Modelling of Electrodynamic Systems

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Introduction

Lecturer:

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Previous knowledge:

- Electrophysics
- Transmission line theory

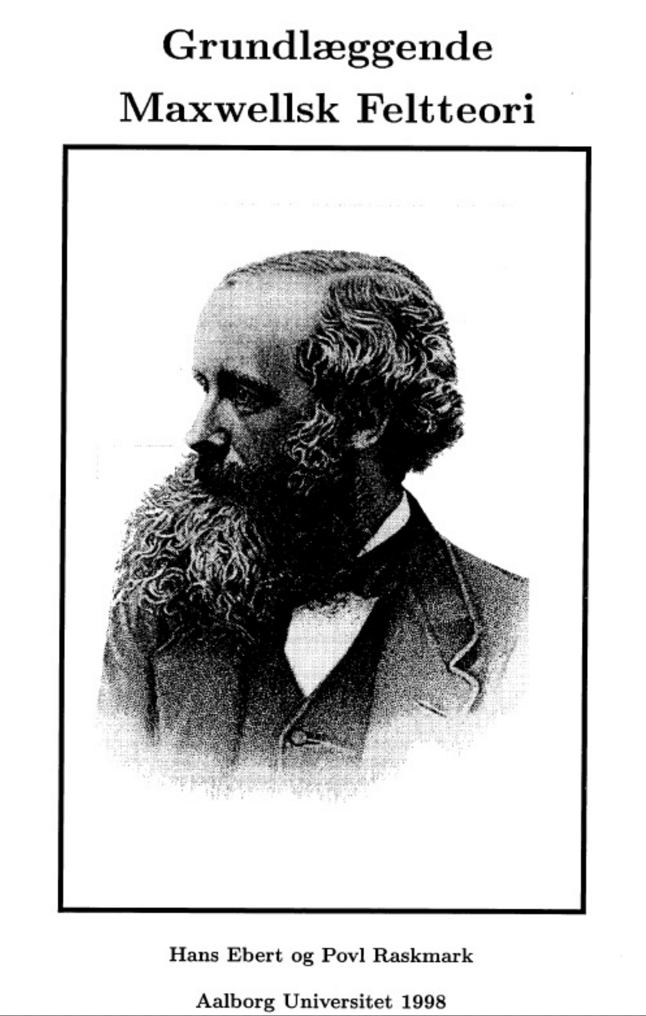
Content on Electromagnetic Field:

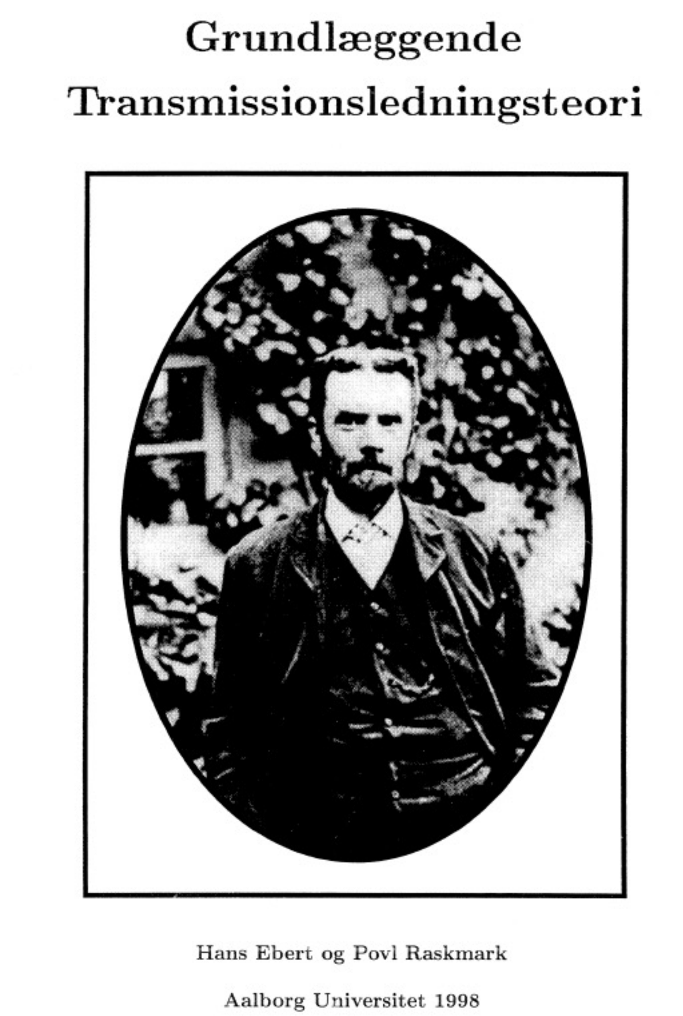
Lecture 5: Maxwell's Field Theory
Lecture 6: Wave Propagation at Good Conductor
Lecture 7: Wave Propagation at Media with Loss
Lecture 8: Oblique Incident Wave

