1 + odds} = \frac{\exp(logit)}{1 + \exp(logit)} = \frac{e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k}}{1 + e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k}}$$

To help you to visualize the relationship between a predictor and the (a) predicted probability, (b) predicted odds, and (c) predicted logit associated with target group membership, I ran a BLR in SPSS with the variable, terminate (coded 0=did not terminate early, 1=terminated early), serving as the outcome of a single predictor, 'avdiscl'. When running the model, I saved the predicted probability of a person falling into the "terminated early" group (the target outcome) and then computed the predicted odds and predicted logits for each person.

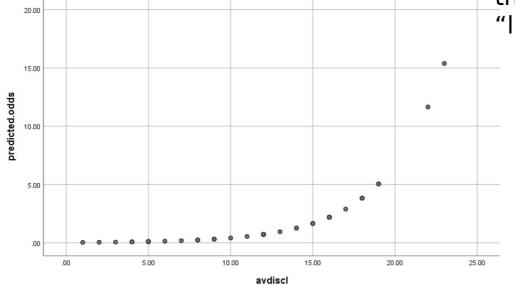
	& terminate	ø avdiscl		🗞 genderid	₫ income			
1	.00	10.00	12.00	1.00	1.00	.29125	.41	89
2	.00	8.00	14.00	.00	1.00	.19051	.24	-1.45
3	.00	6.00	19.00	1.00	1.00	.11878	.13	-2.00
4	.00	9.00	22.00	1.00	1.00	.23722	.31	-1.17
5	.00	12.00	16.00	.00	1.00	.41777	.72	33
6	.00	1.00	18.00	1.00	2.00	.03237	.03	-3.40

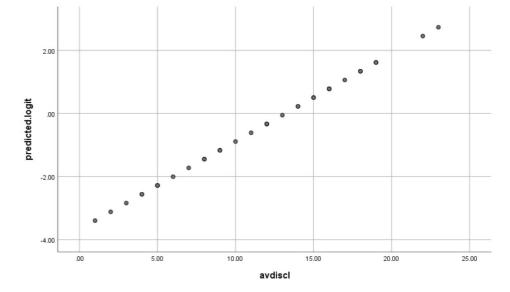


To the left is a plot of the predicted probabilities of target group membership ("terminated early") as a function of the predictor. A plot of the predicted odds of target group members is shown (left) below. In both of these cases, you see non-linear relationships between the fitted values and the predictor.

Below, we have a plot of the predicted logits as a function of the predictor. As you can see, the relationship between the predictor and outcome has been

"linearized".





A couple of final notes

Model estimation:

Unlike OLS regression, BLR uses maximum likelihood (ML) to estimate model parameters. Maximum likelihood estimation is an iterative process aimed at arriving at population (parameter) values that most likely produced the observed (sample) data. In general, this estimation approach assumes large samples and, aside from issues of power, smaller sample sizes can create problems with model convergence and estimation of model parameters. [Side note: With smaller samples, Exact logistic regression or Firth procedure using Penalized Maximum likelihood can be used. Unfortunately, these options are not commonly available in statistics programs.]

Evaluation of model fit:

As with standard OLS regression, evaluation of fit occurs at two levels. Typically, one evaluates the fit of the full-model (i.e., containing the full set of predictors). In BRL, this can be done using a likelihood-ratio chi-square test (which compares the full model with a null, or intercept-only, model), evaluation of various "pseudo-r-squared" indices, evaluation the degree to which the model is able to classify cases into groups on the dependent variable. Following, one evaluates the individual predictors for their contribution to overall model fit. This is done using either the Wald test or likelihood ratio tests (the latter involves comparing the full model with all the predictors against a reduced model with a given predictor removed).

