Volatility modeling

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Abstract

In times of uncertainty, volatility of the stock markets can increase dramatically. It can be challenging to create a precise forecast of the volatility. However, to make investment decisions, it is important to have an accurate prediction of the volatility. This research predicts the volatility using Asymmetric GARCH, Asymmetric EGARCH and Beta-t-EGARCH models, based on data from the S&P500 index. The realised variance, close-to-close log returns and the VIX are available for the S&P500 index. Our models are evaluated using the Akaike Information Criterion and the Bayesian Information Criterion, the forecasts are evaluated using six different loss functions from Hansen and Lunde (2005) and Mincer-Zarnowitz regressions. Furthermore the research discovered that the one component Beta-t-EGARCH performs much better than the market expectations and Asymmetric GARCH model for a time horizon of 21 days. The Asymmetric GARCH model performs the best among all models for a time horizon of 5 days, while performing worse than the other available models for all other time horizons. However, the research discussed that as the model was estimated over the period 2000-2016, market behaviour could have changed and true parameters are different from estimated parameters.

1 Introduction

Investors have tried to predict the stock market for decades. Since the introduction of prediction models, investors strive to make these models as accurate as possible. As volatility of stocks play an important role in making investment decisions, predictive models for forecasting volatility are beneficial for creating and altering investment strategies. This research will evaluate different volatility prediction models, and conclude which volatility prediction model performs the best. The research question is: "What is the most accurate model for predicting the volatility of the S&P500 index?". The dataset describes the close-to-close log returns of the S&P500 index from 04-01-2000 to 22-05-2020, along with the realised variance and closing level of the VIX-index for each day.

Five different models will be evaluated and compared: the one component Asymmetric GARCH model; the two component Asymmetric EGARCH model and the one - and two component Beta-t-EGARCH model. The fifth model predicts the time-varying volatility with only the VIX-index. The parameters for the models are estimated using maximum likelihood. The forecasts of the models are evaluated on their relative quality, accuracy and efficiency for three different time horizons: 1-day, 5-day and 21-days. The quality is evaluated using the Akaike Information Criterion and Bayesian Information Criterion, accuracy is evaluated using two target variables, which are the realised volatility computed from daily and intra-day returns. Finally, forecasts are evaluated based on the Mincer-Zarnowitz regressions and six different loss functions as in Hansen and Lunde (2005). The models are estimated and computed using Matlab, the code is freely available at *www.github.com/nickyMKB*.

This research shows that the Asymmetric GARCH model overestimates the volatility in volatile times, especially during the recovery from volatile levels. The Beta-t-EGARCH model does not seem to have this problem. The Beta-t-EGARCH outperformes the market expectations (VIX) when predicting volatility with a time horizon of 21 days. The research has found presumptive evidence that the Asymmetric GARCH model is superior over Beta-t-EGARCH and market expectations (VIX) when predicting the volatility over a time horizon of 5 days, based on the loss functions from Hansen and Lunde (2005).

The results of this research contribute to improving market efficiency by investigating the existence of arbitrage opportunities using better-than-market-expectations volatility models. The models used in this research can also be used by investors to alter and create an investment strategy according to the predicted volatility.

This paper proceeds as follows. In Section 2 the sub-questions and corresponding hypotheses are presented. Section 3 describes the data set. The model definition, estimation and diagnostics are explained in Section 4. Section 5 illustrates the results of the research. Section 6 concludes.

2 Theory

To be able to answer the research question more efficiently, it is divided into three sub-questions: "Can the volatility of the S&P500 index over a given period be better predicted using an Asymmetric GARCH model or by the one component Beta-t-EGARCH model?", "Can the volatility of the S&P500 index over a given period be better predicted using an Asymmetric EGARCH model or by the two components

Beta-t-EGARCH model?" and "Can either of these models beat market expectations of volatility, as provided by VIX?"

As is stated in the first sub-question, this research aims to investigate which model of the two as previously mentioned is better in predicting the volatility of the S&P500 index. The hypothesis is that the one component Beta-t-EGARCH model is superior or equal in predicting the volatility of the S&P500 index. This is in line with the conclusions of Blazsek and Villatoro (2015). The same procedure will be applied for the second sub-question, the hypothesis is that the two component Beta-t-EGARCH model is superior as supported by Blazsek and Villatoro (2015).

Finally the third sub-question aims to answer whether any of these models will provide better-than-market-expectations predictions of volatility. The hypothesis is that both the Beta-t-EGARCH model with one - and two components provide more accurate forecasts of volatility than the VIX, as supported by Müller and Bayer (2017).

3 Data

The data set consists of four daily time series from 04-01-2000 to 22-05-2020 related to the S&P 500 index: (i) date, (ii) close-to-close log return of the index, (iii) the realised variance and (iv) the closing level of the VIX. Each of these series contains 5114 observations. In this section each of the time series is described in more detail.

In the first time series the dates of trading days in the US is found, these are business days like Monday till Friday, although the markets can be closed on national holidays. The second time series is the close-to-close log return of the S&P 500 index, which is calculated as $r_t = 100 \times log(y_t/y_{t-1})$, where y_t is the level of the index at the end of day t. The next series is the daily measures of the realised variance of the S&P500 index based on intra-day returns at the five minute frequency as obtained from the Oxford library. The last time series is the closing level of the VIX, the volatility index constructed by the Chicago Board Options Exchange. It is constructed from option prices and is intended to measure the expectation among investors of the annualised standard deviation of the return on the S&P 500 index in the next month.

