Analog Electronics

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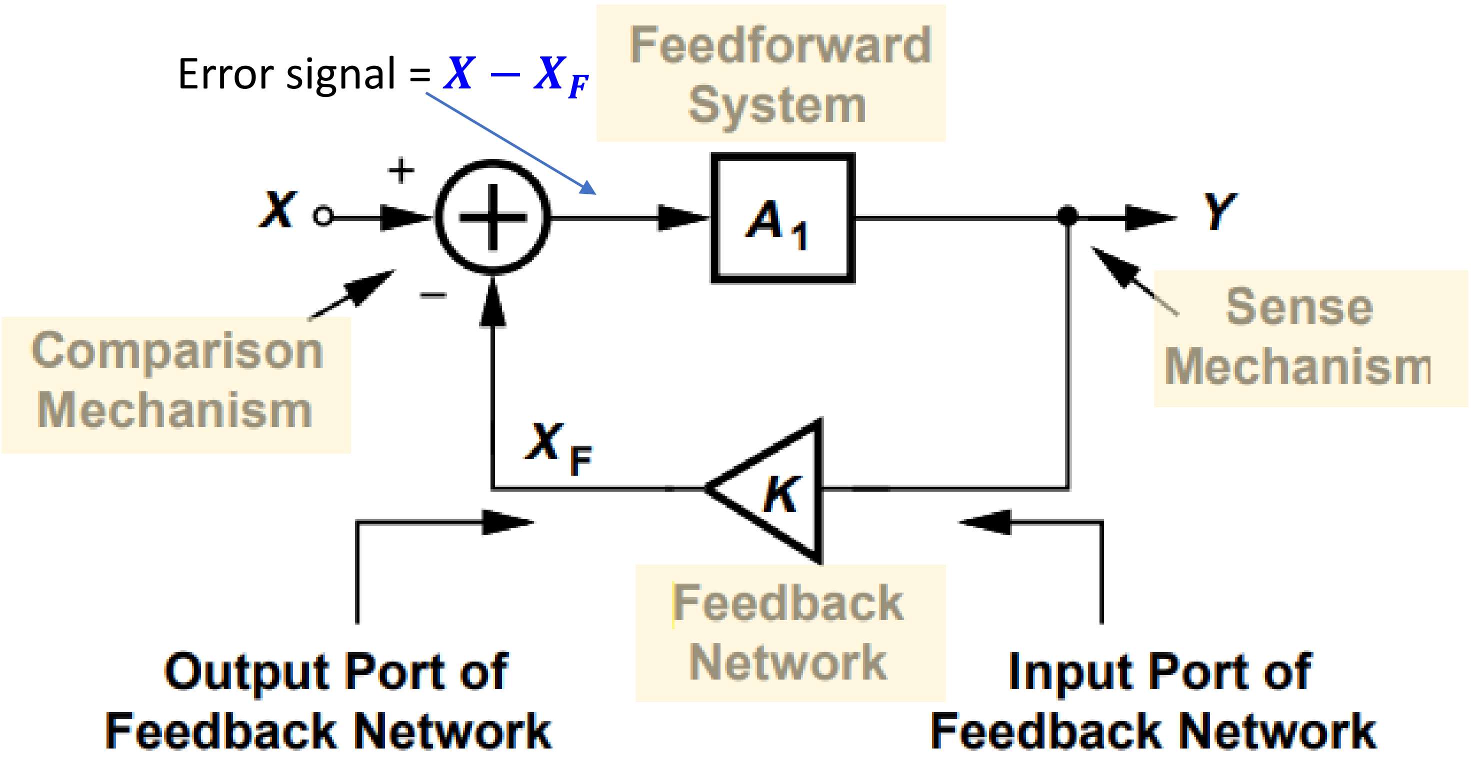
Agenda

- Recap feedback system
- Solutions of the assignments
- Gain optimization and harmonic distortion improvement for BJT
- Introduction to biasing
    - BJT case
    - MOSFET case

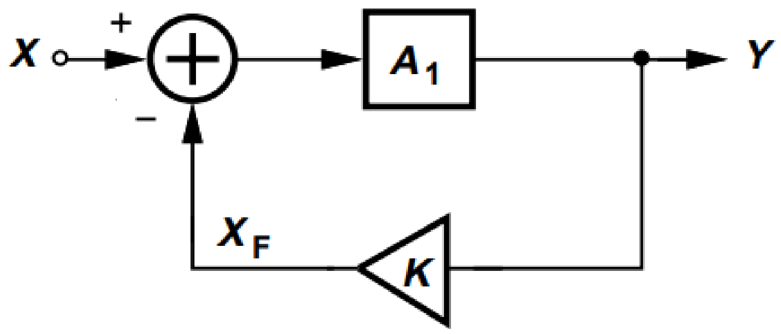
General feedback system



Negative feedback system:

- X and X_F change in the same direction;
- The error signal should be minimized;
- Open-loop system: break the feedback network, $K = 0$
- Closed-loop system: $K \neq 0$

Loop-gain



- Open-loop gain: A_1
- Closed-loop gain: $A_{cL} = \frac{Y}{X} = \frac{A_1}{1+KA_1}$
- Loop-gain: KA_1
  - Procedure to measure loop-gain
    - Set the input X to zero (X is voltage → AC ground; if X is current→ open)
    - Break the loop at an arbitrary point
    - Apply a test signal $\boldsymbol{V_{test}}$ at one terminal and measure the signal $\boldsymbol{V_F}$ at the other terminal
    - Calculate the loop-gain $-\frac{V_F}{V_{test}} = KA_1$
      - $\frac{V_F}{V_{test}} < 0$ → negative feedback
      - $\frac{V_F}{V_{test}} > 0$ → positivie feedback

Properties of negative feedback

- Cons:
  - Smaller gain

- Pros:
  - Gain desensitization
  - Bandwidth extension
  - Linearity improvement
  - Modification of input and output impedance

