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Geodesic Tracking of Retinal Vascular Trees with Optical and TV-Flow Enhancement in SE(2)

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Introduction

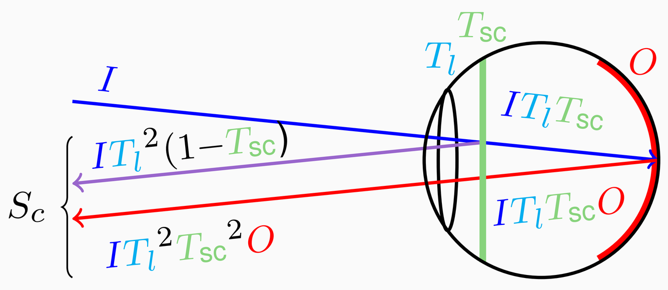
The retinal vasculature is representative of the vasculature throughout the human body. Consequently, it is beneficial to automate an entire vascular tracking (in 1 run) in retinal images. This poster exhibits our results of the incorporation of optical and TV-flow enhancements in metric tensor fields used for geodesic tracking.

Orientation Scores

To minimize tracking mistakes at crossings, we lift the image (via cake-wavelet ψ) to the space of positions and orientations $\mathbb{M}_2 := \mathbb{R}^2 \rtimes S^1$, with $S^1 \equiv \mathbb{R}/(2\pi\mathbb{Z}) \equiv SO(2)$.

Illumination Enhancement

The unprocessed picture S, taken by the ophthalmologist, may deviate from the actual retinal image O, which we aim to recover. For details, see article.



TV-Flow Enhancement on Orientation Scores

TV-flow enhancement is a technique for denoising surfaces but simultaneously preserving sharp edges. TV-flow $U \mapsto W_0(\cdot,t) = \lim_{\varepsilon\to 0} W_\varepsilon(\cdot,t)$ is given by

$$\begin{cases} \frac{\partial W_\varepsilon}{\partial t}(\mathbf{p},t) = \text{div}\left(\frac{\nabla_{\mathcal{G}} W_\varepsilon(\cdot,t)}{\varepsilon^2 + |\nabla_{\mathcal{G}} W(\cdot,t)|^2}\right)(\mathbf{p}), & \mathbf{p} = (\mathbf{x},\mathbf{n}) \in \mathbb{M}_2,\ t \geq 0, \\ W_\varepsilon(\mathbf{p},0) = U(\mathbf{p}), \end{cases}$$

with
$$\mathcal{G}_{\mathbf{p}}(\dot{\mathbf{p}},\dot{\mathbf{p}}) = C(\mathbf{p})^2 \left(\xi^2 |\dot{\mathbf{x}}\cdot\mathbf{n}|^2 + \frac{\xi^2}{\zeta^2}\|\dot{\mathbf{x}}\wedge\mathbf{n}\|^2 + \|\dot{\mathbf{n}}\|^2\right).$$

Metric Tensor Field

The metric tensor field $\mathcal{F}^M$ mixes the asymmetric Reeds-Shepp car model $\mathcal{F}$ (near crossings), and a data-driven variant $\mathcal{F}^U$. For practical details, see article.

Theorem for Asymmetric Geodesic Tracking

The shortest curve $\gamma : [0,1] \to \mathbb{M}_2$ with $\gamma(0) = \mathbf{p}$ and $\gamma(1) = \mathbf{p}_0$ can be computed by steepest descent tracking on distance map $W(\mathbf{p}) = d_{\mathcal{F}^M}(\mathbf{p},\mathbf{p}_0)$

$$\gamma(t) := \gamma^U_{\mathbf{p},\mathbf{p}_0}(t) = \text{Exp}_{\mathbf{p}}(t\, v(W)), \qquad t \in [0,1], \tag{1}$$