Table 1: Key statistics of data

	close-to-close log return	realised intraday variance	closing level VIX
Mean	0.01	1.12	19.75
Standard deviation	1.25	2.68	8.97
Skewness	-0.35	10.66	2.27
Kurtosis	13.80	192.04	10.82

The mean of close-to-close log returns is 0.014 with standard deviation 1.25. this means that on average the log return is positive. The observed volatility is in most observations low, but there a few observation with very high volatility. This can be explained by the difference between the mean, which is 1.210, and the median, which equals 0.473. It's standard deviation is 2.684. The VIX has a mean of 19.749 and a standard deviation of 8.971. The VIX, similarly to the realised variance, has a few observation that are much higher than the mean.

4 Methodology

4.1 Volatility Modelling

Five models are to be considered in this paper: the one component Asymmetric GARCH model as in Glosten, Jagannathan, and Runkle (1993), the one component Beta-t-EGARCH model as first formulated in Harvey and Chakravarty (2008), the two components Asymmetric EGARCH model as in Adrian and Rosenberg (2008), the two components Beta-t-EGARCH model as in section 2.4 in Harvey and Lange (2018) and finally a benchmark model, the VIX-index constructed by the Chicago Board Options Exchange (CBOE). These models for time-varying volatility assume that r_t is generated by a model defined as:

$$r_t = \mu + \sigma_{t|t-1}\varepsilon_t, \; 1 \leq t \leq T, \tag{1}$$

where T denotes the sample size, the ε_t's are i.i.d. 'shocks' with $E[\varepsilon_t] = 0$ and $V[\varepsilon_t] = 1$.

The one component Asymmetric GARCH model splits the α into α_{pos} and α_{neg} , where they describe the sensitivity of volatility with respect to the positive and negative shocks. This is based on the assumption that $\alpha_{pos} \neq \alpha_{neg}$. This results in the following Asymmetric GARCH model:

$$\sigma^2_{t+1|t} = \sigma^2 + \alpha_{pos}\left[\varepsilon_t^2 1_{\varepsilon_t \geq 0} - \frac{1}{2}\right]\sigma^2_{t|t-1} + \alpha_{neg} a\left[\varepsilon_t^2 1_{\varepsilon_t < 0} - \frac{1}{2}\right]\sigma^2_{t|t-1} + \Big(\beta + \frac{\alpha_{pos}}{2} + \frac{\alpha_{neg}}{2}\Big)(\sigma^2_{t|t-1} - \sigma^2), \tag{2}$$

where $1_{\varepsilon_t \geq 0}$ equals one if the shock is positive, and zero otherwise. Likewise, this applies to $1_{\varepsilon_t < 0}$, which is one for a negative shock and zero otherwise.

The one component Beta-t-EGARCH model is specified as follows (defined in Lange, Vermeulen, and Lether (2020)):

$$\text{dynamic scale: } \lambda_{t+1|t} := \lambda(1-\phi) + \phi\lambda_{t|t-1} + \kappa u_t + \tilde{\kappa} v_t, \tag{3}$$

$$\text{robust variance measure: } u_t := \frac{\sqrt{v+3}}{\sqrt{2v}}\Big(\frac{v+1}{v-2+\varepsilon_t^2}\varepsilon_t^2 - 1\Big), \tag{4}$$

$$\text{robust location measure: } v_t := \frac{\sqrt{(v-2)(v+3)}}{\sqrt{v(v+1)}}\frac{v+1}{v-2+\varepsilon_t^2}\varepsilon_t, \tag{5}$$

where ε_t follows a student-t distribution with $v > 2$ degrees of freedom. Furthermore the volatility is modelled as $\sigma_{t+1|t} = \exp(\lambda_{t+1|t})$.

Two component models represent a long-run and short-run component for modelling volatility. The two components Asymmetric EGARCH model is specified as follows:

$$\text{Short-run component: } s_{t+1} := \theta_4 s_t + \theta_5 \varepsilon t + 1 + \theta_6\big(|\varepsilon_{t+1}| - \sqrt{2/\pi}\big), \tag{6}$$

$$\text{Long-term component: } l_{t+1} := \theta_7 + \theta_8 l_t + \theta_9 \varepsilon_{t+1} + \theta_{10}\big(|\varepsilon_{t+1}| - \sqrt{2/\pi}\big), \tag{7}$$

where ε_t follows a student-t distribution with $v > 2$ degrees of freedom, notice that this is different than the formulation in Adrian and Rosenberg (2008) where ε_t follows the normal distribution. This modification is due to empirical evidence of fat tails in market volatility which is discussed in Peiró (1994). Finally the market volatility is modelled as $\sigma_{t+1|t} = \exp(s_{t+1} + l_{t+1})$.

The two components Beta-t-EGARCH model is specified as follows:

$$\text{dynamic scale: } \lambda_{t+1|t} := \omega + \lambda_{1,t|t-1} + \lambda_{2,t|t-1}, \tag{8}$$

$$\text{Short/long-run component: } \lambda_{i,t+1|t} := \phi_i \lambda_{i,t+1|t} + \kappa_i u_t + \widetilde{\kappa}_i v_t \qquad i = 1, 2, \tag{9}$$

where $\phi_1 > \phi_2$ denotes the long-run component. Finally the market volatility is modelled as $\sigma_{t+1|t} = \exp(\lambda_{t+1|t})$, this model is also discussed in Harvey and Lange (2012).