Binary logistic regression example using jamovi

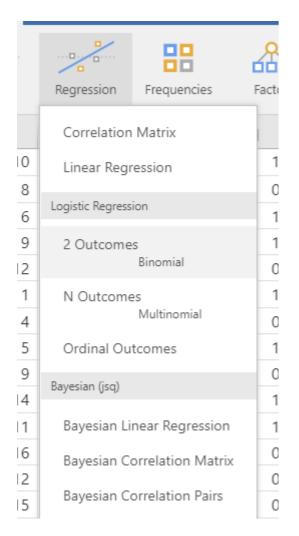
Scenario:

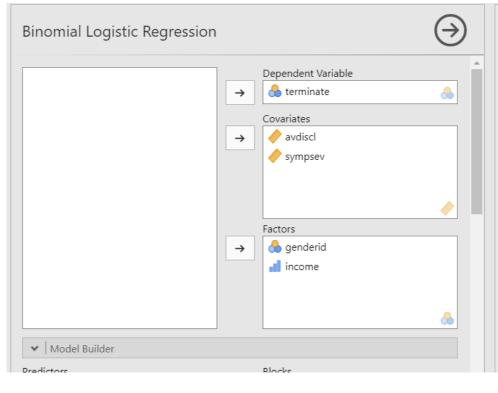
In this example, we are predicting the likelihood of early termination from counseling in a sample of n=45 clients at a community mental health center. The dependent variable in the model is 'terminate' (coded 1=terminated early, 0=did not terminate early), where the "did not terminate" group is the reference (baseline) category and the "terminated early" group is the target category. Two predictors in the model are categorical: gender identification ('genderid', coded 0=identified as male, 1=identified as female) and 'income' (ordinal variable, coded 1=low, 2=medium, 3=high). The reference category for 'genderid' is male identification, whereas the reference category for 'income' is the low income group. Finally, two predictors are assumed continuous in the model: avoidance of disclosure ('avdisc') and symptom severity ('sympsev').

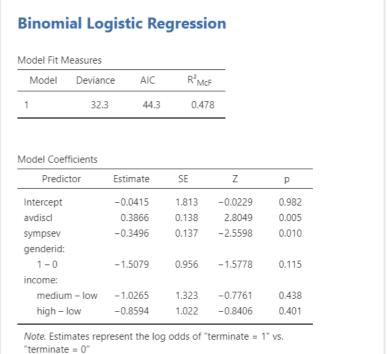
	A terminate	avdiscl	sympsev	A genderid	income
1	0	10	12	1	low
2	0	8	14	0	low
3	0	6	19	1	low
4	0	9	22	1	low
5	0	12	16	0	low
6	0	1	18	1	medium
7	0	4	17	0	low
8	0	5	15	1	low
9	0	9	14	0	low
10	0	14	8	1	low
11	0	11	7	1	medium
12	0	16	9	0	high
13	0	12	12	0	high
14	0	15	15	0	high
15	0	14	17	1	low
16	0	2	18	0	low
17	0	5	14	0	low
18	0	7	13	1	medium
19	0	13	10	1	high
20	0	8	8	0	high
21	0	5	4	1	low
22	0	18	18	0	medium
23	0	3	16	1	high
24	0	4	19	1	high
25	0	15	14	1	high
26	1	22	15	0	low
27	1	18	14	0	low
28	1	16	8	0	high

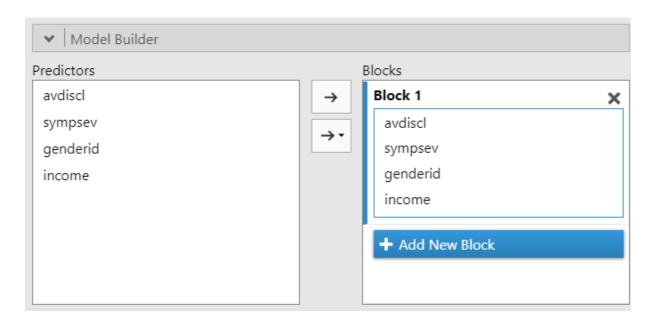
		<i>♦</i> avdiscl	<u> </u>		
	A terminate	avdisci	sympsev	A genderid	income
29	1	5	9	0	high
30	1	19	11	0	medium
31	1	18	10	0	medium
32	1	16	4	0	medium
33	1	12	9	1	low
34	1	19	8	1	low
35	1	23	14	0	low
36	1	16	13	0	medium
37	1	15	12	1	high
38	1	9	8	0	low
39	1	12	4	0	low
40	1	16	6	1	medium
41	1	16	9	0	low
42	1	17	16	1	low
43	1	19	14	0	high
44	1	12	13	0	high
45	1	8	3	0	low

Click on Regression and then 2 Outcomes Binomial. Then move variables from left box into the boxes on the right to designate the DV, covariates (assumed Scale) and factors (nominal or ordinal variables). Technically, 'genderid' could've been included in the Covariates box since it is a binary variable and already dummy coded (0 and 1).









