Analog Electronics

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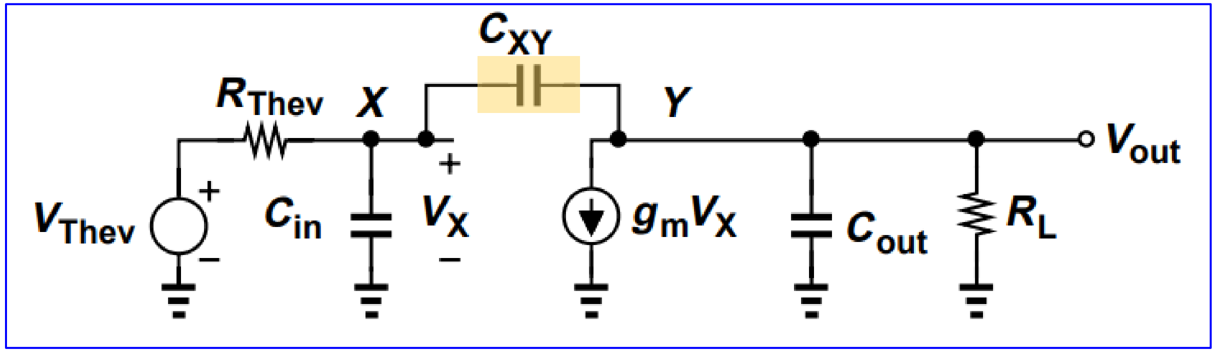
Agenda

- Recap Frequency response analysis and nonlinear systems
- Solutions of the assignments
- Feedback system

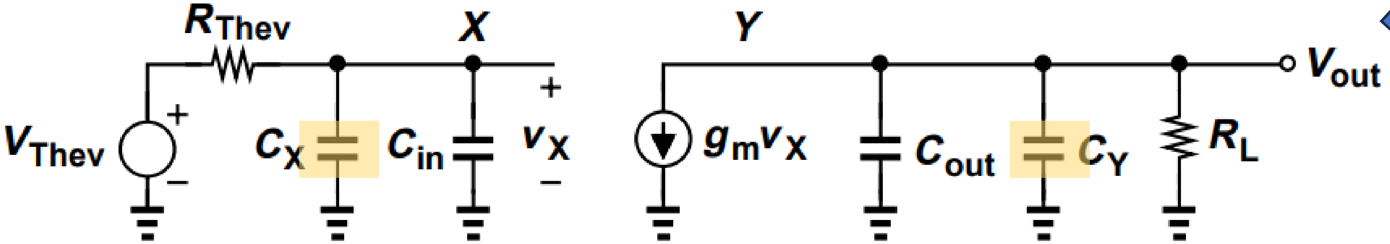
High-frequency response
Approach I: finding poles by inspection



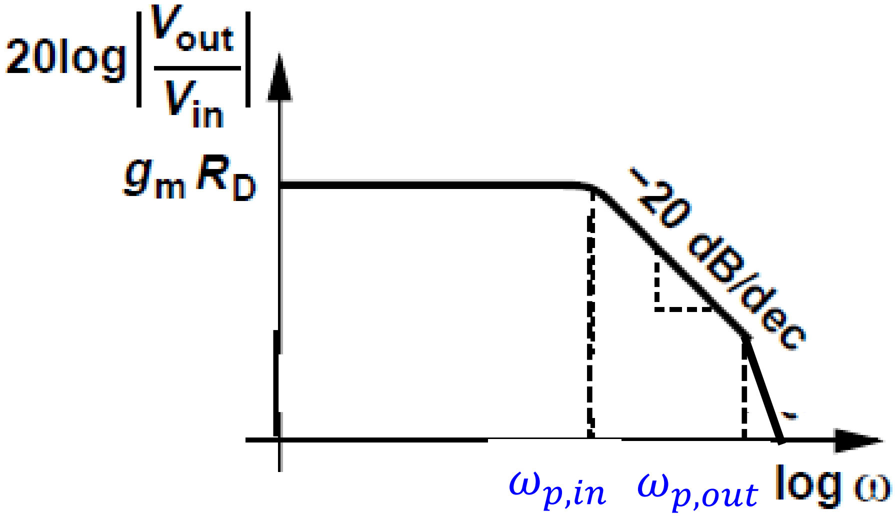
Apply Miller's theorem to $\boldsymbol{C_{XY}}$

$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + C_{XY}(1 + g_m R_L)]}$$

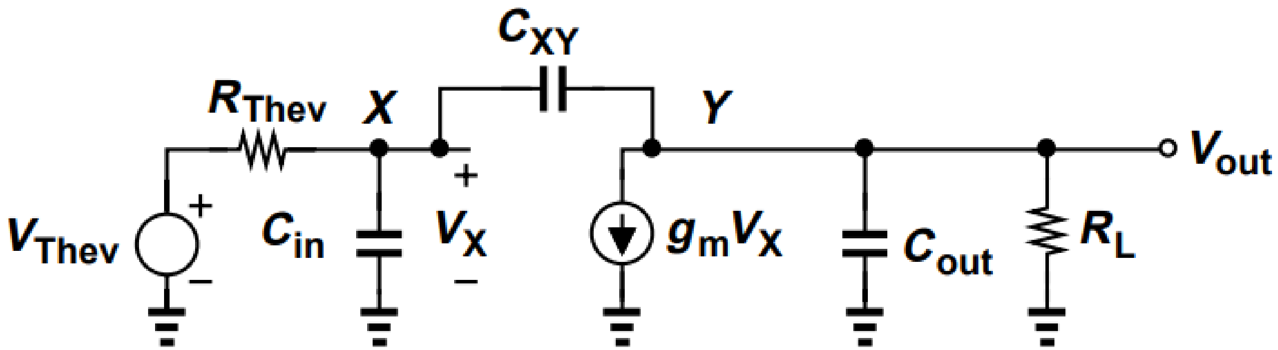
$$|\omega_{p,out}| = \frac{1}{R_L[C_{out} + C_{XY}(1 + \frac{1}{g_m R_L})]}$$



CE Stage	CS Stage
$V_{Thev} = V_{in} \frac{r_\pi}{r_\pi + R_S}$	$V_{Thev} = V_{in}$
$R_{Thev} = R_S \Vert r_\pi$	$R_{Thev} = R_S$
$C_X = C_\mu (1 + g_m R_L)$	$C_X = C_{GD} (1 + g_m R_L)$
$C_Y = C_\mu (1 + \frac{1}{g_m R_L})$	$C_Y = C_{GD} (1 + \frac{1}{g_m R_L})$



Approach II: exact analysis



$$\frac{V_{out}}{\mathrm{V_{Thev}}}(\mathrm{s}) = \frac{(C_{XY}s - g_m)R_L}{R_{Thev}R_L[C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out}]s^2 + [R_{Thev}C_{XY}(1 + g_mR_L) + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})]s + 1}$$

- If $s = 0$, $\frac{V_{out}}{\mathrm{V_{Thev}}}(\mathrm{s}) = -g_mR_L$
- If $s = \infty$, $\frac{V_{out}}{\mathrm{V_{Thev}}}(\mathrm{s}) = 0$, C_{in} and C_{out} as short circuits ➔ $V_{out} = V_{Thev} = 0$
- One zero: $\omega_z = \frac{g_m}{C_{XY}}$, e.g., $g_m \sim 0.01$, $C_{XY} \sim 10^{-12}$ ➔ $\omega_z \sim 10$ GHz, very high frequency, so typically not important
- Two poles: dominant pole approximation, i.e., $\omega_{p_1} \ll \omega_{p_2}$

➔ $\omega_{p_1} = \frac{1}{[R_{Thev}C_{XY}(1+g_mR_L)+R_{Thev}C_{in}+R_L(C_{XY}+C_{out})]}$

Low-frequency response-BJT

$$\frac{V_o}{V_{sig}} = A_M \frac{s}{s+\omega_{p1}} \frac{s+\omega_z}{s+\omega_{pE}} \frac{s}{s+\omega_{p2}}$$

$$\omega_{p1} = \frac{1}{\tau_{c1}} = \frac{1}{C_{C1}(R_B||r_\pi + R_{sig})}$$

$$\omega_{pE} = \frac{1}{\tau_{cE}} = \frac{1}{C_E[R_E||\left(\frac{1}{g_m} + \frac{R_B||R_{sig}}{\beta+1}\right)]}$$ Dominant pole

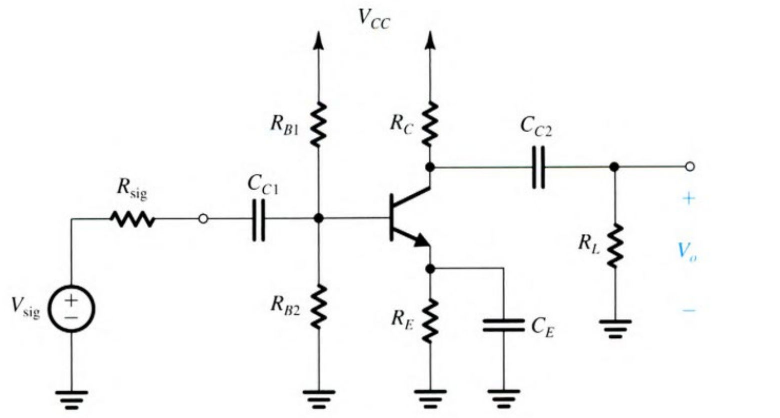
$$\omega_{p2} = \frac{1}{\tau_{c2}} = \frac{1}{C_{C2}(R_C + R_L)}$$