The last model for modelling volatility, the VIX, is the benchmark model and is calculated using the prices of options in order to measure the expectation among investors of the (annualised) standard deviation of the S&P500 return over the next month, this is discussed in Bekaert and Hoerova (2014).

4.2 Estimation

This section discusses the estimation of all models but the VIX using maximum likelihood (ML). As all model's ε_t follow the t-distribution, therefore there is written a generalized estimation method such that:

$$\begin{aligned}\hat{\boldsymbol{\theta}}_{ML} &= argmax_{\boldsymbol{\theta}} \sum_{t=1}^{T} \log(f(r_t|I_{t-1};\boldsymbol{\theta})) = argmax_{\boldsymbol{\theta}} \sum_{t=1}^{T} \Big[-\log(\sigma_{t|t-1}) + \log p\Big(\frac{r_t - \mu}{\sigma_{t|t-1}}\Big)\Big] \\ &= argmax_{\boldsymbol{\theta}} \sum_{t=1}^{T} \Big[-\log(\sigma_{t|t-1}) + \log p\big(\varepsilon_t\big)\Big],\end{aligned} \tag{10}$$

where $\sigma_{t|t-1}$ is as to be defined for the different models as in 4.1 and $p(\cdot)$ is the p.d.f. of the Student's t distribution.

4.3 Prediction

The predicted volatility an h-day horizon is defined as $\sqrt{E_t \sum_{d=1}^{h} r_{t+d}^2} = z_{t,h}$ for simplicity. This predicted volatility for the different models is given by the following equations, defined in Lange, Vermeulen, and Lether (2020):

$$PVol_t^{(1)} := z_{t,h} = \sqrt{(\mu^2 + \sigma^2) + \frac{1 - (\frac{\alpha_{pos}}{2} + \frac{\alpha_{neg}}{2} + \beta)^h}{1 - \frac{\alpha_{pos}}{2} + \frac{\alpha_{neg}}{2} + \beta}(\sigma_{t+1}^2 - \sigma^2)}, \tag{11}$$

$$PVol_t^{(2)} := z_{t,h} \approx \sqrt{h\mu^2 + \sum_{d=1}^{h} \exp\Big\{2\lambda + 2\phi^{d-1}(\lambda_{t+1|t} - \lambda) + 2(\kappa^2 + \tilde{\kappa})^2 \frac{1 - \phi^{2(d-1)}}{1 - \phi^2}\Big\}}, \tag{12}$$

where model (1) indicates the Asymmetric GARCH model and (2) the one component Beta-t-EGARCH model Beta-t-EGARCH model as defined in 4.1.

4.4 Model diagnostics and performance evaluation

To measure the relative quality of a model using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), both BIC and AIC are criterion for model selection that prefer parsimonious models by introducing a penalty term for the number of parameters in the model; the penalty term is larger in BIC than in AIC. The models with the lowest criterions are preferred. These are defined as follows:

$$AIC = \frac{2k}{T} - \frac{2LogL}{T} \qquad\qquad BIC = \frac{k\log(T)}{T} - \frac{2LogL}{T}$$

where k are the number of parameters, T the number of observations and LogL is the log likelihood of the model.

To evaluate the accuracy of the models, we consider two target variables as follows and a benchmark model:

$$RVol_t^{(1)} = \sqrt{\sum_{d=1}^{h} r_{t+d}^2} \qquad RVol_t^{(2)} = \sqrt{\frac{4}{3}\sum_{d=1}^{h} rv5\_ss_{t+d}} \qquad Pvol_t^{(5)} = \sqrt{\frac{h}{250}}\mathrm{VIX}_t$$

where target variable (1) is the realized volatility computed from daily returns and target variable (2) the realized volatility computed from intra-day returns, notice that this variable is scaled up as discussed in Lange, Vermeulen, and Lether (2020). Finally the predicted volatility using the benchmark model (5) as defined in 4.1 is used for diagnostics.

To evaluate the forecasts, Mincer-Zarnowitz (MZ) regressions will be used for the out-of-sample predictions. This is done by estimating the regression

$$\sigma_{t+h} = \alpha + \beta\hat{\sigma}_{t+h|t} + \varepsilon_{t+h|h}, \tag{13}$$

where $\hat{\sigma}_{t+h|t}$ are the forecasts and σ_{t+h} the true value. The joint null hypothesis $\alpha = 0, \beta = 1$ is to be tested. The model is correctly specified if the null hypothesis fails to be rejected.

To further evaluate the predictions, the six loss functions proposed by Hansen and Lunde (2005) are to be evaluated:

$$\begin{aligned} MSE_1 &= T^{-1}\sum_{t=1}^{T}(\sigma_t - h_t)^2, & MSE_2 &= T^{-1}\sum_{t=1}^{T}(\sigma_t^2 - h_t^2)^2, \\ QLIKE &= T^{-1}\sum_{t=1}^{T}(ln(h_t) + \sigma_t^2/h_t^2)^2, & R^2LOG &= T^{-1}\sum_{t=1}^{T}[ln(\sigma_t^2/h_t^2)]^2, \\ MAE_1 &= T^{-1}\sum_{t=1}^{T}|\sigma_t - h_t|, & MAE_2 &= T^{-1}\sum_{t=1}^{T}|\sigma_t^2 - h_t^2|. \end{aligned}$$

where σ_t is the square root of the realized variance and h_t denotes the predicted standard deviation.

