Modelling of Electrodynamic Systems

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Recall from the previous lecture

Differential Form	Integration Form	Meaning
$\nabla \times \bar{E} = -\frac{\partial}{\partial t}\bar{B}$	$\oint_C \bar{E}\,d\bar{l} = -\frac{d}{dt}\phi$	FARADAYS LAW
$\nabla \times \bar{H} = \frac{\partial}{\partial t}\bar{D} + \bar{J}$	$\oint_C \bar{H}\,d\bar{l} = I + \int_S \frac{\partial}{\partial t}\bar{D}\,d\bar{a}$	AMPÈRES LAW
$\nabla \cdot \bar{D} = \rho$	$\oint_S \bar{D}\,d\bar{a} = Q$	GAUSS' LAW
$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B}\,d\bar{a} = 0$	There does not exist magnetic monopole (magnetic field are always closed)

Constructive equation:

$\bar{D} = \varepsilon \bar{E}$

$\bar{B} = \mu \bar{H}$

$\bar{J} = \sigma \bar{E}$

Others:

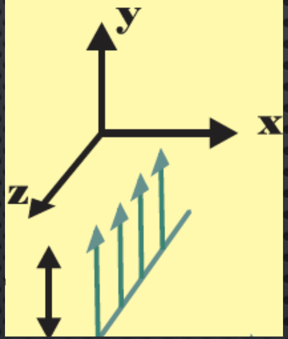
$\nabla \cdot \bar{J} = -\frac{\partial}{\partial t}\rho$

Maxwell's Equations

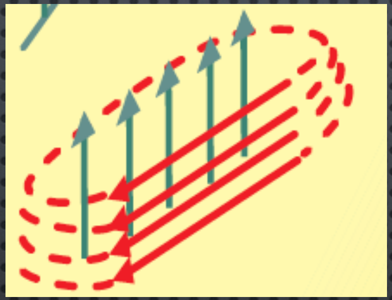
Wave Propagation

$$\nabla \times \bar{H} = \frac{\partial}{\partial t} \varepsilon \bar{E}$$

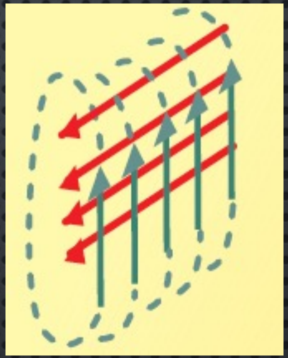
$$\nabla \times \bar{E} = -\frac{\partial}{\partial t} \mu \bar{H}$$



E field in y direction



H field is generated by E field



E field is generated by H field again

There are 90 degrees phase difference between E field and H field, where they are orthogonal to each other.

$$\nabla \times \bar{E} = -\frac{\partial}{\partial t}\bar{B}$$

$$\nabla \times \bar{H} = \frac{\partial}{\partial t}\bar{D} + \bar{J}$$

$$\nabla \bullet \bar{D} = \rho$$

$$\nabla \bullet \bar{B} = 0$$

General form for harmonic signals (KSN)

Maxwell's equations

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$

$$\nabla \times \bar{H} = (j\omega\varepsilon + \sigma)\bar{E}$$

$$\nabla \bullet \bar{B} = 0$$

$$\nabla \bullet \bar{D} = \rho$$

Wave equations

$$\Delta \bar{E} = \gamma^2 \bar{E}$$

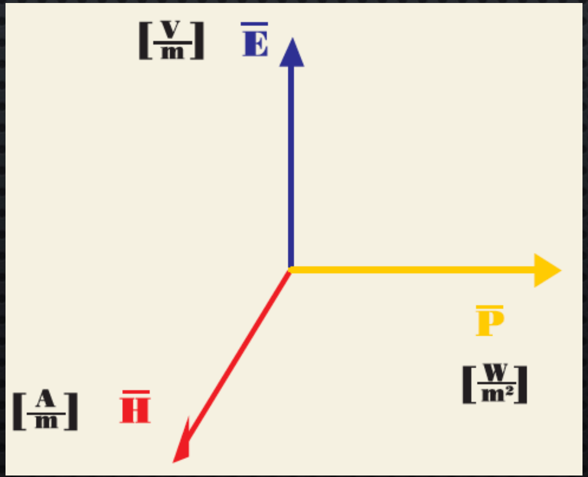
$$\Delta \bar{H} = \gamma^2 \bar{H}$$

Solutions (Wave on YZ plane)