Course plan:

Day	Date	Topic	
Mon	5 Feb.	Lecture 1: Fundamental Vector Calculus	(Vector Calculus)
Mon	12 Feb.	Lecture 2: Vector Differential Calculus	(Vector Calculus)
Mon	19 Feb.	Lecture 3: Vector Integral Calculus – I	(Vector Calculus)
Mon	26 Feb.	Lecture 4: Vector Integral Calculus – II	(Vector Calculus)
Wed	28 Feb.	Lecture 5: Maxwell's Field Theory	(Electromagnetic field)
Wed	13 Mar.	Lecture 6: Wave Propagation at Good Conductor	(Electromagnetic field)
Mon	25 Mar.	Lecture 7: Wave Propagation at Media with Loss	(Electromagnetic field)
Wed	27 Mar.	Lecture 8: Oblique Incident Wave	(Electromagnetic field)
Tue	2 Apr.	Lecture 9: Newton's Laws of Motion	(Mechanics)
Thu	4 Apr.	Lecture 10: Work and Energy	(Mechanics)
Tue	16 Apr.	Lecture 11: Linear momentum and Collisions	(Mechanics)
Thu	18 Apr.	Lecture 12: System of Particles	(Mechanics)
Tue	23 Apr.	Lecture 13: Fixed-axis Rotation	(Mechanics)
Thu	25 Apr.	Lecture 14: Angular Momentum	(Mechanics)

Literature :


Grundlæggende
Maxwellsk Feltteori
Hans Ebert og Povl Raskmark
Aalborg Universitet 1998


Grundlæggende
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Exam of the course:

Oral exam, 20 mins in June 2024.

Students draw two questions out of the question list, one from electromagnetic field and one from mechanics.

The students then present the solution/explanation to each of the questions (within 8 min for each question).

Following that, there will be follow-up questions in other areas of the curriculum (within 4 min).

The question list will be posted in Moodle for preparing (after the course and before the exam).

Modelling of Electrodynamic Systems

MM5. Maxwell’s Field Theory

Maxwell’s Field Theory

Targets:

1. Read “Grundlæggende Maxwellsk Feltteori” (Page 51– 72 with Page 82) (before or after the lecture)

2. Be able to calculate maxwell’s field propagation (calculation over the boundary) (lecture)

3. Finish the exercise (after the lecture)

Recall from the previous semester

Field

$\bar{E}$ $\left[\frac{V}{m}\right]$

$\bar{D}$ $\left[\frac{A \cdot s}{m^2}\right]$

V $[V]$

$\bar{H}$ $\left[\frac{A}{m}\right]$

$\bar{B}$ $\left[\frac{V \cdot s}{m^2}\right]$

ρ $\left[\frac{A \cdot s}{m^3}\right]$

ρ_s $\left[\frac{A \cdot s}{m^2}\right]$

$\bar{J}$ $\left[\frac{A}{m^2}\right]$

$\bar{J}_s$ $\left[\frac{A}{m}\right]$

Medium constant

μ $\left[\frac{H}{m}\right]$

Permeability

σ $\left[\frac{S}{m}\right]$

Conductivity

ε $\left[\frac{F}{m}\right]$

Permittivity

(1) Ampere's law and Faraday's law

Ampere's law

Static or quasi static

magnetomotive force

$$mmf = F = \oint_C \bar{H}\cdot d\bar{l} = \int_S \bar{J}\cdot d\bar{a} = I_{TOT}\,[A]$$

Differential- or point-form

$$\nabla \times \bar{H} = \bar{J}$$

Faraday's law

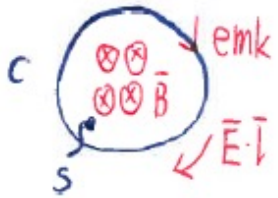
Time-variant

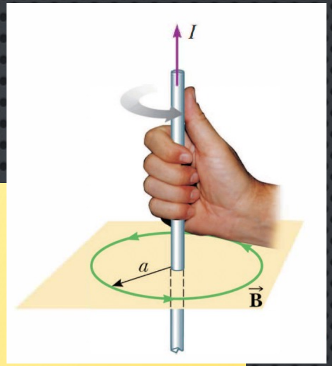
electromotive force

$$[redacted]\ emf = -V = \oint_C \bar{E}\cdot d\bar{l} = \int_S -\frac{\partial}{\partial t}\bar{B}\cdot d\bar{a} = -\frac{\partial}{\partial t}\phi\,[V]$$

