Special Topic

Using ChatGPT to Solve FinMath Problems? Be Careful

Updated on July 19, 2024

nickycanhelp@qq.com

当我们尝试用ChatGPT去求解金融数学问题时, 一定要非常小心, 因为通常你都会看到ChatGPT在"一本正经"的生成错误答案. 例如, 我们采用如下的一道稍微复杂一点的例题测试两遍, ChatGPT都没能生成出正确的结果.[1]

Example 1. *The price of a US stock S and the USD/Euro exchange rate Q follow two GBMs given by, respectively, for $t \in [0,T]$,*

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t^1, \tag{1}$$

$$\frac{dQ_t}{Q_t} = \beta dt + \delta dW_t^2, \tag{2}$$

where W^1 and W^2 are BMs under the probability measure $\mathbb{P}$ with correlation ρ. Find two new BMs W^{1*} and W^{2*} under a new probability measure $\mathbb{P}^*$ such that $e^{-rt}Q_t e^{r_f t}$ and $e^{-rt}S_t$ are two martingales under $\mathbb{P}^*$, where r and r_f are risk-free rates of USD and Euro, respectively.

但ChatGPT的回答并非一无是处, 其中的关键词Girsanov theorem, change of measure, martingale drift term is zero等都能够给我们提供hint和求解的大致方向. 加上自己已储备的FinMath知识和基本逻辑, 我们便能够在ChatGPT的辅助下, 自己写出正确的答案.

Let us first recall the Girsanov theorem in multi-dimensional version.

Theorem 1. *(Girsanov, multi-dimension.) Given $W = (W_1, \dots, W_n)'$ an n-dimensional standard BM on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$. Suppose for any $i = 1, \dots, n$, $\gamma^i = (\gamma_t^i)_{t\in[0,T]}$ is an $\mathbb{F}$-adaptive process such that*

$$\mathbb{E}^{\mathbb{P}}[\exp\{\frac{1}{2}\int_0^T (\gamma_t^i)^2 dt\}] < +\infty, \tag{3}$$

then there exists a probability measure $\mathbb{P}^*$ on $(\Omega, \mathcal{F})$ s.t. $\mathbb{P}^* \sim \mathbb{P}$ and

$$\frac{d\mathbb{P}^*}{d\mathbb{P}} = \exp\{-\frac{1}{2}\int_0^T \sum_i (\gamma_t^i)^2 dt - \sum_i \int_0^T \gamma_t^i dW_t^i\}, \tag{4}$$

and W^* is an n-dimensional standard BM under $\mathbb{P}^*$ with

$$W_t^{i*} = W_t^i + \int_0^t \gamma_s^i ds. \tag{5}$$

[1]你可以自行向ChatGPT询问, 或参考我的chat过程https://chatgpt.com/share/1ada8ccf-9b7b-4e76-be65-1acc7e04e457

本题的正确答案如下:
Ans. Note that the martingale should have driftless SDE form. By Itô formula, we have

$$\begin{aligned} d(e^{-rt}S_t) &= \sigma e^{-rt}S_t(dW_t^1 + \frac{\mu - r}{\sigma}dt) \\ &= \sigma e^{-rt}S_t dW_t^{1*}, \end{aligned} \tag{6}$$

which is driftless hence is a martingale[2] under $\mathbb{P}^*$, where

$$W_t^{1*} = W_t^1 + \gamma_1 t,\ \gamma_1 = \frac{\mu - r}{\sigma}. \tag{7}$$

Let us define a new BM W^0 under $\mathbb{P}$ independent of W^1, thus

$$W_t^2 = \rho W_t^1 + \sqrt{1-\rho^2}W_t^0. \tag{8}$$

Then we have

$$dQ_t = \beta Q_t dt + \rho\delta Q_t dW_t^1 + \sqrt{1-\rho^2}\delta Q_t dW_t^0. \tag{9}$$

By Itô formula, we have

$$\begin{aligned} d(e^{-rt}Q_t e^{r_f t}) &= e^{-(r-r_f)t}Q_t[(\beta - (r - r_f))dt + \rho\delta dW_t^1 + \sqrt{1-\rho^2}\delta dW_t^0] \\ &= e^{-(r-r_f)t}Q_t[\rho\delta dW_t^{1*} + \sqrt{1-\rho^2}\delta dW_t^{0*}] \\ &= e^{-(r-r_f)t}Q_t\delta dW_t^{2*}, \end{aligned} \tag{10}$$

which is driftless hence is a martingale under $\mathbb{P}^*$, where

$$W_t^{0*} = W_t^0 + \gamma_0 t,\ \gamma_0 = \frac{\beta - (r - r_f) - \delta\rho\frac{\mu-r}{\sigma}}{\delta\sqrt{1-\rho^2}}, \tag{11}$$

$$\begin{aligned} W_t^{2*} &= \rho W_t^{1*} + \sqrt{1-\rho^2}W_t^{0*} \\ &= W_t^2 + \rho\gamma_1 t + \sqrt{1-\rho^2}\gamma_0 t. \end{aligned} \tag{12}$$

Therefore, by Girsanov theorem, W^{1*} and W^{2*} are two BMs with the same correlation ρ under $\mathbb{P}^*$ which is defined via

$$\frac{d\mathbb{P}^*}{d\mathbb{P}} = \exp\{-\frac{1}{2}(\gamma_1^2 + \gamma_0^2)T - (\gamma_1 W_T^1 + \gamma_0 W_T^0)\}. \tag{13}$$

□

而在ChatGPT的回答中,其并没有考虑到correlation ρ所产生的影响, 以及Girsanov theorem中所使用的multi-dimesional的BM是standard BM which has independent components.

◊ 关于使用ChatGPT辅助求解金融数学相关题目的建议:
(1) 尽量使用英文与ChatGPT沟通;
(2) ChatGPT目前为止更多的是为我们提供了解题思路和大致方向, 切不可完全依赖其生成的结果, 正确的答案最终是建立在我们对金融数学知识充分理解和掌握的基础之上的.

[2]In general, driftless SDE w.r.t. BM leads to local martingale but in this case it is a true martingale.

End.