Analog Electronics

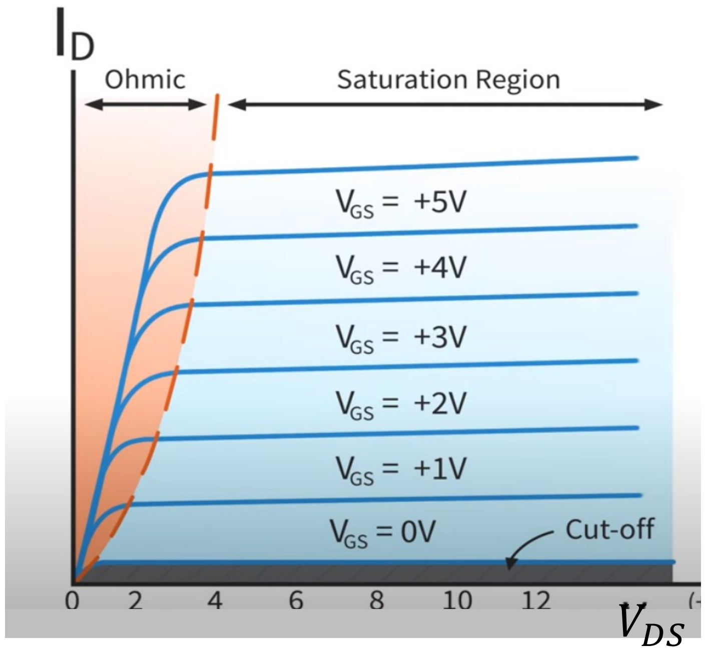
Fengchun Zhang

fz@es.aau.dk

Agenda

- Recap MOSFET and Frequency response
- Computation of frequency response
- Nonlinear system
  - General nonlinear system and harmonic distortion
  - Nonlinear system example—BJT
  - Nonlinear system example--MOSFET

Operation regions of MOSFET



MOSFET transconductance parameter:

$$k_n = \mu_n C_{ox} W/L$$

- μ_n: mobility of the electrons at the surface of the channel
- C_{ox}: oxide capacitance
- W & L: width & length of the channel

Overdrive voltage: $V_{ov} = V_{GS} - V_{TH}$
Threshold voltage: V_{TH}----0.3 V ~ 1V

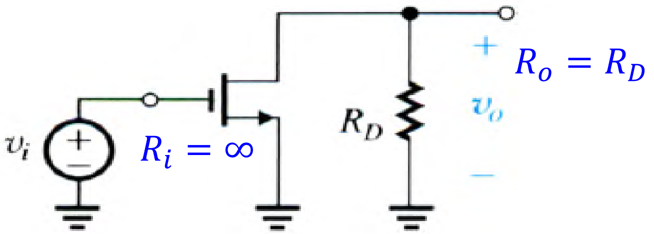
$$I_D = \begin{cases} 0, & \text{cut–off: } V_{GS} < V_{TH} \\ k_n[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2], & \text{triode: } V_{GD} > V_{TH} \\ \frac{1}{2}k_n(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}), & \text{saturation: } V_{GD} < V_{TH} \end{cases}$$

Triode and cut-off region: switching devices

Saturation region: amplifier

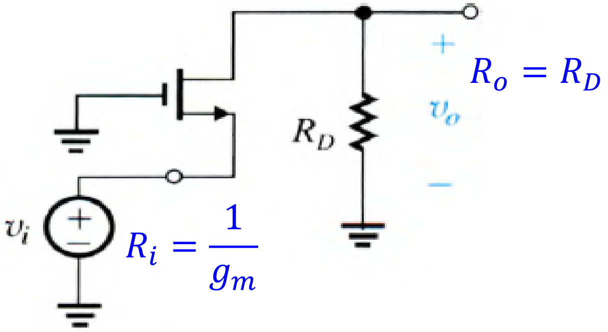
Three basic configurations: MOSFET VS. BJT

MOSFET



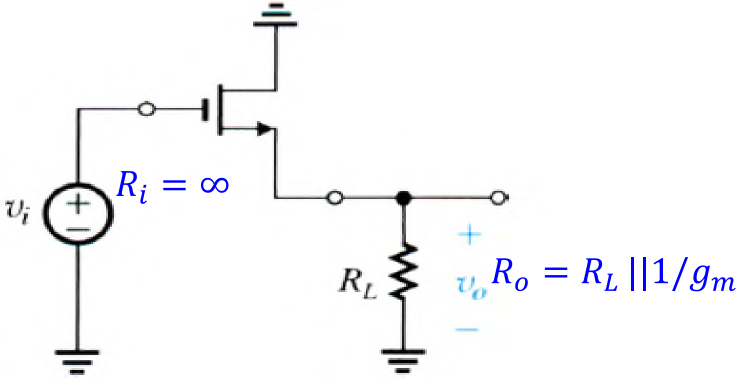
(a) Common-Source (CS)

$A_v = -g_m R_D$



(b) Common-Gate (CG)

$A_v = g_m R_D$

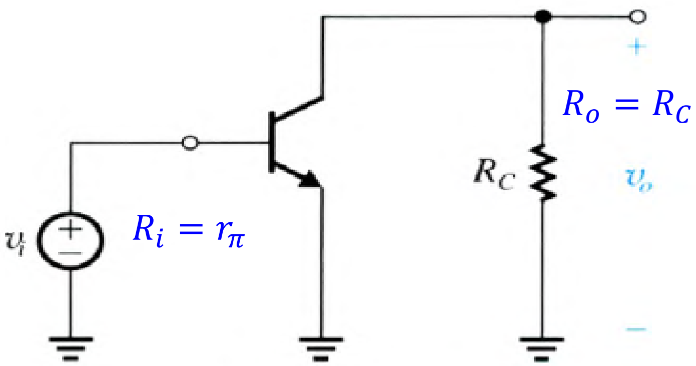


(c) Common-Drain (CD)
or Source Follower

$$A_v = \frac{R_L}{R_L + 1/g_m}$$

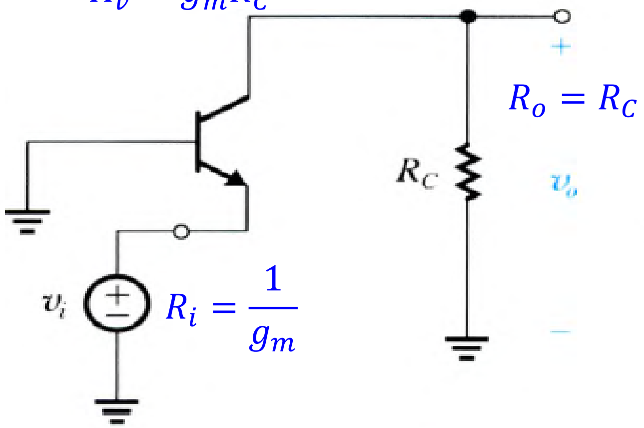
BJT

$A_v = -g_m R_C$

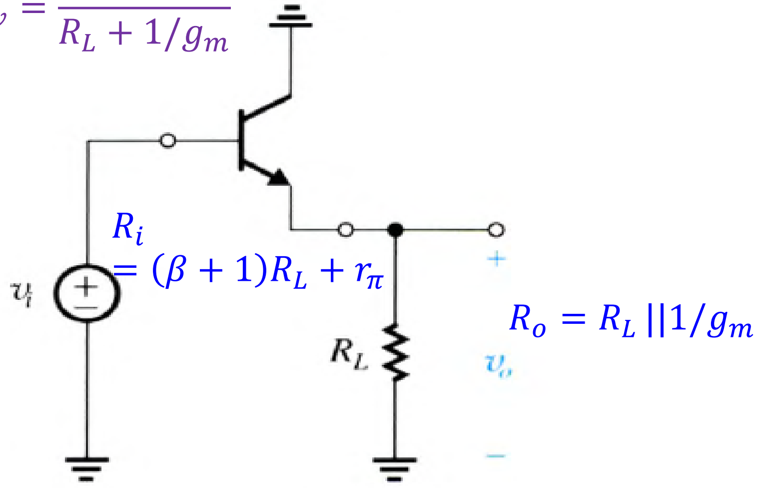


(d) Common-Emitter (CE)

$A_v = g_m R_C$



(e) Common-Base (CB)



(f) Common-Collector (CC)
or Emitter Follower

Transfer function and frequency response

- The transfer function of a circuit:

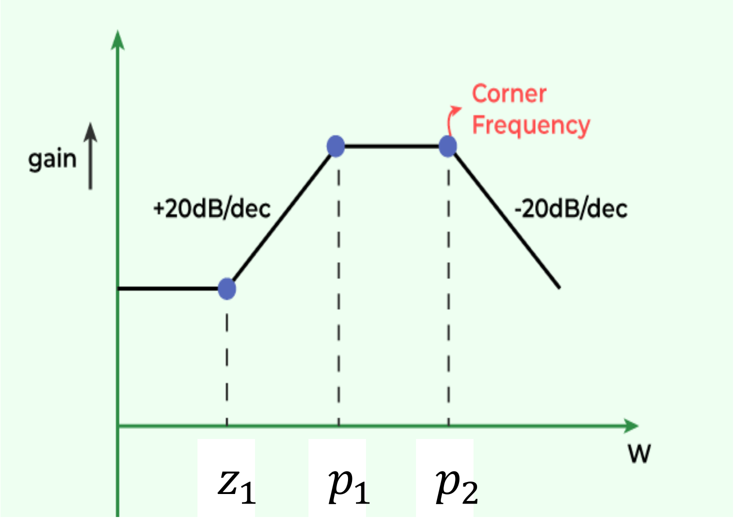
$$H(s) = \frac{Y(s)}{X(s)} = A_0 \frac{(s - z_1)(s - z_2) \dots (s - z_M)}{(s - p_1)(s - p_2) \dots (s - p_N)}$$

$$A(\omega) = |H(j\omega)| = |A_0 \frac{(j\omega - z_1)(j\omega - z_2) \dots (j\omega - z_M)}{(j\omega - p_1)(j\omega - p_2) \dots (j\omega - p_N)}|$$