Differential- or point-form

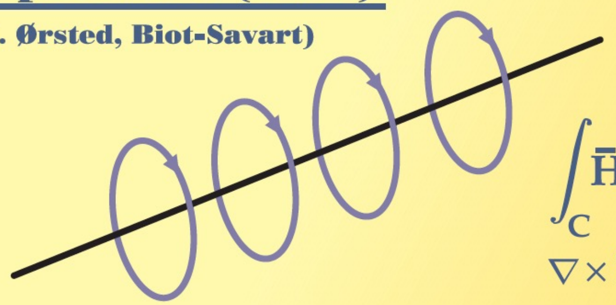
$$\nabla \times \bar{E} = -\frac{\partial}{\partial t}\bar{B}$$





Ampères lov (1820)

(H. C. Ørsted, Biot-Savart)



$$\int_C \bar{H}\bullet d\bar{a} = I \quad [A]$$

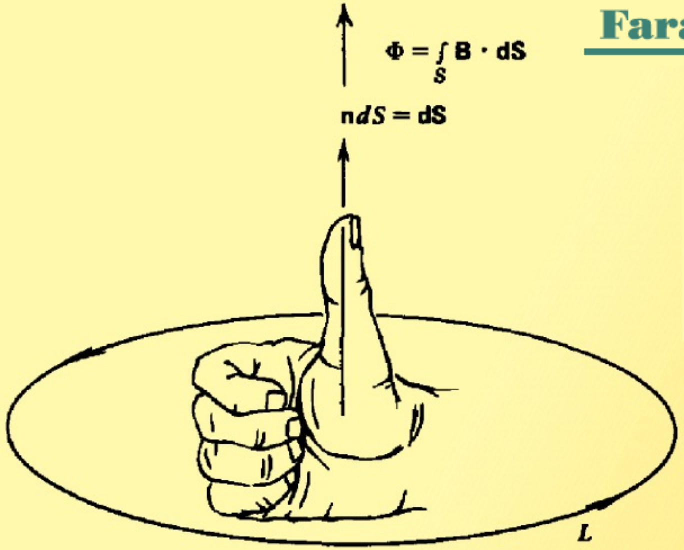
$$\nabla \times \bar{H} = \bar{J} \quad [\frac{A}{m^2}]$$

$$H_\varphi \cdot 2\pi r = I \quad [A]$$

$$H_\varphi = \frac{I}{2\pi r} \quad [\frac{A}{m}]$$

$$I = \int_S \bar{J}\bullet d\bar{a}$$

Faraday (1831)

