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IT Mathematics

Assignment 2

IT Maths - B00058026 - Nicky Randles

Question 1

(a) $(A+B)C = AC + BC$

$$(A+B)C = \left(\begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 2 & -2 \\ -1 & 1 & -1 \\ 3 & 2 & 3 \end{bmatrix} \right) \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)+(-2) & 2+2 & (-2)+(-2) \\ 0+(-1) & 1+1 & (-2)+(-1) \\ 0+3 & (-1)+3 & 3+3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & -4 \\ -1 & 2 & -3 \\ 3 & 2 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} [-3(2)+4(-1)-4(3)] & [-3(0)+4(-1)-4(2)] \\ [-1(2)+2(-1)-3(3)] & [-1(0)+2(-1)-3(2)] \\ [3(2)+2(-1)+6(3)] & [3(0)+1(-1)+6(2)] \end{bmatrix}$$

$$= \begin{bmatrix} [-6-4-12] & [0+4-8] \\ [-2-2-9] & [0-2-6] \\ [6-2+18] & [0-1+12] \end{bmatrix}$$

$$= \begin{bmatrix} -22 & -12 \\ -13 & -8 \\ 22 & 11 \end{bmatrix} \Big/ \begin{bmatrix} -22 & -12 \\ -13 & -8 \\ 23 & 11 \end{bmatrix} = \begin{bmatrix} -22 & -12 \\ -13 & -8 \\ 22 & 11 \end{bmatrix}$$

= True

$$AC+BC \begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 2 & -2 \\ -1 & 1 & -1 \\ 3 & 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & -1 \\ -3 & 2 \end{bmatrix}$$

$$AC = \begin{bmatrix} [-1(2)+2(-1)-2(3)] & [-1(0)+2(-1)-2(2)] \\ [0(2)+1(-1)-2(3)] & [0(0)+1(-1)-2(2)] \\ [0(2)-1(-1)+3(3)] & [0(0)-1(-1)+3(2)] \end{bmatrix}$$

$$= \begin{bmatrix} [-2-2-6] & [0-2-4] \\ [0-1-6] & [0-1-4] \\ [0+1+9] & [0+1+6] \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -6 \\ -7 & -5 \\ 10 & 7 \end{bmatrix}$$

$$BC = \begin{bmatrix} -2(2)+2(-1)-2(3) & -2(0)+2(-1)-2(2) \\ -1(2)+1(-1)-1(3) & -1(0)+1(-1)-1(2) \\ 3(2)+2(-1)+3(3) & 3(0)+2(-1)+3(2) \end{bmatrix}$$

$$= \begin{bmatrix} [-4-2-6] & [0-2-4] \\ [-2-1-3] & [0-1-2] \\ [6-2+9] & [0-2+6] \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -6 \\ -6 & -3 \\ 13 & 4 \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} (-10)+(-12) & (-6)+(-6) \\ (-7)+(-6) & (-5)+(-3) \\ 10+(13) & 7+4 \end{bmatrix} = \begin{bmatrix} -22 & -12 \\ -13 & -8 \\ 23 & 11 \end{bmatrix}$$

(b) $A^T + B^T = (A+B)^T$

$$A^T + B^T = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 1 & -1 \\ -2 & -2 & 3 \end{bmatrix} + \begin{bmatrix} -2 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+(-2) & 0+(-1) & 0+3 \\ 2+2 & 1+1 & (-1)+2 \\ (-2)+(-2) & (-2)+(-1) & 3+3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & 3 \\ 4 & 2 & 1 \\ -4 & -3 & 6 \end{bmatrix}$$

$$(A+B^T) = \left(\begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 2 & -2 \\ -1 & 1 & -1 \\ 3 & 2 & 3 \end{bmatrix} \right)^T$$

$$= \left(\begin{bmatrix} (-1)+(-2) & 2+2 & (-2)+(-2) \\ 0+(-1) & 1+1 & (-2)+(-1) \\ 0+3 & (-1)+2 & 3+3 \end{bmatrix} \right)^T$$

$$= \left(\begin{bmatrix} -3 & 4 & -4 \\ -1 & 2 & -3 \\ 3 & 1 & 6 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} -3 & -1 & 3 \\ 4 & 2 & 1 \\ -4 & -3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & 3 \\ 4 & 2 & 1 \\ -4 & -3 & 6 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 3 \\ 4 & 2 & 1 \\ -4 & -3 & 6 \end{bmatrix} = \text{True}$$