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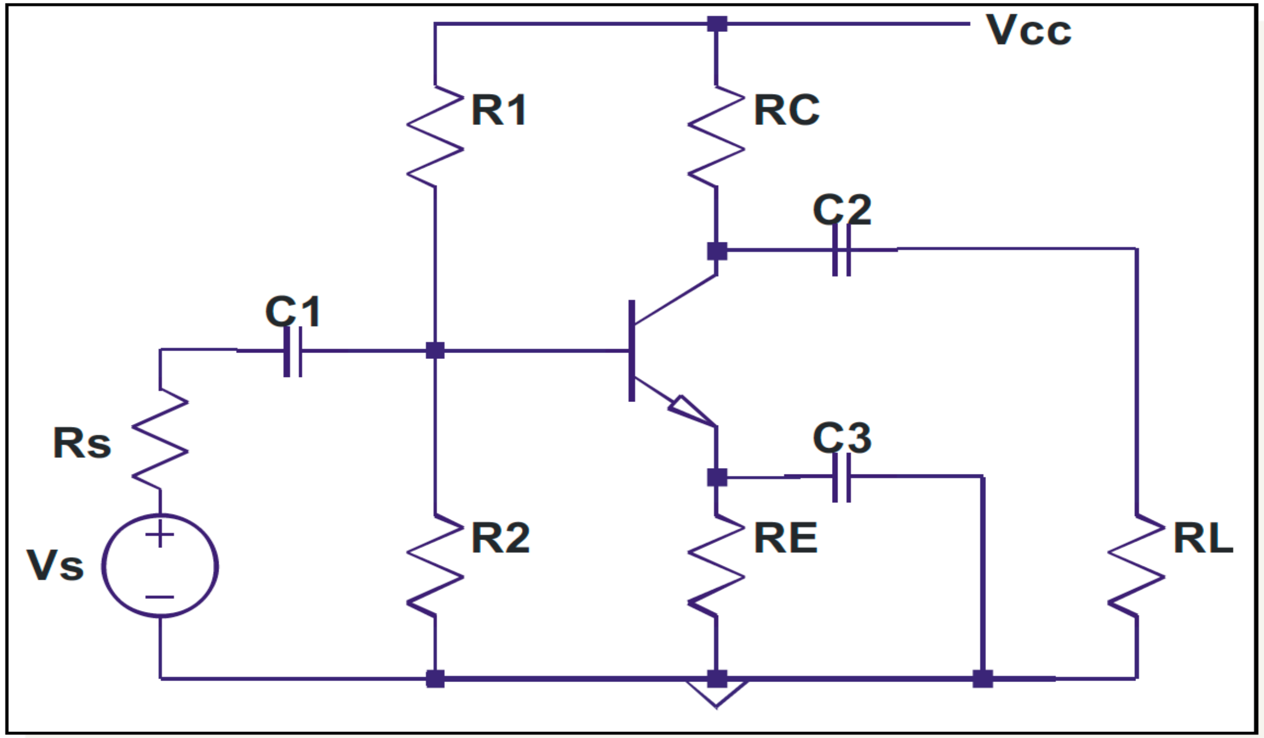
Gain optimization for BJT



Definition & assumption:

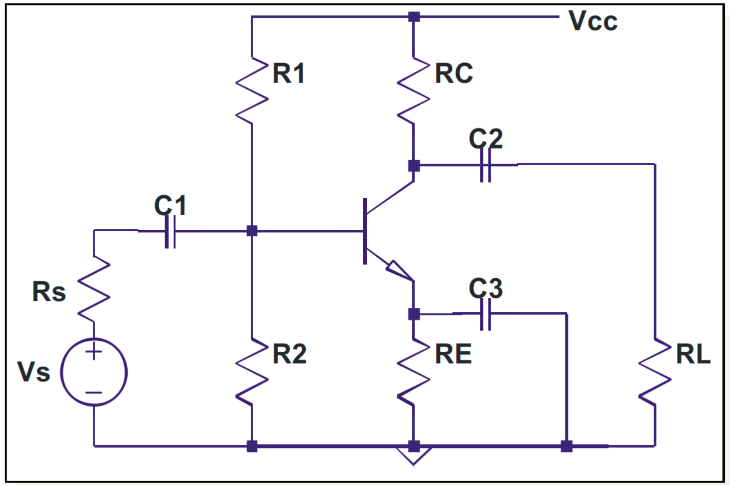
$$v_S' = \frac{R_B}{R_B + R_S} v_S$$

$$R_S' = R_S \parallel R_B$$

$$r_o >> R_C \parallel R_L$$

$$R_B = R_1 || R_2$$

Optimize R_C to maximize gain for the fixed V_{R_C}



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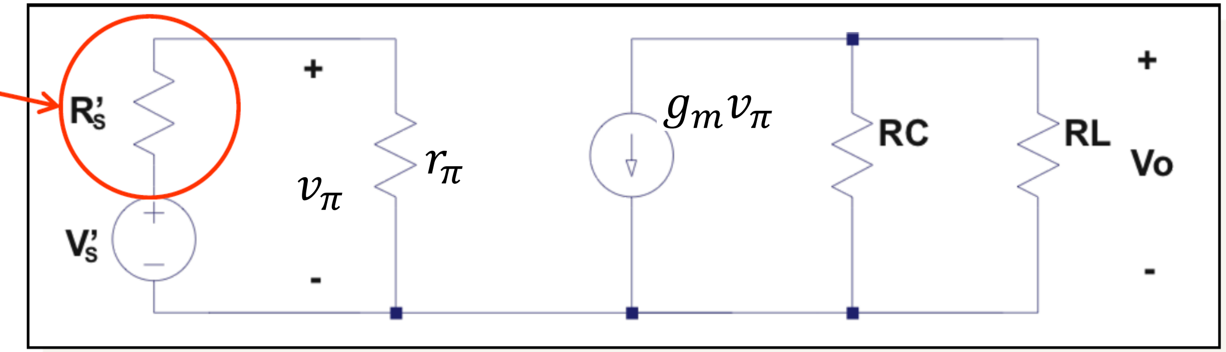


$$A_v = \frac{v_o}{v_s} = \frac{v_o}{v_s'} \cdot \frac{v_s'}{v_s} = A_v' \cdot \frac{R_B}{R_B + R_S}$$

$$A_v' = -\frac{g_m (R_C || R_L)}{1 + \frac{R_S'}{r_\pi}} = -\frac{1}{\left(\frac{1}{g_m} + \frac{R_S'}{g_m r_\pi}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)}$$

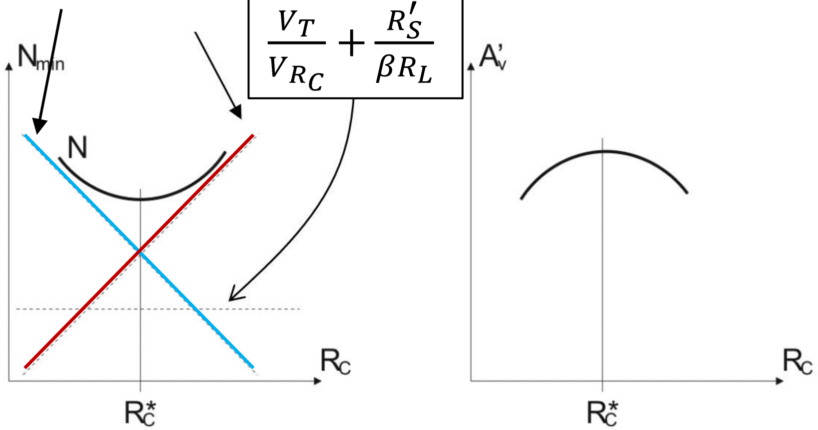
$g_m = \frac{I_C}{V_T}$ and $\beta = g_m r_\pi$

$$\Rightarrow A_v' = -\frac{1}{\left(\frac{V_T}{I_C} + \frac{R_S'}{\beta}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)} = -\frac{1}{\frac{V_T}{I_C R_C} + \frac{R_S'}{\beta R_C} + \frac{V_T}{I_C R_L} + \frac{R_S'}{\beta R_L}}$$

$$= -\frac{1}{\frac{V_T}{V_{R_C}} + \frac{R_S'}{\beta R_C} + \frac{R_C V_T}{V_{R_C} R_L} + \frac{R_S'}{\beta R_L}} = -\frac{1}{N}$$

Minimize $\frac{R_S'}{\beta R_C} + \frac{R_C V_T}{V_{R_C} R_L}$ → *minimize* N → *maximize* A_v'

$$\frac{V_T}{V_{R_C}} + \frac{R_S'}{\beta R_L}$$



Optimized R_C

$$\frac{\partial N}{\partial R_C} = 0 \Rightarrow R_C^* = \sqrt{\frac{R_S' \cdot R_L \cdot V_{R_C}}{\beta \cdot V_T}} \rightarrow \left|A_{v,\max}'\right| = \frac{1}{\left(\sqrt{\frac{V_T}{V_{R_C}}} + \sqrt{\frac{R_S'}{R_L \cdot \beta}}\right)^2}$$