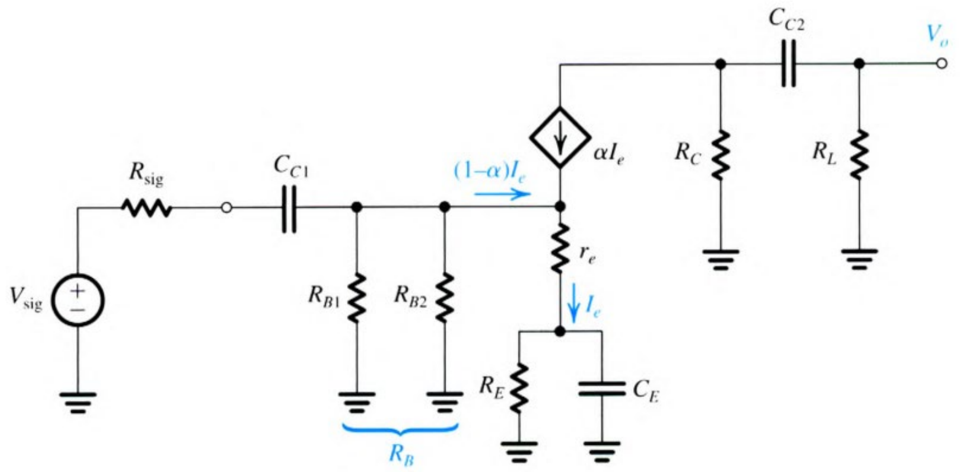
$$\omega_z = \frac{1}{C_E R_E}$$

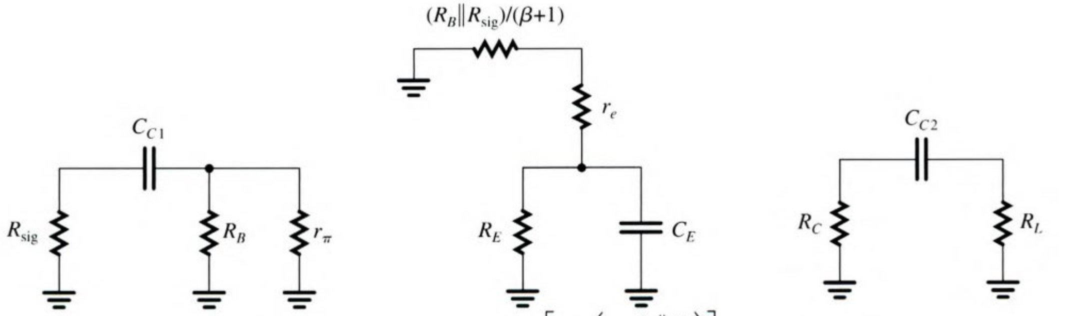
$R_B = R_{B1}||R_{B2}$ $\qquad r_\pi = \frac{\beta}{g_m} = \beta r_e$

Short-circuit time constant method:

- Short other capacitors
- turn off sources:
    - Voltage source → short circuit
    - Current source →open circuit







$\tau_{C1} = C_{C1}[(R_B \| r_\pi) + R_{sig}]$ $\qquad \tau_{CE} = C_E\left[R_E \| \left(r_e + \frac{R_B \| R_{sig}}{\beta+1}\right)\right]$ $\qquad \tau_{C2} = C_{C2}(R_C + R_L)$

The role of capacitors

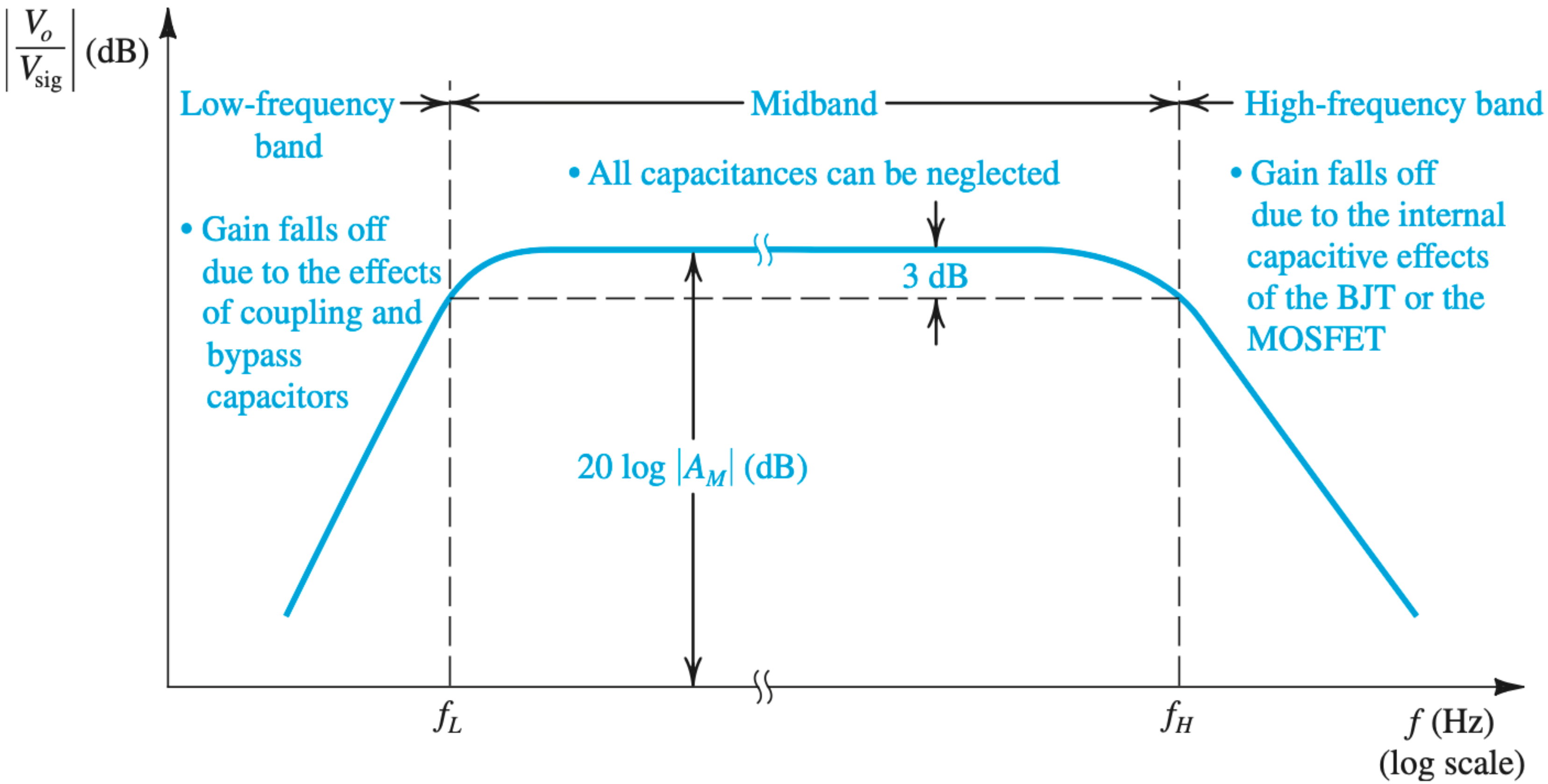
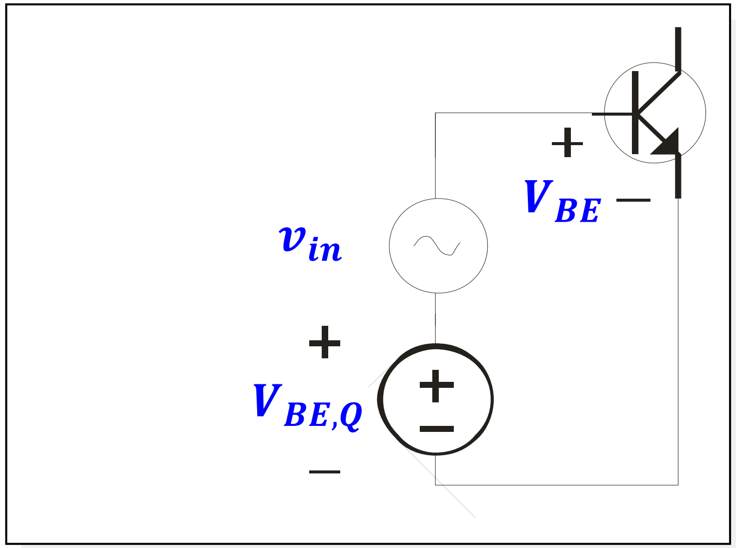


Figure 10.1 Sketch of the magnitude of the gain of a discrete-circuit BJT or MOS amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

Nonlinear system example--BJT



Taylor series expansion:

$$e^{x/a} = 1 + \frac{1}{a}x + \frac{1}{2!*a^2}x^2 + \cdots + \frac{1}{n!*a^n}x^n + \cdots$$

$$I_C = I_s\, e^{V_{BE}/V_T} = I_s e^{(V_{BE,Q}+v_{in})/V_T} = I_S\, e^{V_{BE,Q}/V_T}\ e^{v_{in}/V_T} = I_{CQ}\, e^{v_{in}/V_T}$$

$$= I_{CQ}\left(1 + \frac{1}{V_T}v_{in} + \frac{1}{2!*V_T^2}v_{in}^2 + \frac{1}{3!*V_T^3}v_{in}^3 + \ldots\right)$$

$$= I_{CQ} + \frac{I_{CQ}}{V_T}v_{in} + \frac{I_{CQ}}{2!*V_T^2}v_{in}^2 + \frac{I_{CQ}}{3!*V_T^3}v_{in}^3 + \ldots$$

$$= \alpha_0 + \alpha_1 v_{in} + \alpha_2 v_{in}^2 + \alpha_3 v_{in}^3 + \ldots$$

DC:

$$\beta_0 = \alpha_0 + \alpha_2\frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2\frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A + \alpha_3\frac{3A^3}{4} \approx \alpha_1 A$$

3rd Harmonic:

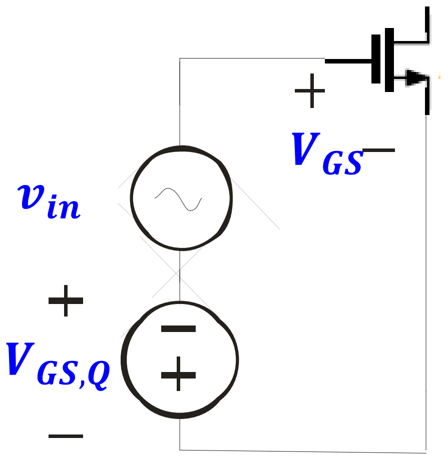
$$\beta_3 = \alpha_3\frac{A^3}{4}$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

$$= \sqrt{\left(\frac{\beta_2}{\beta_1}\right)^2 + \left(\frac{\beta_3}{\beta_1}\right)^2 + \left(\frac{\beta_4}{\beta_1}\right)^2 + \cdots}$$

Nonlinear system example--MOSFET



$$I_D = \frac{1}{2}k_n(V_{GS} - V_{TH})^2 = \frac{1}{2}k_n\left(V_{GS,Q} - V_{TH} + v_{in}\right)^2$$

$$= \frac{1}{2}k_n\left(V_{GS,Q} - V_{TH}\right)^2\left(1 + \frac{v_{in}}{V_{GS,Q} - V_{TH}}\right)^2$$

$$= I_{DQ}\left(1 + \frac{2}{V_{GS,Q} - V_{TH}}v_{in} + \frac{1}{\left(V_{GS,Q} - V_{TH}\right)^2}v_{in}^2\right)$$

$$= I_{DQ} + \frac{2I_{DQ}}{V_{GS,Q} - V_{TH}}v_{in} + \frac{I_{DQ}}{\left(V_{GS,Q} - V_{TH}\right)^2}v_{in}^2$$

$$= \alpha_0 + \alpha_1 \boldsymbol{v_{in}} + \boldsymbol{\alpha_2 v_{in}^2}$$

DC:

$$\beta_0 = \alpha_0 + \alpha_2\frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2\frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A$$

3rd Harmonic:

$$\beta_3 = 0$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

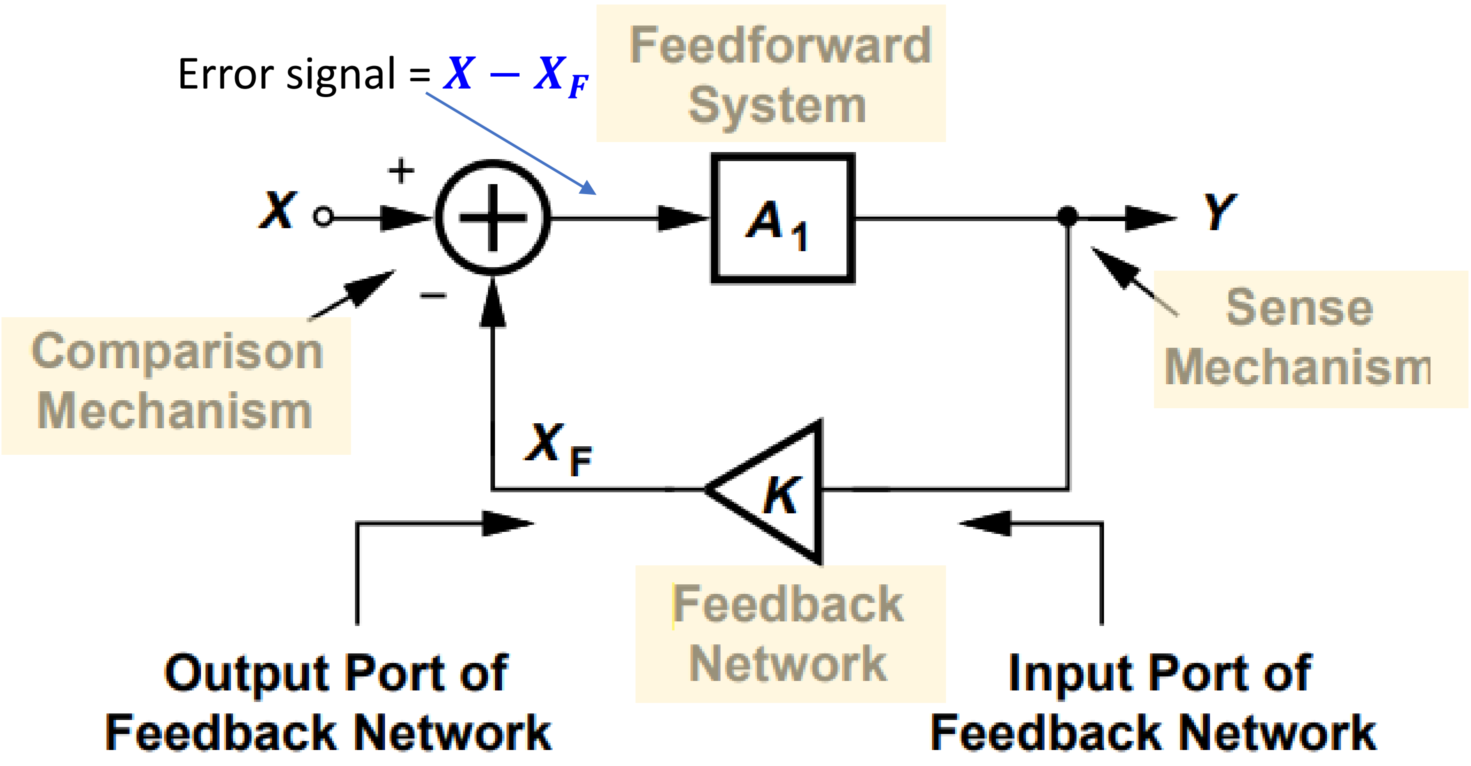
$$= \frac{\beta_2}{\beta_1} = HD_2$$

Some practical problems of amplifiers

- Gain variation: e.g. $A_v = -g_m R_1$
    - Temperature: $g_m = I_{C,Q}/V_T$
    - Supply voltage: $g_m = I_{C,Q}/V_T$
    - Manufacture: R_1 varies from sample to sample
- Nonlinear distortion
- Bandwidth
- Input and output impedance

- Some applications, e.g. analog-to-digital converters, require very precise voltage gain e.g. $A_v = 2.000$

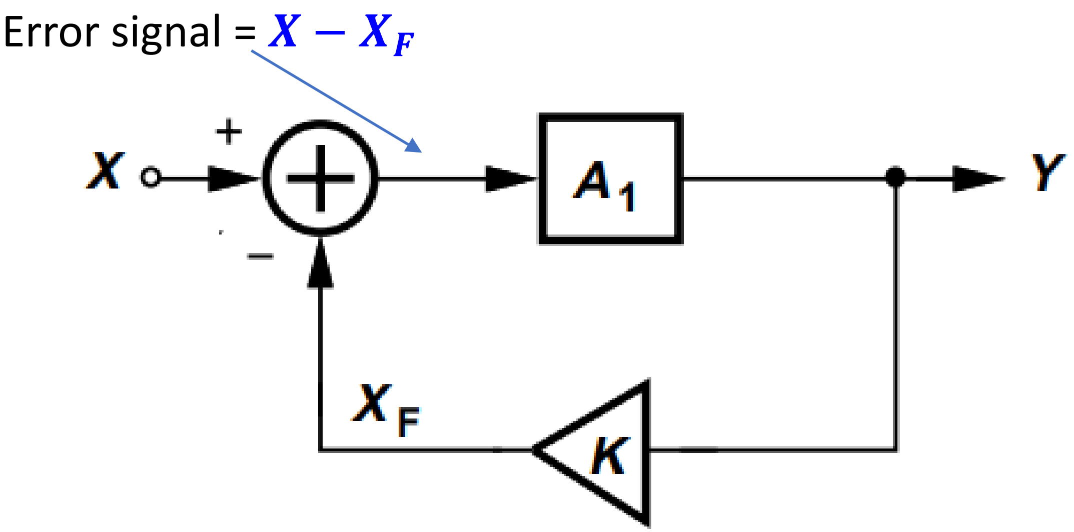
General feedback system



Negative feedback system:

- X and X_F change in the same direction;
- The error signal should be minimized;
- Open-loop system: break the feedback network, $K = 0$
- Closed-loop system: $K \neq 0$

Transfer function of closed-loop system



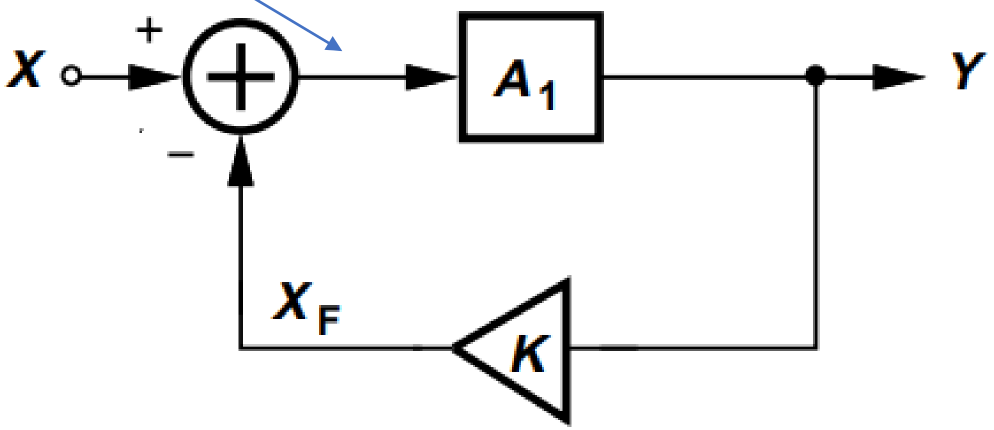
$$A_{cL} = \frac{Y}{X} = \frac{A_1}{1+KA_1}$$ --- closed-loop gain