5 Results

We consider the daily returns as defined in Section 3 from 4 January 2000 to 22 May 2020 (5114 observations), the first set of estimates in Table 2 are from fitting a one-component Asymmetric GARCH model. As opposed to the small sensitivity of volatility to positive news, since α_{pos} is small and insignificant, negative news has a high sensitivity on the volatility as α_{neg} is positive, relatively large and significant. β. Furthermore as $\alpha_{pos}/2 + \alpha_{neg}/2 + \beta = 0.99$, $\sigma^2_{t|t-1}$ rather quickly mean-reverts to σ^2.

Table 2: One-component Asymmetric GARCH model estimates, with their standard errors (in the next row) for daily SP500 close-to-close log returns from 4 January 2000 to 22 May 2020 (5114 observations)

Volatility			Mean		Shape	Fit		
α_{pos}	α_{neg}	β	μ	ω	ν	Log L	AIC	BIC
.000	.2263	.876	.040	.017	6.633	-6776.2	2.6524	2.6601
.115	.035	.056	.012	.0025	.512			

The second set of estimates in Table 3 are from fitting a one-component Beta-t-EGARCH model. The news impact curve is asymmetric even though $\widetilde{\kappa}$ is small, it is not insignificant.

Table 3: One-component Beta-t-EGARCH model estimates, with their standard errors (in the next row) for daily SP500 close-to-close log returns from 4 January 2000 to 22 May 2020 (5114 observations)

Volatility				Mean	Shape	Fit		
κ	$\widetilde{\kappa}$	ϕ	λ	μ	ν	Log L	AIC	BIC
.0473	-.0944	.9749	.000	.0537	12.054	-6769.0	2.6496	2.6573
.0117	.0779	.0069	.5095	.1010	1.7452			

The third set of estimates in Table 4 are from fitting a two-component Asymmetric EGARCH model. To detect if l_t is truly the slowly mean-reverting, long-run component and l_t the quickly mean-reverting, short-run component is recognized $\theta_4 < \theta_8$. The terms $|\varepsilon_{t+1}| - \sqrt{2/\pi}$ in equation 6 and 7 are the shocks to volatility components, equal-sized positive or negative news result in the same volatility change, although as $\theta_6 < \theta_{10}$ sudden shocks immediately transfer over to the long-term component. Furthermore as both θ_5 and θ_9 (the asymmetric effects) are significant and negative, negative news result in a higher positive volatility change while positive news result in lower negative volatility change.

Table 4: Two-component Asymmetric EGARCH model estimates, with their standard errors (in the next row) for daily SP500 close-to-close log returns from 4 January 2000 to 22 May 2020 (5114 observations)

Volatility						Mean		Shape	Fit		
θ_4	θ_8	θ_6	θ_{10}	θ_5	θ_9	θ_7	μ	ν	Log L	AIC	BIC
.8768	.9906	-.0271	.0670	-0.0989	-.0296	.0006	.0375	7.1525	-6723.5	2.6330	2.6445
.0349	.0028	.0157	.0097	.0098	.0103	.0014	.0102	.7041			

The fourth set of estimates in Table 5 are from fitting a two-component Beta-t-EGARCH model. The long-term news impact curve is nearly symmetric as $\widetilde{\kappa}_1$ is small and insignificant. As opposed to the the symmetric long-term news impact curve, the short-term news impact curve implies asymmetry as $\widetilde{\kappa}_2$ is smaller than κ_2. This is derived using equation 4, which implies that when $\widetilde{\kappa} < \kappa < 0$ negative shocks result in higher absolute volatility change compared to positive shocks, important to note is that the volatility change during negative shocks is positive (and higher in absolute terms) while during positive shocks negative (but lower in absolute terms).

A positive (negative) shock, will result in the two components moving in opposite directions as $\kappa_1 = .0412 > \kappa_2 = -.1014$ with the initial net effect being a lowering (increase) of total volatility as the short-term component initially dictates the playing field.

Table 5: Two-component Beta-t-EGARCH model estimates, with their standard errors (in the next row) for daily SP500 close-to-close log returns from 4 January 2000 to 22 May 2020 (5114 observations)

Volatility							Mean	Shape	Fit		
κ_1	$\widetilde{\kappa_1}$	ϕ_1	κ_2	$\widetilde{\kappa_2}$	ϕ_2	ω	μ	ν	Log L	AIC	BIC
.0412	-.0291	.9884	-.0059	-.1014	.8919	.6758	.0313	9.5235	-6722.9	2.6327	2.6442
.0208	.0150	.0076	.0245	.0089	.0571	.0578	.0111	.5833			

The two component Beta-t-EGARCH is superior to the other four models according to AIC and BIC, as the respective values are the lowest among all models. This is remarkable as this model is not very parsimonious as it has 9 parameters.

The difference between the AIC and the BIC values of the two component Beta-t-EGARCH model and the two component Asymmetric EGARCH model is very small, approximately $3e^{-4}$ for both the AIC and the BIC values.

5.1 Out-of-sample volatility prediction

The daily returns as defined in Section 3 from 4 January 2000 to 20 May 2016 (4112 observations) are considered, which is approximately 80% of the data and leaves us with 20% for out-of-sample volatility prediction. The loss functions and Mincer-Zarnowitz regression proposed in 4.4 will be used to evaluate the out-of-sample volatility predictions. As the derivations of the formulas for predicted volatility of longer step horizons for model (3) and (4) are beyond the scope of this paper we resort to one-step ahead forecasts of these models, furthermore these models do not perform significantly better in the Mincer-Zarnowitz regression than the Asymmetric GARCH and Beta-t-EGARCH with one component which is why they're excluded in Table 6.