Gain optimization

- Max gain based *exclude* $r_o = \frac{V_A}{I_C}$

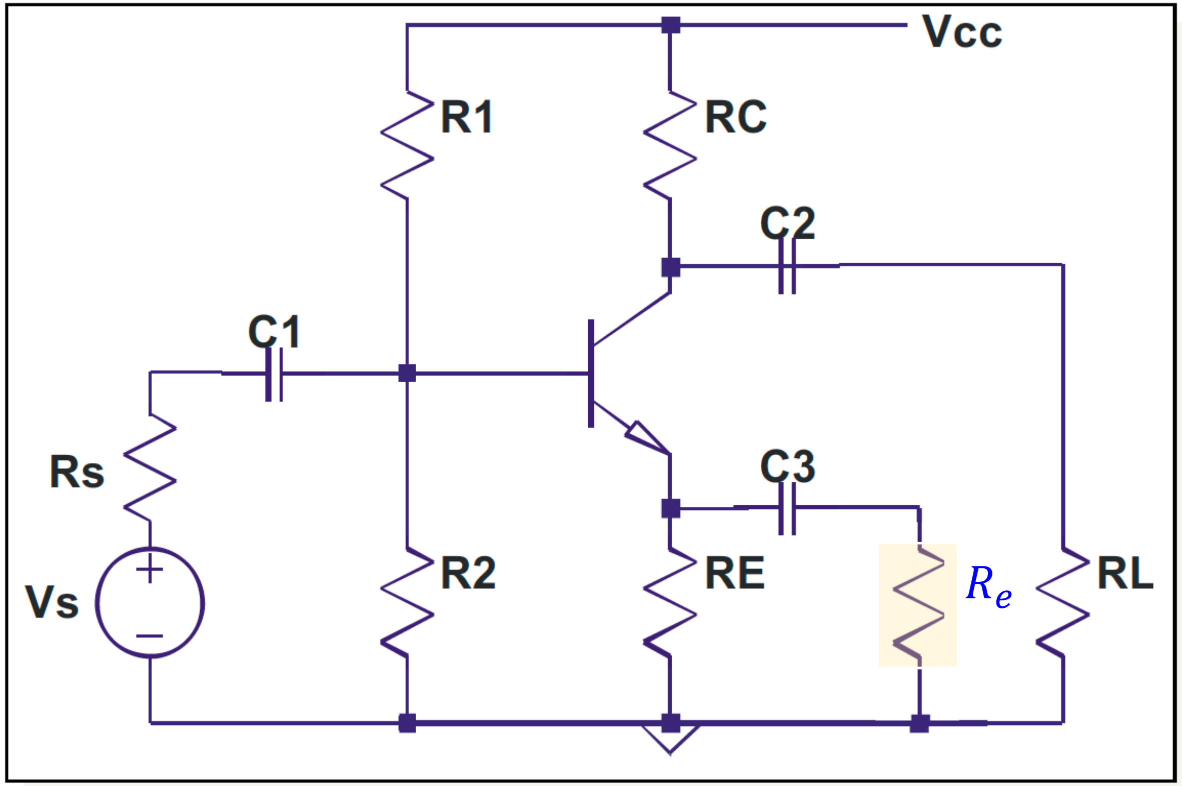
$$\left|A'_{v,\max}\right| = \frac{1}{\left(\sqrt{\frac{V_T}{V_{R_C}}} + \sqrt{\frac{R'_S}{R_L \cdot \beta}}\right)^2} \Rightarrow R_C^* = \sqrt{\frac{R'_S \cdot R_L \cdot V_{R_C}}{\beta \cdot V_T}}$$

- Max gain based *include* $r_o = \frac{V_A}{I_C}$

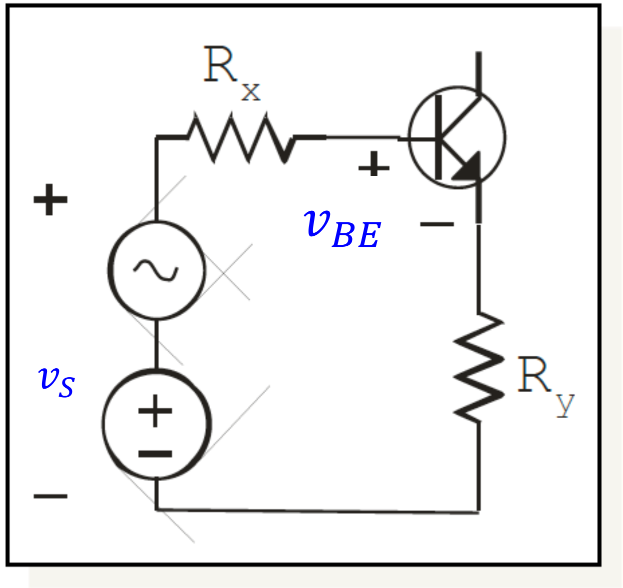
$$\left|A'_v\right| = \frac{r_o \parallel R_C \parallel R_L}{\frac{1}{g_m} + \frac{R'_S}{\beta}} \Rightarrow R_C^* = \sqrt{\frac{R'_S \cdot R_L \cdot V_{R_C}}{\beta \cdot V_T}\left(1 + \frac{V_{R_C}}{V_A}\right)}$$

- Adding emitter resistor R_e reduce the gain

$$\left|A'_v\right| = \frac{r_o \parallel R_C \parallel R_L}{R'_e + \frac{1}{g_m} + \frac{R'_S}{\beta}} \Rightarrow R'_e = \frac{r_o \parallel R_C \parallel R_L}{A'_v} - \frac{1}{g_m} - \frac{R'_S}{\beta} = R_e \parallel R_E$$



Improving the THD for a BJT



Large-signal model

$v_S = V_S + \Delta V_S$

$i_C = I_C + \Delta I_C$

- $i_c = f(v_s)$ must be found and then $f^{(1)}$and $f^{(2)}$are found at the operating point given by I_C
- Determine $f^1$and f^2 by implicitly differentiation

$$v_S = \frac{i_C}{\beta} R_x + v_{BE} + i_C R_y$$

$$= \left(\frac{R_x}{\beta} + R_y \right) i_C + V_T \ln \left(\frac{i_C}{I_S} \right)$$

$$\boxed{i_C = I_S \cdot e^{\left(\frac{v_{BE}}{V_T}\right)}}$$

Improving the THD for a BJT

Determination of $f^{(1)}$:

$$v_S = \left(\frac{R_x}{\beta} + R_y\right) i_C + V_T \ln\left(\frac{i_C}{I_S}\right)$$
$$\Downarrow$$
$$1 = \left(\frac{R_x}{\beta} + R_y\right)\frac{\partial i_C}{\partial v_S} + 0 + V_T \frac{1}{\frac{i_C}{I_S}} \frac{1}{I_S} \frac{\partial i_C}{\partial v_S}$$
$$\Downarrow$$
$$1 = \left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right)\frac{\partial i_C}{\partial v_S} = \left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right) f^1$$
$$\Downarrow$$
$$\boxed{f^{(1)}(i_C) = \frac{1}{\left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right)}}$$

