

机器人导论——作业3.15

D-H 参数表

| 关节i | α_i | a_i | θ_i | d_i |
|-----|------------|-------|------------|--------|
| 1 | 0 | 0 | θ_1 | 0 |
| 2 | 0 | l_1 | θ_2 | 0 |
| 3 | 0 | l_2 | 0 | $-d_3$ |
| 4 | 0 | 0 | θ_4 | 0 |

正解

```
clear all
% 定义D-H参数表
syms l1 l2 c1 c2 c4 d3
alpha1=0
alpha2=0
alpha3=0
alpha4=0
a1=0
a2=l1
a3=l2
a4=0
theta1 = c1
theta2 = c2
theta3 = 0
theta4 = c4
dis1=0
dis2=0
dis3=-d3
dis4=0
% 计算4个A矩阵
A1 = [cos(theta1),-sin(theta1),0,a1;
      sin(theta1)*cos(alpha1),cos(theta1)*cos(alpha1),-sin(alpha1),-
      dis1*sin(alpha1);

      sin(theta1)*sin(alpha1),cos(theta1)*sin(alpha1),cos(alpha1),dis1*cos(alpha1);
      0,0,0,1]
A2 = [cos(theta2),-sin(theta2),0,a2;
      sin(theta2)*cos(alpha2),cos(theta2)*cos(alpha2),-sin(alpha2),-
      dis2*sin(alpha2);

      sin(theta2)*sin(alpha2),cos(theta2)*sin(alpha2),cos(alpha2),dis2*cos(alpha2);
      0,0,0,1]
A3 = [cos(theta3),-sin(theta3),0,a3;
      sin(theta3)*cos(alpha3),cos(theta3)*cos(alpha3),-sin(alpha3),-
      dis3*sin(alpha3);

      sin(theta3)*sin(alpha3),cos(theta3)*sin(alpha3),cos(alpha3),dis3*cos(alpha3);
```

```

0,0,0,1]
A4 = [cos(theta4),-sin(theta4),0,a4;
      sin(theta4)*cos(alpha4),cos(theta4)*cos(alpha4),-sin(alpha4),-
dis4*sin(alpha4);

      sin(theta4)*sin(alpha4),cos(theta4)*sin(alpha4),cos(alpha4),dis4*cos(alpha4);
      0,0,0,1]
%正解
T=simplify(A1*A2*A3*A4)

```

解得

```

T =
[ cos(c1 + c2 + c4), -sin(c1 + c2 + c4), 0, l2*cos(c1 + c2) + l1*cos(c1)]
[ sin(c1 + c2 + c4),  cos(c1 + c2 + c4), 0, l2*sin(c1 + c2) + l1*sin(c1)]
[          0,          0, 1,          -d3]
[          0,          0, 0,          1]

```

即

```

nx=cos(c1 + c2 + c4)
ny=sin(c1 + c2 + c4)
ox=-sin(c1 + c2 + c4)
oy=cos(c1 + c2 + c4)
az=1
px=l2*cos(c1 + c2) + l1*cos(c1)
py=l2*sin(c1 + c2) + l1*sin(c1)
pz=-d3

```

其余为0

逆解

```

syms nx ny nz ox oy oz ax ay az px py pz
T40 = [nx ox ax px;
      ny oy ay py;
      nz oz az pz;
      0 0 0 1]

```

```

PX=l2*cos(c1 + c2) + l1*cos(c1)
PY=l2*sin(c1 + c2) + l1*sin(c1)
PZ=-d3
simplify(PX^2+PY^2)

```

$$l_1^2 + 2 * \cos(c_2) * l_1 * l_2 + l_2^2 = px^2 + py^2$$

解得

$$d3=-pz$$

$$c2 = \arccos((px^2 + py^2 - l1^2 - l2^2)/2l1l2)$$

```
simplify(inv(A1)*T40)
simplify(A2*A3*A4)
```

$$py * \cos(c1) - px * \sin(c1) = l2 * \sin(c2)$$

解得

$$c1 = \arcsin\left(\frac{l2\sin(c2)}{\sqrt{(px^2+py^2)}} - \arctan\left(\frac{py}{-px}\right)\right) \quad \text{将上面c2带入可得最终结果，过于繁琐，在此处不展开}$$

```
simplify(inv(A2)*inv(A1)*T40)
simplify(A3*A4)
```

$$\cos(c4) = nx * \cos(c1 + c2) + ny * \sin(c1 + c2)$$

解得

$$c4 = \arccos(nx * \cos(c1 + c2) + ny * \sin(c1 + c2)) \quad \text{将上面的c1,c2带入即可得最终结果，此处不展开}$$

综上，c1,c2,c4,d3均解得，逆解完成