Table 6: Comparison of one-, five- and twenty one steps predictions of of the total realised volatility computed from SP500 daily returns and intra-day returns using Mincer-Zarnowitz regressions. Non-rejected null-hypothesis are shown in bold. For the coefficients α, β the first row correspond to the coefficients and the second row to the std. error, for the F-statistic the first row correspond to the value while the second row correspond to it's probability.

		Daily returns			Intra-day returns		
Asymmetric GARCH		1	5	21	1	5	21
	α	-.085	.044	1,243	.048	.291	1,448
		.036	.071	.177	.019	.052	.138
	β	.796	.902	.604	.706	.708	.461
		.028	.024	.03	.015	.018	.024
	F	92,980	14,723	10.856	375.570	181.800	336.590
		.000	.000	.000	.000	.000	.000
	R^2	.448	.577	.286	.703	.608	.277
	RMSE	.771	1.479	3.475	.401	1.088	2.714
Beta-t-EGARCH		1	5	21	1	5	21
	α	-.209	-.350	.313	-.062	-.023	.698
		.038	.076	.214	.019	.055	.165833
	β	1.061	1.237	.903	.941	.974	.699
		.036	.032	.045	.018	.023	.035
	F	23.412	30.780	2.664	49.400	3.123	58.020
		.000	.000	**.070**	.000	.045	.000
	R^2	.470	.606	.291	.737	.641	.290
	RMSE	.755	1.427	3.463	.377	1.041	2.690
VIX		1	5	21	1	5	21
	α	-.744	-1.268	-.119	-.0480	-.717	0.426
		.045	.093	.23	.022	.069	.177
	β	1.386	1.424	0.883	1.173	1.109	.670
		.039	.035	.042	.018	.026	.033
	F	178.139	93.725	22.902	470.283	109.220	137.365
		.000	.000	.000	.000	.000	.000
	R^2	0.561	.618	.30	.802	.64	.291
	RMSE	.688	1.405	3.431	.327	1.042	2.687

According to the Mincer-Zarnowitz regressions the VIX benchmark model is the clear winner with the overall highest R^2. The Beta-t-EGARCH model however, fails to reject the null-hypothesis of $\alpha = 0$ and $\beta = 1$ while predicting volatility over 21 days using daily returns which indicates a correct model.

According to the six loss functions as in Hansen and Lunde (2005), Beta-t-EGARCH performs better than the other models in 3 out of 6 loss functions for one step ahead forecasts using daily returns as target variable. As the VIX performs better in the remaining loss functions, we cannot argue that

Table 7: Comparison of one-, five- and twenty one steps predictions of of the total realised volatility computed from S&P500 daily returns using six loss functions as in Hansen and Lunde (2005). For each loss function (and horizon) the winner is shown in bold.

	MAE_1	MAE_2	MSE_1	MSE_2	QLIKE	R^2LOG
One step ahead (T=1003)						
Asymmetric GARCH	.55	1.61	.70	43.56	6.36	9.01
Beta-t-EGARCH	**.49**	**1.37**	**.60**	43.11	7.28	8.23
EGARCH	.51	1.44	.65	44.63	1.24	8.41
Beta-t-EGARCH(2 comp.)	.50	1.41	.621	43.15	9.06	8.26
VIX	.59	1.48	.64	**4.87**	**1.54**	**1.43**
Five steps ahead (T=998)						
Asymmetric GARCH	**.17**	**1.37**	**.42**	**157.37**	**1.63**	**.23**
Beta-t-EGARCH	.72	5.58	2.09	812.74	11.74	1.03
VIX	.91	6.38	2.22	833.21	4.74	1.48
Twenty one steps ahead (T=982)						
Asymmetric GARCH	1.99	32.95	12.58	11682.49	24.93	1.10
Beta-t-EGARCH	**1.74**	**25.54**	10.87	8553.11	38.27	**1.03**
VIX	1.91	27.30	**10.30**	**8268.15**	**16.86**	1.10

Table 8: Comparison of one-, five- and twenty one steps predictions of the total realised volatility computed from S&P500 intra-day returns using six loss functions as in Hansen and Lunde (2005). For each loss function (and horizon) the winner is shown in bold.

	MAE_1	MAE_2	MSE_1	MSE_2	QLIKE	R^2LOG
One step ahead (T=1003)						
AS	.33	1.02	.28	11.83	1.44	.96
Beta-t-EGARCH	**.25**	**.65**	**.17**	5.92	1.64	**.68**
EGARCH	.29	.82	.23	8.57	4.20	.84
Beta-t-EGARCH (2 comp.)	.27	.77	.20	8.21	2.78	.73
VIX	.38	.82	.21	**5.06**	**.64**	1.50
Five steps ahead (T=998)						
Asymmetric GARCH	**.14**	**1.10**	**.30**	**68.30**	**.82**	**.13**
Beta-t-EGARCH	.56	3.85	1.02	204.72	5.39	.51
VIX	.78	4.59	1.19	208.28	2.94	.92
Twenty one steps ahead (T=982)						
Asymmetric GARCH	1.85	27.97	10.38	831.12	12.38	1.00
Beta-t-EGARCH	**1.55**	**19.47**	**6.88**	**3335.29**	17.69	**.88**
VIX	1.83	22.27	7.19	3493.12	**9.01**	1.06

it performs significantly better for one-step ahead forecasts than market expectations. This is further strengthened by the huge difference in the the R^2LOG between the VIX and Beta-t-EGARCH, as this loss function assesses the goodness-of-fit of the out-of-sample forecasts based on the regressions of Mincer-Zarnowitz. This conclusion is in line with the conclusions in Table 6. By contrast, the R^2log is lower for Beta-t-EGARCH than VIX using intra-day returns variance as target variable.