Diff. With v_S on both sides

(fg)’=(f)’g+f(g)’

(f(g))’=(f(g))’g’

Determination of $f^{(2)}$:

$$1 = \left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right)\frac{\partial i_C}{\partial v_s}$$
$$\Downarrow$$
$$0 = \left(\frac{R_x}{\beta} + R_y\right)\frac{\partial^2 i_C}{\partial v_s^2} + \frac{V_T}{i_C}\frac{\partial^2 i_C}{\partial v_s^2} + \frac{\partial i_C}{\partial v_s}\left(-\frac{V_T}{i_C^2}\right)\frac{\partial i_C}{\partial v_s}$$
$$\Downarrow$$
$$0 = \left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right)\frac{\partial^2 i_C}{\partial v_s^2} - \left(\frac{\partial i_C}{\partial v_s}\right)^2 \left(\frac{V_T}{i_C^2}\right)$$
$$\Downarrow$$
$$\boxed{f^{(2)}(i_C) = \frac{\left(f^1\right)^2 \frac{V_T}{i_C^2}}{\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}}}$$

Diff. With v_S on both sides one more time

(fg)’=(f)’g+f(g)’

(f(g))’=(f(g))’g’

Improving the THD for a BJT

We have now determined $f^{(1)}$ and of $f^{(2)}$ and are thus able to assess which effect *Rx* and *Ry* might have on, for example, HD2

$$\Downarrow$$

$$f^{(1)}(i_C) = \frac{1}{\left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right)}$$

$$\Downarrow$$

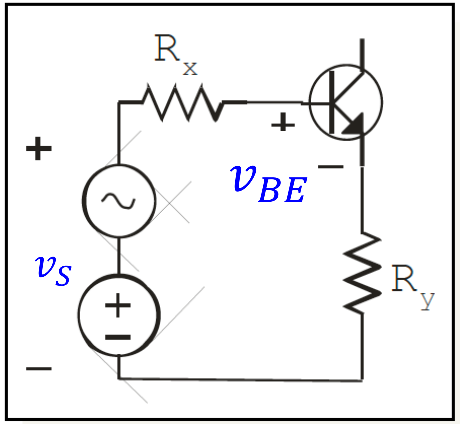
$$f^{(2)}(i_C) = \frac{\left(f^{1}\right)^2 \frac{V_T}{i_C^2}}{\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}}$$

$$HD_2 = \frac{1}{4}\left|\frac{f^{(2)}}{f^{(1)}}\right| A = \frac{1}{4}\frac{\left(f^{(1)}\right)\frac{V_T}{i_C^2}}{\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}} A = \frac{1}{4}\frac{\frac{V_T^2}{i_C^2}}{\left(\frac{R_x}{\beta} + R_y + \frac{V_T}{i_C}\right)^2}\frac{A}{V_T}$$

$$= \frac{1}{4}\frac{1}{\left(\frac{i_C}{V_T}\left(\frac{R_x}{\beta} + R_y\right) + 1\right)^2}\frac{A}{V_T} = \frac{\frac{1}{4}\frac{A}{V_T}}{F^2}$$

$$i_C = I_C + \Delta I_C \approx I_C$$

$$F = \frac{i_C}{V_T}\left(\frac{R_x}{\beta} + R_y\right) + 1 = g_m\left(\frac{R_x}{\beta} + R_y\right) + 1$$



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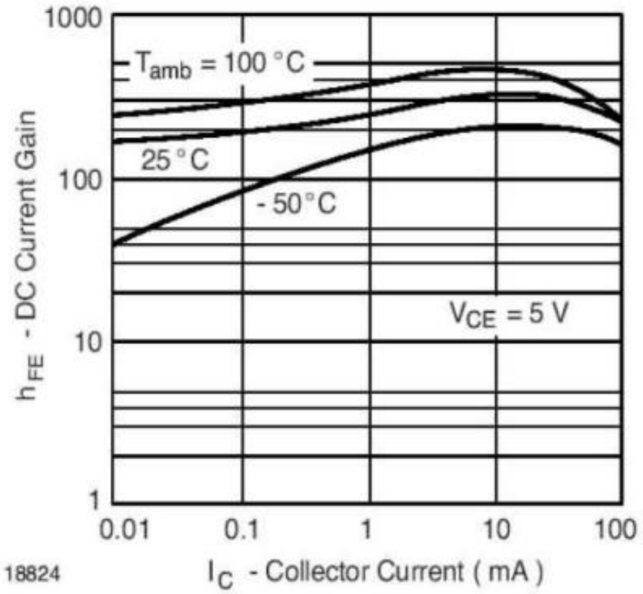
Dispersion/spread -BJT

- Key parameter:
  - V_{BE}: $\pm 10\%$ spread; vary with temperature $-2\ \mathrm{mV}/^{\mathrm{o}}\mathrm{C}$
  - β: typical β_{DC} and β_{AC} are given, variation of $\pm 50\%$ is normal; vary with temperature $-0.5\%\ /^{\mathrm{o}}\mathrm{C}$