The 'Model' menu fills in automatically as you add in predictors above. This box also allows you to specify interaction terms for your model & allows you to perform hierarchical regression analyses by adding variables in successive blocks.

For this demo, we are entering the full set of predictors simultaneously in a single block.

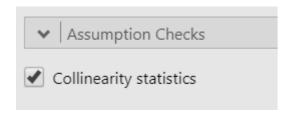


This input box allows you to establish the reference category (or baseline category) for each of the categorical variables in the model, which includes the dependent variable and any factors you include.

In this demo, the reference category for 'terminate' is group 0 (did not terminate early). This is the group against which group 1 (did terminate early) is compared.

The reference category for 'genderid' is 0 (identified as male) and is the group against which group 0 (identified as female) is compared.

The reference category for income is the 'low' group (coded 1) and is the category against which the medium (coded 2) and high (coded 3) groups are compared. This is done by including dummy variables in the regression model, with regression coefficients representing differences in the probability of early termination between the medium and low groups (dummy variable 1) and between the high and low groups (dummy variable 2).



Assumption Checks

Collinearity Statistics

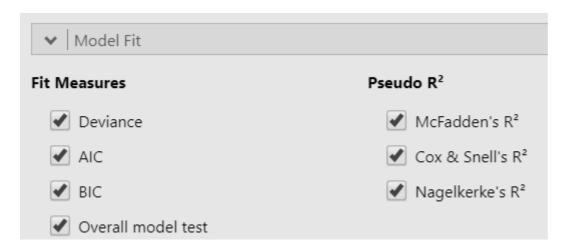
	VIF	Tolerance		
avdiscl	1.24	0.805		
sympsev	1.19	0.841		
genderid	1.07	0.939		
income	1.06	0.944		
income	1.00	0.944		

[3]

By clicking on 'Collinearity statistics', you are able to obtain the Variance Inflation Factor (VIF) and Tolerance who are used to evaluate for the presence of multicollinearity among your predictors.

The VIF associated with each predictor indexes the degree of variance inflation in the standard error (of the regression coefficient) due to collinearity among the predictors. Typically, VIF values > 10 are taken as a signal that multicollinearity among predictors may be problematic and should be investigated further. [By the same token, Tolerance values < .10 are suggestive of the need for further investigation. VIF=1/Tolerance]

These options give you various indices to evaluate overall model fit.



Binomial Logistic Regression

Model Fit Measures

							Ove	rall Model	l Test	
Model	Deviance	AIC	BIC	R ² _{McF}	R ² CS	R ² N	χ²	df	р	
1	32.3	44.3	55.1	0.478	0.481	0.644	29.5	5	< .001	

Binomial Logistic Regression

Model Fit Measures

							Overall Model Test			
Model	Deviance	AIC	BIC	R^2_{MCF}	R ² CS	R^2_N	χ²	df	р	
1	32.3	44.3	55.1	0.478	0.481	0.644	29.5	5	< .001	

This likelihood ratio chi-square test shown here is used to evaluate whether the fit of the model containing the full set of predictors is a significant improvement in fit over a null (i.e., intercept only) model. In effect, it can be considered an omnibus test of the null hypothesis that the regression slopes for all predictors in the model are equal to zero (Pituch & Stevens, 2016). If significant (as is the case in this example), these results are indicating that the model is a better fit to the data than the null model.