(c.) $B^T A^T = (A+B)^T$

$$B^T A^T = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 2 & 1 & -1 \\ -2 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} [-2(-1)-1(2)+3(-2)] & [-2(0)-1(1)+3(-2)] & [-2(0)-1(-1)+3(3)] \\ [2(-1)+1(2)-2(-2)] & [2(0)+1(1)+2(-2)] & [2(0)+1(-1)+2(3)] \\ [-2(-1)-1(2)+3(-2)] & [-2(0)-1(1)+3(-2)] & [-2(0)-1(-1)+3(3)] \end{bmatrix}$$

$$= \begin{bmatrix} [2-2-6] & [0-1-6] & [0+1+9] \\ [-2+2+4] & [0+1-4] & [0-1+6] \\ [2-2-6] & [0-1-6] & [0+1+9] \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -7 & 10 \\ 4 & -3 & 5 \\ -6 & -7 & 10 \end{bmatrix}$$

$$(A+B)^T = \left(\begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 2 & -2 \\ -1 & 1 & -1 \\ 3 & 2 & 3 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} [-1(-2)+2(-1)-2(3)] & [-1(2)+2(1)-2(2)] & [-1(-2)+2(-1)-2(3)] \\ [0(-2)+1(-1)-2(3)] & [0(2)+1(1)-2(2)] & [0(-2)+1(-1)-2(3)] \\ [0(-2)-1(-1)+3(3)] & [0(2)-1(1)+3(2)] & [0(-2)-1(-1)+3(3)] \end{bmatrix}^T$$

$$= \left(\begin{bmatrix} [2-2-6] & [-2+2-4] & [2-2-6] \\ [0-1-6] & [0+1-4] & [0-1-6] \\ [0+1+9] & [0-1+6] & [0+1+9] \end{bmatrix} \right)^T$$

$$= \left(\begin{bmatrix} -6 & -4 & -6 \\ -7 & -3 & -7 \\ 10 & 5 & 10 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} -6 & -7 & 10 \\ -4 & -3 & 5 \\ -6 & -7 & 10 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -7 & 10 \\ -4 & -3 & 5 \\ -6 & -7 & 10 \end{bmatrix} = \begin{bmatrix} -6 & -7 & 10 \\ -4 & -3 & 5 \\ -6 & -7 & 10 \end{bmatrix} = \text{True}$$

(d) $A(BC) = (AB)C$

$$A(BC) = \begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix} \left(\begin{bmatrix} -2 & 2 & -2 \\ -1 & 1 & -1 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix} \right)$$

$$BC = \begin{bmatrix} [-2(2)+2(-1)-2(3)] & [-2(0)+2(-1)-2(2)] \\ [-1(2)+1(-1)-1(3)] & [-1(0)+1(-1)-1(2)] \\ [3(2)+2(-1)+3(3)] & [3(0)+2(-1)+3(2)] \end{bmatrix}$$

$$= \begin{bmatrix} [-4-2-6] & [0-2-4] \\ [-2-1-3] & [0-1-2] \\ [6-2+9] & [0-2+6] \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -6 \\ -6 & -3 \\ 13 & 4 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} -12 & -6 \\ -6 & -3 \\ -13 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} [-1(-12)+2(-6)-2(13)] & [-1(-6)+2(-3)-2(4)] \\ [0(-12)+1(-6)-2(13)] & [0(-6)+1(-3)-2(4)] \\ [0(-12)-1(-6)+3(13)] & [0(-6)-1(-3)+3(4)] \end{bmatrix}$$

$$= \begin{bmatrix} [12-12-26] & [6-6-8] \\ [0-6-26] & [0-3-8] \\ [0+6+39] & [0+3+12] \end{bmatrix}$$

$$= \begin{bmatrix} -26 & -8 \\ -32 & -11 \\ 45 & 15 \end{bmatrix} \Big/ \begin{bmatrix} -26 & -8 \\ -32 & -11 \\ 45 & 15 \end{bmatrix} = \begin{bmatrix} -26 & -8 \\ -32 & -11 \\ 45 & 15 \end{bmatrix}$$

= True

$$(AB)C = \left(\begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 2 & -2 \\ -1 & 1 & -1 \\ -3 & 2 & 3 \end{bmatrix} \right) \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} [-1(-2)+2(-1)-2(3)] & [-1(2)+2(1)-2(2)] & [-1(-2)+2(-1)-2(3)] \\ [0(-2)+1(-1)-2(3)] & [0(2)+1(1)-2(2)] & [0(-2)+1(-1)-2(3)] \\ [0(-2)-1(-1)+3(3)] & [0(2)-1(1)+3(2)] & [0(-2)-1(-1)+3(3)] \end{bmatrix}$$