$$E = E^+ e^{-\gamma x} + E^- e^{+\gamma x}$$

$$H = H^+ e^{-\gamma x} + H^- e^{+\gamma x}$$

$$= \frac{1}{\eta}\left(E^+ e^{-\gamma x} - E^- e^{+\gamma x}\right)$$



Transmissionline Analysis

$$K_L = \frac{\eta_L - \eta}{\eta_L + \eta} \quad [\cdot]$$

$$K(x) = K_L \cdot e^{2\gamma x} \quad [\cdot]$$

$$\eta(x) = \eta \cdot \frac{1 + K(x)}{1 - K(x)} \quad [\Omega]$$

$$K(x) = \frac{E^-(x)}{E^+(x)} = -\frac{H^-(x)}{H^+(x)} \quad [\cdot]$$

Modelling of Electrodynamic Systems

MM6. Wave Propagation at Good Conductor

Wave Propagation at Good Conductor

Targets:

1. Read "Grundlæggende Maxwellsk Feltteori" (Page 83 – 105) (before or after the lecture)

2. Be able to calculate attenuation and penetration depth at good conductor (lecture)

3. Finish the exercise (after the lecture)

Wave Propagation at Good Conductor

Blackboard (1)

Identiteter

$$\nabla \times (\nabla F) = \bar{O}$$

Rotation of a scaler field's gradient is null vector.

$$\nabla \bullet (\nabla \times \bar{A}) = 0$$

Divergence of a vector field's rotation is null.

$$\nabla \bullet (\bar{A} \times \bar{B}) = \bar{B} \bullet (\nabla \times \bar{A}) - \bar{A} \bullet (\nabla \times \bar{B})$$

Laplace

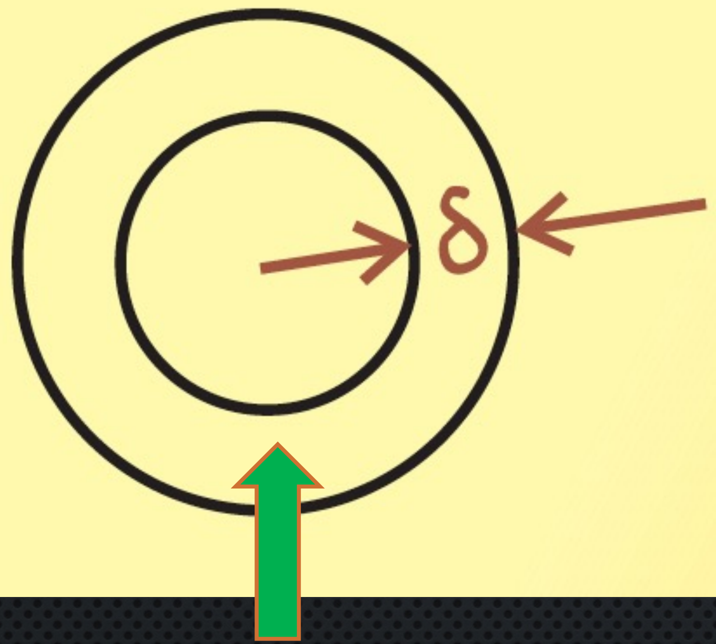
$$\Delta \bar{A} = \nabla^2 \bar{A} = \nabla \bullet (\nabla \bar{A})$$

Nabla i anden er diver-gensen af gradienten

$$\Delta \bar{A} = \nabla (\nabla \bullet \bar{A}) - \nabla \times \nabla \times \bar{A}$$

