Geodesic Tracking of Retinal Vascular Trees with Optical and TV-Flow Enhancement in SE(2)

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Introduction

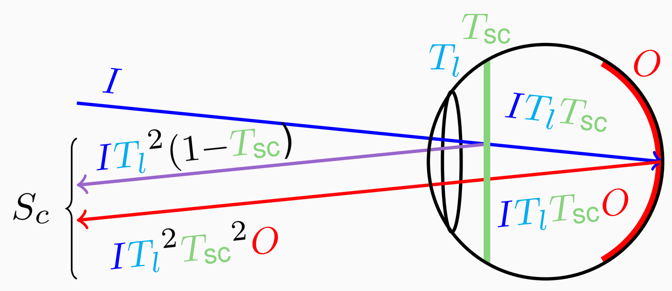
The retinal vasculature is representative of the vasculature throughout the human body. Consequently, it is beneficial to automate an entire vascular tracking (in 1 run) in retinal images. This poster exhibits our results of the incorporation of optical and TV-flow enhancements in metric tensor fields used for geodesic tracking.

Orientation Scores

To minimize tracking mistakes at crossings, we lift the image (via cake-wavelet ψ) to the space of positions and orientations $\mathbb{M}_2 := \mathbb{R}^2 \rtimes S^1 \cong \mathbb{R}^2 \rtimes SO(2) =: SE(2)$.

Illumination Enhancement of Image

The unprocessed picture S, taken by the ophthalmologist, may deviate from the actual retinal image O, which we aim to recover.



TV-Flow via PDEs on Orientation Scores

TV-flow enhancement is a technique for denoising surfaces but simultaneously preserving sharp edges. TV-flow $U = W_\phi f \mapsto W_0(\cdot,t) = \lim_{\varepsilon \to 0} W_\varepsilon(\cdot,t)$ is given by

$$\begin{cases} \frac{\partial W_\varepsilon}{\partial t}(\mathbf{p},t) = \text{div}\left(\frac{\nabla_{\mathcal{G}} W_\varepsilon(\cdot,t)}{\sqrt{\varepsilon^2+|\nabla_{\mathcal{G}} W(\cdot,t)|^2}}\right)(\mathbf{p}), & \mathbf{p}=(\mathbf{x},\mathbf{n}) \in SE(2),\ t \geq 0, \\ W_\varepsilon(\mathbf{p},0) = U(\mathbf{p}), \end{cases}$$

with metric $$\mathcal{G}_{\mathbf{p}}(\dot{\mathbf{p}},\dot{\mathbf{p}}) = C(\mathbf{p})^2\left(\xi^2 \underbrace{|\dot{\mathbf{x}}\cdot\mathbf{n}|^2}_{\text{forward}} + \frac{\xi^2}{\zeta^2}\underbrace{\|\dot{\mathbf{x}}\wedge\mathbf{n}\|^2}_{\text{sideways}} + \underbrace{\|\dot{\mathbf{n}}\|^2}_{\text{angular}}\right),$$

where $U(\mathbf{p})$ and $W_0(\mathbf{p},t)$ denote the (enhanced) orientation score.

Metric Tensor Field

The metric tensor field $\mathcal{F}^M$ mixes the asymmetric Reeds-Shepp car model $\mathcal{F}$ (near crossings) with extra constraint $\dot{\mathbf{x}}\cdot\mathbf{n} \geq 0$, and a data-driven variant $\mathcal{F}^U$.

Theorem for Asymmetric Geodesic Tracking

The shortest curve $\gamma : [0,1] \to SE(2)$ with $\gamma(0) = \mathbf{p}$ and $\gamma(1) = \mathbf{p}_0$ can be computed by steepest descent tracking on distance map $W(\mathbf{p}) = d_{\mathcal{F}^M}(\mathbf{p},\mathbf{p}_0)$