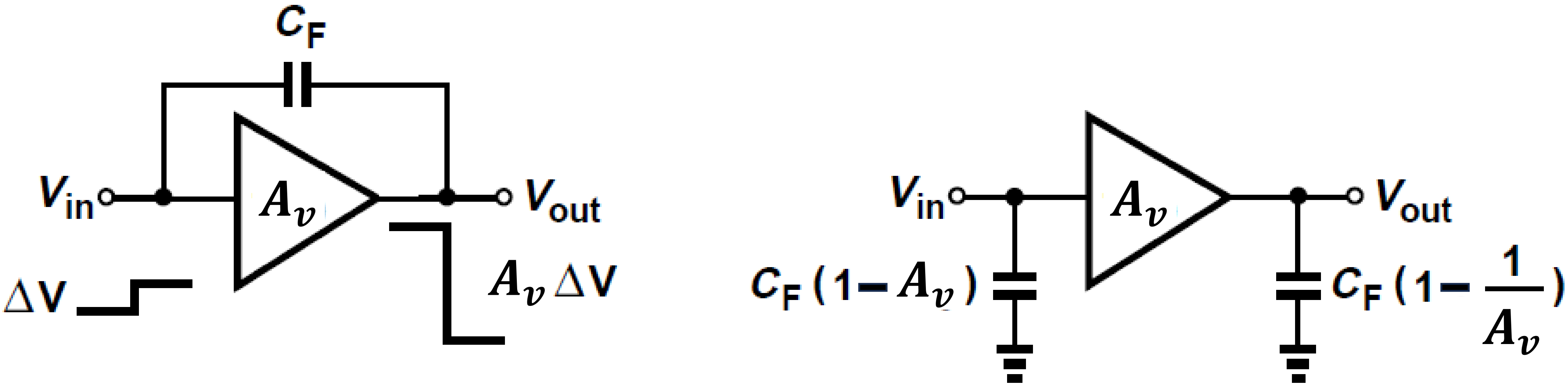
$$\omega = 2\pi f$$

z_m and p_n are zeros and poles

Bode's rules to fast construct $A(\omega)$ plot approximately:
As ω passes a **pole** / **zero**, the slope $A(\omega)$ **decreases** / **increases** by 20 dB/dec.



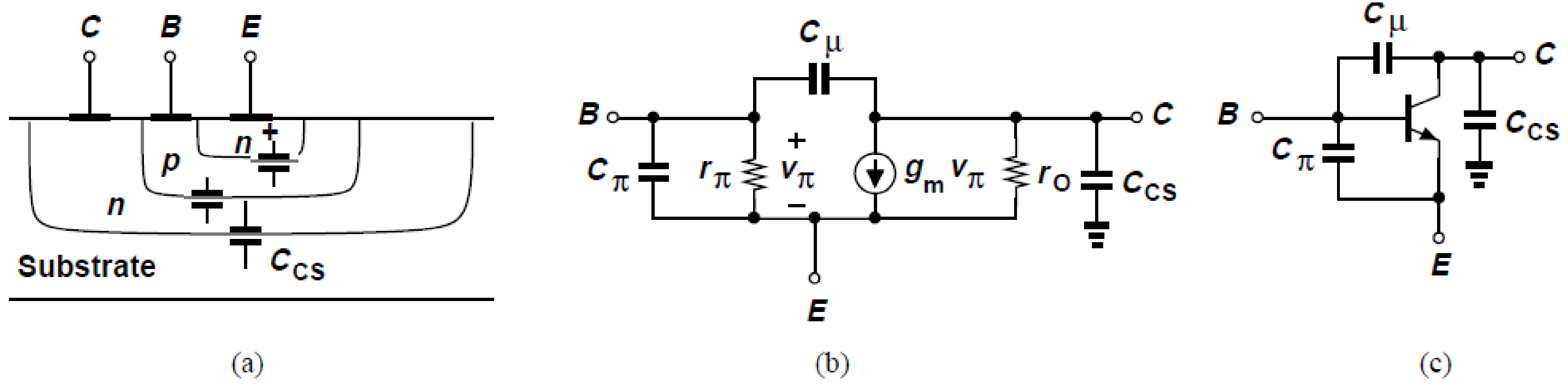
Miller’s theorem

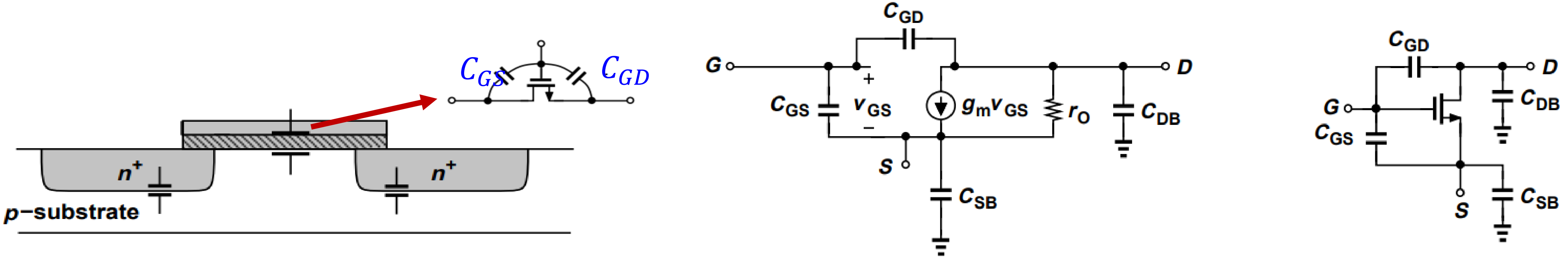


$$A_v = V_{out}/V_{in}$$

Decompose a floating capacitor into two grounded capacitors.

High-frequency models of BJT and MOSFET





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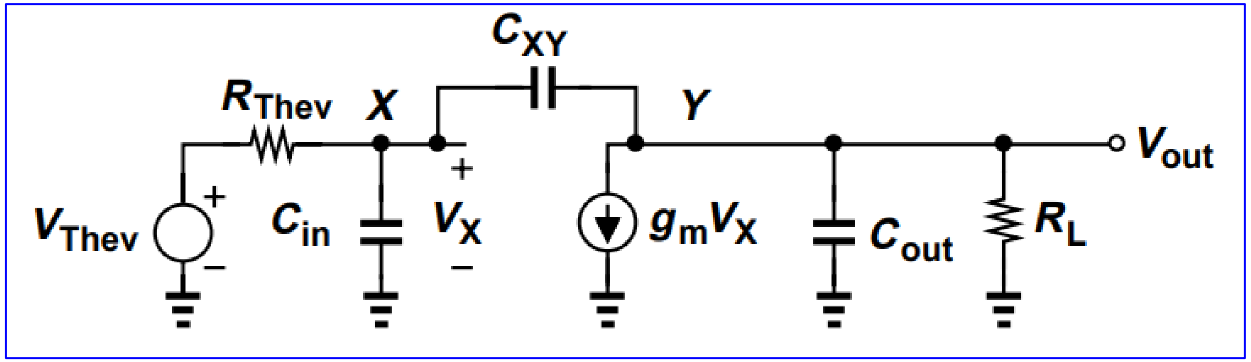
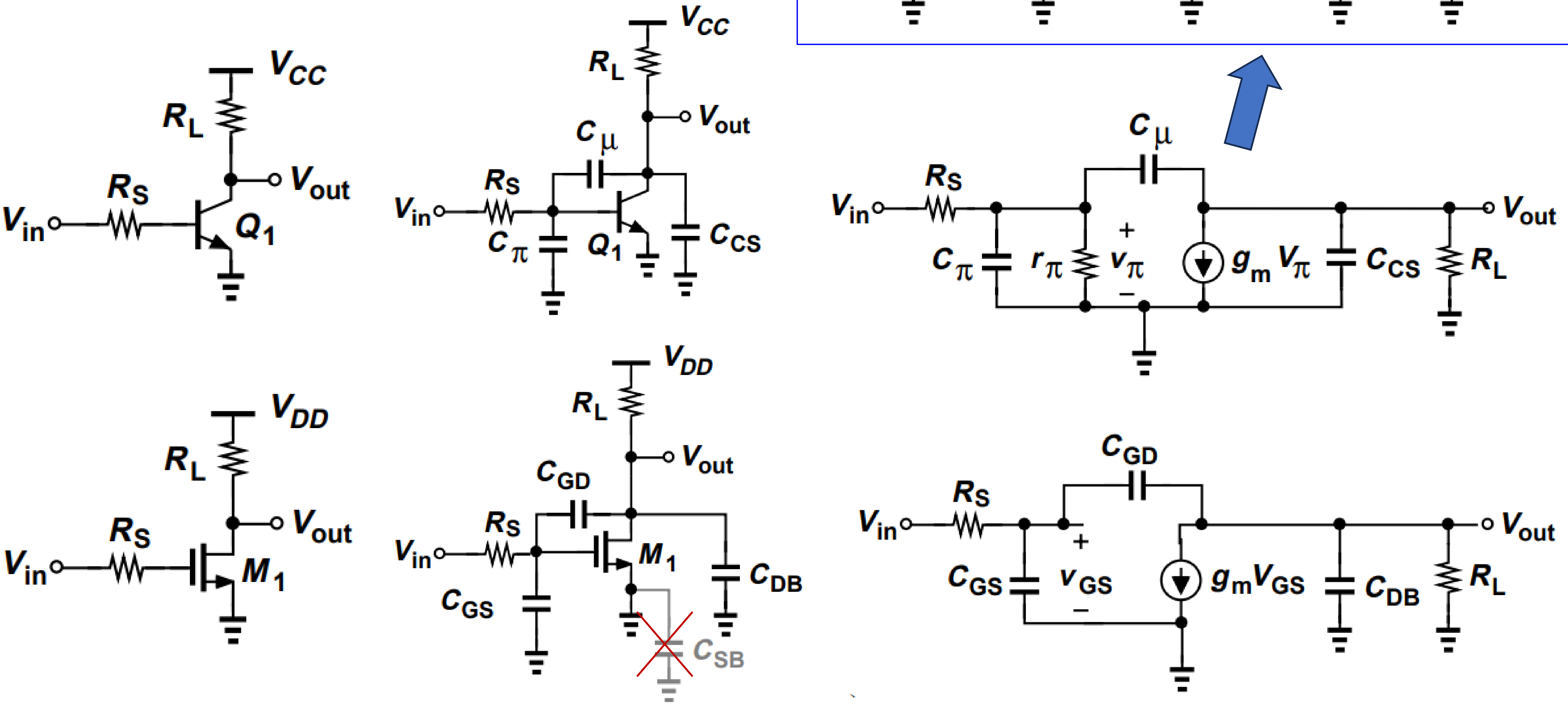
Computation of frequency response