Binomial Logistic Regression

Model Fit N	1easures									
							Ove	Overall Model Test		
Model	Deviance	AIC	BIC	R^2_{McF}	R^2_{CS}	R ² N	χ²	df	р	
1	32.3	44.3	55.1	0.478	0.481	0.644	29.5	5	< .001	

These are pseudo-R-square values. They are not computed in the same manner as R-square in OLS regression. They are interpreted as analogous to R-square. R^2_{MCF} is McFadden's pseudo-R-square. R^2_{CS} is the Cox and Snell pseudo-R-square. And R^2_{N} is the Nagelkerke pseudo-R-square.

According to Osborne (2015), the literature provides little guidance on how to interpret different variants of pseudo-R-square (this leading him to reject offering suggestions on their use; see p. 51). Similarly, Lomax & Hahs-Vaugn (2012) note that there is "no consensus" on "which is best". As such, researchers often choose not to report on them. Notably, reviews of pseudo-R-square indices by Tabachnick & Fidell (2013) and UCLA: Statistical consulting group (https://stats.idre.ucla.edu/other/mult-pkg/faq/general/faq-what-are-pseudo-r-squareds/) largely provide mathematical treatments of the topic. And the IBM Knowledge Center addressed pseudo-R-square values (

https://www.ibm.com/support/knowledgecenter/en/SSLVMB_23.0.0/spss/tutorials/plum_germcr_rsquare.html

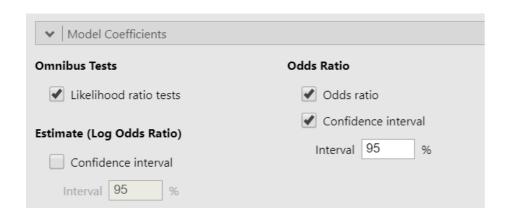
) by suggesting they are most useful when comparing different competing models.

Binomial Logistic Regression

Model Fit	Measures						Ove	rall Model	Test
Model	Deviance	AIC	BIC	R ² _{McF}	R ² CS	R ² N	χ²	df	р
1	32.3	44.3	55.1	0.478	0.481	0.644	29.5	5	< .001

The deviance statistic is an indicator of "badness of fit" and is particularly useful in logistic regression when one is comparing nested models (i.e., in circumstances where one model is a subset of another model). The AIC and BIC can be used with both nested and non-nested models (so are applicable in comparing nested models in the same fashion as the deviance). The difference is that these indices include a penalty for model complexity, which is a function of the number of parameters that are estimated (see Pituch & Stevens, 2016).

In general, when multiple models are being tested the one with the lowest deviance, AIC, or BIC value is preferred. [Side note: Sometimes it is the case that one may obtain AIC or BIC values that are negative. When this occurs, the model with AIC or BIC values that are "most negative" is preferred. In other words, if you can imagine a number line that runs from negative infinity to positive infinity, models with AIC or BIC values that are closer to negative infinity (relative to others) are preferred.]



Omnibus Likelihood Ratio Tests

χ^2	df	р
15.531	1	< .001
9.811	1	0.002
2.764	1	0.096
0.979	2	0.613
	9.811 2.764	15.531 1 9.811 1 2.764 1

[3]

Model Coefficients

						95% Confidence Interva	
Predictor	Estimate	SE	Z	р	Odds ratio	Lower	Upper
Intercept	-0.0415	1.813	-0.0229	0.982	0.959	0.0274	33.532
avdiscl	0.3866	0.138	2.8049	0.005	1.472	1.1235	1.928
sympsev	-0.3496	0.137	-2.5598	0.010	0.705	0.5394	0.921
genderid:							
1 – 0	-1.5079	0.956	-1.5778	0.115	0.221	0.0340	1.441
income:							
medium – Iow	-1.0265	1.323	-0.7761	0.438	0.358	0.0268	4.787
high – low	-0.8594	1.022	-0.8406	0.401	0.423	0.0571	3.141

Note. Estimates represent the log odds of "terminate = 1" vs. "terminate = 0"