Blackboard (2) (3)

Skin Effect



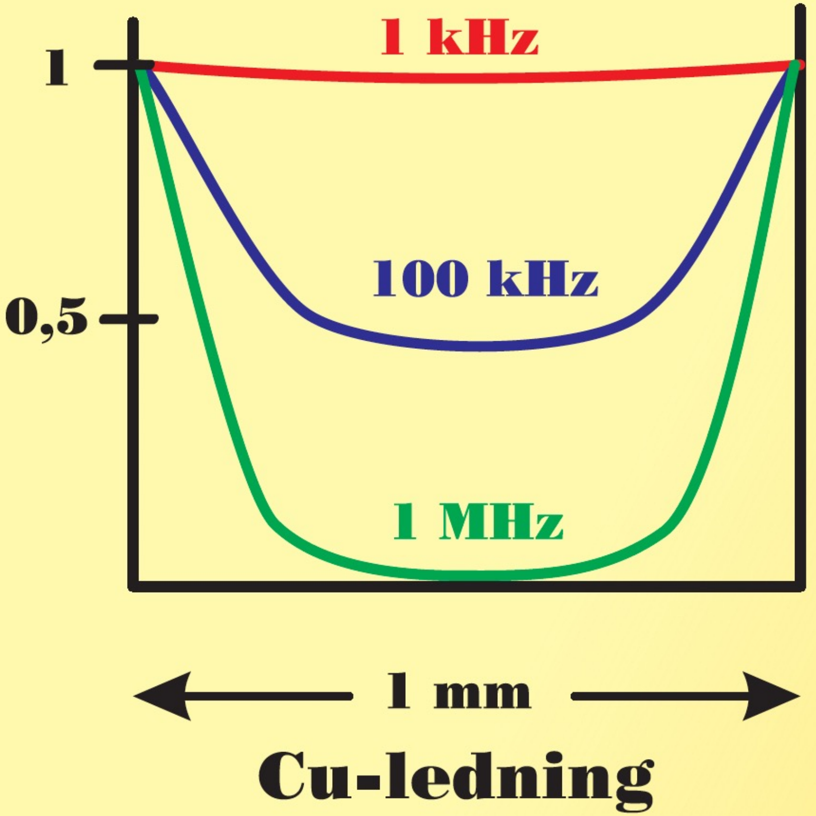
$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} \quad [m]$$