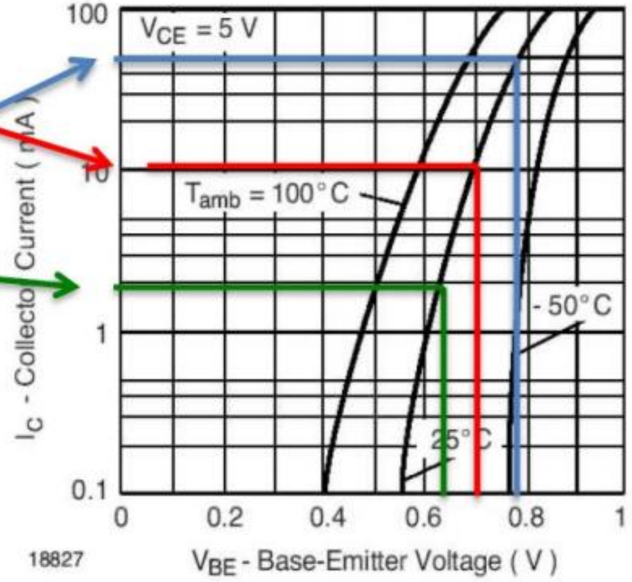
$$[V_{BE}, I_C] = [0.7V, 10mA]$$

$$+10\% \rightarrow 80mA$$

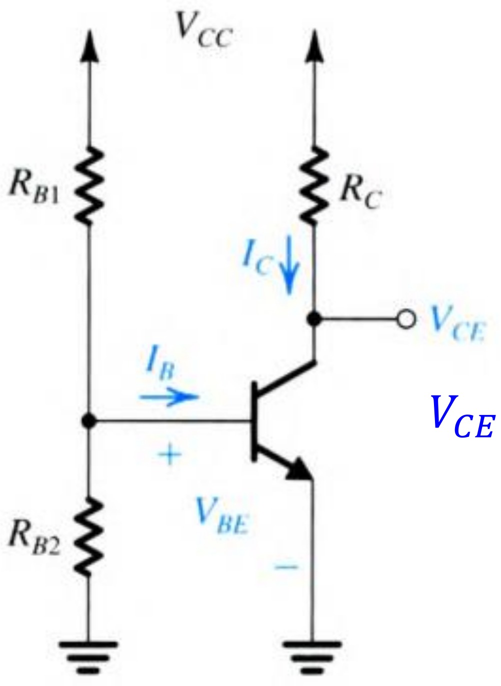
$$-10\% \rightarrow 2mA$$

$$-80\% \leftrightarrow +70\%$$

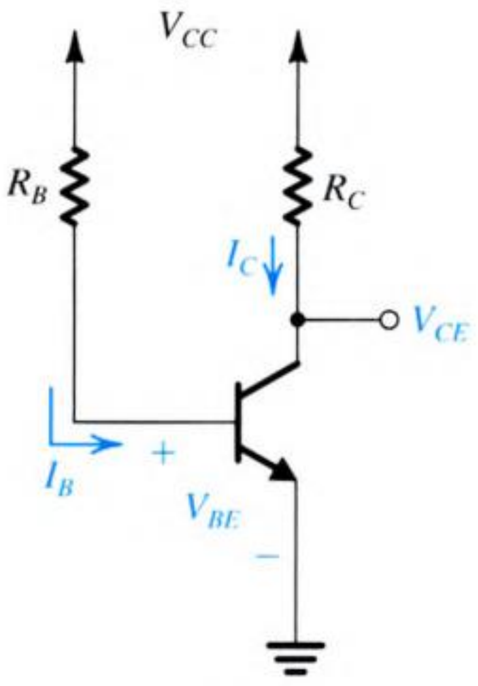
I_C tolerance



Bias circuit – BJT



$$V_{CE} = V_{CC} - I_C R_C$$



Fix V_{BE} is not a good design:

$$I_C = I_s\, e^{\frac{V_{BE}}{V_T}}$$

- Small variation in V_{BE} ➔ large variation in I_C
- I_s and V_T is temperature dependent

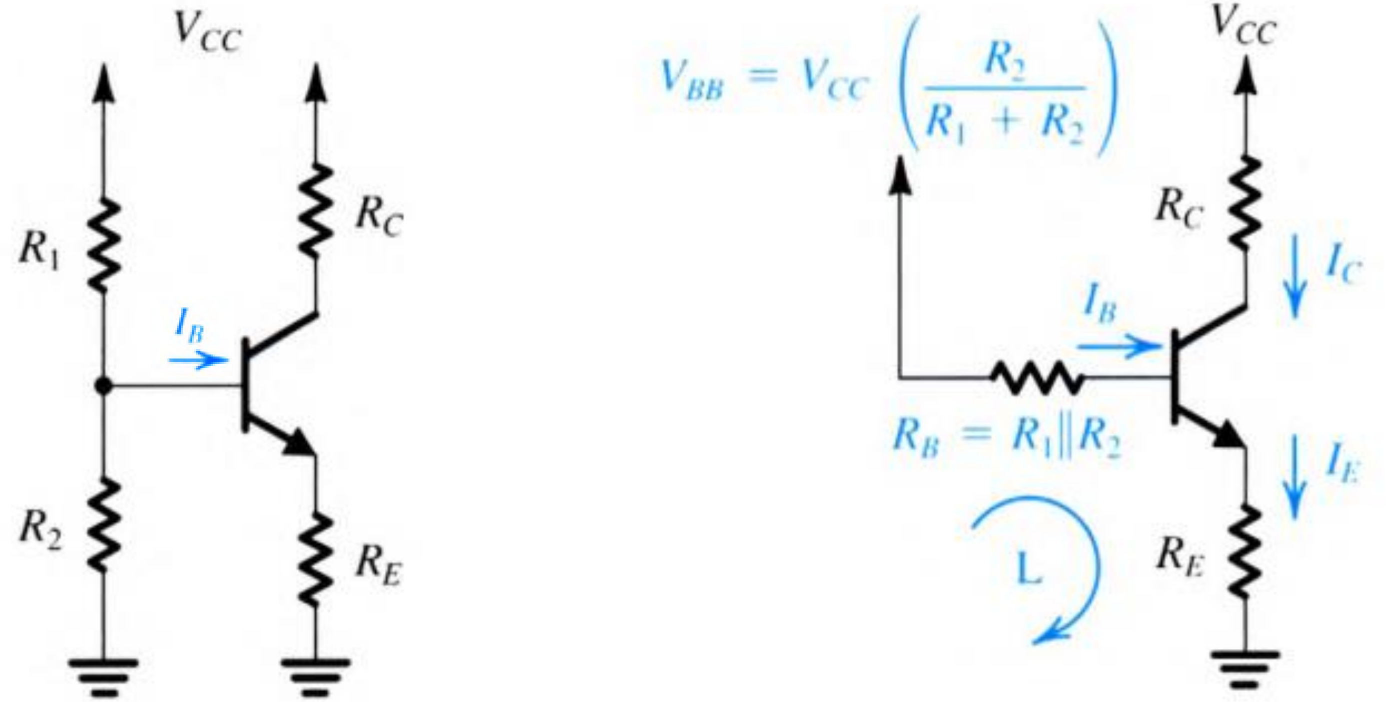
Fix I_B is not a good design:

$$I_C = \beta I_B$$

- Large variation in β among units of the same device type ➔ large variation in I_C

The classical discrete circuit bias design--BJT

Single-power supply



(a) (b)

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2}$$

$$R_B = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

To make I_E insensitive to

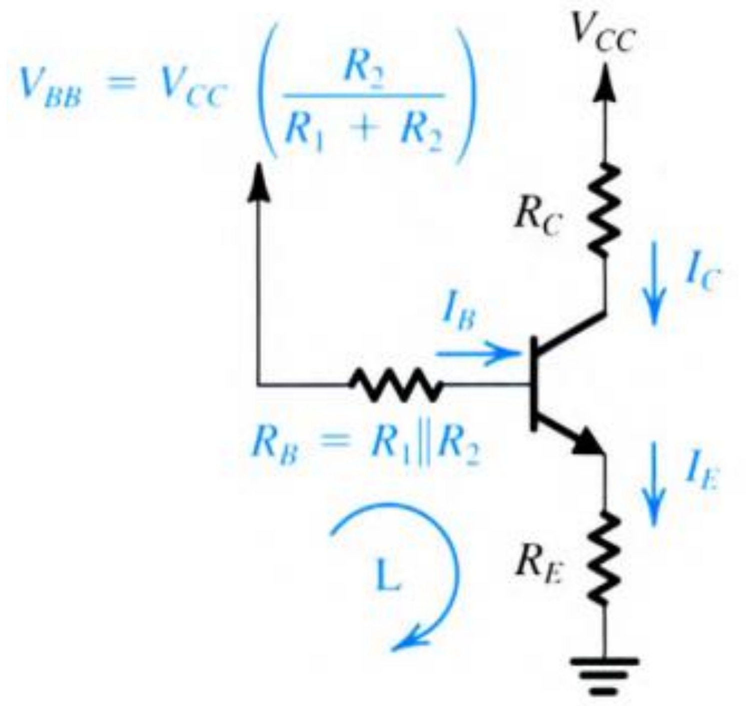
- Temperature (V_{BE}) ➔ $V_{BB} \gg \mathrm{V_{BE}}$
- β ➔ $R_E \gg \frac{R_B}{\beta+1}$