A_1 is the open-loop gain
K is the feedback factor

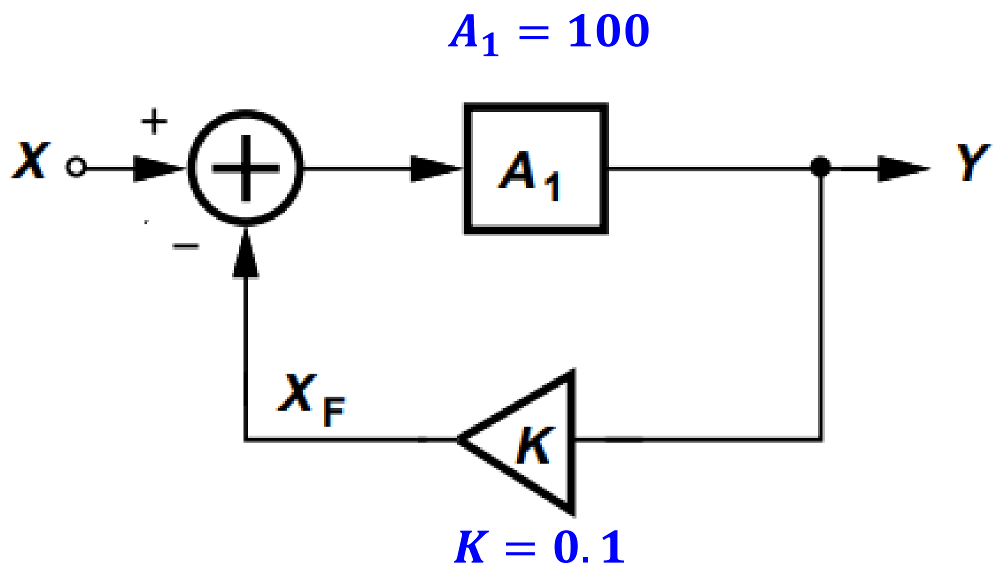
$KA_1 > 0 \Rightarrow |A_{cL}| < |A_1|$

Quiz: determine the error signal in terms of X

Error signal = $\boldsymbol{X - X_F}$



Example



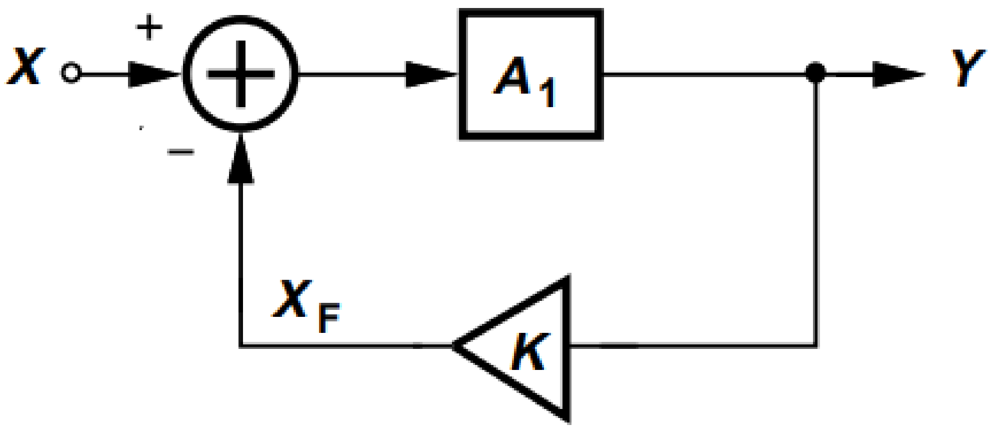
Nominal gain of an amp: $A_1 = 100$
Actual gain in application: $A'_1 = 50$

How much does A_1 change?
How much does the closed-loop gain A_{cL} change?

A_{cL} = A_1 /(1+K A_1) = 100/11 = 9.09
A_{cL}’ = A_1 ’/(1+K A_1 ’) = 50/6 = 8.33

A_{cL} change: 8.3%

Feedback loop system



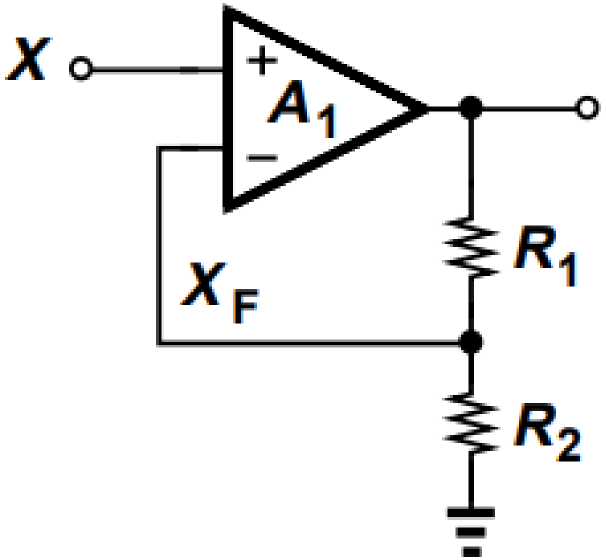
Closed-loop gain

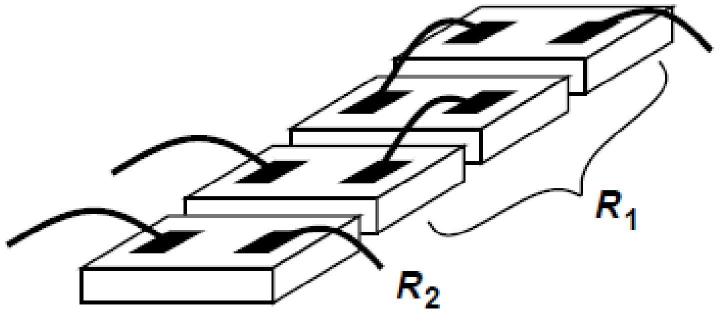
$A_{cL} = \frac{Y}{X} = \frac{A_1}{1+KA_1} \approx \frac{1}{K}$ if $KA_1 \gg 1$

➔Closed-loop gain A_{cL} is relatively independent of the open-loop gain A_1

We still need $A_{cL} > 1$ ➔ $0 < K \leq 1$

Feedback system example





The op amp A1 performs two functions:

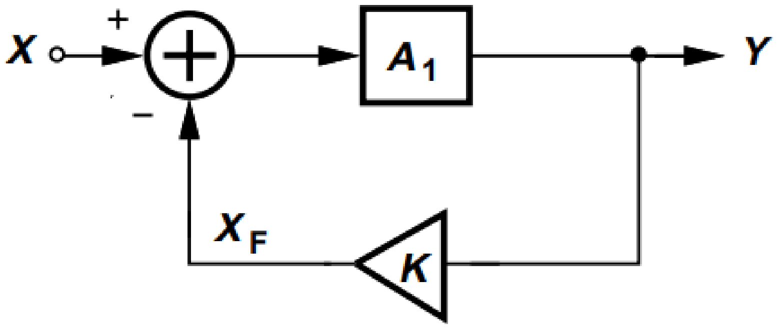
- substraction
- Amplification

What is the closed-loop gain?

$$A_{cL} = \frac{Y}{X} = \frac{A_1}{1 + KA_1} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

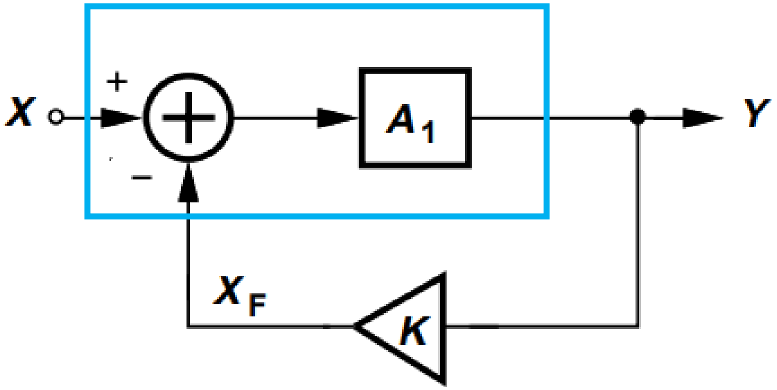
$$A_{cL} \approx \frac{R_1+R_2}{R_2} = 1 + \frac{R_1}{R_2}, \text{ if } \frac{R_2}{R_1+R_2} A_1 \gg 1$$

Loop-gain

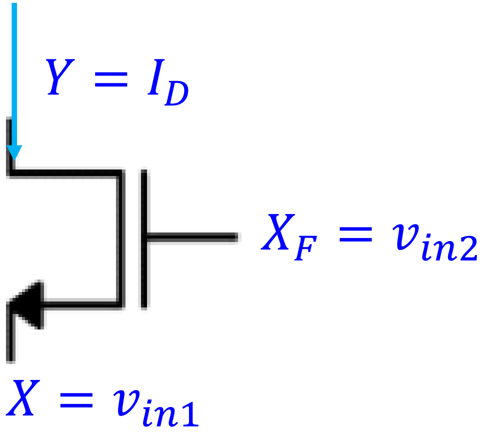


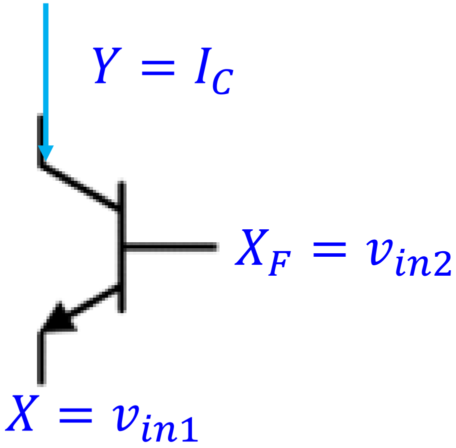
- Open-loop gain: A_1
- Closed-loop gain: $A_{cL} = \frac{Y}{X} = \frac{A_1}{1+KA_1}$
- Loop-gain: KA_1
  - Procedure to measure loop-gain
    - Set the input X to zero (X is voltage → AC ground; if X is current→ open)
    - Break the loop at an arbitrary point
    - Apply a test signal $\boldsymbol{V_{test}}$ at one terminal and measure the signal $\boldsymbol{V_F}$ at the other terminal
    - Calculate the loop-gain $-\frac{V_F}{V_{test}}= KA_1$
      - $\frac{V_F}{V_{test}} < 0$ → negative feedback
      - $\frac{V_F}{V_{test}} > 0$ → positivie feedback