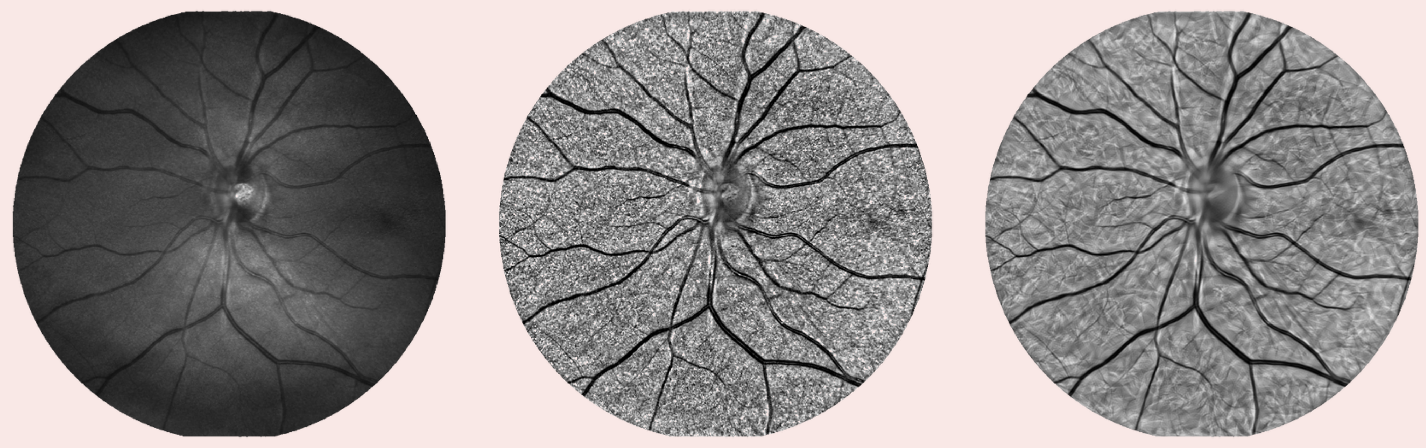
$$\gamma(t) := \gamma^U_{\mathbf{p},\mathbf{p}_0}(t) = \text{Exp}_{\mathbf{p}}(t\,v(W)), \qquad t \in [0,1], \tag{1}$$

where Exp integrates the following vector field on $SE(2)$: $v(W) := -W(\mathbf{p})\nabla_{\mathcal{F}^M} W$ and where W is the viscosity solution of the eikonal PDE system

$$\begin{cases} \mathcal{F}^*_M(\mathbf{p}, \mathrm{d}W(\mathbf{p})) = 1 & \mathbf{p} \in SE(2), \\ W(\mathbf{p}_0) = 0, \end{cases} \tag{2}$$

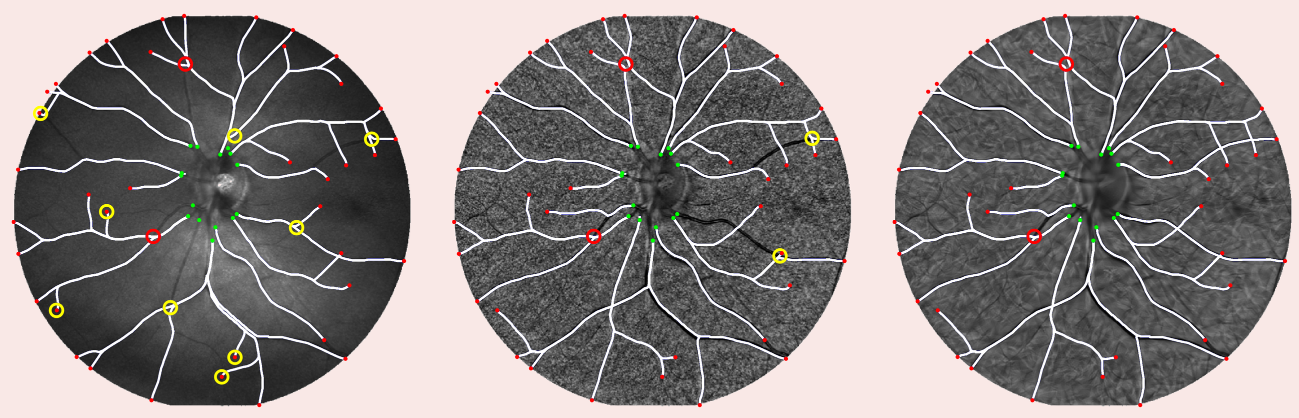
with dual Finsler function $\mathcal{F}^*_M(\mathbf{p},\hat{\mathbf{p}}) := \max\{\langle \hat{\mathbf{p}},\dot{\mathbf{p}}\rangle \mid \dot{\mathbf{p}} \in T_{\mathbf{p}}(SE(2)) \text{ with } \mathcal{F}^M(\mathbf{p},\dot{\mathbf{p}}) \leq 1\}$.

