Lecture 6.

(1) Conductor / Isolator ?

Traditional classification:

- conductor (metal)
- Isolator (air, glass)
- Semi conductor (water, silicon)

The classification is made from:

$R\ [\Omega \cdot m]$ or $\sigma = \frac{1}{\rho}\ [\frac{S}{m}]$

specific resistance — specific conductivity

New conductor / isolator model

We have:

$$\nabla \times \bar{H} = \bar{J}_c + \bar{J}_D$$
$$= \sigma \bar{E} + j\omega\varepsilon\bar{E}$$

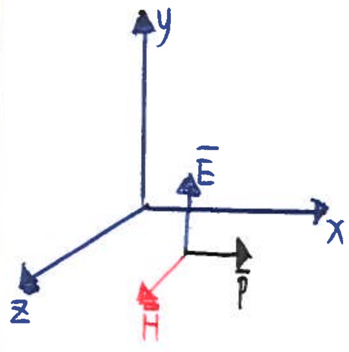
For a conductor, σ is large,

so $|\bar{J}_c| \gg |\bar{J}_D|$. We set $\bar{J}_D = \bar{0}$

We have

$$\nabla \times \bar{H} = \bar{J}_c$$

Propagation in good conductor



$$\rho = 0\ [\frac{C}{m^3}] \quad \nabla \cdot \bar{E} = 0$$
$$\sigma \neq 0\ [\frac{S}{m}] \quad \bar{J}_c = \sigma \bar{E}$$

(good conductor condition)

$\bar{E} = E_y \hat{y}\ [\frac{V}{m}]$ $\bar{H} = H_z \hat{z}\ [\frac{A}{m}]$

$$\frac{\partial \bar{E}}{\partial y} = \frac{\partial \bar{E}}{\partial z} = \frac{\partial \bar{H}}{\partial y} = \frac{\partial \bar{H}}{\partial z} = 0$$

- $\varepsilon_r, \mu, \sigma$ are HiLS

We apply KSN.

Maxwell's equations:

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad (1)$$
$$\nabla \times \bar{H} = \sigma\bar{E} \quad (2)$$
$$\nabla \cdot \bar{B} = 0$$
$$\nabla \cdot \bar{D} = 0$$

(2) Maxwell's equations in good conductor

Take the rotation of (1)

$$\nabla\times\nabla\times\bar{E} = -j\omega\mu(\nabla\times\bar{H}) \quad (3)$$

Insert (2) to (3)

$$\nabla\times\nabla\times\bar{E} = -j\omega\mu\sigma\bar{E} \quad (4)$$

$$\Delta\bar{E} = \nabla^2\bar{E} =$$

$$= \nabla(\nabla\cdot\bar{E}) - \nabla\times\nabla\times\bar{E}$$

$$= -\nabla\times\nabla\times\bar{E} \quad \text{for } \nabla\cdot\bar{E}=0$$

So we have:

$$\Delta\bar{E} = j\omega\mu\sigma\bar{E} \quad (5)$$

(5) are 3-dimentional wave equation and we also have:

$$\Delta\bar{D} = j\omega\mu\sigma\bar{D}$$

$$\Delta\bar{J} = j\omega\mu\sigma\bar{J}$$

$$\Delta\bar{B} = j\omega\mu\sigma\bar{B}$$

General wave equations:

$$\boxed{\Delta\bar{E} = \gamma^2\bar{E}}$$

With oure direction:

$$\frac{\partial^2 E_y}{\partial x^2} = \gamma^2 E_y$$

Propagation constant

We have:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{j}\cdot\sqrt{\omega\mu\sigma}$$

$$= (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

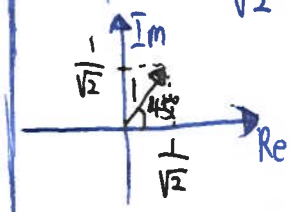
$$= (1+j)\frac{1}{\delta}$$

where $\delta \triangleq \sqrt{\frac{2}{\omega\mu\sigma}}$ [m]

$$j = 1\angle 90^\circ = 1e^{j90^\circ}$$

$$\sqrt{j} = \sqrt{1\angle 90^\circ} = \sqrt{1e^{j90^\circ}}$$

$$= 1\angle 45^\circ = \frac{1+j}{\sqrt{2}}$$

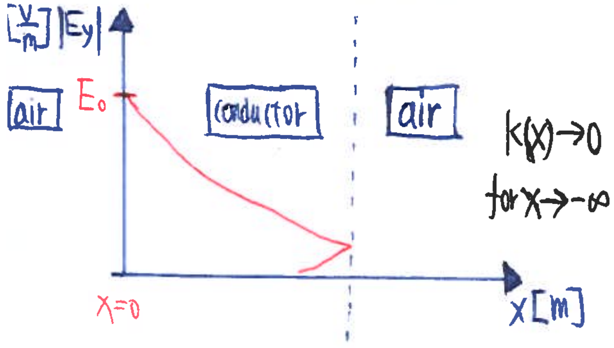


Penetration depth: $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$ [m]