Apply feedback to transistor amplifiers

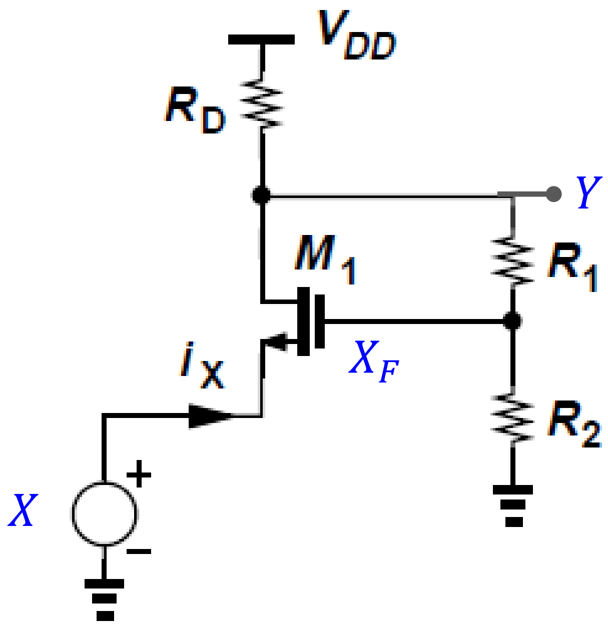


- Two input, one output
- Generates the output that proportional to the deviation of the two inputs.





Quiz: $\frac{Y}{X} = ?$



Assume R_1 and R_2 are very large

➔ $A_1 = g_m R_D$

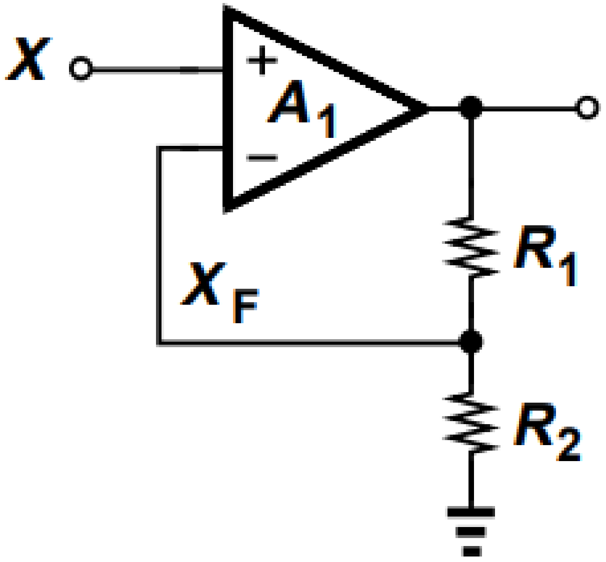
Summary of negative feedback concept

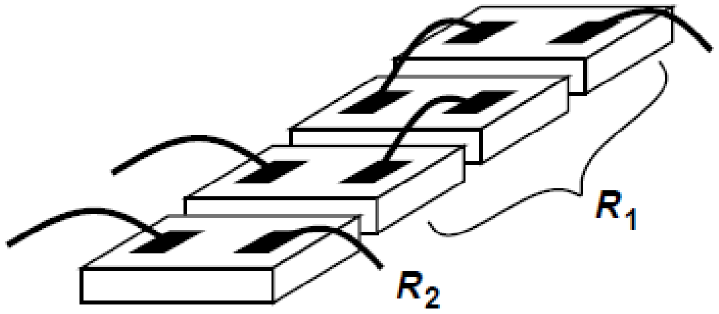
- Sacrifice the open-loop gain A_1 to benefit from negative feedback
- The feedback signal X_F is a good copy of input signal X
- The feedback factor K is normally independent to frequency
  - Make Y a good (scaled by $1/\mathrm{K}$) copy of X
    - Wider frequency band
    - Better linearity
- If loop-gain $KA_1 \gg 1$ ➔ $A_{cL} \approx \frac{1}{K}$, relatively independent of A_1
  - Factors that cause A_1 to vary have less impact on the closed-loop gain
  - Factors: temperature, supply voltage, frequency, load impedance

Properties of negative feedback

- Gain desensitization
- Bandwidth extension
- Linearity improvement
- Modification of input and output impedance

Gain desensitization example



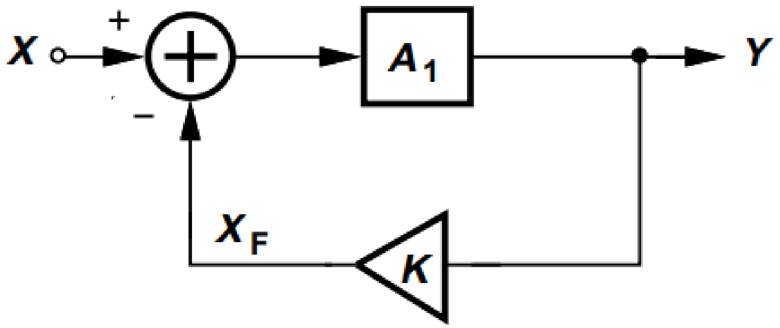


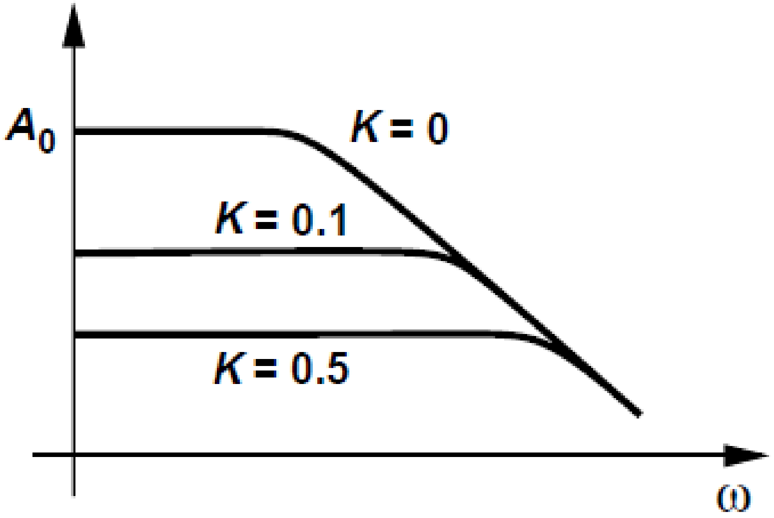
Assume the nominal gain of A_1=100 and $\frac{R_1}{R_2} = 3$. Due to e.g., temperature, supply voltage, frequency and loading imdedance, A_1 drops to 50.

- How does the closed-loop gain A_{cL} change?

$$A_{cL} = \frac{Y}{X} = \frac{A_1}{1 + KA_1} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