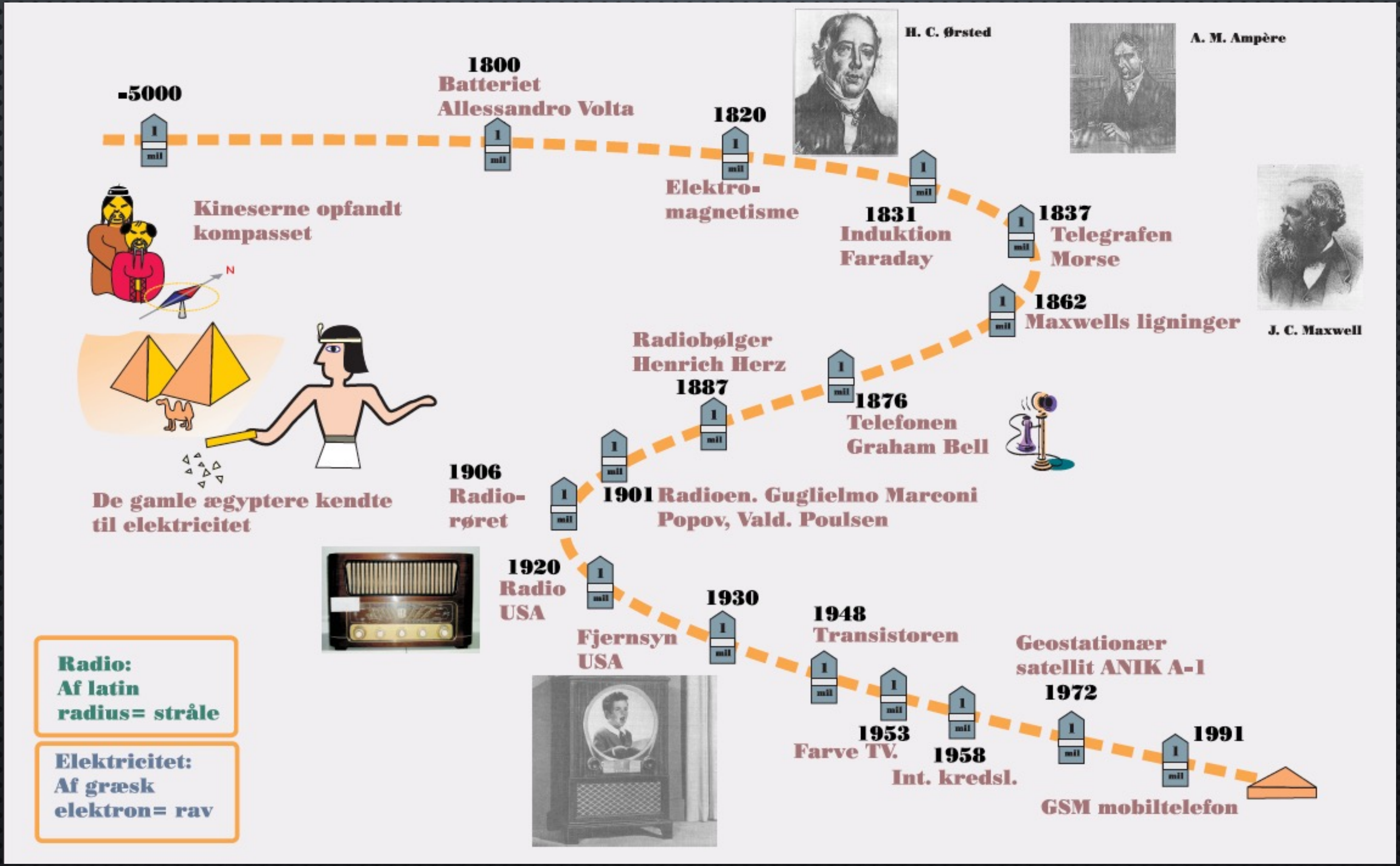


$$\mathcal{V} = -\frac{d\Phi}{dt} \qquad (V)$$

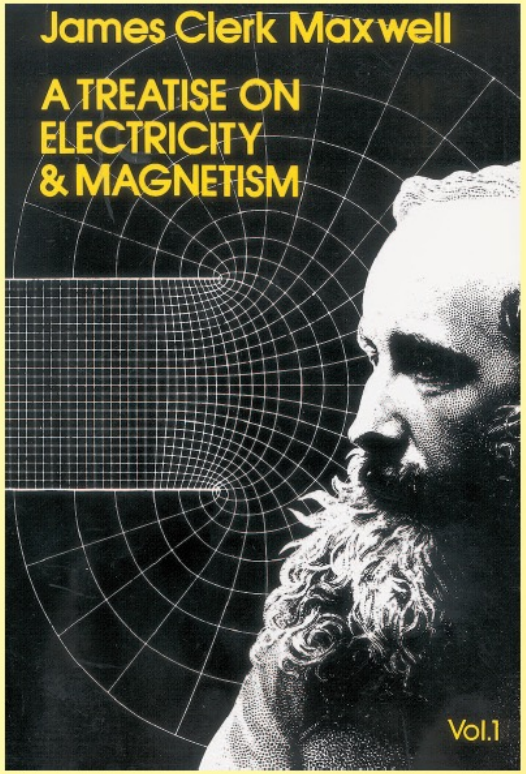
$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S}$$

Maxwell's Equations



Maxwell's Equations



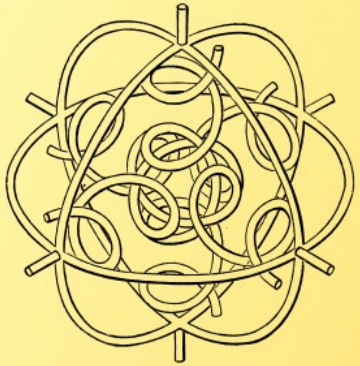


Fig. 25.

Relations between the positive directions of motion and of rotation indicated by three right-handed screws.

