Practice exercise of Lecture_3

1. Please calculate the double integrals

   a. $\int_0^2 \int_x^{2x} (x+y)^2 dydx$

   b. $\int_0^1 \int_{x^2}^{x} (1-2xy) dydx$

2. Find the volume of the given region in space: the region beneath $z = 4x^2 + 9y^2$ and above the rectangle with vertices (0, 0), (3, 0), (3, 2), (0, 2) in the xy-plane.

3. Evaluate the vector line integral $\int_C \mathbf{F}(\mathbf{r}) \bullet d\mathbf{r}$ counterclockwise around the boundary C of the region R by Green's theorem, where

   a. $\mathbf{F} = [y, -x]$, C the circle $x^2 + y^2 = 1/4$

   b. $\mathbf{F} = [6y^2, 2x - 2y^4]$, R the square with vertices (2, 2), (-2, -2), (2, -2), (-2, 2).

   c. $\mathbf{F} = [x^2 + y^2, x^2 - y^2]$, R: $1 < y < 2 - x^2$

4. Calculate surface integral of the vector function $\mathbf{F} = [x^2, 0, 3y^2]$ and S is the portion of the plane $x + y + z = 1$ in the first octant.