Fix I_E is a good design: $I_E \approx \frac{V_{BB}}{R_E}$

The classical discrete circuit bias design -BJT

Stable operating point despite of uncertainty in V_{BE} & β

$$\boxed{I_E = \frac{V_{BB}-V_{BE}}{R_E+\frac{R_B}{\beta+1}} \approx \frac{V_{BB}}{R_E}}$$



Cond 1: $V_{BB} \gg V_{BE}$

$V_{CC} = V_{R_C} + V_{CB} + V_{BB}$

- $V_{BB} \uparrow$ → $(V_{R_C} + V_{CB})\downarrow$
- V_{R_C} needs be large to have a large $A_v \approx -g_m R_C = -\frac{I_C}{V_T} R_C = -\frac{V_{RC}}{V_T}$ (CE, R_E bypassed in AC)
- V_{CB} needs to be large
  → large signal swing

➔ *trade-off* in designing
V_{R_C} & V_{CB} & V_{BB}

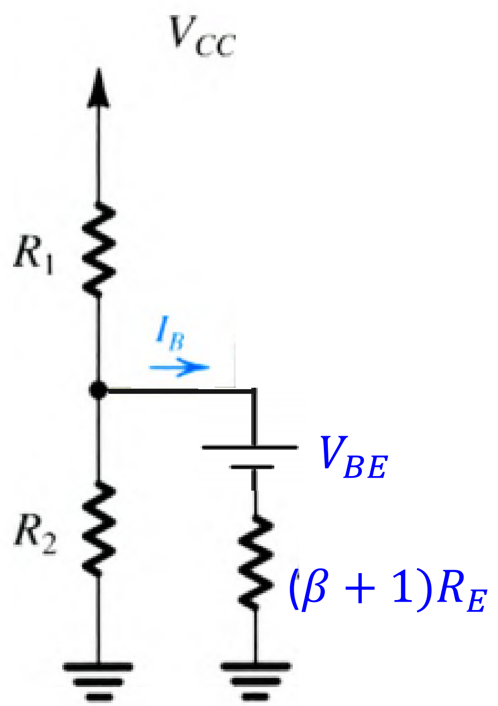
Cond 2: $R_E \gg \frac{R_B}{\beta+1}$

- small R_B → small R_1 & R_2 :
  - large bias current drained from V_{CC}
  - Smaller input impedance

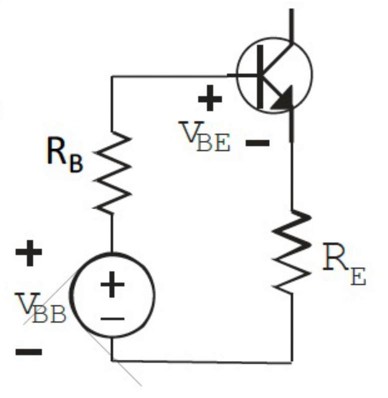
➔ *trade-off* in design
R_1 & R_2

Rules of thumb for practical bias design: BJT amplifiers

Using these rules, a design based on data for a **BC547b** transistor can ensure ~ 3% spread at operating point due to V_{BE} and β tolerances



$$V_{BB} >> V_{BE} \Downarrow V_{BB} = 5 \cdot V_{BE}$$



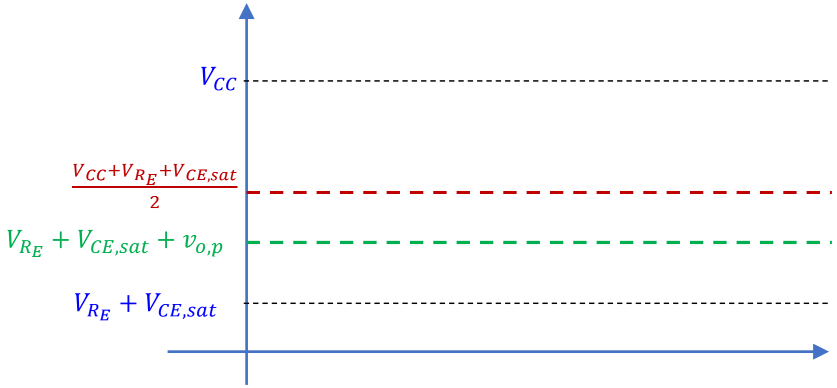
$$V_{R_B} \approx 0$$
$$\Downarrow$$
$$V_{R_E} = V_{BB} - V_{BE} = 5 \cdot V_{BE} - V_{BE} = 4 \cdot V_{BE}$$
$$\Downarrow$$
$$\boxed{V_{R_E} \approx [2.4V, 3V]}$$

$$R_E >> \frac{R_B}{\beta} \Downarrow R_E \geq 10 \cdot \frac{R_B}{\beta}$$

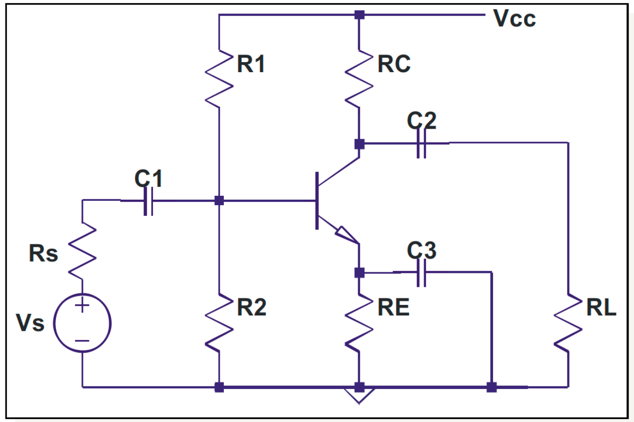
$$\boxed{R_B \leq \frac{\beta \cdot R_E}{10}}$$

$$\boxed{R_B = R_1 \parallel R_2 \Rightarrow \begin{cases} I_{R_1} = 10 \cdot I_B \\ I_{R_2} = 9 \cdot I_B \end{cases}}$$

Design procedure - BJT



- Determine V_C or V_{R_C}: $V_{R_E}+V_{CE,sat} \leq V_C \leq V_{CC}$
  - *Maximum output swing case*: $V_C = \frac{V_{CC}+V_{R_E}+V_{CE,sat}}{2} \rightarrow V_{R_C} = V_{CC} - V_C = \frac{V_{CC}-(V_{R_E}+V_{CE,sat})}{2}$
  - *Known $v_{o,p}$ for highest A_v case*: $V_C = V_{CE,sat} + V_{R_E} + v_{o,p} \rightarrow V_{R_C} = V_{CC} - V_C = V_{CC} - V_{CE,sat} - V_{R_E} - v_{o,p}$
- Determine R_C to achieve maximum gain: $R_C = \sqrt{\frac{R_S' \cdot R_L \cdot V_{R_C}}{\beta V_T}} \approx \sqrt{\frac{R_S \cdot R_L \cdot V_{R_C}}{\beta V_T}}$, as $R_S \ll R_B$
- Determine $I_C = V_{R_C}/R_C$
- Determine $R_E = \frac{V_{R_E}}{I_C} = \frac{3V}{I_C}$
- Determine $R_B = \beta R_E/10$ -- to approx $R_E \gg R_B/\beta$
- Calculate $V_{BB} = I_C R_E + V_{BE} + \frac{I_C}{\beta} R_B$
- Determine R_1 and R_2: $R_B = \frac{R_1 R_2}{R_1+R_2}$ & $V_{BB} = V_{CC}\frac{R_2}{R_1+R_2}$
- Calculate the gain and compare with requirement
- Check harmonic distortion



