Practice exercise of Lecture_1

1. Is the cross product associative?

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C}\overset{?}{\equiv}\vec{A}\times\left(\vec{B}\times\vec{C}\right)$$

If so, prove it; if not, provide a counterexample (the simpler the better).

2. Use the cross product to find the components of the unit vector ***n*** perpendicular to the shaded plane in Fig. 1.

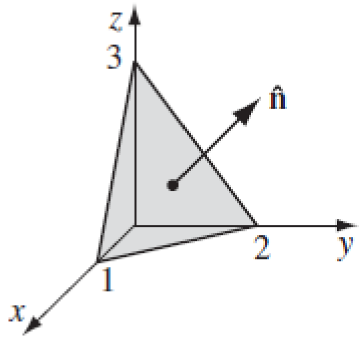

Fig. 1

3. For each of the following, find a vector which satisfies the given conditions.

   (a). A unit vector which is in the opposite direction of $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$;

   (b). A unit vector which is in the same direction as the vector from $P_1(1, 0, 5)$ to $P_2(3, -1, 2)$;

   (c). A vector which is in the opposite direction of $\mathbf{v} = [1, 2, 3]$ and whose magnitude is half that of $\mathbf{v}$;

   (d). A vector which is in the same direction of $\mathbf{w} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and which has a length of 2;

   (e). A vector in 2-space which makes an angle of $\theta = \pi / 6$ with the positive x-axis and which has a magnitude of 4.

4. A weight of 200 Newtons (N) is being supported by two wires, as shown in Fig. 2. Find the tension in each wire. (Assume that the system is in equilibrium)

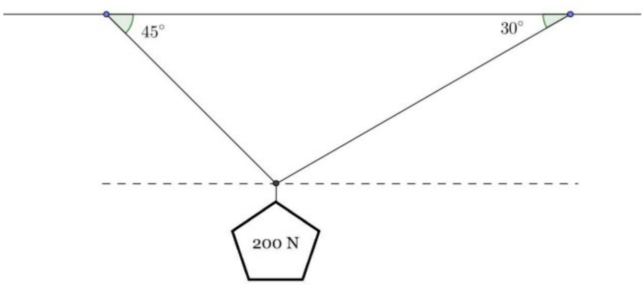

Fig. 2

5. For each of the following, compute $\mathbf{u} \cdot \boldsymbol{\upsilon}$ based on the given information.

   (a). $\mathbf{u} = [4, -5, 1]$; $\boldsymbol{\upsilon} = [3, 6, -1]$

   (b). $\| \mathbf{u} \| = 3$; $\| \boldsymbol{\upsilon} \| = 4$; the angle between $\mathbf{u}$ and $\boldsymbol{\upsilon}$ is $\pi / 4$

6. Compute the area of the triangle with vertices A (1, 2, 3), B (6, 4, 7), and C (1, 5, 2)

7. Let $\mathbf{u} = [3, -4, 1]$ and $\boldsymbol{\upsilon} = [2, -2, 3]$. Compute $\mathbf{u} \times \boldsymbol{\upsilon}$ and verify that it is orthogonal to both $\mathbf{u}$ and $\boldsymbol{\upsilon}$

8. Consider the triangle, shown in Fig. 3, with vertices A (1, -2, 6), B (3, 0, -1), and C (-2, 1, 0), please compute all three angles of the triangle.

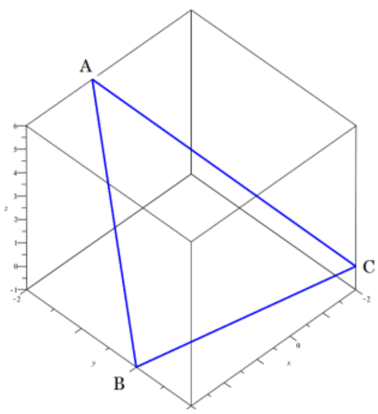

Fig. 3

9. Express the following vectors in Cartesian, cylindrical, and spherical coordinates:

a. $\vec{A} = \overrightarrow{a_\rho} \rho z \sin\phi + \overrightarrow{a_\phi} 3\rho \cos\phi + \overrightarrow{a_z} \rho \cos\phi \sin\phi$

b. $\vec{B} = \overrightarrow{a_r} r^2 + \overrightarrow{a_\phi} \sin\theta$

c. $\vec{C} = \overrightarrow{a_x} 3x + \overrightarrow{a_y} xy^2 + \overrightarrow{a_z} yz$