

Practice exercise of Lecture_2

1. Find the gradient of a scalar function of $f(x, y, z) = 4x^2 + 2y^2 + 3z^3$, and also find its directional derivative at $P: (1, 3, 2)$ in the direction of $\mathbf{a} = [0, 1, -1]$.

2. Find $\text{div } \mathbf{v}$ and its value at P .

$$\mathbf{v} = [x^2, 4y^2, 9z^2], \quad P: (-1, 0, 0.5)$$

$$\mathbf{v} = [0, \cos(xyz), \sin(xyz)], \quad P: (2, \frac{1}{2}\pi, 0)$$

3. calculate $\nabla^2 f$

$$f(x, y, z) = e^{xyz}$$

$$f(\rho, \phi, z) = 2\rho^2 + 3\rho \cos \phi + \rho z^3$$

$$f(r, \phi, \theta) = 2r^2 \sin \theta + 3r \cos \phi + 3 \sin \theta$$

4. Please prove the divergence of the curl of a twice continuously differentiable vector function \mathbf{v} is zero:

$$\text{div} (\text{curl } \mathbf{v}) = 0$$