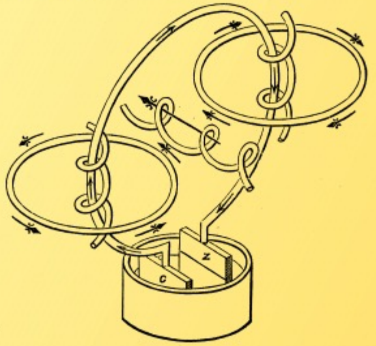


Fig. 24.

Relation between the electric current and the lines of magnetic induction indicated by a right-handed screw.

$$\nabla \times (\nabla F) = \bar{0}$$

Rotation of a scaler field's gradient is null vector.

$$\nabla \bullet (\nabla \times \bar{A}) = 0$$

Divergence of a vector field's rotation is null.

Blackboard (1)

Maxwell's Equations

Differential Form	Integration Form	Meaning
$\nabla \times \bar{E} = -\frac{\partial}{\partial t}\bar{B}$	$\oint_C \bar{E}\, d\bar{l} = -\frac{d}{dt}\phi$	FARADAYS LOV
$\nabla \times \bar{H} = \frac{\partial}{\partial t}\bar{D} + \bar{J}$	$\oint_C \bar{H}\, d\bar{l} = I + \int_S \frac{\partial}{\partial t}\bar{D}\, d\bar{a}$	AMPÈRES LOV
$\nabla \cdot \bar{D} = \rho$	$\oint_S \bar{D}\, d\bar{a} = Q$	GAUSS' LOV
$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B}\, d\bar{a} = 0$	There does not exist magnetic monopole (magnetic field are always closed)