Attenuation constant: $\alpha = \frac{1}{\delta} = \sqrt{\frac{\omega\mu\sigma}{2}}$ [Np/m]

Phase propagation constant: $\beta = \alpha = \frac{1}{\delta} = \sqrt{\frac{\omega\mu\sigma}{2}}$ [rad/m]

(3) New calculation model

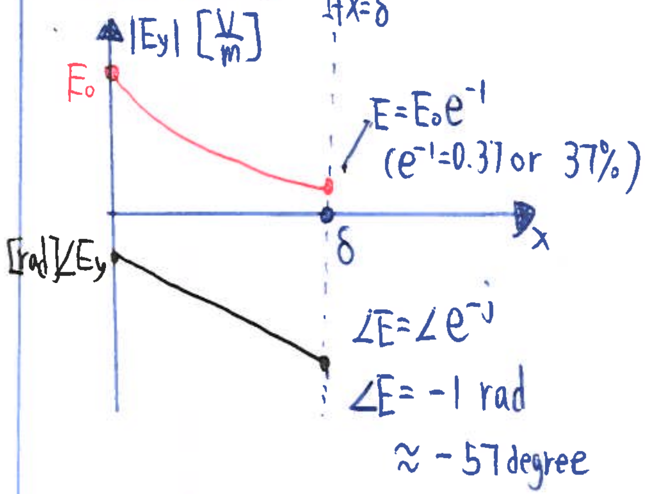


$k(x) \to 0$ for $x \to -\infty$

$$E_y(x) = E^+ e^{-\gamma x} + E^- e^{+\gamma x}$$
$$= E^+(x)(1+k(x))$$
$$\simeq E^+ e^{-\gamma x}$$

$$E_y(x) = E_0 e^{-\gamma x} = E_0 e^{-(1+j)\frac{x}{\delta}}$$
$$= E_0 \cdot \underbrace{e^{-\frac{x}{\delta}}}_{\text{attenuation}} \cdot \underbrace{e^{-j\frac{x}{\delta}}}_{\text{phase change}}$$

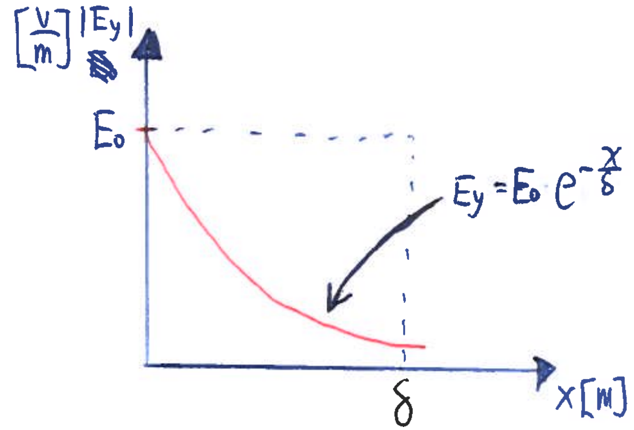
New model



We also see:

$$\left.\begin{aligned} \lambda_g &= \frac{2\pi}{\beta} = 2\pi\delta \ [m] \\ v &= \frac{\omega}{\beta} = \omega \cdot \delta \ [\tfrac{m}{s}] \end{aligned}\right\} \text{small number when } \delta \text{ is very small}$$

Good conductor / ideal conductor



$$\int_0^\infty E_0 \cdot e^{-\frac{x}{\delta}} \cdot dx = \left[E_0 \cdot e^{-\frac{x}{\delta}}(-\delta)\right]_0^\infty = E_0$$

We see for δ

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} \begin{cases} \text{for } \omega \to \infty \text{ have } \delta \to \\ \text{for } \sigma \to \infty \text{ have } \delta \to \end{cases}$$

For ideal conductor $\delta \approx 0$ m