Remarkably the Asymmetric GARCH model's prediction for five day performs the best overall for both daily returns and intra-day return's variance as target variables. This is in line with the Mincer-Zarnowitz regressions as the F-statistic is lower than that of the other models.

Using visual representation of the predictions we notice that the Asymmetric GARCH model, especially in volatile times, does not adjust to lower volatility levels quickly enough which makes the model overestimate the volatility during the recovery of volatile levels. The Beta-t-EGARCH model does not seem to have this problem and stays on target even in volatile times. Furthermore visual representation supports the hypothesis of the VIX often overestimating the realized volatility (and generally staying on the same level in non-volatile times), while realized volatility swings more often and between different levels.

The VIX still seems to outperform other models in 3 out of 6 loss functions for twenty one day forecasts using daily returns as target variable, as the VIX has the lowest MSE values for we can conclude here that if the other models have errors they tend to be more extreme than the error of the VIX (overall), which is further supported by the visual representation of the models. Remarkably, the Beta-t-EGARCH model performs the best in 5 out of 6 loss functions using variance computed from intra-day returns as a target variable. We could argue that variance computed from intra-day returns is closer to real-life volatility as it is computed using short intervals and error-free which makes Beta-t-EGARCH outperform market expectations for twenty one day forecasts.

6 Conclusions

The research question is "What is the most accurate model for predicting the volatility of the S&P500 index?", this research investigated five different models. These are respectively the one component

Asymmetric GARCH model as in Glosten, Jagannathan, and Runkle (1993), the one component Beta-t-EGARCH model as first formulated in Harvey and Chakravarty (2008), the two components Asymmetric EGARCH model as in Adrian and Rosenberg (2008), the two components Beta-t-EGARCH model as in section 2.4 in Harvey and Lange (2018) and finally a benchmark model, the VIX-index constructed by the Chicago Board Options Exchange.

The provocative question raised by Hansen and Lunde (2005): "Does anything beat a GARCH(1,1)" is in some agreement with our research as the differences for one steps are not statistically significant (using Diebold-Mariano tests) between the Beta-t-EGARCH and the Asymmetric GARCH, furthermore the Asymetric GARCH model is the clear winner for predicting the volatility for 5 days using just the loss functions. By contrast, looking at the R^2 in the Mincer-Zarnowitz regression of both models however, we find higher R^2 in the Beta-t-EGARCH for any horizon and target variable except when the horizon is 1 day and the daily returns as target variable excluding the VIX benchmark model. Furthermore, respectively 3 out of 5, 4 out of 5 and 5 out of 6 loss functions for time horizons 1, 5 and 21 prefer the Beta-t-EGARCH model when using variance from intra-day returns as target variable. As argued in Section 5 the squared variance computed from intra-day returns is close to real-life volatility. Hence, we still conclude that the Beta-t-EGARCH model is superior in predicting the volatility of the S&P500 index as in line with our first hypothesis, yet also concluding that the Beta-t-EGARCH model performs better at predicting volatility than market expectations according to the previously mentioned loss-functions. This conclusion is also in correspondence with our third hypothesis.

Using the loss functions and the Mincer-Zarnowitz regressions we conclude that the two component Beta-t-EGARCH model performs better at predicting volatility over a one day horizon than the two components EGARCH model, which is in line with our second hypothesis. As we have no closed analytical expressions for predicting more-than-one-day horizons, it is hard to compare these models to the one component models and determine if they perform better at predicting volatility than the market expectations (VIX). It is beyond the scope of this paper to further evaluate the performance of these models and compare them to market expectations.

As the out-of-sample period is quite large (1000 observations) and the models were not re-estimated, we could argue that any bad behaviour (bad predicting) of the models is due to market behaviour changing over time (from 2016 to 2020) and thus true parameters changing over time. This can be supported by the lower AIC&BIC of both the 2 components Beta-t-EGARCH and the 2 component EGARCH model, as these models are to be preferred using these criteria but not when using the loss-functions and Mincer-Zarnowitz regressions during out-of-sample forecasting. Hence, we recommend further research of the previously mentioned models, using moving windows and test for significant changing market behaviour in these parameters. Furthermore we recommend deriving the equations to predict volatility over longer horizons ($h > 1$) for these models, so the models can be better compared.

The results of this research contribute to improving market efficiency by investigating arbitrage opportunities using better-than-market-expectations volatility models.