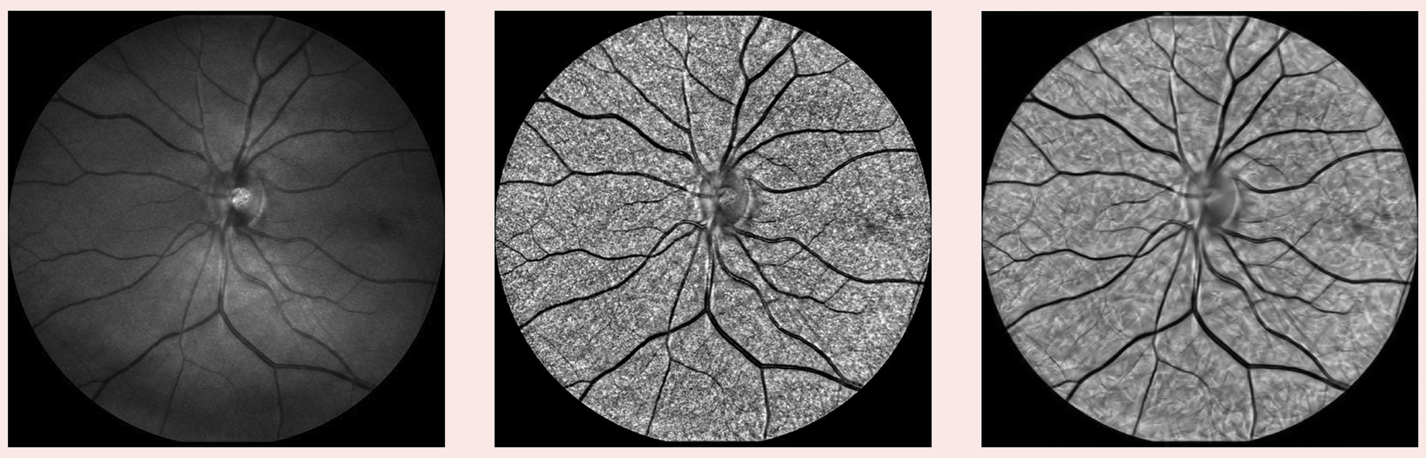
where Exp integrates the following vector field on $\mathbb{M}_2$: $v(W) := -W(\mathbf{p})\nabla_{\mathcal{F}^M} W$ and where W is the viscosity solution of the eikonal PDE system

$$\begin{cases} \mathcal{F}^*_M(\mathbf{p}, \mathrm{d}W(\mathbf{p})) = 1 & \mathbf{p} \in \mathbb{M}_2, \\ W(\mathbf{p}_0) = 0, \end{cases} \tag{2}$$

assuming $\mathbf{p}$ is neither a 1st Maxwell-point nor a conjugate point, with dual Finsler function $\mathcal{F}^*_M(\mathbf{p},\hat{\mathbf{p}}) := \max\{\langle\hat{\mathbf{p}},\dot{\mathbf{p}}\rangle |\ \dot{\mathbf{p}} \in T_{\mathbf{p}}(\mathbb{M}_2)$ with $\mathcal{F}^M(\mathbf{p},\dot{\mathbf{p}}) \leq 1\}$. As $v(W)$ is data-driven left invariant, the geodesics carry the desired symmetry

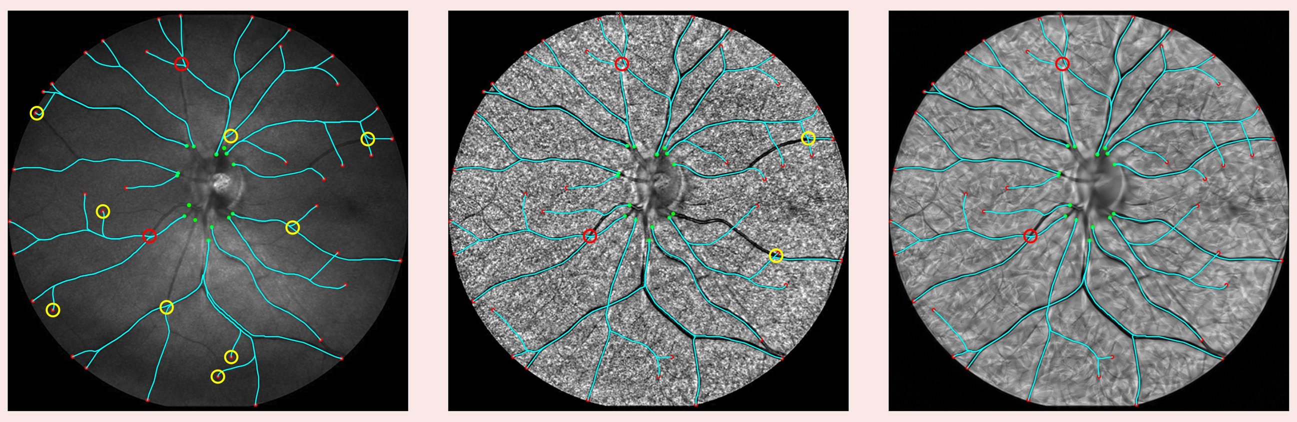
$$\gamma^{\mathcal{L}_g U}_{g\cdot\mathbf{p}, g\cdot\mathbf{p}_0}(t) = g\, \gamma^U_{\mathbf{p},\mathbf{p}_0}(t) \text{ for all } g \in SE(2), \mathbf{p},\mathbf{p}_0 \in \mathbb{M}_2, t \in [0,1]. \tag{3}$$

Retinal image, optically enhanced image excl. and incl. TV-flow on $\mathbb{M}_2$ resp.