- General procedure:
  - Draw the circuit;
  - Draw all of the device internal capacitors;
  - Remove or merge capacitors;
  - Write the transfer function $\boldsymbol{H(s)}$;
  - Plot the frequency response $|\boldsymbol{H(s=j\omega)}|$.

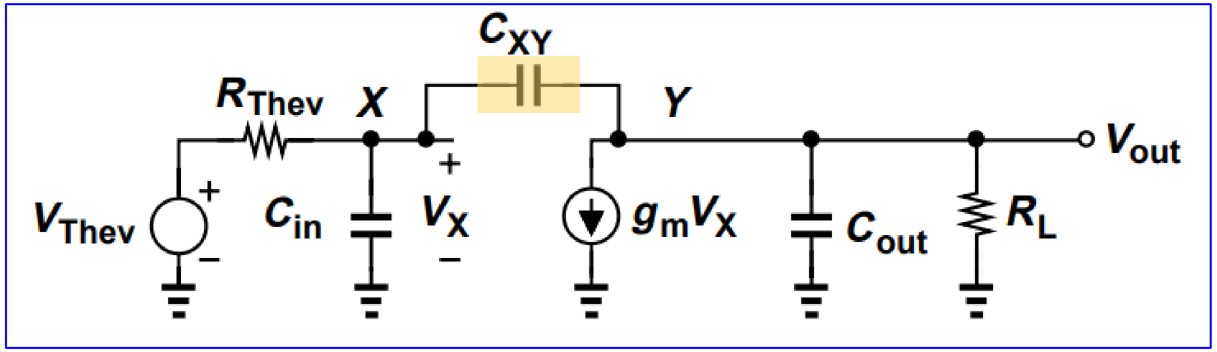
Or approximation: find the poles by inspection

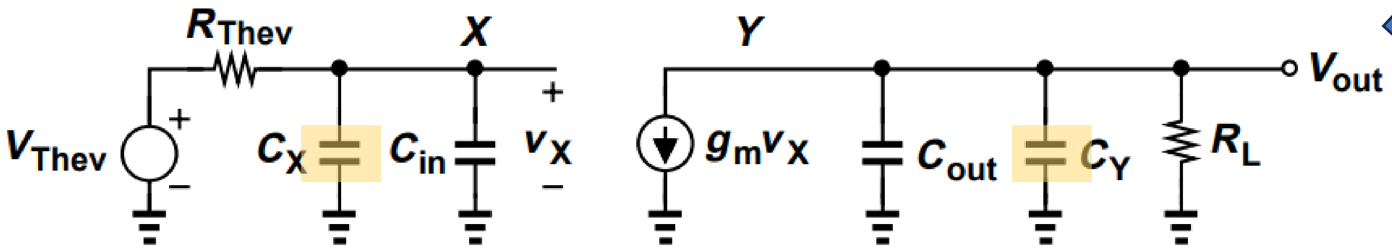
CE/CS frequency response





Approach I: finding poles by inspection





Apply Miller's theorem to $\boldsymbol{C_{XY}}$

$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + C_{XY}(1 + g_m R_L)]}$$

$$|\omega_{p,out}| = \frac{1}{R_L[C_{out} + C_{XY}(1 + \frac{1}{g_m R_L})]}$$

CE Stage

$$V_{Thev} = V_{in}\frac{r_\pi}{r_\pi + R_S}$$

$$R_{Thev} = R_S \| r_\pi$$

$$C_X = C_\mu(1 + g_m R_L)$$

$$C_Y = C_\mu\left(1 + \frac{1}{g_m R_L}\right)$$

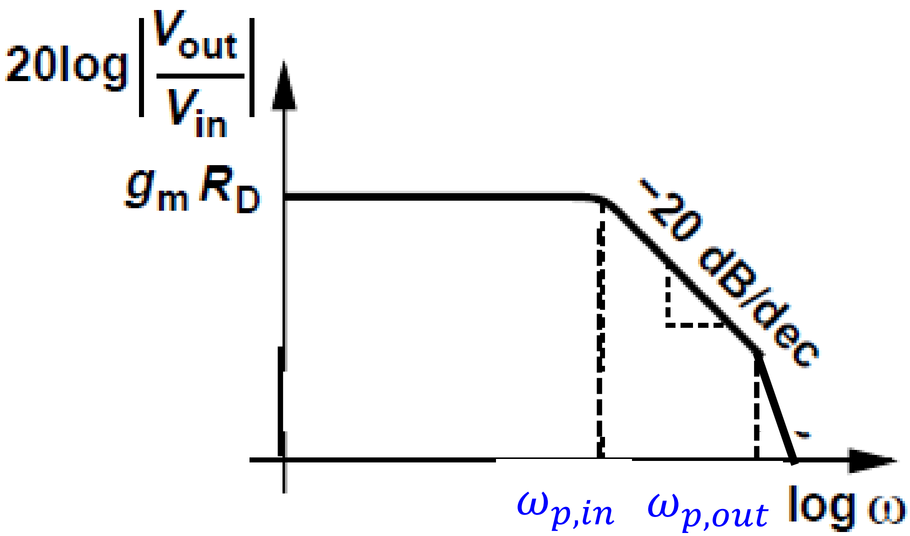
CS Stage

$$V_{Thev} = V_{in}$$

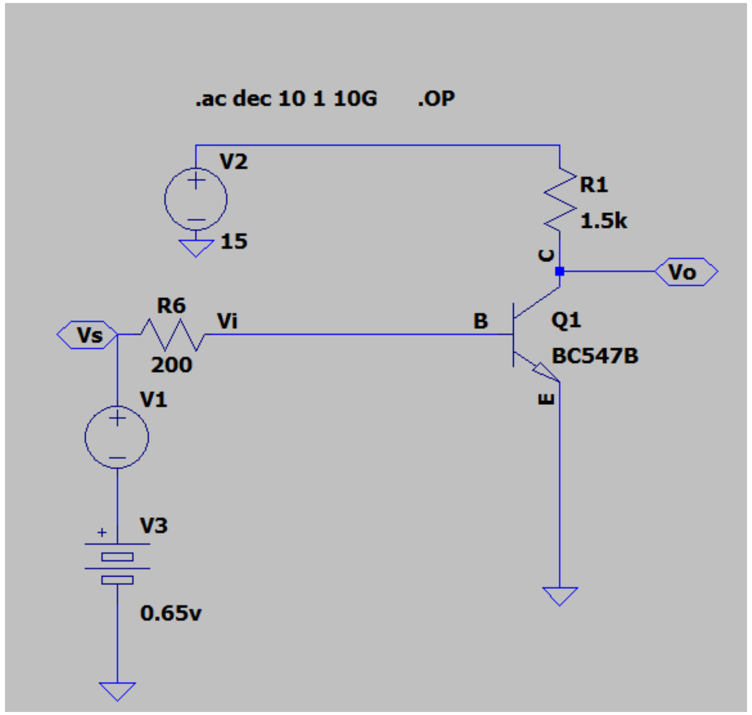
$$R_{Thev} = R_S$$

$$C_X = C_{GD}(1 + g_m R_L)$$

$$C_Y = C_{GD}\left(1 + \frac{1}{g_m R_L}\right)$$



Ltspice simulation example



Name:	q1
Model:	bc547b
Ib:	5.13e-06
Ic:	1.72e-03
Vbe:	6.49e-01
Vbc:	-1.18e+01
Vce:	1.24e+01
BetaDC:	3.35e+02
Gm:	6.52e-02
Rpi:	5.10e+03
Rx:	1.00e+00
Ro:	4.27e+04
Cbe:	7.87e-11
Cbc:	8.41e-13
Cjs:	0.00e+00
BetaAC:	3.32e+02
Cbx:	5.17e-13
Ft:	1.29e+08