$$= \begin{bmatrix} [2-2-6] & [-2+2-4] & [2-2-6] \\ [0-1-6] & [0+1-4] & [0-1-6] \\ [0+1+9] & [0-1+6] & [0+1+9] \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -4 & -6 \\ -7 & -3 & -7 \\ 10 & 5 & 10 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} -6 & -4 & -6 \\ -7 & -3 & -7 \\ 10 & 5 & 10 \end{bmatrix} \right) \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} [-6(2)-4(-1)-6(3)] & [-6(0)-4(-1)-6(2)] \\ [-7(2)-3(-1)-7(3)] & [-7(0)-3(-1)-7(2)] \\ [10(2)+5(-1)+10(3)] & [10(0)+5(-1)+10(2)] \end{bmatrix}$$

$$= \begin{bmatrix} [-12+4-18] & [0+4-12] \\ [-14+3-21] & [0+3-14] \\ [20-5+30] & [0-5+20] \end{bmatrix}$$

$$= \begin{bmatrix} -26 & -8 \\ -32 & -11 \\ 45 & 15 \end{bmatrix}$$

Question 3

$$1.5x + 3.1y = 10.8$$
$$2.5x + 4.1y = 14.8$$

$$\begin{bmatrix} 1.5 & 3.1 \\ 2.5 & 4.1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10.8 \\ 14.8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 10.8 \\ 14.8 \end{bmatrix}$$

$$\det(A) = 1.5 \times 4.1 - 2.5 \times 3.1 = 6.15 - 7.75 = -1.6$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 4.1 & -2.5 \\ -3.1 & 1.5 \end{bmatrix} = \frac{1}{1.6} \begin{bmatrix} 4.1 & -2.5 \\ -3.1 & 1.5 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = A^{-1} \begin{bmatrix} 10.8 \\ 14.8 \end{bmatrix} = \frac{1}{1.6} \begin{bmatrix} 4.1 & -2.5 \\ -3.1 & 1.5 \end{bmatrix} \begin{bmatrix} 10.8 \\ 14.8 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1.6} \begin{bmatrix} 7.28 \\ -23.68 \end{bmatrix} = \begin{bmatrix} \frac{1}{1.6} \times 7.28 \\ \frac{1}{1.6} \times (-23.68) \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4.55 \\ -14.8 \end{bmatrix}$$

Therefore, $x = 4.55$, $y = -14.8$.

Question 4

$$AB = \begin{bmatrix} 4.0 & 3.3 & 2.9 \\ 3.3 & 4.5 & 3.4 \\ 2.9 & 3.1 & 3.9 \end{bmatrix} \begin{bmatrix} 7 & -3 & -2.8 \\ -4 & 7.2 & -2.8 \\ -2 & -4 & 7.1 \end{bmatrix}$$

$$= \begin{bmatrix} [4.0(7)+3.3(-4)+2.9(-2)] & [4.0(-3)+3.3(7.2)+2.9(-4)] & [4.0(-2.8)+3.3(-2.8)+2.9(7.1)] \\ [3.3(7)+4.5(-4)+3.4(-2)] & [3.3(-3)+4.5(7.2)+3.4(-4)] & [3.3(-2.8)+4.5(-2.8)+3.4(7.1)] \\ [2.9(7)+3.1(-4)+3.9(-2)] & [2.9(-3)+4.5(7.2)+3.4(-4)] & [2.9(-2.8)+3.1(-2.8)+3.9(7.1)] \end{bmatrix}$$

$$= \begin{bmatrix} [28-13.2-5.8] & [-12+23.76-11.6] & [-11.2-9.24+20.59] \\ [23.1-18-6.8] & [-9.9+32.4-13.6] & [-9.24-12.6+24.14] \\ [20.3(4)-12.4-7.8] & [-8.7+22.32-15.6] & [-8.12(4)-8.68+27.69] \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0.2 & 0.2 \\ -1.7 & 8.6 & 2.3 \\ 0.1 & -2 & 10.9 \end{bmatrix}$$

Calculating the determinant by using diagonal method:

$$\begin{bmatrix} 9 & 0.2 & 0.2 & 9 & 0.2 & 0.2 \\ -1.7 & 8.6 & 2.3 & -1.7 & 8.6 & 2.3 \\ 0.1 & -2 & 10.9 & 0.1 & -2 & 10.9 \end{bmatrix}$$

Downward diagonals: $9 \times 8.6 \times 10.9 + 0.2 * (\text{illegible}) 2.3 \times 0.1 + 0.2 \times (-1.7) \times (-2) = 843.66 + 0.046 + 0.68 = 844.386$

Upward diagonals: $0.1 \times 8.6 \times 0.2 + (-2) \times 2.3 \times 9 + 10.9 \times (-1.7) \times 0.2 = 0.172 + (-41.4) + -3.706 = -44.934$

$$\det(A) = 844.386 - (-44.934) = 889.32$$

A^{-1} does not exist.