Lecture 5

(1) Ampere's and Faraday's Law

Point-form:

Ampere's law: $\nabla \times \bar{H} = \bar{J}$

Faraday's law: $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

[$\nabla \cdot (\nabla \times \bar{A}) = 0$]

Take both sides for A's law:

$\nabla \cdot (\nabla \times \bar{H}) = 0$ (left side)

$\nabla \cdot (\bar{J}) = -\frac{\partial \rho}{\partial t}$ (right side)

Something wrong!!!

A's law is wrong in general.

Repairation of A's law

$\nabla \cdot \bar{J} = -\frac{\partial}{\partial t}\rho \quad (1)$

$\nabla \cdot \bar{D} = \rho \quad (2)$

Insert (2) to (1) and have:

$\nabla \cdot \bar{J} = -\frac{\partial}{\partial t}\rho = -\frac{\partial}{\partial t}(\nabla \cdot \bar{D})$

$= \nabla \cdot (-\frac{\partial}{\partial t}\bar{D})$

$\Rightarrow \nabla \cdot \bar{J} - \nabla \cdot (-\frac{\partial}{\partial t}\bar{D}) = 0$

$\Rightarrow \nabla \cdot (\bar{J} + \frac{\partial}{\partial t}\bar{D}) = 0$

We have A's law, version 2.0:

$\nabla \times \bar{H} = \bar{J} + \frac{\partial}{\partial t}\bar{D}$

Displacement current

Now we have 2 types of currents:

$\nabla \times \bar{H} = \bar{J} + \frac{\partial}{\partial t}\bar{D}$

$= \bar{J}_c + \bar{J}_D$

- Conduction Current: $\bar{J}_c = \sigma \bar{E}$

  $[\frac{A}{m^2} = \frac{A}{V \cdot m} \cdot \frac{V}{m}]$

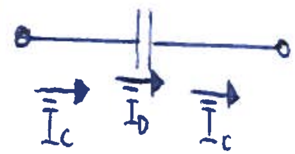
- Displacement Current: $\bar{J}_D = \frac{\partial}{\partial t}\varepsilon \cdot \bar{E}$

  $[\frac{A}{m^2} = \frac{1}{s} \cdot \frac{A \cdot s}{V \cdot m} \cdot \frac{V}{m}]$

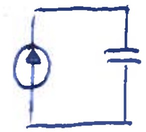
KSN: $\bar{J}_D = j\omega \varepsilon_0 \varepsilon_r \bar{E}$

(time-harmonic)

Displacement current, continue:



For constant current:

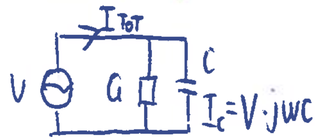


$V = \frac{1}{C}\int I\,dt$

$= \frac{It}{C}$

For harmonic current: (KSN)

Technical formula



$I_{TOT} = V \cdot (G + j\omega C)$

$\bar{J} = \bar{E} \cdot (\sigma + j\omega \varepsilon)$

Maxwell's equations

$$\boxed{\begin{aligned} \nabla \times \bar{E} &= -\frac{\partial}{\partial t}\bar{B} \\ \nabla \cdot \bar{D} &= \rho \\ \nabla \times \bar{H} &= \bar{J} + \frac{\partial}{\partial t}\bar{D} \\ \nabla \cdot \bar{B} &= 0 \end{aligned}}$$

Constructive equation:

$\bar{D} = \varepsilon \bar{E}$

$\bar{B} = \mu \bar{H}$

$\bar{J} = \sigma \cdot \bar{E}$

Others:

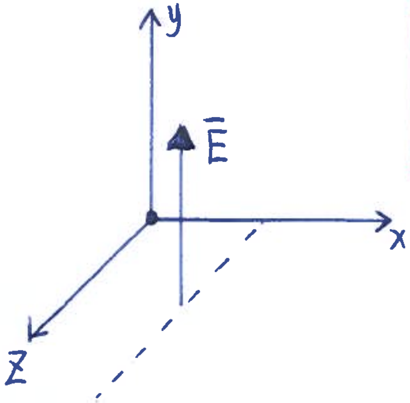
$\nabla \cdot \bar{J} = -\frac{\partial}{\partial t}\rho$

(8 equations)

(2) Propagation in lossless medium

We have a medium with $\sigma = 0\ S/m$.