Bandwidth extension





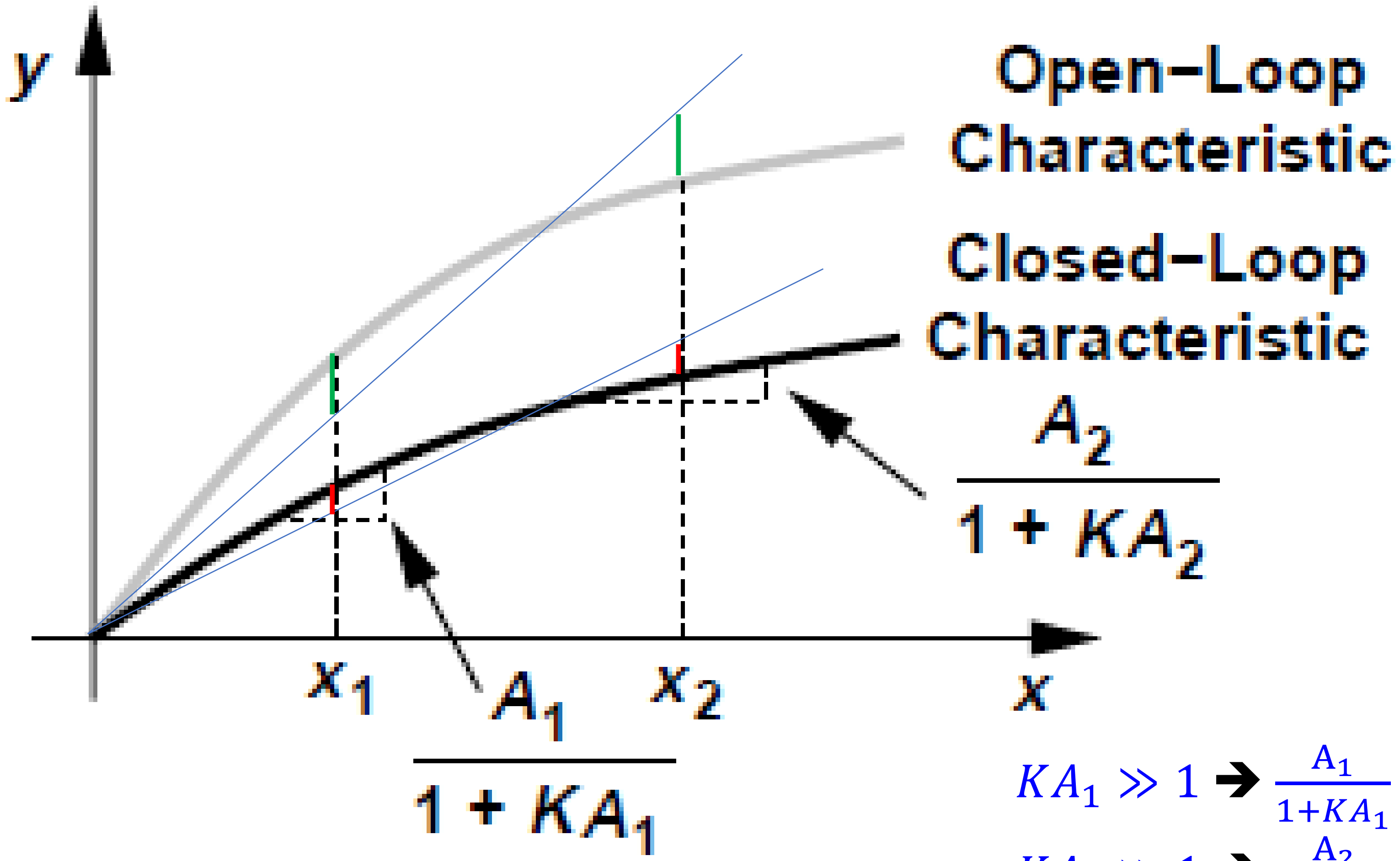
$$A_1(s) = \frac{A_0}{1 + s/\omega_0}$$

$$A_{cL}(s) = \frac{A_1(s)}{1 + KA_1(s)}$$

Closed-loop gain = ?
Closed-loop bandwidth = ?

Gain x Bandwidth is constant!

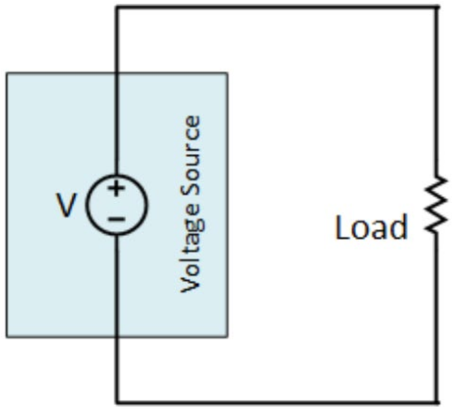
Linearity improvement



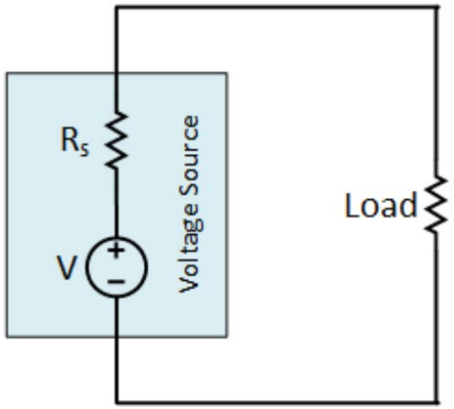
$KA_1 \gg 1 \rightarrow \frac{A_1}{1+KA_1} \approx K, when\ KA_1 \gg 1$

$KA_2 \gg 1 \rightarrow \frac{A_2}{1+KA_2} \approx K, when\ KA_2 \gg 1$

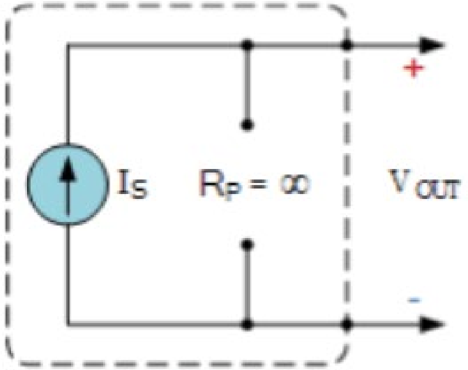
Ideal Vs. real sources



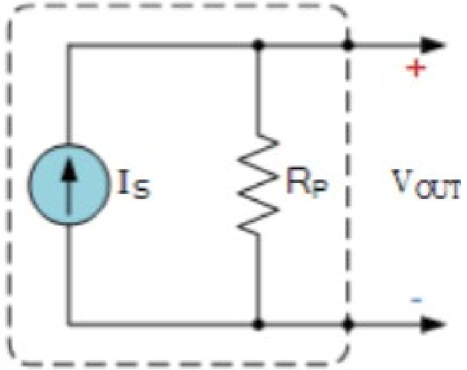
Ideal Voltage Source



Practical Voltage Source

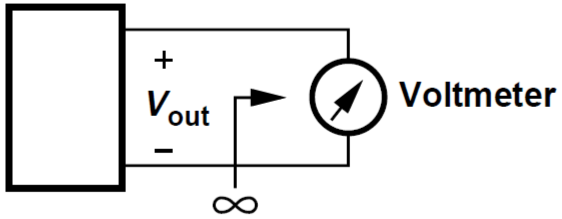


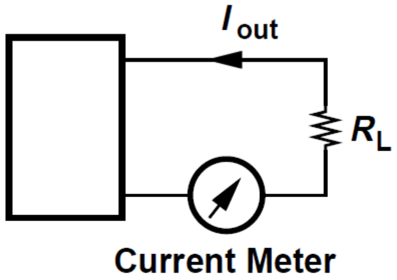
Ideal Current Source



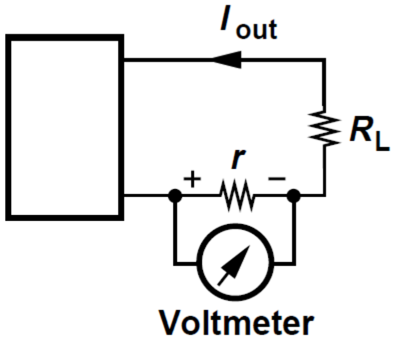
Practical Current Source

How to sense/measure voltage or current

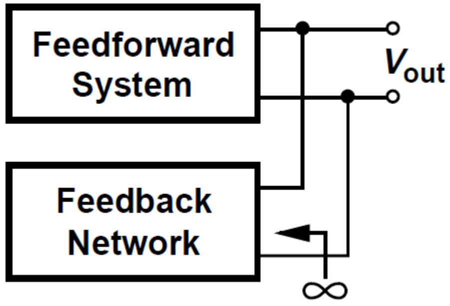


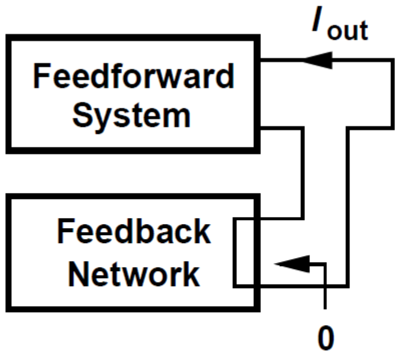


(a)



(b)





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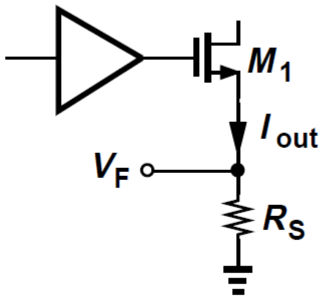
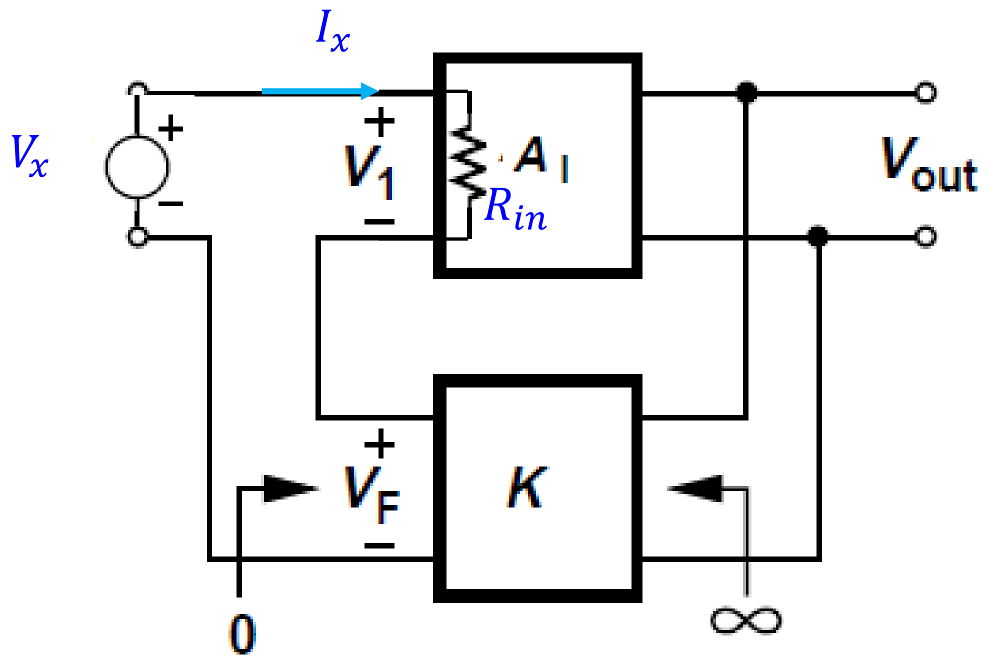