6,6 μm for Cu @ 100 MHz

E, H, B, D, J

Skin Effect

Current Density



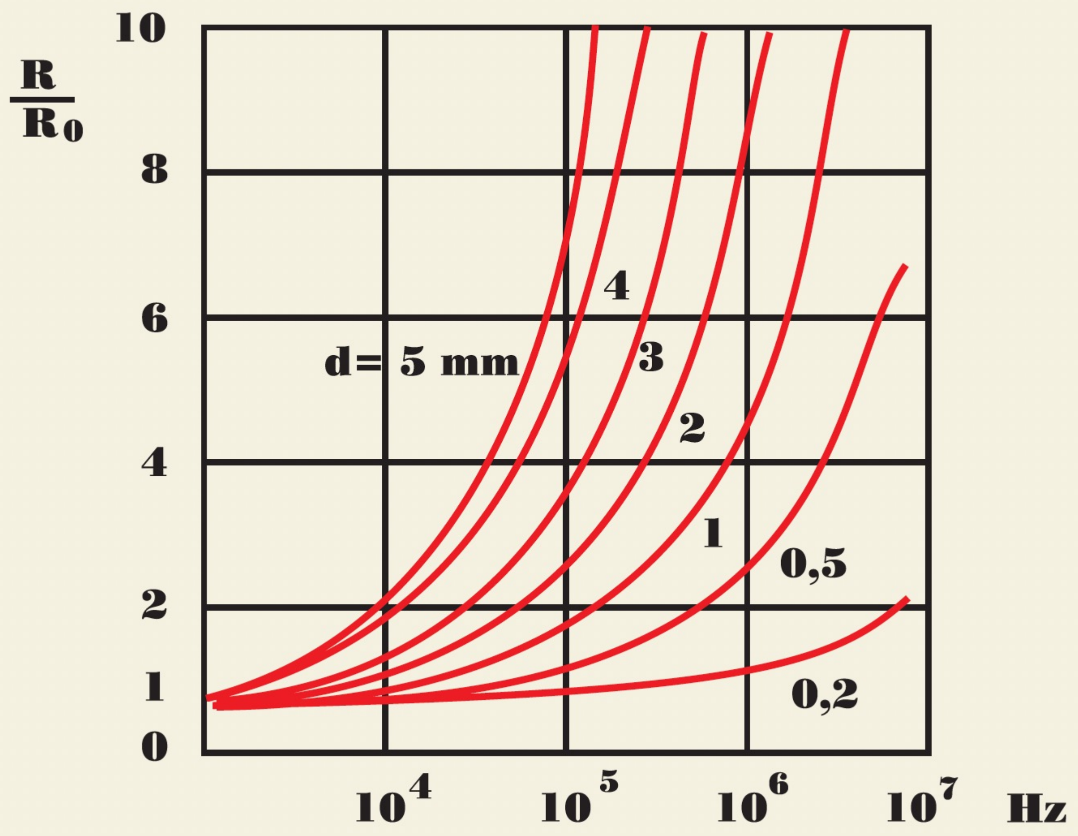
$$\frac{R_{HF}}{R_{DC}} = \frac{\frac{1}{\delta\sigma} \cdot \frac{\ell}{2\pi r}}{\frac{1}{\sigma} \cdot \frac{\ell}{\pi r^2}} = \frac{r}{2\delta}$$

For Cu, 100 MHz, r= 0,5 mm

$$\frac{R_{HF}}{R_{DC}} = \frac{0{,}5\,\text{mm}}{2 \cdot 6{,}6\,\mu\text{m}} = 40\times$$

For very high frequency, dielectric waveguide is preferred, e.g., optical fiber.

Skin Effect



Resistance increase for different copper wire diameter

(0,2 - 5 mm Ø)

Skin Effect

When an electromagnetic wave passes through a medium, its amplitude decreases exponentially. This decay occurs because the currents, induced in the medium, produce ohmic losses which are converted to heat in the material. From this it follows that:

$$E_1 = E_o e^{-t/\delta} \qquad (9.3)$$

$$H_1 = H_o e^{-t/\delta} \qquad (9.4)$$

where E_1, H_1 is the wave intensity at a distance t within the medium and δ is the so-called skin depth. At a distance $t = \delta$ inside the medium the fields are attenuated to 37% of its original value E_o, H_o at the surface of the medium. The skin depth is defined by the following equation:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{meters} \qquad (9.5)$$

Relative conductivity

where, σ is the conductivity of the medium. Usually conductivity is measured relative to the conductivity of copper, i.e. $\sigma = \sigma_r \cdot \sigma_{cu}$. Table 9.1 lists the relative conductivity σ_r for various materials, relative to the conductivity of copper:

$$\sigma_{cu} = 5.82 \cdot 10^{7} \text{ mhos/m} \qquad (9.6)$$

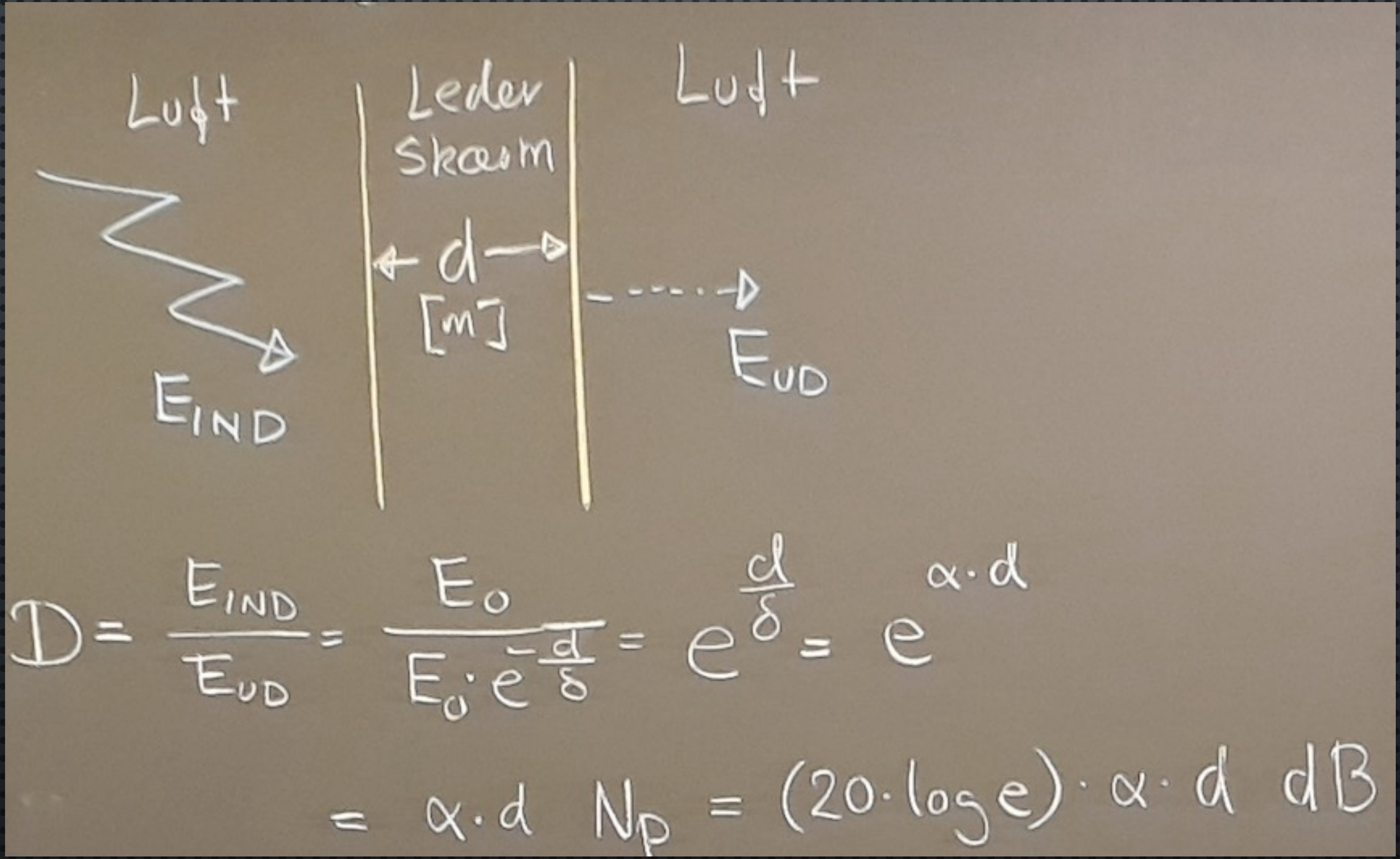
$$\mu_o = 4\pi \cdot 10^{-7} \text{ H/m}$$

$$\epsilon_o = 8.86 \cdot 10^{-12} \text{ F/m} \approx \frac{1}{36\pi} \cdot 10^{-9} \text{ F/m}$$