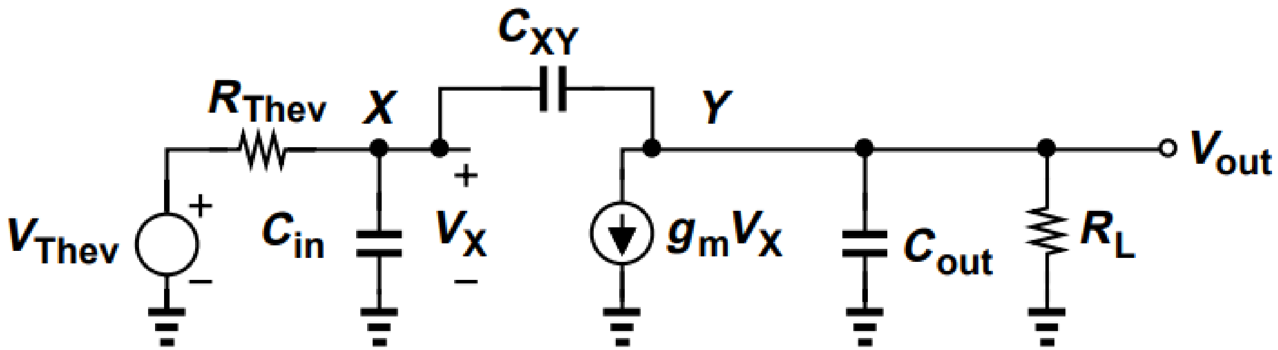
$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + C_{XY}(1 + g_m R_L)]}$$

$f_{p,in}$ =5.1e6 Hz

$$|\omega_{p,out}| = \frac{1}{R_L[C_{out} + C_{XY}(1 + \frac{1}{g_m R_L})]}$$

$f_{p,out}$ =1.2e8 Hz

Approach II: exact analysis

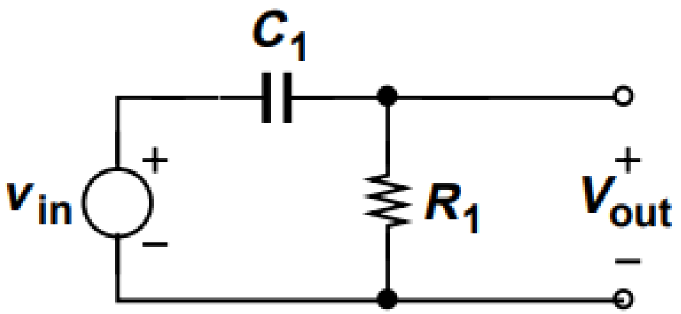


$$\frac{V_{out}}{\mathrm{V_{Thev}}}(\mathrm{s}) = \frac{(C_{XY}s - g_m)R_L}{R_{Thev}R_L[C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out}]s^2 + [R_{Thev}C_{XY}(1 + g_mR_L) + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})]s + 1}$$

- If $s = 0$, $\frac{V_{out}}{\mathrm{V_{Thev}}}(\mathrm{s}) = -g_mR_L$
- If $s = \infty$, $\frac{V_{out}}{\mathrm{V_{Thev}}}(\mathrm{s}) = 0$, C_{in} and C_{out} as short circuits ➔ $V_{out} = V_{Thev} = 0$
- One zero: $\omega_z = \frac{g_m}{C_{XY}}$, e.g., $g_m \sim 0.01$, $C_{XY} \sim 10^{-12}$ ➔ $\omega_z \sim 100$ GHz, very high frequency, so typically not important
- Two poles: dominant pole approximation, i.e., $\omega_{p_1} \ll \omega_{p_2}$

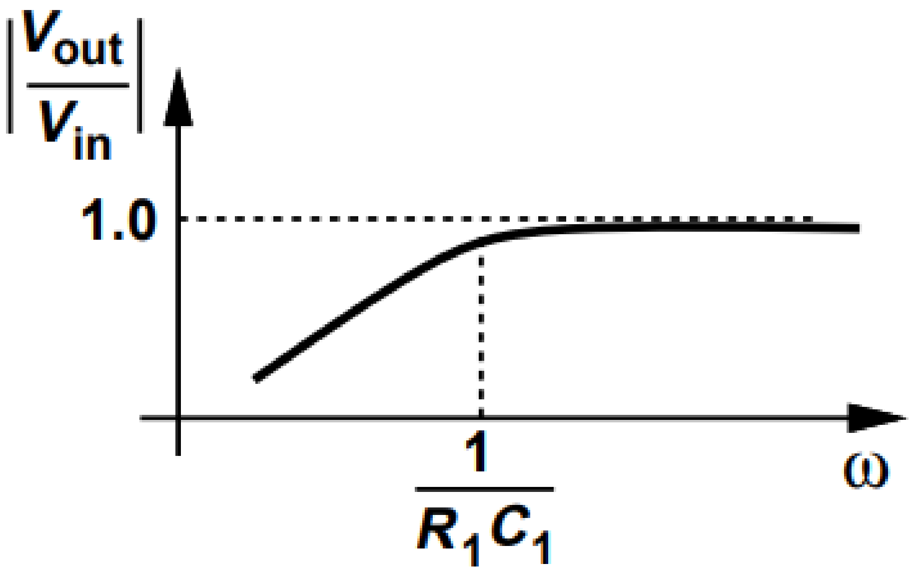
➔ $\omega_{p_1} = \frac{1}{[R_{Thev}C_{XY}(1+g_mR_L)+R_{Thev}C_{in}+R_L(C_{XY}+C_{out})]}$

Low-frequency response



$$\frac{V_{out}}{V_{in}}(s) = \frac{R_1}{R_1 + \frac{1}{C_1 s}} = \frac{R_1 C_1 s}{R_1 C_1 s + 1},$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1 C_1 \omega}{\sqrt{R_1^2 C_1^2 \omega_1^2 + 1}}.$$



Low-frequency response-BJT

$$\frac{V_o}{V_{sig}} = A_M \frac{s}{s+\omega_{p1}} \frac{s+\omega_Z}{s+\omega_{pE}} \frac{s}{s+\omega_{p2}}$$

$$\omega_{p1} = \frac{1}{\tau_{c1}} = \frac{1}{C_{C1}(R_B||r_\pi + R_{sig})}$$

$$\omega_{pE} = \frac{1}{\tau_{cE}} = \frac{1}{C_E[R_E||\left(\frac{1}{g_m} + \frac{R_B||R_{sig}}{\beta+1}\right)]}$$ Dominant pole

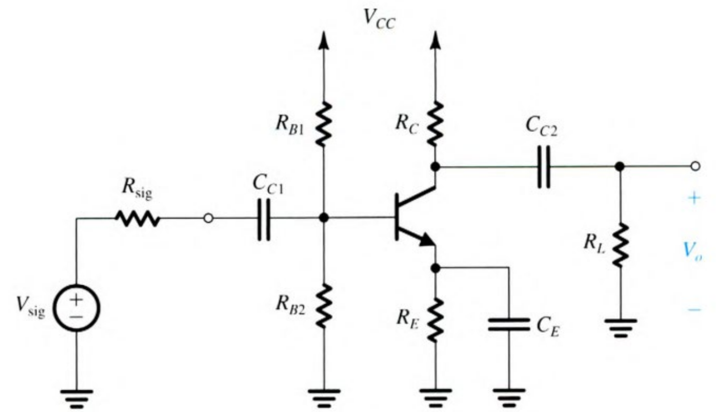
$$\omega_{p2} = \frac{1}{\tau_{c2}} = \frac{1}{C_{C2}(R_D + R_L)}$$

