

Discrete time systems and z-transform

Exercises lektion 11

1.

A stable LTI system is characterized by the following difference equation:

$$\frac{1}{2}y[n-1] - \frac{9}{4}y[n] + y[n+1] = x[n]$$

- Calculate the system function $H(z)$.
- Calculate the impulse response $h[n]$.

2.

The output of a LTI system is $y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$, when the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1].$$

- Calculate the system function $H(z)$ and its ROC.
- Find the system impulse response $h[n]$.
- Write the difference equation that characterizes the system.
- Is the system stable and/or causal? Motivate your response.

3.

When the input is $x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1]$, the z-transform of the output signal of an LTI system is $Y(z) = \frac{1-z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$. Calculate the impulse response $h[n]$ of the system.

4.

Given the following system function

$$H(z) = \frac{1 - 3z^{-1}}{(1 - z^{-1})(1 - 2z^{-1})}, \quad |z| > 2$$

Determine the impulse response $h_i(n)$ of the inverse system.

5.

Determine the impulse response $h_i(n)$ of an unstable LTI system whose system function is given by

$$H(z) = \frac{21}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})(1 - 4z^{-1})}$$

Is there a single possible $h_i(n)$?