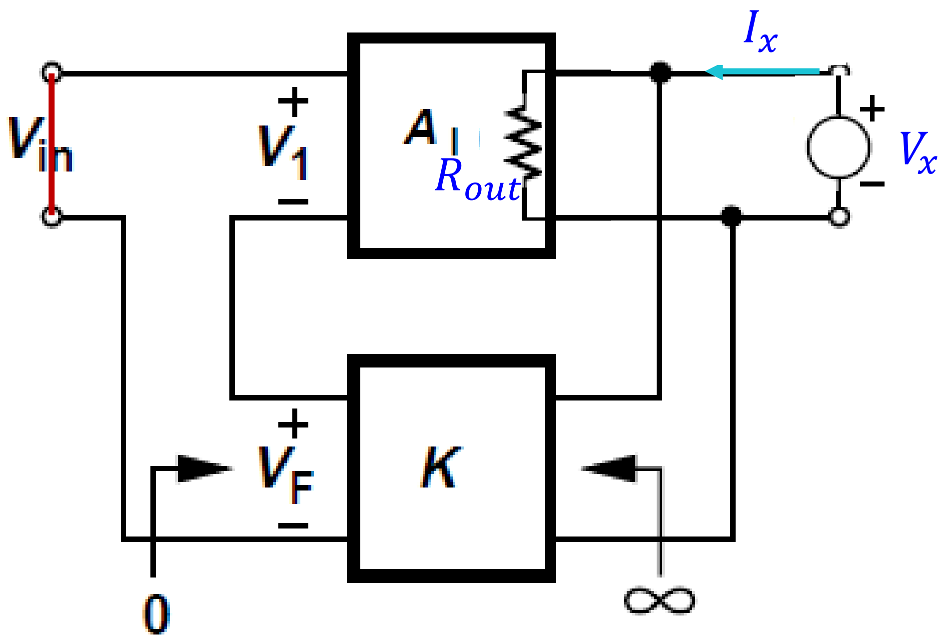
(d)

Modification of input and output impedance
Voltage-voltage FB



$V_1 = I_x R_{in}$
$V_{out} = A_1 I_x R_{in}$
$V_F = K A_1 I_x R_{in}$
$V_x = V_1 + V_F$
➔ $R_{in,CL} = V_x / I_x = R_{in}(1 + KA_1)$

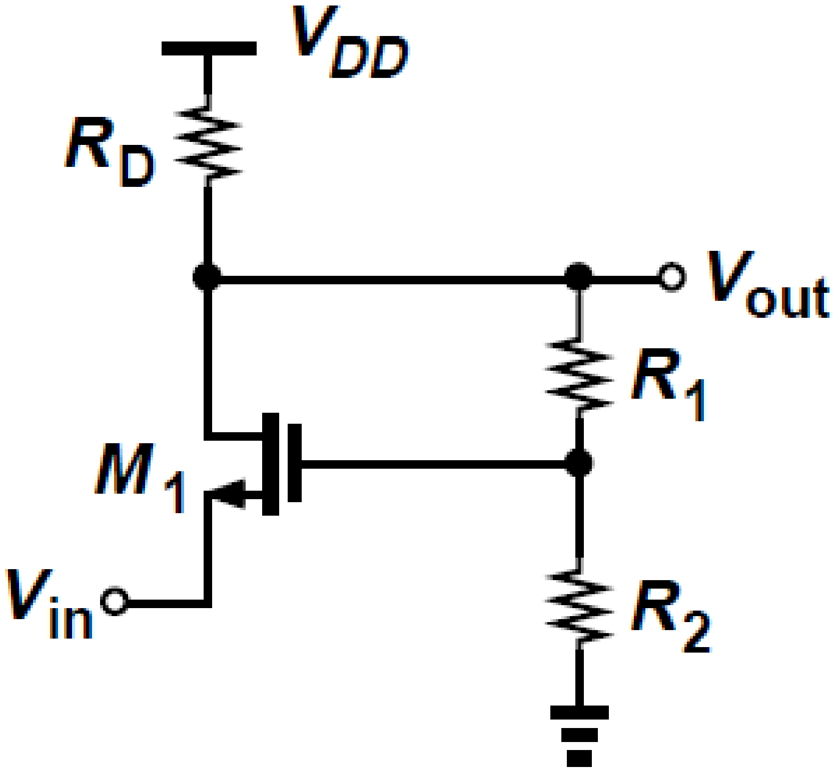
A better voltage sensor



$V_x = I_x R_{out}$
$V_1 = -V_F = -KV_x$
➔ $R_{out,CL} = V_x / I_x = R_{out}/(1 + KA_1)$

A better voltage source

Modification of input and output impedance

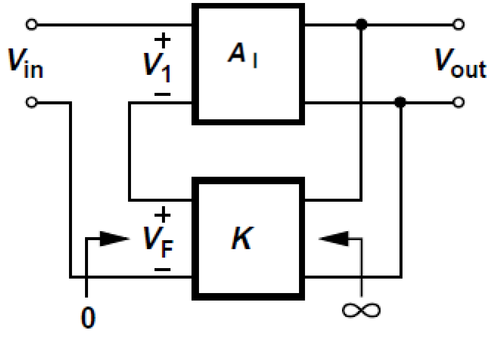


Assume $R_1 + R_2 >> R_D$

$$A_1 = g_m R_D$$
$$K = \frac{R_2}{R_1 + R_2}$$
$$A_{CL} = \frac{A_1}{1 + KA_1}$$
$$R_{in} = 1/g_m$$
$$R_{out} = R_D$$
$$R_{in,CL} = R_{in}(1 + KA_1)$$
$$R_{out,CL} = R_{out}/(1 + KA_1)$$

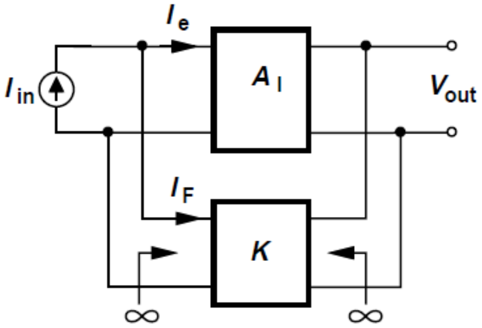
Feedback topologies

$V_o - V_i$



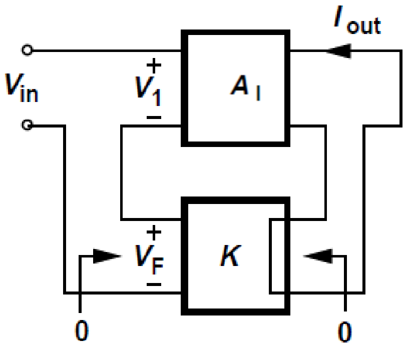
$$R_{in,CL} = R_{in}(1 + KA_1)$$
$$R_{out,CL} = R_{out}/(1 + KA_1)$$

$V_o - I_i$



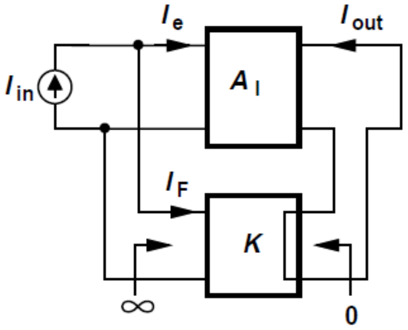
$$R_{in,CL} = R_{in}/(1 + KA_1)$$
$$R_{out,CL} = R_{out}/(1 + KA_1)$$

$I_o - V_i$



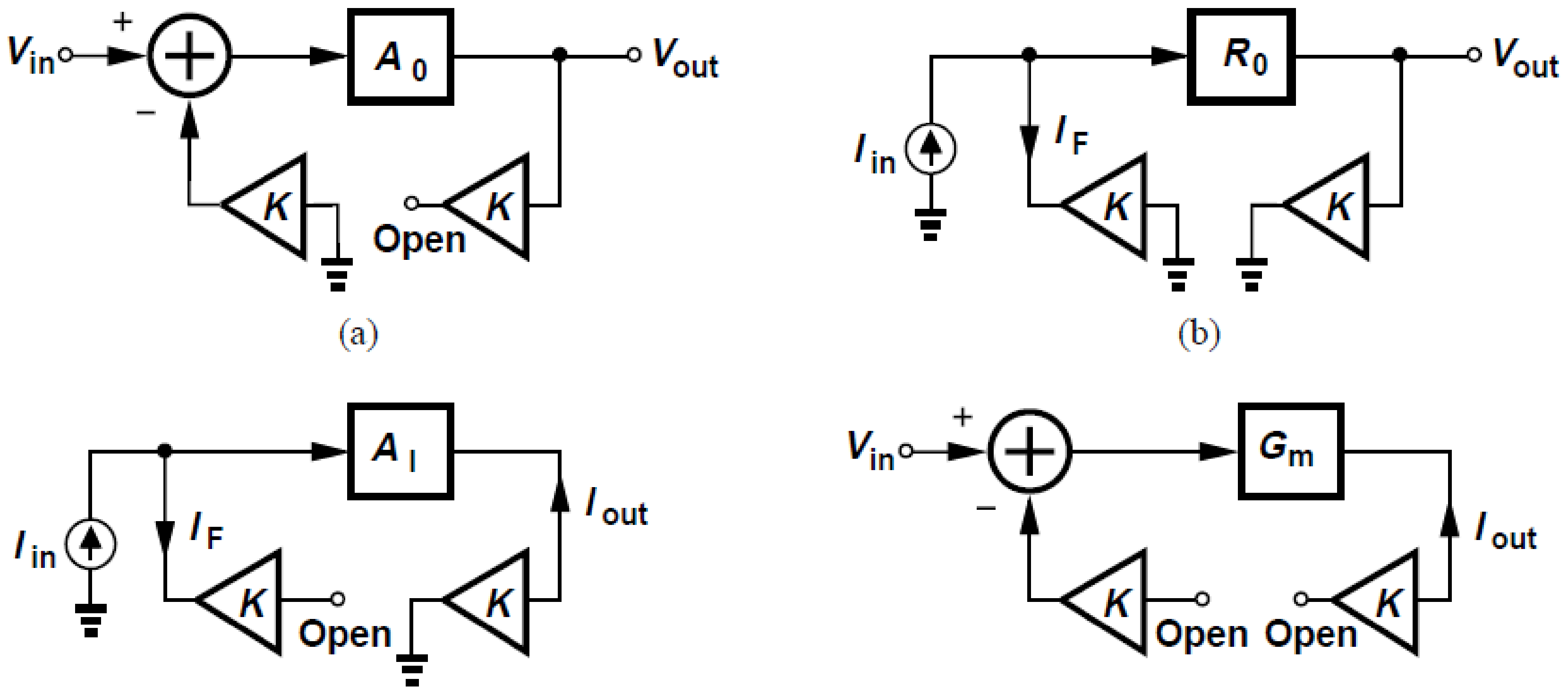
$$R_{in,CL} = R_{in}(1 + KA_1)$$
$$R_{out,CL} = R_{out}(1 + KA_1)$$