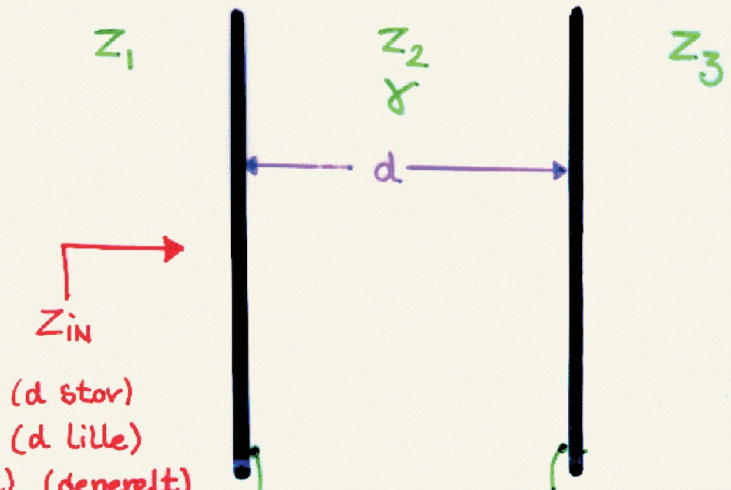
Material	Relative Conductivity σ_r	Relative Permeability μ_r
Silver	1.05	1
Copper	1.00	1
Gold	0.70	1
Aluminium	0.61	1
Zink	0.29	1
Brass	0.26	1
Nickel	0.20	1
Bronze	0.18	1
Iron	0.17	1000
Tin	0.15	1
Steel (SAE 1045)	0.10	1000
Lead	0.08	1
Mu-Metal	0.03	80000
Stainless Steel (430)	0.02	500

Relative Conductivity and Permeability of Various Materials at 100 kHz

Absorption Attenuation



Reflection Attenuation



$$= \begin{cases} Z_2 & (d \text{ stor}) \\ Z_3 & (d \text{ lille}) \\ Z(-d) & (\text{generelt}) \end{cases}$$

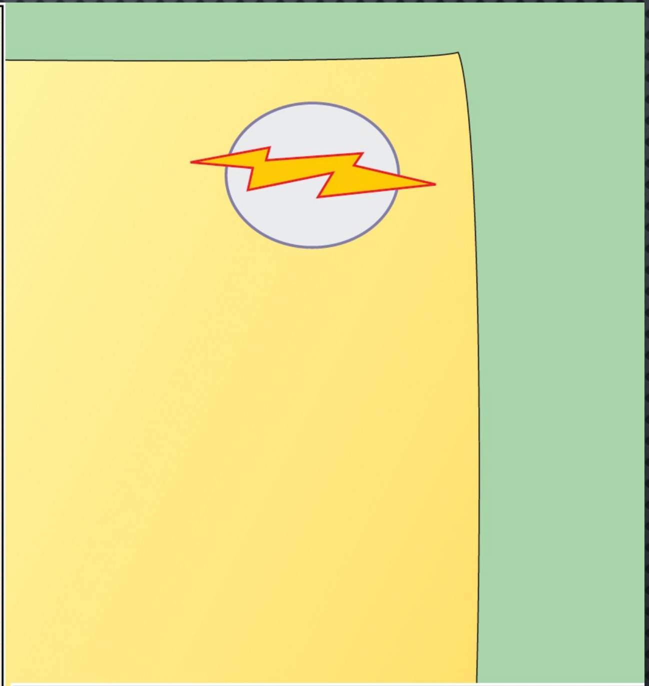
$$K_L = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$