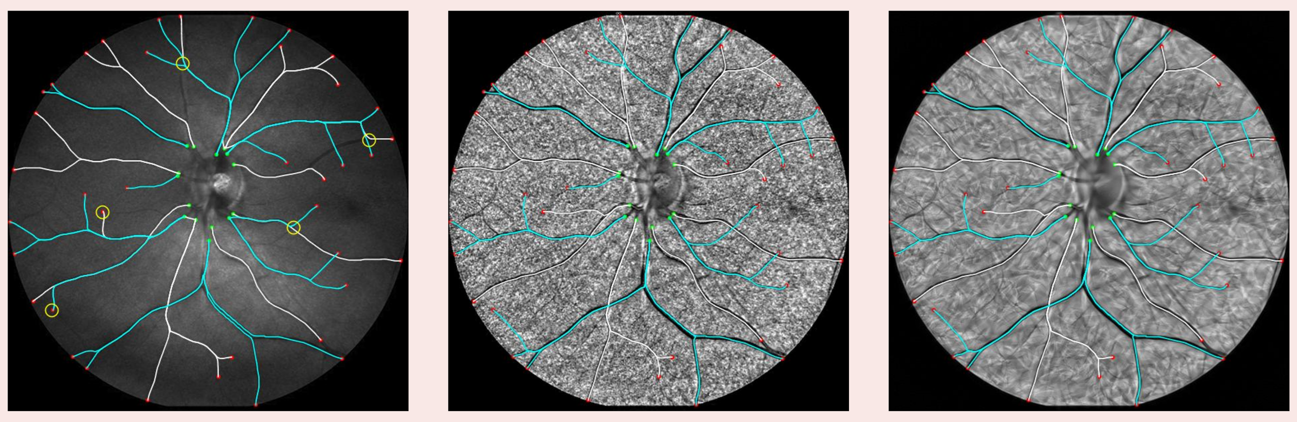


Experimental Results

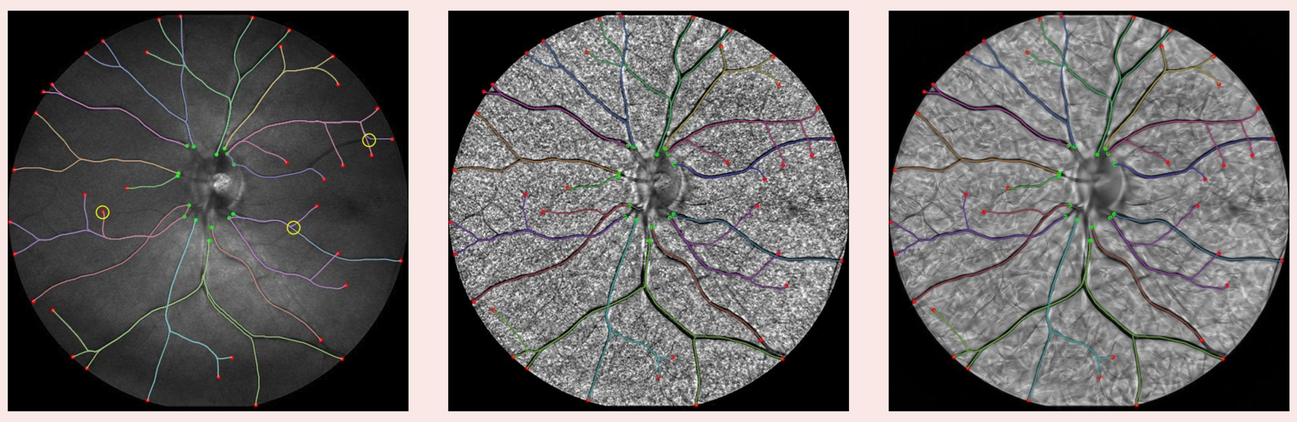
Single Run



Per Type (Artery/Vein)



Per Seed (per vascular tree)



Error measure	Original image	Optically enhanced image	Optically enhanced image with $SE(2)$ TV-flow
Single Run	0.34	0.23	0.20
Per Type (A/V)	0.25	0.12	0.10
Per Seed	0.23	0.09	0.09

Conclusion

We developed a new asymmetric, data-driven left-invariant Finsler geometric model that includes contextual contrast enhancement via TV-flows on $SE(2)$. The new model reduces many errors and performs very well on both realistic and challenging low-quality retinal images where *entire vascular trees* are computed from a *single* asymmetric Finslerian distance map. Although we have shown that both the contrast enhancement and the TV-flow on $SE(2)$ in the new Finslerian model are highly beneficial, there are still exceptional cases where vessel tracts take the wrong exit, in particular at places where both a crossing and a bifurcation occur.

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