$I_o - I_i$



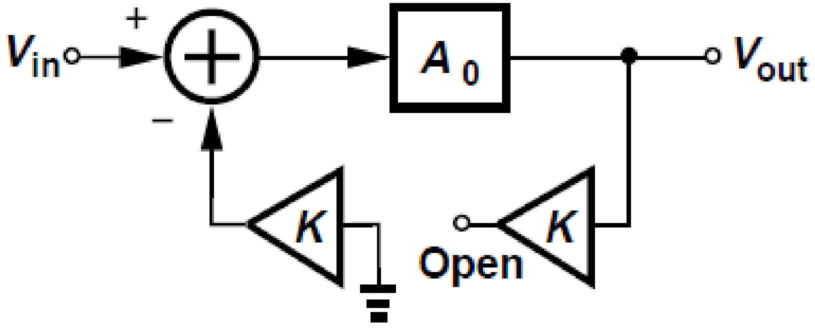
$$R_{in,CL} = R_{in}/(1 + KA_1)$$
$$R_{out,CL} = R_{out}(1 + KA_1)$$

Rules for breaking the feedback network(self-study)

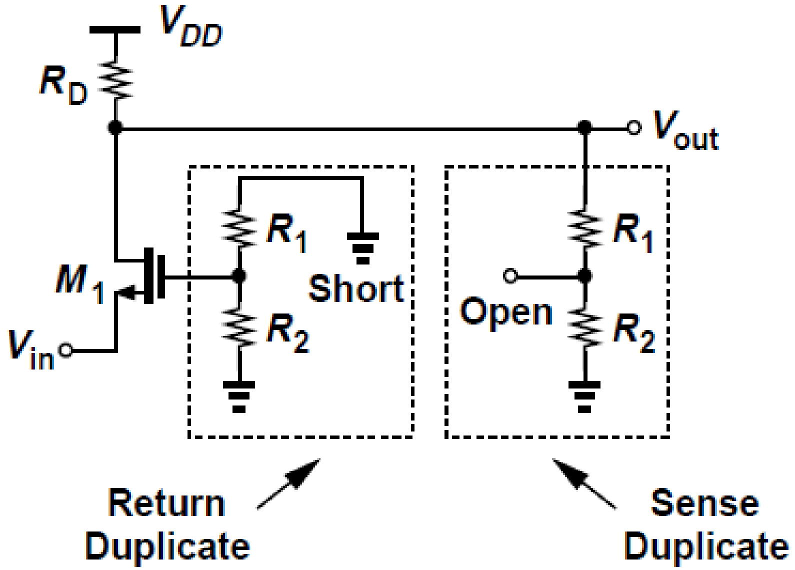


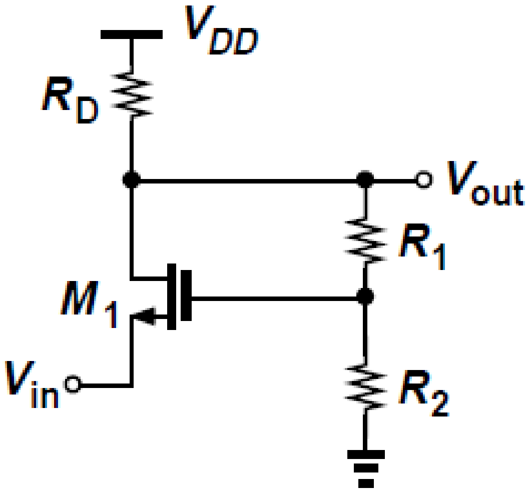
(a) (b) (c) (d)

Rules for breaking the feedback network



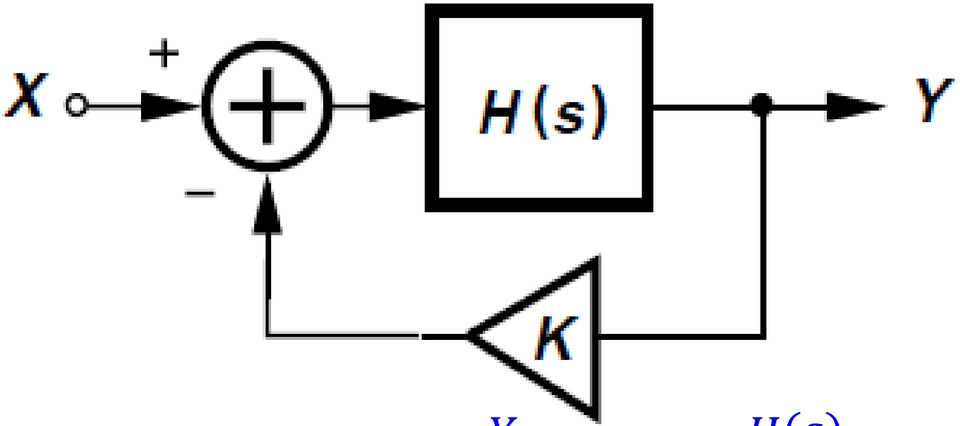
(a)





$$K = \frac{R_2}{R_1 + R_2}$$
$$A_1 = g_m[R_D||(R_1 + R_2)]$$
$$R_{in} = 1/g_m$$
$$R_{out} = R_D\,||(R_1 + R_2)$$

Instability in FB system



Loop transmission: $\frac{Y}{X}(s) = \frac{H(s)}{1+KH(s)}$

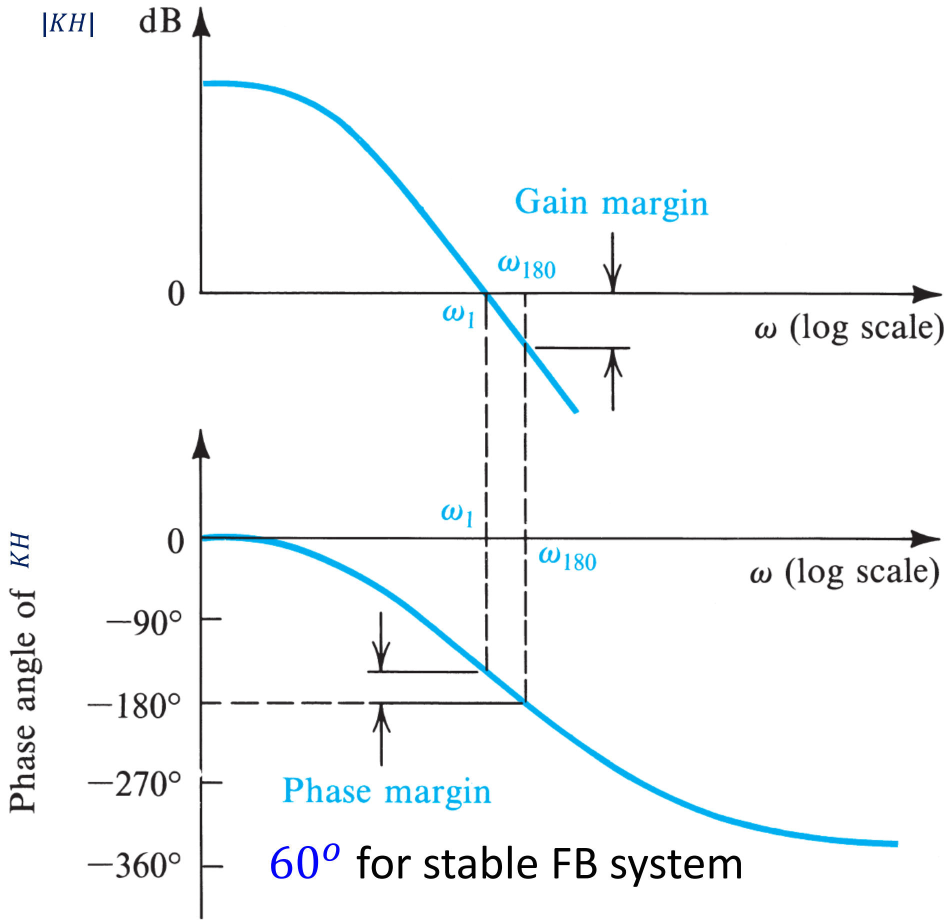
For a certain input frequency ω_1,

$$KH(j\omega_1) = -1$$

➔ $\frac{Y}{X}(j\omega_1) = \infty$

Even X is very small, Y can be extremely large.

- Acting like an oscillator
- Bad for building amplifiers



60^o for stable FB system

Unstable system:

$|KH(j\omega_1)| \geq 1$

$\angle KH(j\omega_1) = 180^o$

Stability/compensation

- Reduce $K \rightarrow \omega_1$ smaller→ more stable system but less control over the system
- Change the phase of $H(s)$