Definitioner/antagelser:

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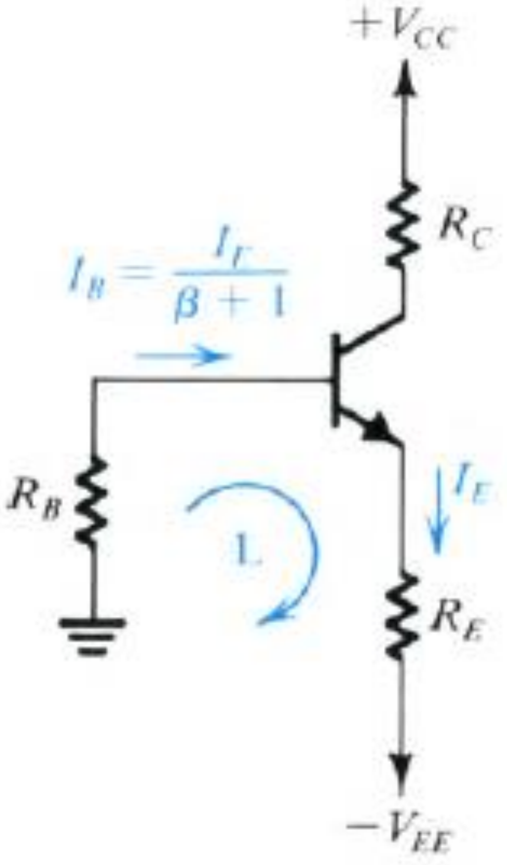
$$R_S' = R_S \parallel R_B$$

$$r_o >> R_C \parallel R_L$$

$A_v = -\frac{R_{in}}{R_{in}+R_S} g_m(R_C||R_L)$ -- $R_{in} = R_1||R_2||r_\pi$

Or $A_v = A_v' \frac{R_B}{R_B+R_S}$ with $A_v' = \frac{R_C||R_L}{\frac{1}{g_m}+\frac{R_S'}{\beta}}$

Classical configuration with two power supplies



$$I_E = \frac{V_{EE} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

Cond 1: $V_{EE} \gg V_{BE}$

$V_{EE} \geq 5V_{BE}$

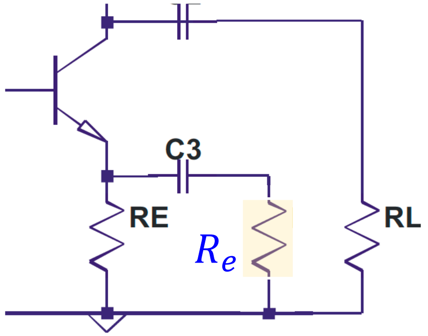
Cond 2: $R_E \gg \frac{R_B}{\beta+1}$

$R_E \geq 10\frac{R_B}{\beta}$

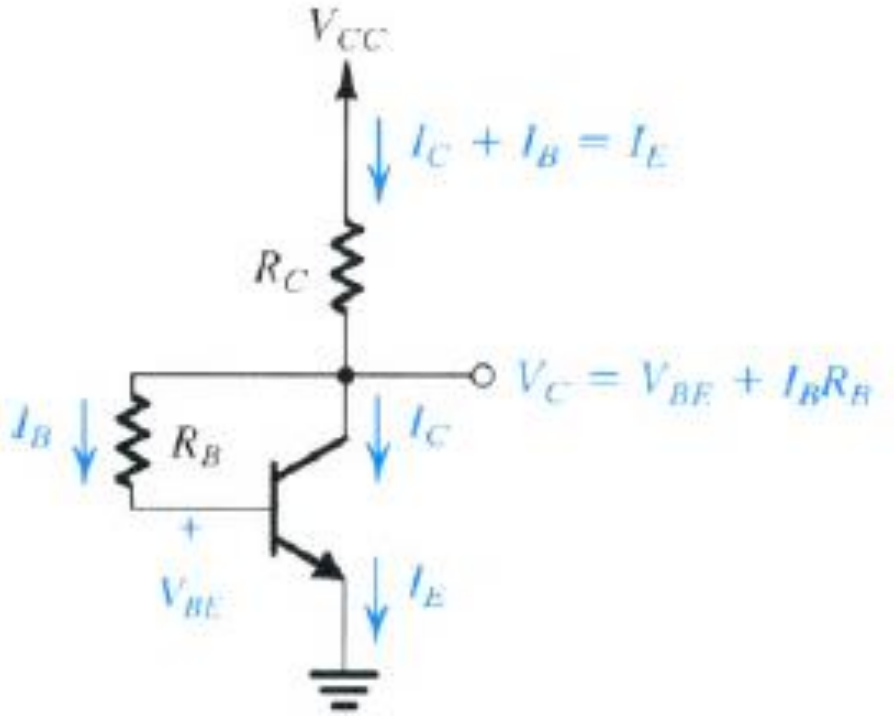
A good configuration:
a stable operating point that is robust to changes in V_{BE}, β and temperature.

$A_v = -g_m R_C$ -- without AC R_e

$A_v = -\frac{R_C}{\frac{1}{g_m}+R_e'}$ -- with AC R_e & $R_e' = R_E||R_e$



Biasing with collector-to-base feedback R_B



$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

Cond 1: $V_{CC} \gg V_{BE}$

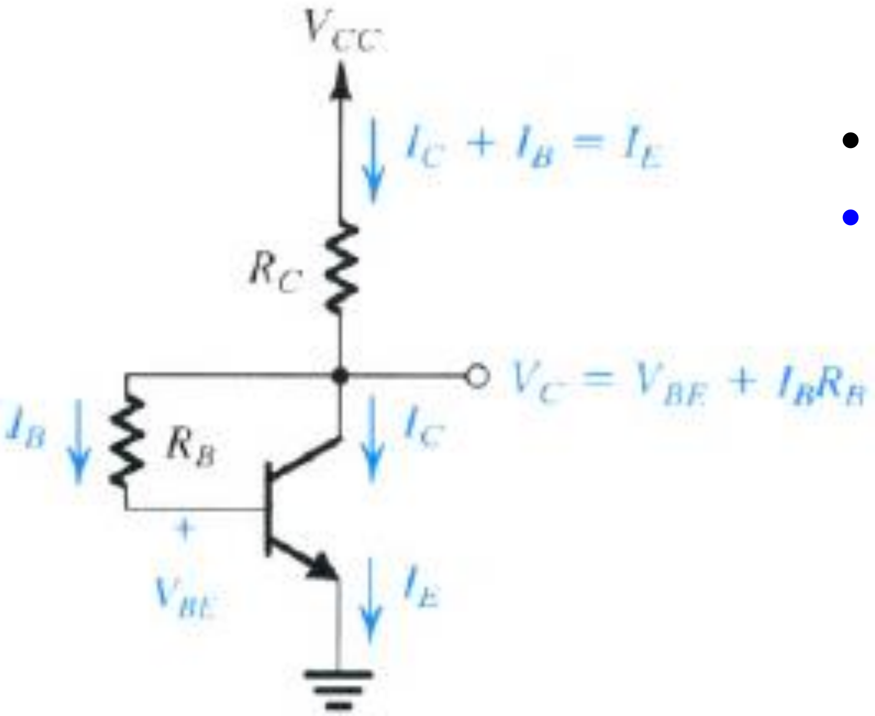
Cond 2: $R_C \gg \frac{R_B}{\beta+1}$

$$R_C \geq 10\frac{R_B}{\beta}$$

- Small R_B → small signal swing, as $V_{CB} = I_B R_B$
- Large R_C → large V_{CC}

$A_v = -g_m(R_C||R_B)$ -- without AC R_e
Gain drops due to the feedback

Quiz:



- The small-signal model of the circuit for CE stage?
- A_v = ?