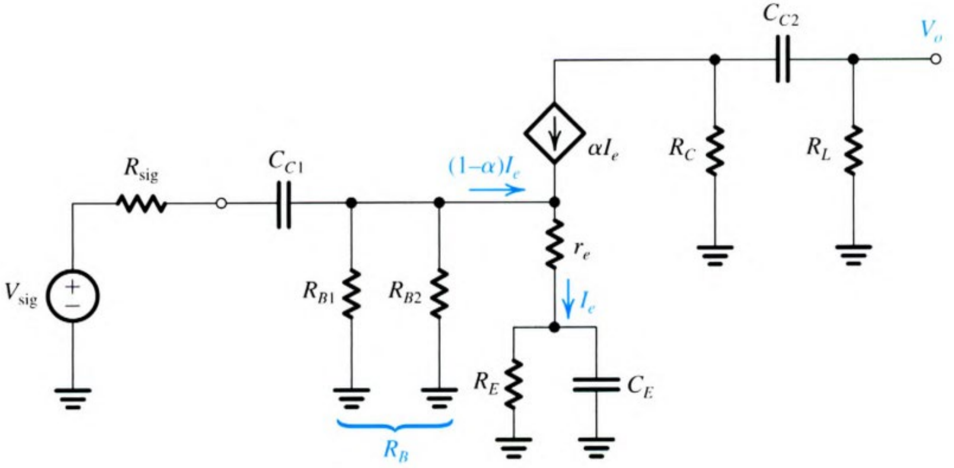
$$\omega_Z = \frac{1}{C_E R_E}$$

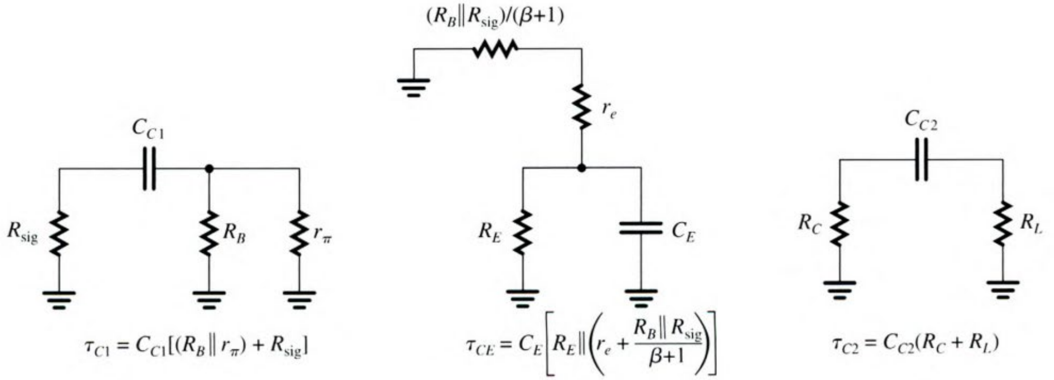
$R_B = R_{B1}||R_{B2}$ $\qquad r_\pi = \frac{\beta}{g_m} = \beta r_e$

Short-circuit time constant method:

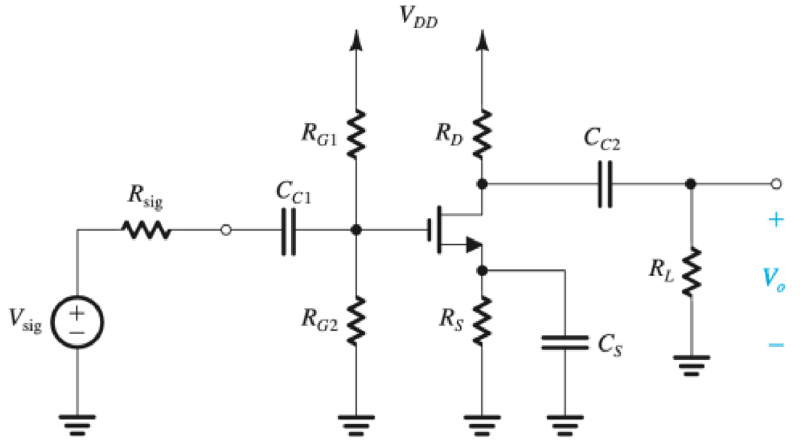
- Short other capacitors
- turn off sources:
  - Voltage source → short circuit
  - Current source →open circuit

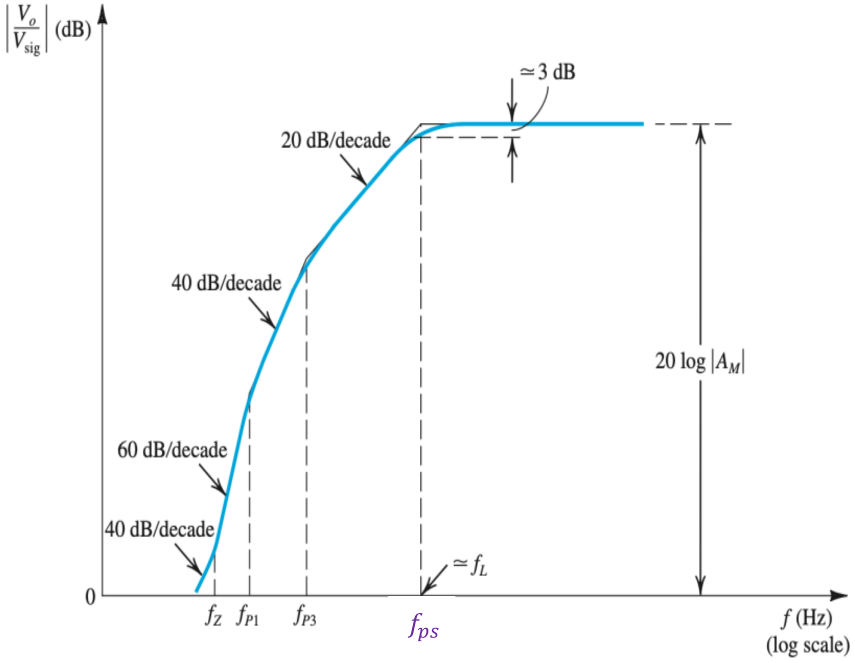






Low-frequency response--MOSFET





Dominant pole

$$\frac{V_o}{V_{sig}} = A_M \frac{s}{s+\omega_{p1}} \frac{s+\omega_z}{s+\omega_{ps}} \frac{s}{s+\omega_{p2}}$$

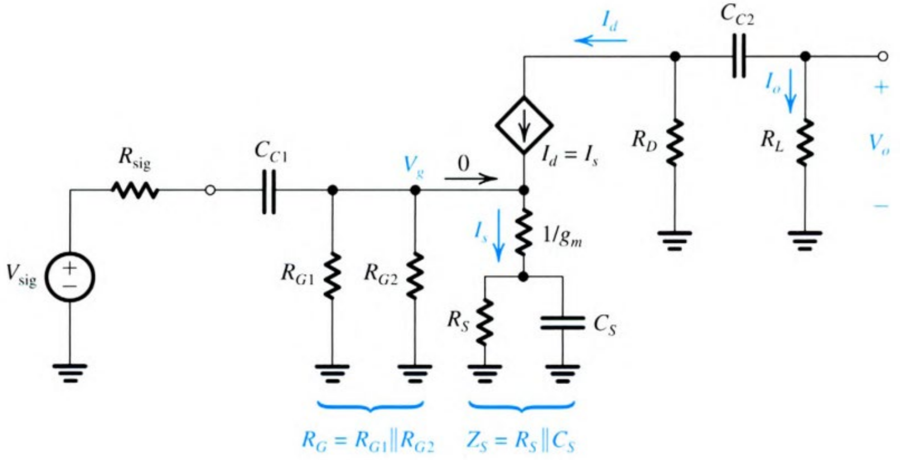
$$\omega_{p1} = \frac{1}{\tau_{c1}} = \frac{1}{C_{C1}(R_G + R_{sig})}$$

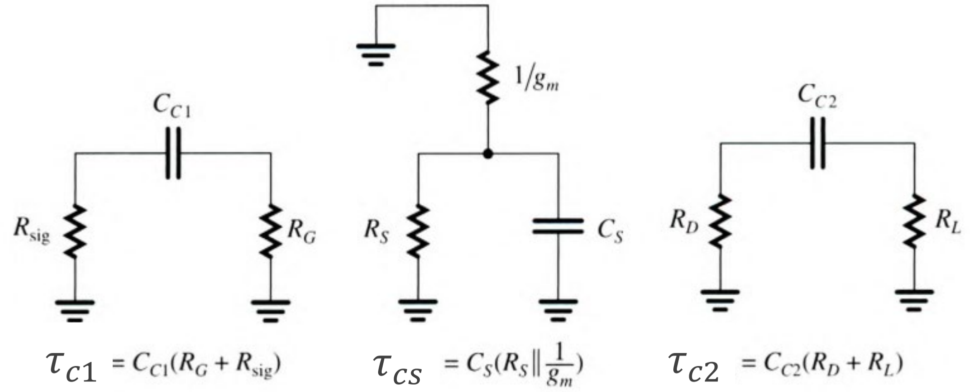
$$\omega_{ps} = \frac{1}{\tau_{cs}} = \frac{1}{C_s(R_s||\frac{1}{g_m})}$$

