Power series

Exercises Lektion 4

A. Find center and radius of convergence of the following series

1. $\sum_{n=0}^{\infty} 4^n \left(z + 1 \right)^n$

2. $\sum_{n=1}^{\infty} \frac{n^n}{n} \left(z - \pi i \right)^n$

3. $\sum_{n=0}^{\infty} \frac{n(n-1)}{3^n} \left(z - i \right)^{2n}$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n} \left(z + 3 \right)^n$

5. $\sum_{n=1}^{\infty} \frac{2^n}{n} \left(4z - 8 \right)^n$

6. $\sum_{n=0}^{\infty} n! \left(2z + 1 \right)^n$

7. $\sum_{n=1}^{\infty} \frac{(z - 2i)^n}{n^n}$

8. $\sum_{n=0}^{\infty} \left(\frac{1-i}{2+3i} \right)^n z^n$

9. $\sum_{n=0}^{\infty} 16^n \left(z + i \right)^{4n}$

10. $\sum_{n=0}^{\infty} \frac{3n}{2^n n^2} z^n$

B. Calculate McLaurin series of the following functions

1. $sin\left(2z^2\right)$

2. $\frac{1}{2+z^4}$

3. $\frac{1}{1+5iz}$

4. $cos^2\left(\frac{z}{2}\right)$

5. $sin^2\left(z\right)$

C. Calculate the Taylor series with center z_0 of the following functions:

1. $\frac{1}{z}$, $z_0 = i$

2. $\frac{1}{1-z}$, $z_0 = i$

3. $sin(z)$, $z_0 = \frac{\pi}{2}$

4. e^{-z}, $z_0 = -2$

5. $2sin\left(\pi z + \frac{\pi}{2}\right)$, $z_0 = 0$