Lossless medium with a plane wave:



$$\boxed{\begin{matrix} \sigma = 0\left[\frac{S}{m}\right] & \bar{J}_c = \sigma\bar{E} = 0 \\ \rho = 0\left[\frac{C}{m^3}\right] & \nabla\cdot\bar{D} = 0 \end{matrix}}$$ (lossless condition)

$\bar{E} = E_y \cdot \hat{y}\ \left[\frac{V}{m}\right]$

$\frac{\partial \bar{E}}{\partial y} = \frac{\partial \bar{E}}{\partial z} = 0$ (plane wave)

ϵ and μ are HiLS

Maxwell's rotation's equations:

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\frac{\partial}{\partial t}\mu \cdot \bar{H} \qquad (3)$$

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial}{\partial t}\bar{D} = \frac{\partial}{\partial t}\epsilon\bar{E} \qquad (4)$$

From (3) we have:

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -\frac{\partial}{\partial t}\mu\bar{H}$$

$$\Rightarrow \frac{\partial}{\partial x}E_y\hat{z} = -\frac{\partial}{\partial t}\mu H_z\hat{z} \qquad (5)$$

From (4) we have

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{z} \\ 0 & 0 & H_z \end{vmatrix} = \frac{\partial}{\partial t}\epsilon\bar{E}$$

$$\Rightarrow -\frac{\partial}{\partial x}H_z\hat{y} = \frac{\partial}{\partial t}\epsilon E_y \cdot \hat{y} \qquad (6)$$

Add $\frac{\partial}{\partial x}$ at both sides for (5)

$$\frac{\partial^2}{\partial x^2}E_y = -\frac{\partial^2}{\partial t \partial x}\mu H_z \qquad (7)$$

Add $\frac{\partial}{\partial t}$ at both sides for (6)

$$\frac{\partial^2}{\partial x \partial t}H_z = -\frac{\partial^2}{\partial t^2}\epsilon E_y \qquad (8)$$

From (7) and (8), we have:

$$\frac{\partial^2}{\partial x^2}E_y = \epsilon\mu \cdot \frac{\partial^2}{\partial t^2}E_y \qquad (9)$$

This is one-dimension wave equation.

[general form: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$]

(3) Propagation in lossless medium, analysis

Transmissionline analysis

Wave equation in transmissionline:

$$\frac{\partial^2 V}{\partial x^2} = LC \cdot \frac{\partial^2}{\partial t^2} V$$

$$\frac{\partial^2 I}{\partial x^2} = LC \cdot \frac{\partial^2}{\partial t^2} I$$

They are similar to (9), So:

We can use transmissionline method to analyze the propagation of plane wave.

★

Field	Transmissionline
$E\,[\frac{V}{m}]$	$V\ [V]$
$H\,[\frac{A}{m}]$	$I\ [A]$
$\mu\,[\frac{H}{m}]$	$L\ [\frac{H}{m}]$
$\varepsilon\,[\frac{F}{m}]$	$C\ [\frac{F}{m}]$
$v\,[\frac{m}{s}]$	$v\ [\frac{m}{s}]$
$\eta = \sqrt{\frac{\mu}{\varepsilon}}\ [\Omega]$	$Z_0 = \sqrt{\frac{L}{C}}$

Intrinsic impedance

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\ [\Omega] \quad \text{(eta)}$$

Solution to harmonic signal

$\boxed{KSN}$

$$E_y = E^+ e^{-j\beta x} + E^- e^{+j\beta x}\ [\frac{V}{m}]$$

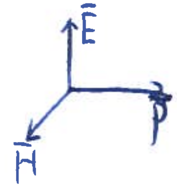
$$H_z = H^+ e^{-j\beta x} + H^- e^{+j\beta x}\ [\frac{A}{m}]$$

$$= \frac{1}{\eta}[E^+ e^{-j\beta x} - E^- e^{+j\beta x}]$$

where:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{E^+}{H^+} = -\frac{E^-}{H^-}\ [\Omega]$$

E and H are orthogonal to each other, and 90° phase difference.