						95% Confidence Interval		
Predictor	Estimate	SE	Z	р	Odds ratio	Lower	Upper	
Intercept	-0.0415	1.813	-0.0229	0.982	0.959	0.0274	33.532	
avdiscl	0.3866	0.138	2.8049	0.005	1.472	1.1235	1.928	
sympsev	-0.3496	0.137	-2.5598	0.010	0.705	0.5394	0.921	
genderid:								
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income:								
medium – low	-1.0265	1.323	-0.7761	0.438	0.358	0.0268	4.787	
high – low	-0.8594	1.022	-0.8406	0.401	0.423	0.0571	3.141	

The 'Estimate' column contains the regression coefficients. For each predictor, the regression slope is the predicted change in the log odds of falling into the target group (as compared to the reference group on the dependent variable) per one unit increase on the predictor (controlling for the remaining predictors). [Note: A common misconception is that the regression coefficient indicates the predicted change in probability of target group membership per unit increase on the predictor – i.e., p(Y=1|X's). This is WRONG! The coefficient is the predicted change in log odds per unit increase on the predictor]. Nevertheless, you can generally interpret a positive regression coefficient as indicating the probability (loosely speaking) of falling into the target group increases as a result of increases on the predictor variable; and that a negative coefficient indicates that the probability (again, loosely speaking) of target membership decreases with increases on the predictor. If the regression coefficient = 0, this can be taken to indicate changes in the probability of being in the target group as scores on the predictor increase.

						95% Confidence Interval	
Predictor	Estimate	SE	Z	р	Odds ratio	Lower	Upper
Intercept	-0.0415	1.813	-0.0229	0.982	0.959	0.0274	33.532
avdiscl	0.3866	0.138	2.8049	0.005	1.472	1.1235	1.928
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genderid:							
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high – low	-0.8594	1.022	-0.8406	0.401	0.423	0.0571	3.141

The Odds Ratio (OR) column contains values that are interpreted as the multiplicative change in odds for every one unit increase on a predictor. In general, an odds ratio (OR) > 1 indicates that as scores on the predictor increase, there is an increasing probability of the case falling into the target group on the dependent variable. An odds ratio (OR) < 1 can be interpreted as decreasing probability of being in the target group as scores on the predictor increase. If the OR=1, then this indicates no change in the probability of being in the target group as scores on the predictor change.

The 95% confidence interval for the Odds ratio can also be used to test the observed OR to determine if it is significantly different from the null OR of 1.0. If 1.0 falls between the lower and upper bound for a given interval, then the computed odds ratio is not significantly different from 1.0 (indicating no change as a function of the predictor).

						95% Confide	ence Interval
Predictor	Estimate	SE	Z	р	Odds ratio	Lower	Upper
Intercept	-0.0415	1.813	-0.0229	0.982	0.959	0.0274	33.532
avdiscl	0.3866	0.138	2.8049	0.005	1.472	1.1235	1.928
sympsev	-0.3496	0.137	-2.5598	0.010	0.705	0.5394	0.921
genderid:							
1 – 0	-1.5079	0.956	-1.5778	0.115	0.221	0.0340	1.441
income:							
medium – low	-1.0265	1.323	-0.7761	0.438	0.358	0.0268	4.787
high – low	-0.8594	1.022	-0.8406	0.401	0.423	0.0571	3.141

Avoidance of disclosure is a positive and significant (b=.3866, s.e.=.138, p=.005) predictor of the probability of early termination, with the OR indicating that for every one unit increase on this predictor the odds of early termination change by a factor of 1.472 (meaning the odds are increasing).

Symptom severity is a negative and significant (b=-.3496, s.e.=.137, p=.010) predictor of the probability of early termination. The OR indicates that for every one unit increment on the predictor, the odds of terminating change by a factor of .705 (meaning that the odds are decreasing).

Genderid is a non-significant predictor of early termination (b=-1.5079, s.e.=.0956, p=.115). [Had the predictor been significant, then the negative coefficient would be taken as an indicator that females (coded 1) are less likely to terminate early than males.]