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Appendix

A: Time series

Data

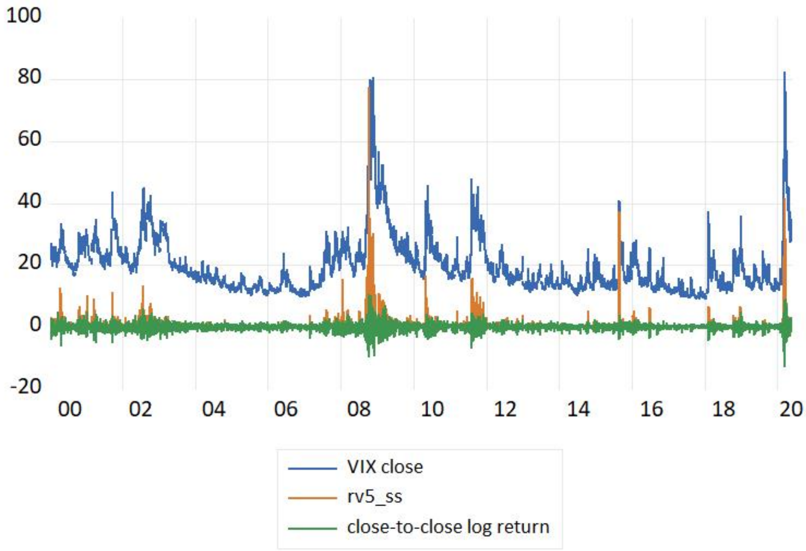


Figure 1: Time series of data

Forecasts

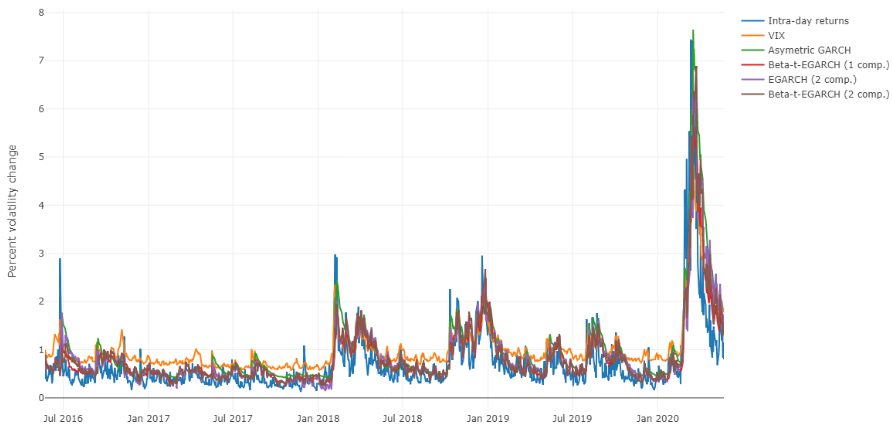


Figure 2: One day forecast of volatility

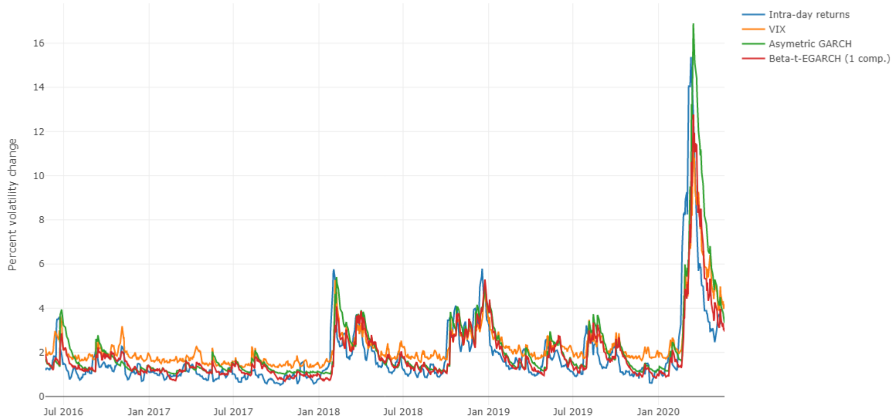


Figure 3: Five day forecast of volatility

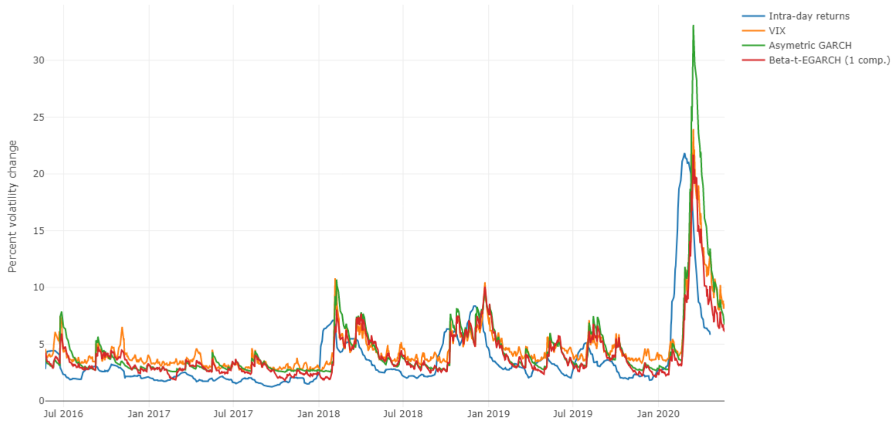


Figure 4: Twenty one day forecast of volatility

B: NIC

The difference between $\sigma^2_{t+1|t}$ and $\sigma^2_{t-1|t}$ can be seen as the the effect of the news received on day t. To quantify this effect there is calculated the so-called 'news impact curve' (NIC).

$$\mathrm{NIC}_t^{(1)} = \alpha_{pos}\left[\varepsilon_t^2 1_{\varepsilon_t \geq 0} - \tfrac{1}{2}\right] + \alpha_{neg}\left[\varepsilon_t^2 1_{\varepsilon_t < 0} - \tfrac{1}{2}\right],$$