$\bar{P} = \bar{E} \times \bar{H}$ (Poynting vector)

↓ power density

Different forms

$$\beta = w\sqrt{\mu\varepsilon}\ [\frac{rad}{m}]$$

$$v = \frac{w}{\beta} = \frac{1}{\sqrt{\varepsilon\mu}}\ [\frac{m}{s}]$$

$$\beta \cdot \lambda = 2\pi = \frac{w}{v} \cdot \lambda$$

$$\Rightarrow \lambda = \frac{v}{f}\ [m]$$

Electrical length angle

$$\phi = \beta l = \frac{2\pi}{\lambda} \cdot l$$

$$= 2\pi \cdot \frac{l}{\lambda}\ [rad]$$

For vaccum

$\mu = \mu_0 \quad \varepsilon = \varepsilon_0$

Speed in vaccum

$$V = C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \begin{cases} 3E8\ [m/s] \\ 300E6\ [m/s] \\ 300\ [m/\mu s] \end{cases}$$ light speed

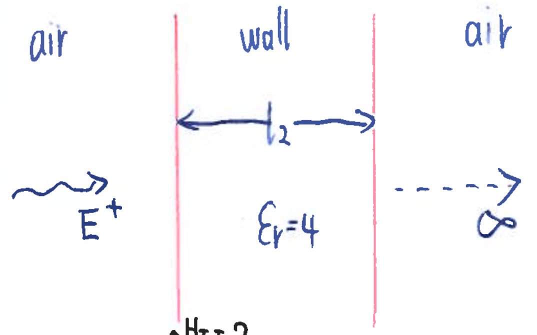
Impedance in vaccum

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377\ [\Omega]$$ free space impeda

note:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_0 \cdot \frac{1}{\sqrt{\varepsilon_r}}$$ (for lossless

(4) Calculation over the boundary



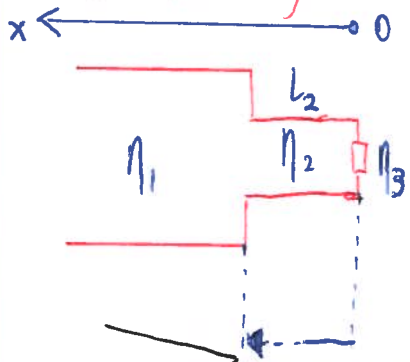
$\uparrow H_{TOT}, E_{TOT}$?

Layer	M_1	M_2	M_3
Constant	$\varepsilon_1 \mu_1$	$\varepsilon_2 \mu_2$	$\varepsilon_3 \mu_3$

Calculate η

$(\mu_1 = \mu_2 = \mu_3 = 1)$

Transmissionline analysis :



$$k_L = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}$$

$$k(-l_2) = k_L e^{j2\beta_2 l_2}$$

$$k_{L12} = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1}$$

$$\eta_{in} = \eta(-l_2) = \eta_2 \frac{1 + k(-l_2)}{1 - k(-l_2)}$$

$$E_{TOT} = E^+ (1 + k_{L12}) \quad \left[\frac{V}{m}\right]$$

$$H_{TOT} = \frac{E_{TOT}}{\eta(-l_2)} \quad \left[\frac{A}{m}\right]$$

notes

$\eta_1 = \eta_0 = 120\pi \ \Omega$

$\eta_3 = 120\pi \ \Omega$

$\eta_2 = 120\pi \cdot \frac{1}{\sqrt{4}} = 60\pi \Omega$

$$k_L = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} = \frac{\eta_0 - \eta_0 \frac{1}{\sqrt{\varepsilon_r}}}{\eta_0 + \eta_0 \frac{1}{\sqrt{\varepsilon_r}}} = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}$$