Maxwell's Equations

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$

$$\nabla \times \bar{H} = (j\omega\varepsilon + \sigma)\bar{E}$$

$$\nabla \bullet \bar{B} = 0$$

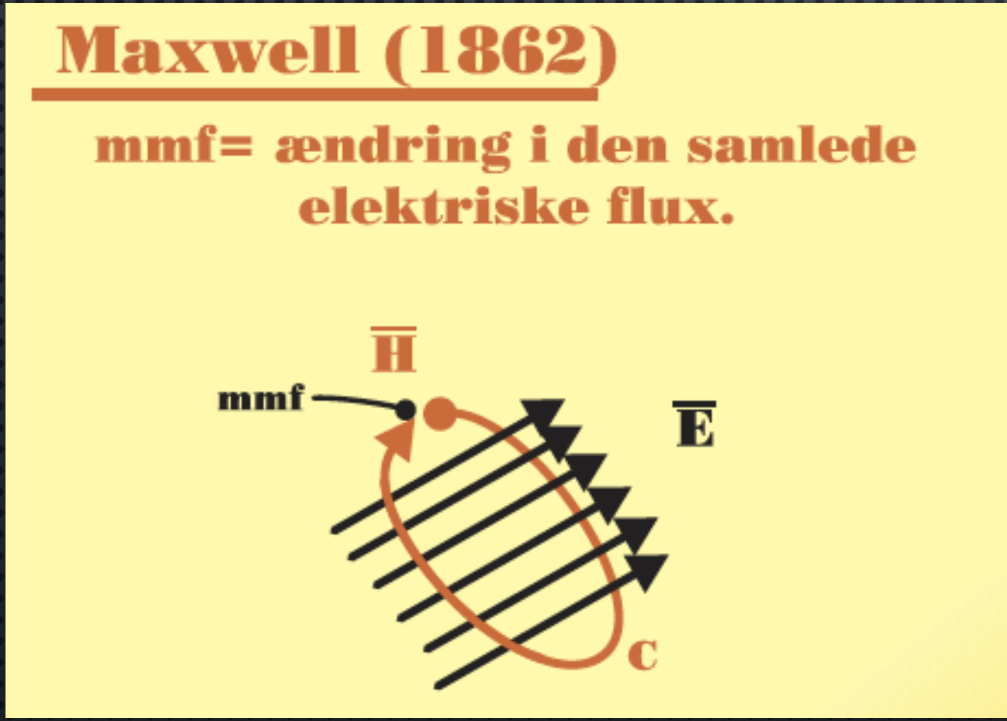
$$\nabla \bullet \bar{D} = \rho$$

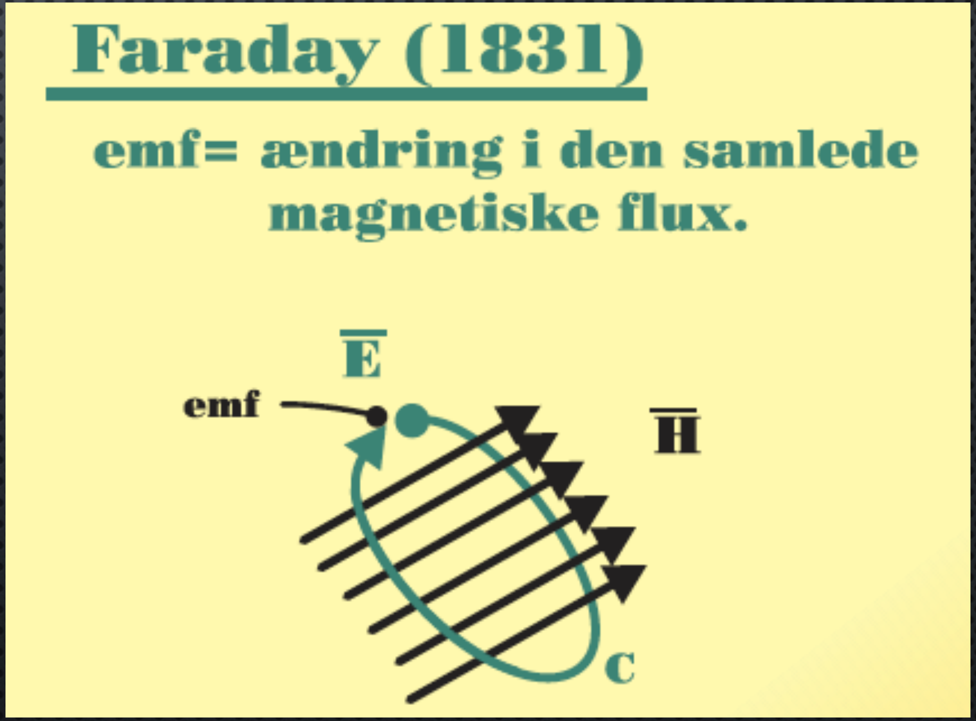
KSN (for time harmonic, sin, cos signals)

KSN
Komplex Symbolsk Notation

Wave Propagation

Blackboard (2)

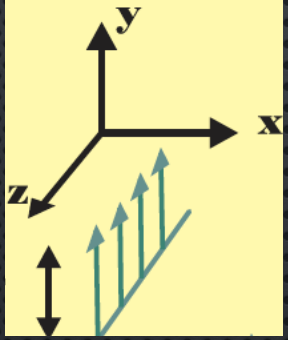




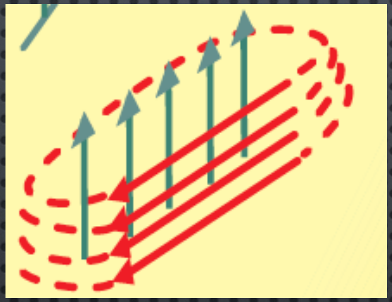
Wave Propagation

$$\nabla \times \bar{H} = \frac{\partial}{\partial t}\varepsilon \bar{E}$$

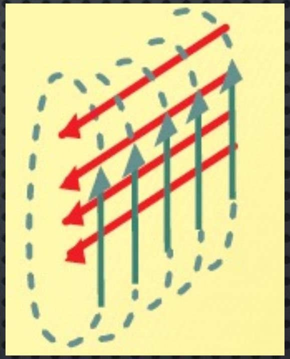
$$\nabla \times \bar{E} = -\frac{\partial}{\partial t}\mu \bar{H}$$



E field in y direction



H field is generated by E field



E field is generated by H field again

There are 90 degrees phase difference between E field and H field, where they are orthogonal to each other.

Wave Propagation in Lossless Medium

Blackboard (3)

General form for harmonic signals (KSN)

Maxwell's equations

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$
$$\nabla \times \bar{H} = (j\omega\varepsilon + \sigma)\bar{E}$$
$$\nabla \bullet \bar{B} = 0$$
$$\nabla \bullet \bar{D} = \rho$$

Wave equations

$$\Delta \bar{E} = \gamma^2 \bar{E}$$
$$\Delta \bar{H} = \gamma^2 \bar{H}$$

Solutions (Wave on YZ plane)

$$E = E^+ e^{-\gamma x} + E^- e^{+\gamma x}$$
$$H = H^+ e^{-\gamma x} + H^- e^{+\gamma x}$$
$$= \frac{1}{\eta}\left(E^+ e^{-\gamma x} - E^- e^{+\gamma x}\right)$$

Blackboard(4)