$$\omega_{p2} = \frac{1}{\tau_{c2}} = \frac{1}{C_{C2}(R_D + R_L)}$$

$$\omega_z = \frac{1}{C_s R_s}$$

$R_G = R_{G1}||R_{G2}$

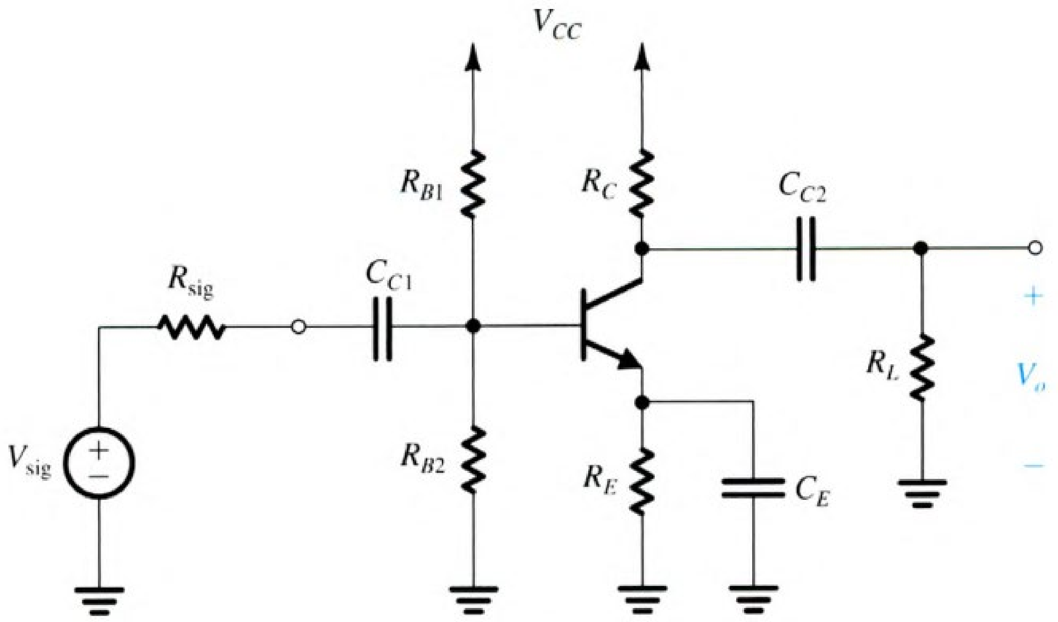




Low frequency design

- Design the C_{C1}, C_{CE} and C_{C1} to achieve a given f_L

$$f_{C1} = f_{C2} = 0.1 f_L$$
$$f_{CE} = 0.8 f_L$$



- $\omega_{p1} = 2\pi f_{C1} = \dfrac{1}{C_{C1}(R_B || r_\pi + R_{sig})}$
- $\omega_{pE} = 2\pi f_{CE} = \dfrac{1}{C_E[R_E || (\frac{1}{g_m} + \frac{R_B || R_{sig}}{\beta+1})]}$
- $\omega_{p2} = 2\pi f_{C2} = \dfrac{1}{C_{C2}(R_D + R_L)}$

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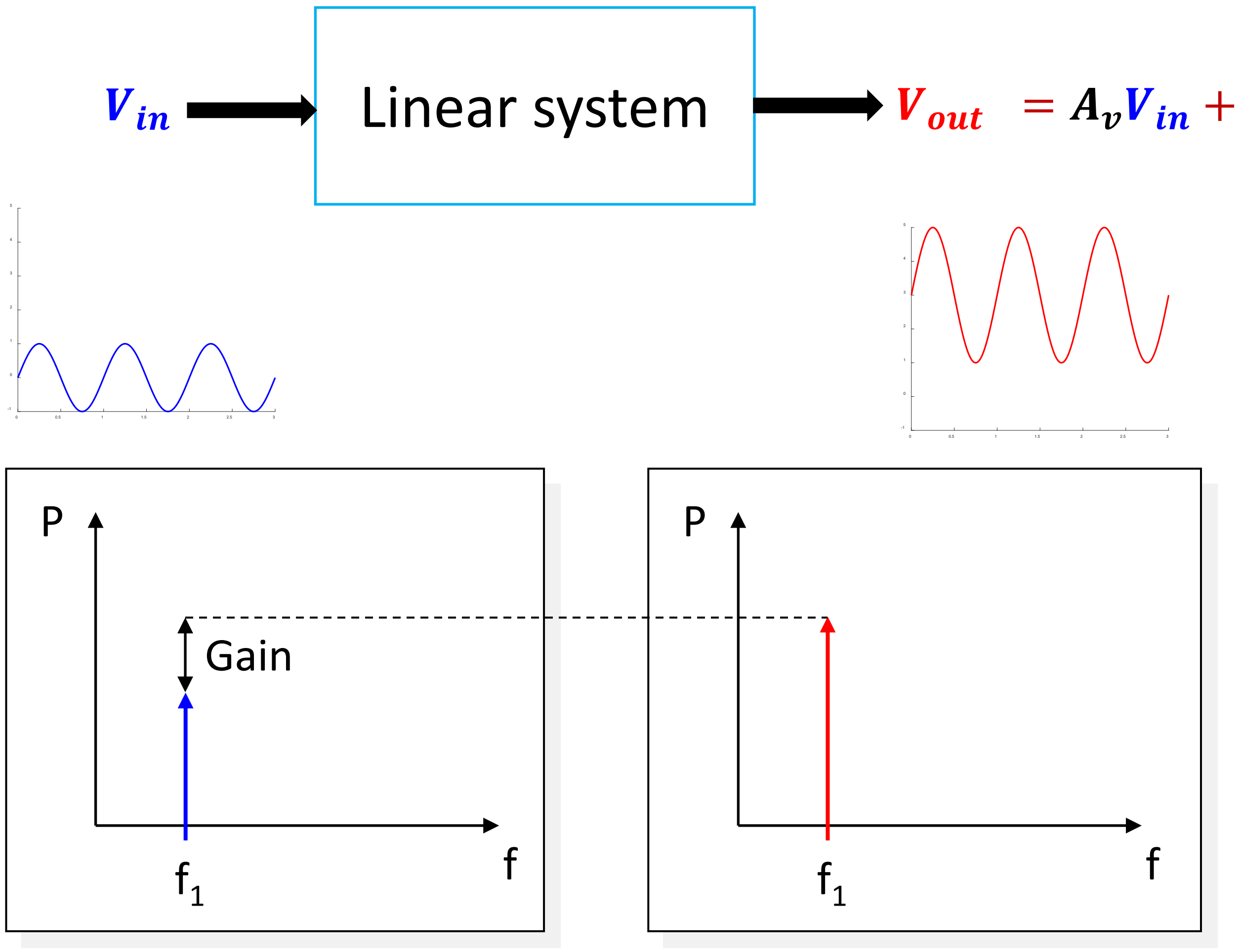
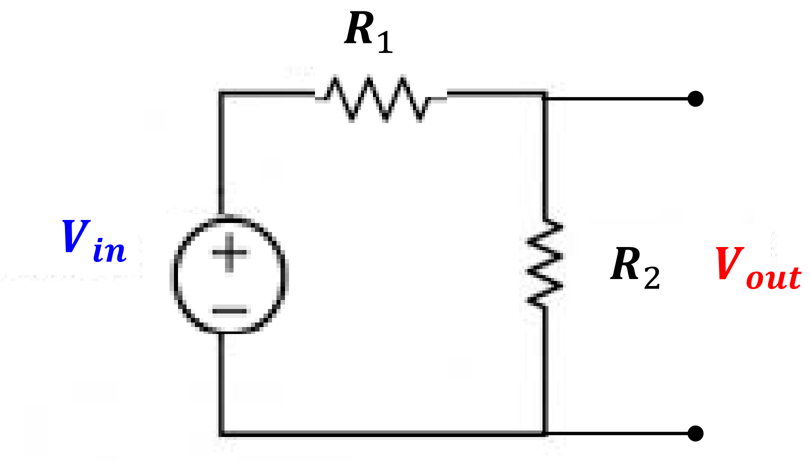
Linear system