						95% Confide	ence Interval
Predictor	Estimate	SE	Z	р	Odds ratio	Lower	Upper
Intercept	-0.0415	1.813	-0.0229	0.982	0.959	0.0274	33.532
avdiscl	0.3866	0.138	2.8049	0.005	1.472	1.1235	1.928
sympsev	-0.3496	0.137	-2.5598	0.010	0.705	0.5394	0.921
genderid:							
1 – 0	-1.5079	0.956	-1.5778	0.115	0.221	0.0340	1.441
income:							
medium – low	-1.0265	1.323	-0.7761	0.438	0.358	0.0268	4.787
high – low	-0.8594	1.022	-0.8406	0.401	0.423	0.0571	3.141

Income is represented by two dummy variables:

The first dummy variable is a comparison of the medium (coded 1 on the variable) and low (reference category coded 0 on the variable) income groups. The negative coefficient suggests that persons in the medium income category were less likely to terminate early than those in the low income category. Nevertheless, the difference is not significant (b=-1.0265, s.e.=1.323, p=1.438). Similarly, the second dummy variable compares the high income group (coded 1 on the variable) and the low income group (again, the reference category; coded 0). The difference between the groups is not significant, however (b=1.8954, s.e.=1.022, p=1.401).

						95% Confide	ence Interval
Predictor	Estimate	SE	Z	р	Odds ratio	Lower	Upper
Intercept	-0.0415	1.813	-0.0229	0.982	0.959	0.0274	33.532
avdiscl	0.3866	0.138	2.8049	0.005	1.472	1.1235	1.928
sympsev	-0.3496	0.137	-2.5598	0.010	0.705	0.5394	0.921
genderid:							
1 – 0	-1.5079	0.956	-1.5778	0.115	0.221	0.0340	1.441
income:							
medium – low	-1.0265	1.323	-0.7761	0.438	0.358	0.0268	4.787
high – low	-0.8594	1.022	-0.8406	0.401	0.423	0.0571	3.141

The tests carried out in this table are Wald tests. One downside of this test is that it can be overly conservative (leading to increased likelihood of Type 2 error).

Note. Estimates represent the log odds of "terminate = 1" vs. "terminate = 0"

Omnibus Likelihood Ratio Tests

Predictor	χ²	df	р
avdiscl	15.531	1	< .001
sympsev	9.811	1	0.002
genderid	2.764	1	0.096
income	0.979	2	0.613

The likelihood ratio chi-square test is generally viewed as "more superior" to Wald tests (Tabachnick & Fidell, 2013, p. 461). As you can see here, although the tests do not suggest any differences in interpretation concerning the significance of the predictors, notice that the p-values are smaller. [Also note that the 'income' variable is testing the overall effect of group income group differences]

✔ Prediction		
Cut-Off	Predictive Measures	ROC
Cut-off plot	Classification table	ROC curve
Cut-off value 0.5	✓ Accuracy	AUC
	Specificity	
	Sensitivity	

The results here provide further information concerning the adequacy of the model in predicting group membership on the dependent variable. The classification table (upper right) gives you cell frequencies representing the correspondence between Observed and Predicted group membership (the latter as a function of the model). We see that 100%*23/(23+2) = 92% of cases that were observed not terminate early were correctly predicted (by the model) not to terminate early. Of the 20 cases observed to terminate early, 100%*15/(5+15) = 75% were correctly predicted by the model to terminate early. [As you can see, sensitivity refers to accuracy of the model in predicting target group membership, whereas specificity refers to the accuracy of a model to predict non-target group membership.] The overall classification accuracy based on the model was 84.4%.

Prediction

Classification Table - terminate

	Predi	cted	
Observed	0	1	% Correct
0	23	2	92.0
1	5	15	75.0

Note. The cut-off value is set to 0.5

Predictive Measures

Accuracy	Specificity	Sensitivity
0.844	0.920	0.750

Note. The cut-off value is set to 0.5

For an additional nice discussion of sensitivity and specificity, try:

https://www.theanalysisfactor.com/sensitivity-and-specificity/

Prediction

Classification Table - terminate

Observed	0	1	% Correct
0	23	2	92.0
1	5	15	75.0

Note. The cut-off value is set to 0.5

Predictive Measures

Accuracy	Specificity	Sensitivity
0.844	0.920	0.750

Note. The cut-off value is set to 0.5

References and resources:

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