Retinal image, illumination enhanced image excl. and incl. TV-flow on $SE(2)$

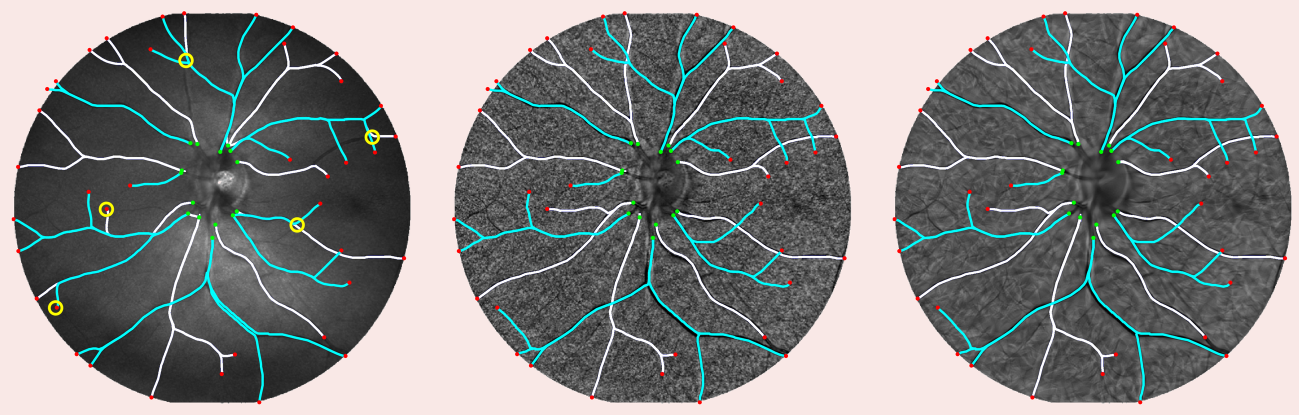


Experimental Results

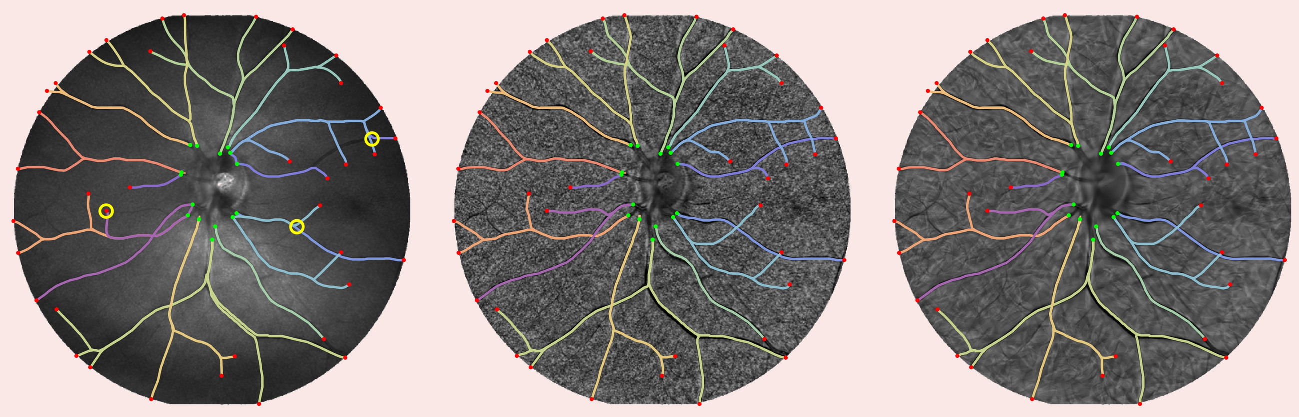
Single Run



Per Type (Artery/Vein)



Per Seed (per vascular tree)



Error measure	Original image	Illumination enhanced image	Illumination enhanced image with $SE(2)$ TV-flow
Single Run	0.34	0.23	0.20
Per Type (A/V)	0.25	0.12	0.10
Per Seed	0.23	0.09	0.09

Conclusion

We developed a new asymmetric, data-driven left-invariant Finsler geometric model that includes contextual contrast enhancement via TV-flows on $SE(2)$. The new model reduces many errors and performs very well on both realistic and challenging low-quality retinal images where *entire vascular trees* are computed from a *single* asymmetric Finslerian distance map.