Bais design - MOSFET

$$I_D = \frac{1}{2} k_n (V_{GS} - V_{TH})^2 = \frac{1}{2} \mu_n C_{ox} W/L \, (V_{GS} - V_{TH})^2$$

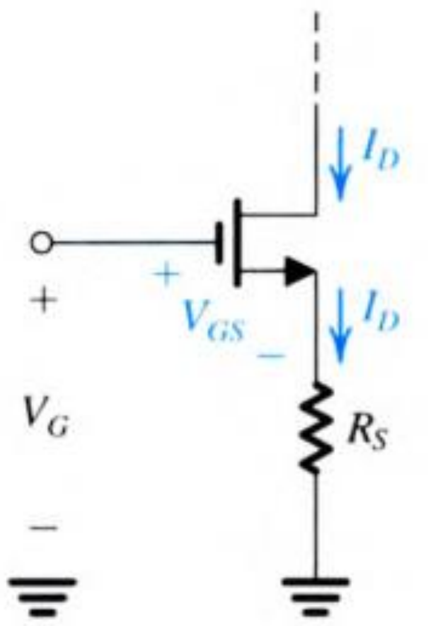
- μ_n: mobility of the electrons at the surface of the channel
- C_{ox}: oxide capacitance
- W & L: width & length of the channel

I_D variation:

- For different devices: V_{TH} , C_{ox} and W / L vary among devices even for devices with the same nominal values due to fabrication.
- For the same device due to temperature

Fix V_{GS} is not good.

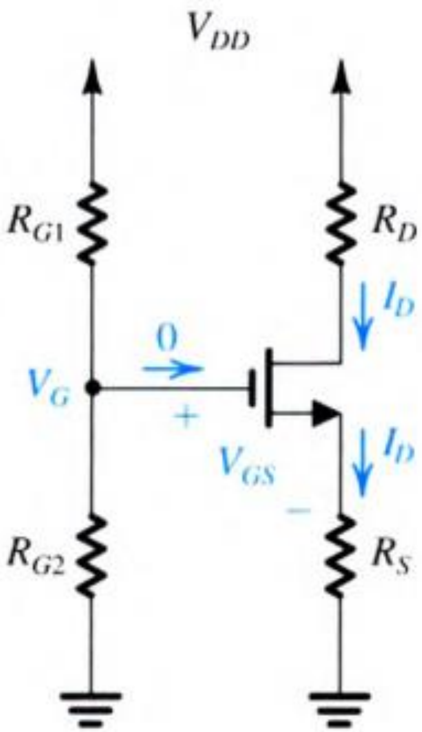
Bias design - MOSFET



Fix V_G and connecting R_S in source lead

$V_G = V_{GS} + I_D R_S \approx I_D R_S$

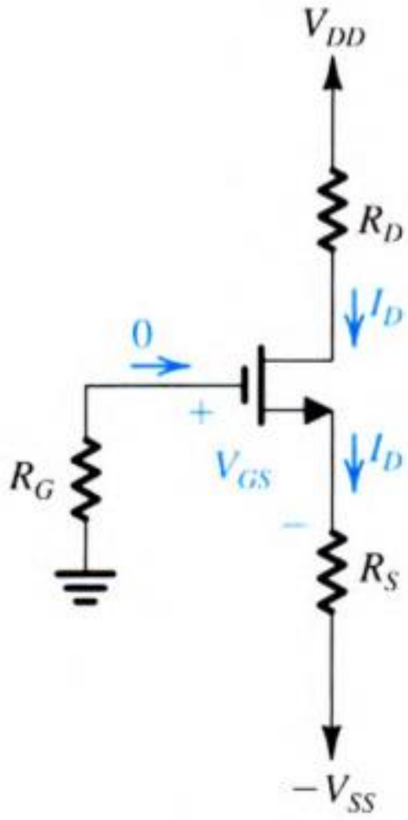
Cond : $V_G \gg V_{GS}$



$$V_G = V_{DD} \frac{R_{G_2}}{R_{G_1} + R_{G_2}}$$

$$V_G = V_{GS} + I_D R_S$$

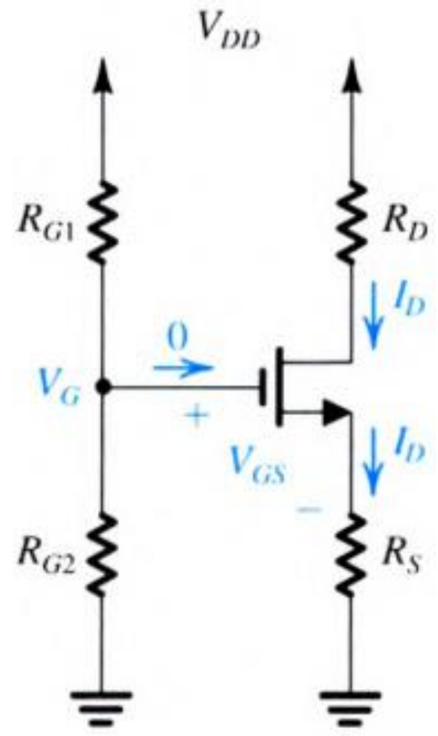
Cond : $V_G \gg V_{GS}$



$$V_{SS} = V_{GS} + I_D R_S \approx I_D R_S$$

Cond : $V_{SS} \gg V_{GS}$

Bias design procedure - MOSFET



- Determine V_{GS} , for a required $THD = HD_2 = \frac{A}{4(V_{GS}-V_{TH})}$, A input amplitude
- Determine $\boldsymbol{I_D} = \frac{1}{2} k_n (V_{GS} - V_{TH})^2$, k_n and V_{TH} from datasheet
- Determine $V_G = 5\ V_{GS}$ to approx $V_G \gg V_{GS}$
- Determine $\mathrm{R_S} = \frac{\mathrm{V_G - V_{GS}}}{\mathrm{I_D}}$
- Determine V_D or V_{R_D}: $V_G \le V_D \le V_{DD}$
  - *Maximum output swing case*: $V_D = \frac{V_{DD}+V_G}{2}$ ➔ $V_{R_D} = V_{DD} - V_D = \frac{V_{DD}-V_G}{2}$
  - *Known $v_{o,p}$ for highest A_v case*: $V_D = V_G + v_{o,p}$ ➔ $V_{R_D} = V_{DD} - V_D = V_{DD} - V_G - v_{o,p}$
- Determine $R_D = \frac{V_{R_D}}{I_D}$
- Determine R_{G_1} and R_{G_1} in $\mathrm{M\Omega}$ *range (provide high input impedance) using* $V_G = V_{DD} \frac{R_{G_2}}{R_{G_1}+R_{G_2}}$
- Calculate the gain and compare with requirement
- Check harmonic distortion