$$K(-d) = K_L \cdot e^{-2\gamma d}$$

$$Z(-d) = Z_2 \cdot \frac{1 + K(-d)}{1 - K(-d)}$$

For $K(-d) \approx 0$ (d eller γ meget stor)
$Z(-d) \approx Z_2$

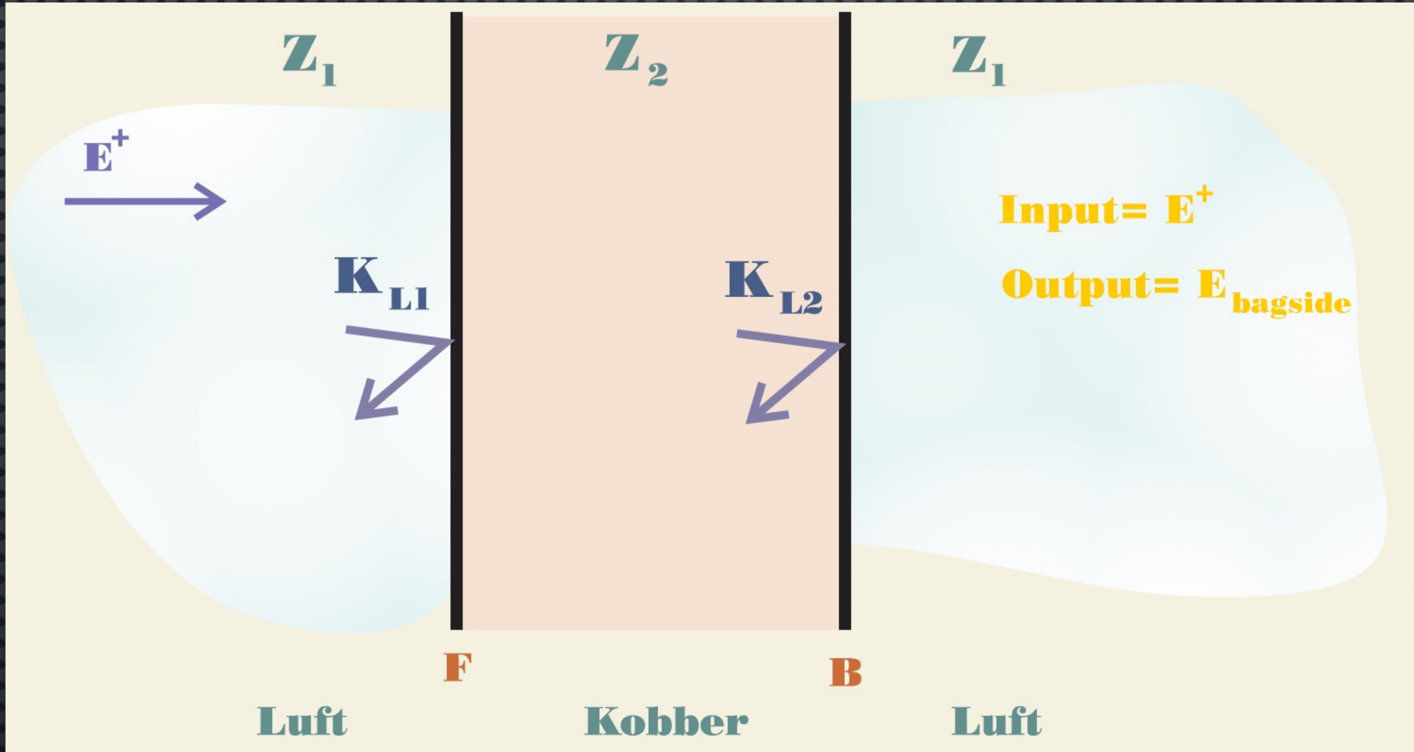
For $K(-d) \approx K_L$ (d eller γ meget lille)
$Z(-d) \approx Z_3$



$$\begin{aligned} Z(-d) &= Z_2 \cdot \frac{1 + K(-d)}{1 - K(-d)} \\ &\approx Z_2 \cdot \frac{1 + K_L}{1 - K_L} \\ &= Z_2 \cdot \frac{1 + (Z_3 - Z_2)/Z_3 + Z_2}{1 - (Z_3 - Z_2)/Z_3 + Z_2} \\ &= Z_3 \end{aligned}$$

Approximation

Reflection Attenuation



$$K_{L1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$K_{L2} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -K_{L1}$$

$$D = \frac{E^+}{E_B} = \frac{1}{1 - K_L^2}$$

$$E_F = E^+(1 + K_{L1})$$

$$E_B = E_F(1 + K_{L2})$$

$$= E^+(1 + K_{L1})(1 + K_{L2})$$

$$= E^+(1 + K_{L1})(1 - K_{L1})$$

$$= E^+(1 - K_L^2)$$

Boundary Condition