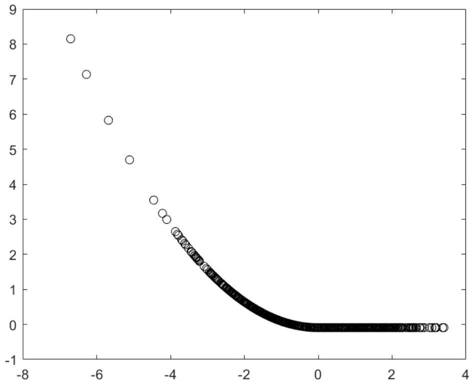
$$\mathrm{NIC}_t^{(2)} = 2\kappa u_t + \tilde{\kappa} v_t,$$

$$\mathrm{NIC}_{t,i}^{(4)} = 2\kappa_i u_t + \tilde{\kappa_i} v_t \text{ for i=1,2,}$$

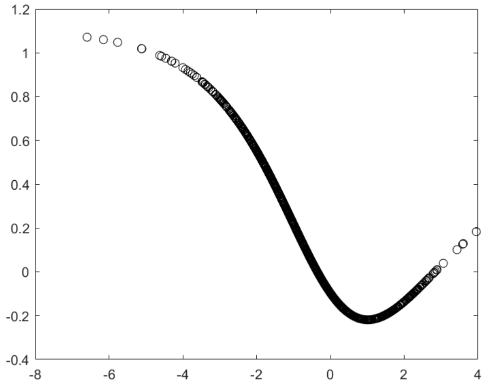
where model (1) is Asymmetric GARCH, (2) is Beta-t-EGARCH and (4) is Beta-t-EGARCH with two components. Notice that in $NIC_{t,i}^{(4)}$, the long-term news impact curve is computed if i=1 when $\phi_1 > \phi_2$

and the short-term news impact curve if i=2. Notice that the news impact curve for EGARCH (model 3) is missing as this is beyond the scope of the paper to derive this equation.

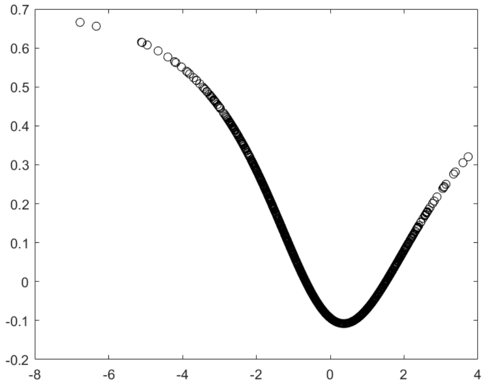
The news impact curve is plotted (on the vertical axis) as a function of the implied residual (on the horizontal axis).



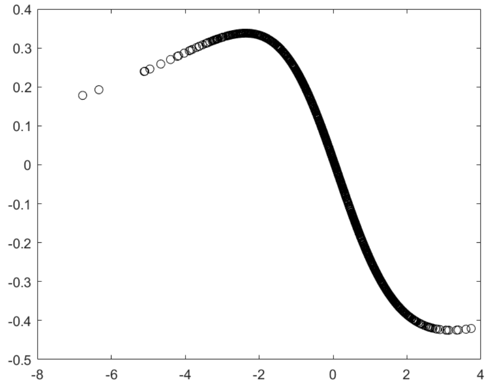
(a) NIC: Asymmetric GARCH



(b) NIC: Beta-t-EGARCH



(c) Long-term NIC: Beta-t-EGARCH (2 comp.)



(d) Short-term NIC: Beta-t-EGARCH (2 comp.)

Figure 5:
NIC for different models

C: Additional methods

$p(\cdot)$ is the p.d.f. of the Student's t distribution as seen in Equation 10, as to be defined as:

$$p(\varepsilon_t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi(\nu-2)}}\left(1+\frac{\varepsilon_t^2}{\nu-2}\right)^{-(\nu+1)/2},\ \nu > 2. \tag{14}$$

Given a set of P 1-step ahead forecasts for t = T, ... , $T+P-1$, the Diebold-Mariano statistic is to be defined as:

$$DM = \frac{\bar{d}}{\sqrt{V(\hat{d}_{t+1})/P}} \overset{H_0}{\sim} N(0,1) \tag{15}$$

where $d_t = e^2_{i,t|t-1} - e_{j,t|t-1}$, where the squared residuals are the loss functions obtained from respectively model i and j. $\bar{d}$ is the sample mean of the loss differentials, $V(\hat{d}_{t+1})$ an estimate of the variance of the loss differential.

D:

Table 9: Comparison of one steps predictions of of the total realised volatility computed from SP500 daily returns and intra-day returns using Mincer-Zarnowitz regressions. Non-rejected null-hypothesis are shown in bold. For the coefficients α, β the first row correspond to the coefficients and the second row to the std. error, for the F-statistic the first row correspond to the value while the second row correspond to it's probability.

		Daily returns (1 day)	Intra-day returns (1 day)
EGARCH			
	α	.119	.029
		.038	.021
	β	.911	.795
		.033	.018
	F	35.800	127.000
		.000	.000
	R^2	.437	.663
	RMSE	.779	.427
Beta-t-EGARCH (2 comp.)			
	α	-.128	.021
		.037	.019
	β	.940	.820
		.032	.017
	F	29.590	109.600
		.000	.000
	R^2	.459	.696
	RMSE	.763	.405