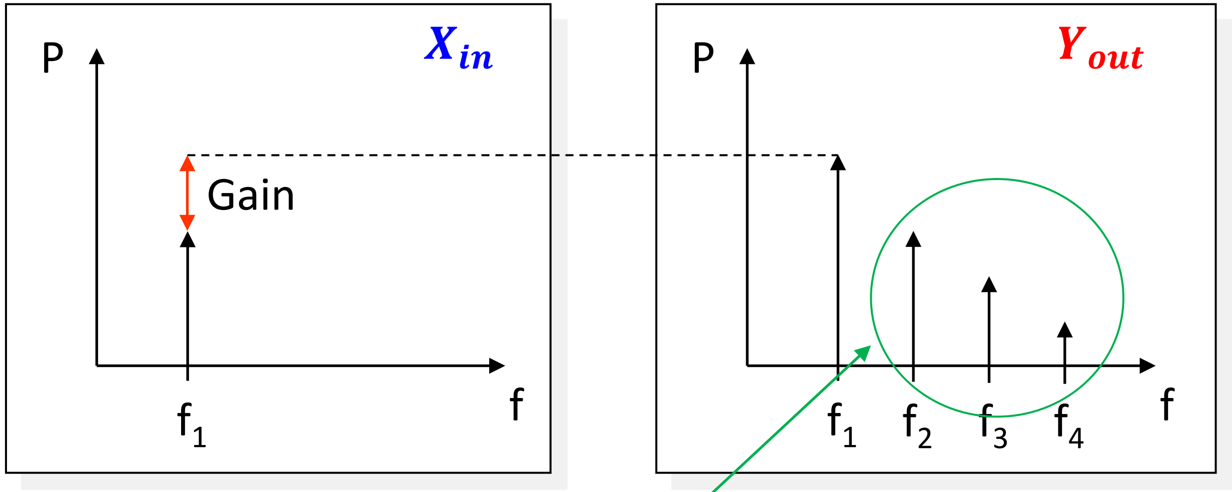
$$= A_v V_{in} + DC$$



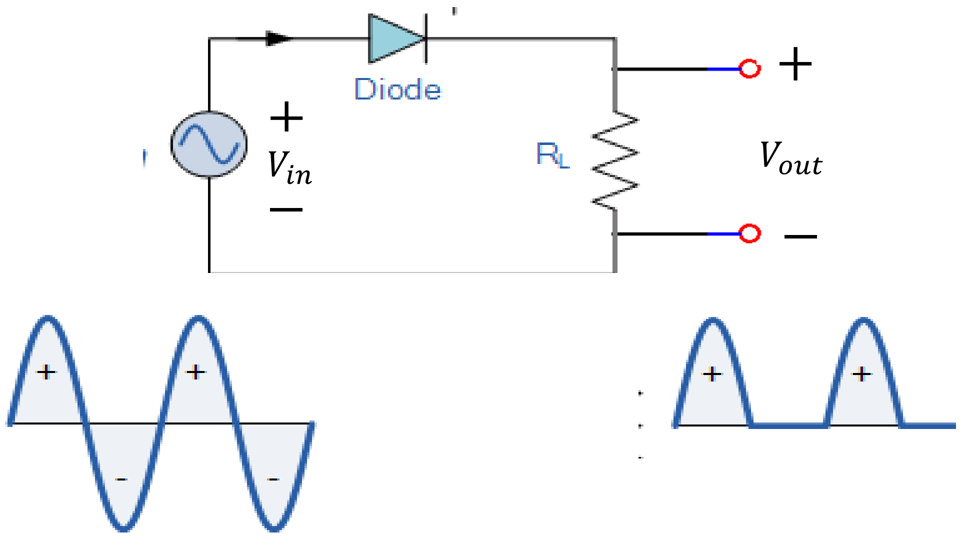
$$V_{out} = \frac{R_2}{R_1+R_2} V_{in}$$

Nonlinear system

- $Y_{out} = f(X_{in})$: nonlinear function
  - X and Y could be **V** or **I**
  - $Y_{out} = \alpha_1 X_{in} + \alpha_2 X_{in}^2$, $Y_{out} = e^{\frac{X_{in}}{a}}$ ...
  - Capacitors, diodes, transistor



Undesirable frequency components



General nonlinear system

$$Y_{out} = f(X_{in}) = f(a) + f'(a)(X_{in} - a) + \frac{f''(a)}{2!}(X_{in} - a)^2 + \ldots$$

$$X_{in} = X_{in,Q} + x_{in}$$

In small-signal model, assume x_{in} is a sin wave centered at $a = 0$

$$Y_{out} = f(0) + f'(0)x_{in} + \frac{f''(0)}{2!}x_{in}^2 + \frac{f'''(0)}{3!}x_{in}^3 + \ldots$$

$$= \alpha_0 + \alpha_1 x_{in} + \alpha_2 x_{in}^2 + \alpha_3 x_{in}^3 + \ldots$$

General nonlinear system

$$Y_{out} = \alpha_0 + \alpha_1 x_{in} + \alpha_2 x_{in}^2 + \alpha_3 x_{in}^3 + \ldots$$

- $x_{in} = A\cos(\omega t)$
- $x_{in}^2 = A^2 \cos^2(\omega t) = A^2 \frac{1+\cos(2\omega t)}{2}$
- $x_{in}^3 = A^3 \cos^3(\omega t) = A^3 \frac{3\cos(\omega t)+\cos(3\omega t)}{4}$

DC:

$$\beta_0 = \alpha_0 + \alpha_2 \frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2 \frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A + \alpha_3 \frac{3A^3}{4} \approx \alpha_1 A$$

3rd Harmonic:

$$\beta_3 = \alpha_3 \frac{A^3}{4}$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

Harmonic distortion (HD):

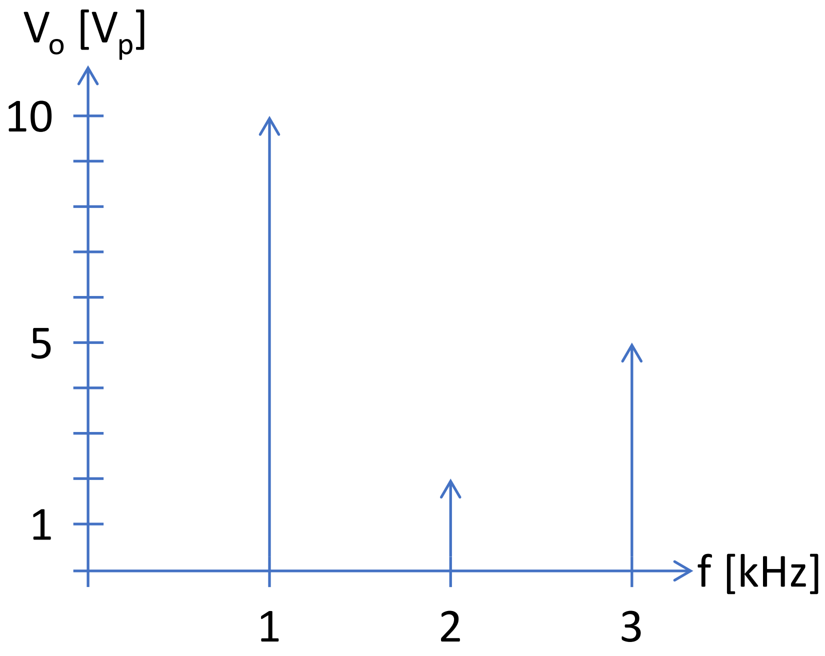
2nd Harmonic:

$$HD_2 = \frac{\beta_2}{\beta_1} \approx \frac{\alpha_2}{2\alpha_1} A$$

3rd Harmonic:

$$HD_3 = \frac{\beta_3}{\beta_1} \approx \frac{\alpha_3}{4\alpha_1} A^2$$

Example for harmonic distortion calculation

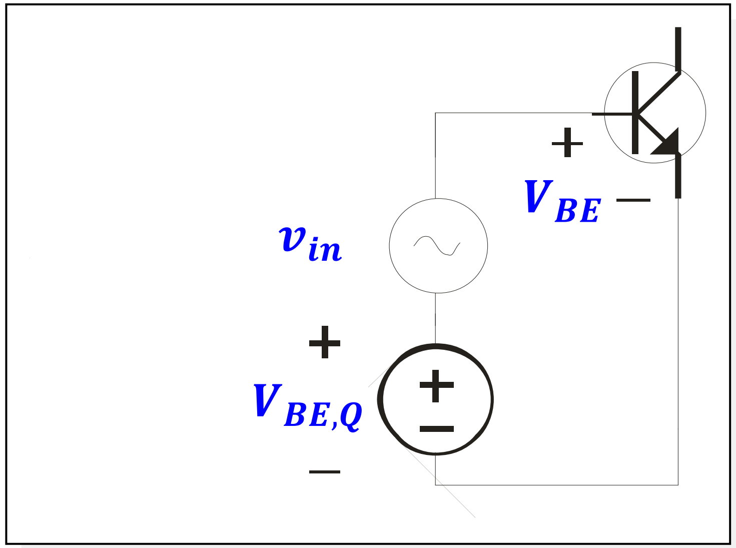


$$HD_2 = \frac{\beta_2}{\beta_1} = \frac{2}{10} = 0.2 \dashrightarrow 20\%$$

$$HD_3 = \frac{\beta_3}{\beta_1} = \frac{5}{10} = 0.5 \dashrightarrow 50\%$$

$$THD = \sqrt{HD_2^2 + HD_3^2} = \sqrt{0.2^2 + 0.5^2} = 0.54 \rightarrow 54\%$$

Nonlinear system example--BJT



Taylor series expansion:

$$e^{x/a} = 1 + \frac{1}{a}x + \frac{1}{2!*a^2}x^2 + \cdots + \frac{1}{n!*a^n}x^n + \cdots$$

$$I_C = I_s\, e^{V_{BE}/V_T} = I_s e^{(V_{BE,Q}+v_{in})/V_T} = I_S\, e^{V_{BE,Q}/V_T}\ e^{v_{in}/V_T} = I_{CQ}\, e^{v_{in}/V_T}$$

$$= I_{CQ}\left(1 + \frac{1}{V_T}v_{in} + \frac{1}{2!*V_T^2}v_{in}^2 + \frac{1}{3!*V_T^3}v_{in}^3 + \ldots\right)$$

$$= I_{CQ} + \frac{I_{CQ}}{V_T}v_{in} + \frac{I_{CQ}}{2!*V_T^2}v_{in}^2 + \frac{I_{CQ}}{3!*V_T^3}v_{in}^3 + \ldots$$

$$= \alpha_0 + \alpha_1 v_{in} + \alpha_2 v_{in}^2 + \alpha_3 v_{in}^3 + \ldots$$

DC:

$$\beta_0 = \alpha_0 + \alpha_2\frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2\frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A + \alpha_3\frac{3A^3}{4} \approx \alpha_1 A$$

3rd Harmonic:

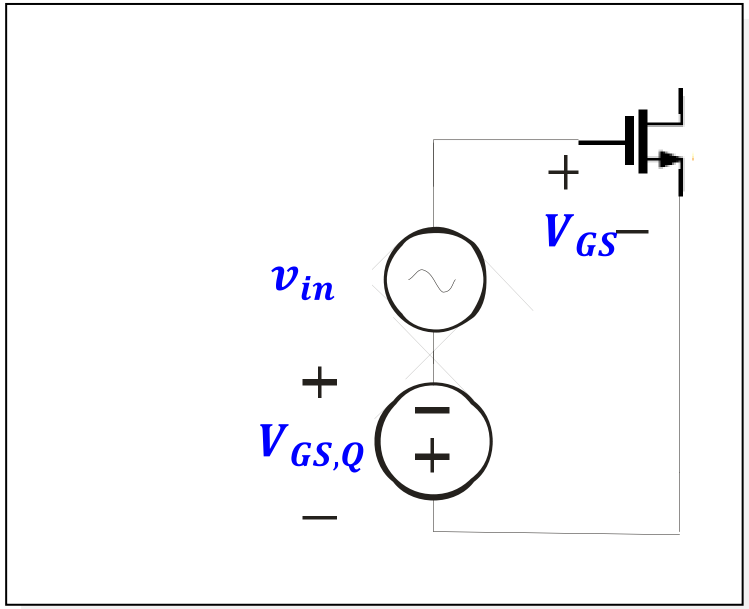
$$\beta_3 = \alpha_3\frac{A^3}{4}$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

$$= \sqrt{\left(\frac{\beta_2}{\beta_1}\right)^2 + \left(\frac{\beta_3}{\beta_1}\right)^2 + \left(\frac{\beta_4}{\beta_1}\right)^2 + \cdots}$$

Nonlinear system example--MOSFET



$$I_D = \frac{1}{2}k_n(V_{GS} - V_{TH})^2 = \frac{1}{2}k_n\left(V_{GS,Q} - V_{TH} + v_{in}\right)^2$$

$$= \frac{1}{2}k_n\left(V_{GS,Q} - V_{TH}\right)^2\left(1 + \frac{v_{in}}{V_{GS,Q} - V_{TH}}\right)^2$$

$$= I_{DQ}\left(1 + \frac{2}{V_{GS,Q} - V_{TH}}v_{in} + \frac{1}{\left(V_{GS,Q} - V_{TH}\right)^2}v_{in}^2\right)$$

$$= I_{DQ} + \frac{2I_{DQ}}{V_{GS,Q} - V_{TH}}v_{in} + \frac{I_{DQ}}{\left(V_{GS,Q} - V_{TH}\right)^2}v_{in}^2$$

$$= \alpha_0 + \alpha_1 v_{in} + \alpha_2 v_{in}^2$$

DC:

$$\beta_0 = \alpha_0 + \alpha_2\frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2\frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A$$

3rd Harmonic:

$$\beta_3 = 0$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

$$= \frac{\beta_2}{\beta_1} = HD_2$$

Sound samples—harmonic distortions

400 Hz

800 Hz

1200 Hz

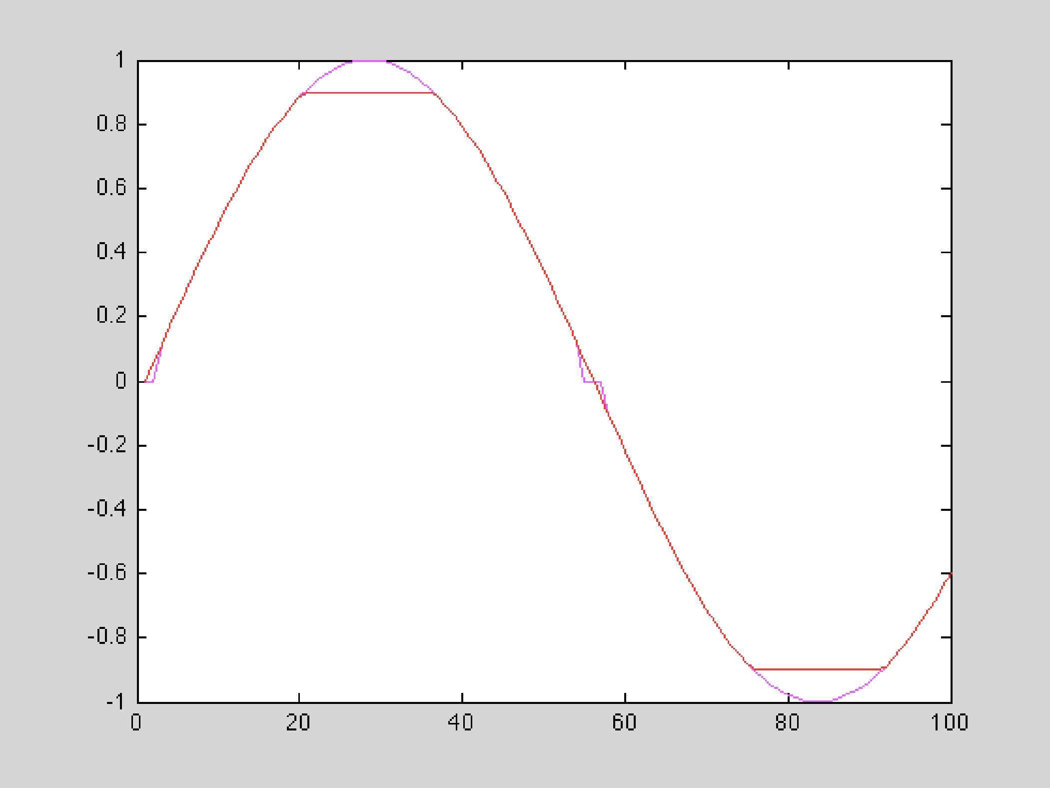
First 400 Hz then all three tones together

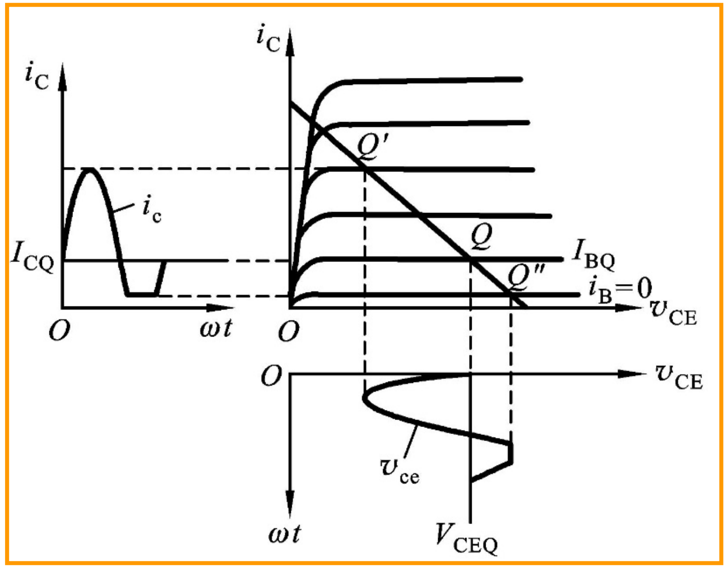
Sound samples—clip

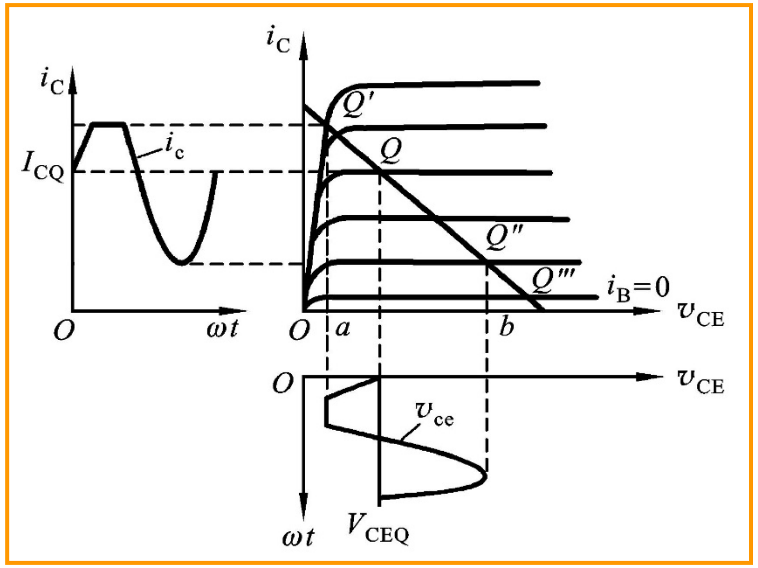
orginal

Zero-clip

Clip







Design consideration

- Properly bias
  - BJT
    - Active forward region ➔ $\boldsymbol{V_{BE} \geq 0.7\,V}$ & $\boldsymbol{V_{CE} \geq V_{BE}}$
  - MOSFET
    - Saturation ➔ $\boldsymbol{V_{DS} > V_{GS} - V_{TH}}$
- Select suitable Q point (clip distortion)
- Small-signal assumption
  - BJT
    - Small-signal assumption ➔ $v_{in} < 0.2\ V_T$
  - MOSFET
    - Small-signal assumption ➔ $v_{in} < 0.2\ (V_{GS,Q} - V_{TH})$