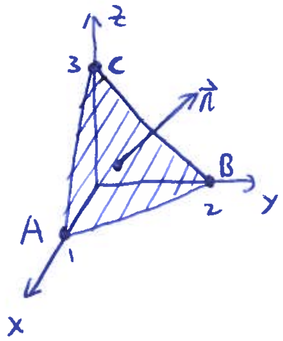
Answers of exercise - 1

1. A counterexample :

$$\vec{A} = (2, 0, 0), \quad \vec{B} = (4, 0, 0), \quad \vec{C} = (0, 1, 0)$$

2. 

$A(1,0,0)$, $B(0,2,0)$, $C(0,0,3)$

$$\vec{AB} = (-1, 2, 0) = -\vec{i} + 2\vec{j}$$

$$\vec{AC} = (-1, 0, 3) = -\vec{i} + 3\vec{k}$$

$$\vec{AB} \times \vec{AC} = (-\vec{i} + 2\vec{j}) \times (-\vec{i} + 3\vec{k}) = 6\vec{i} + 3\vec{j} + 2\vec{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7$$

$$\boxed{\vec{n} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{6}{7}\vec{i} + \frac{3}{7}\vec{j} + \frac{2}{7}\vec{k}}$$

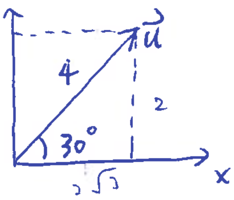
3.

(a). $\vec{u} = -\vec{v} = -(3\vec{i} + 4\vec{j}) = -3\vec{i} - 4\vec{j}$

(b). $\vec{P_1P_2} = (2, -1, -3)$, $\vec{n} = \dfrac{\vec{P_1P_2}}{|\vec{P_1P_2}|} = \dfrac{1}{\sqrt{14}}(2, -1, -3)$

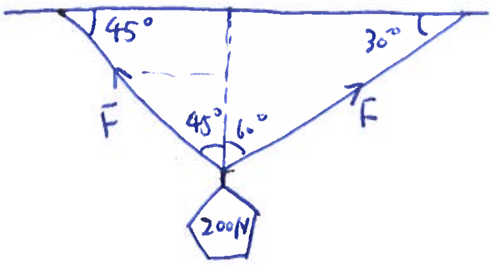
(c). $\vec{u} = -\frac{1}{2}\vec{v} = -\frac{1}{2}[1, 2, 3] = [-\frac{1}{2}, -1, -\frac{3}{2}]$

(d). $\vec{u} = \frac{\sqrt{14}}{7}(\vec{i} - 2\vec{j} + 3\vec{k})$

(e). 

$\vec{u} = 2\sqrt{3}\,\vec{i} + 2\vec{j}$

4.



$$F \cdot \cos 45^\circ + F \cdot \sin 30^\circ = 200N$$

$$F = \frac{200}{\cos 45^\circ + \sin 30^\circ} N$$

$$\approx 165.7 N$$

5.

(a) $\vec{u} = 4\vec{i} - 5\vec{j} + \vec{k}$, $\vec{v} = 3\vec{i} + 6\vec{j} - \vec{k}$

$$\vec{u} \cdot \vec{v} = (4\vec{i} - 5\vec{j} + \vec{k}) \cdot (3\vec{i} + 6\vec{j} - \vec{k})$$

$$= 12 - 30 - 1$$

$$= -19$$

(b). $\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos \frac{\pi}{4}$

$$= 3 \times 4 \times \frac{\sqrt{2}}{2}$$

$$= 6\sqrt{2}$$

6. $\vec{AB} = (5, 2, 4)$, $\vec{AC} = (0, 3, -1)$

$$S_{\triangle} = \frac{1}{2} \vec{AB} \cdot \vec{AC} = \frac{1}{2}(5, 2, 4) \cdot (0, 3, -1) = \frac{1}{2}(6-4) = 1$$

7. $\vec{u} \times \vec{v} = (3\vec{i} - 4\vec{j} + \vec{k}) \times (2\vec{i} - 2\vec{j} + 3\vec{k})$

$$= -10\vec{i} - 7\vec{j} + 2\vec{k}$$

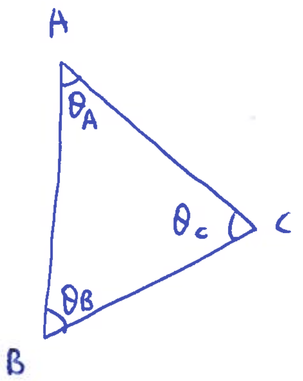
$$(\vec{u} \times \vec{v}) \cdot \vec{u} = (-10\vec{i} - 7\vec{j} + 2\vec{k}) \cdot (3\vec{i} - 4\vec{j} + \vec{k})$$

$$= -30 + 28 + 2$$

$$= 0$$

$$(\vec{u} \times \vec{v}) \cdot \vec{v} = (-10\vec{i} - 7\vec{j} + 2\vec{k}) \cdot (2\vec{i} - 2\vec{j} + 3\vec{k})$$

8.



$\vec{AC} = (-3, 3, -6)$, $\vec{BA} = (-2, -2, 7)$, $\vec{CA} = (3, -3, 6)$

$\vec{AB} = (2, 2, -7)$, $\vec{BC} = (-5, 1, 1)$, $\vec{CB} = (5, -1, -1)$

$|\vec{AC}| = 3\sqrt{6}$, $|\vec{AB}| = \sqrt{57}$, $|\vec{BC}| = \sqrt{27} = 3\sqrt{3}$

$$\cos\theta_A = \frac{|\vec{AC}|^2 + |\vec{AB}|^2 - |\vec{BC}|^2}{2|\vec{AC}||\vec{AB}|} = \frac{54 + 57 - 27}{2 \times 3\sqrt{6} \times \sqrt{57}} = \frac{84}{6\sqrt{6} \times \sqrt{57}}$$

$$= 0.57$$

$$\cos\theta_B = \frac{|\vec{BA}|^2 + |\vec{BC}|^2 - |\vec{AC}|^2}{2|\vec{AB}||\vec{BC}|} = \frac{57 + 27 - 54}{2 \times \sqrt{57} \times 3\sqrt{3}} = \frac{30}{6\sqrt{3} \times \sqrt{57}}$$

$$= 0.146$$

$$\cos\theta_C = \frac{|\vec{CA}|^2 + |\vec{CB}|^2 - |\vec{AB}|^2}{2|\vec{CA}||\vec{CB}|} = \frac{54 + 27 - 57}{2 \times 3\sqrt{6} \times 3\sqrt{3}} = \frac{24}{54\sqrt{2}}$$

$$= 0.1$$

9.

(a). $\vec{A} = \vec{a}_\rho \, \rho z \sin\phi + \vec{a}_\phi \, 3\rho\cos\phi + \vec{a}_z \, \rho\cos\phi\sin\phi$

$A_\rho = \rho z \sin\phi$, $A_\phi = 3\rho\cos\phi$, $A_z = \rho\cos\phi\sin\phi$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$\cancel{A_x = A_\rho\cos\phi \quad A_\phi\sin\phi + A_\rho\sin\phi + A_\phi\cos\phi + A_z}$

$A_x = A_\rho\cos\phi - A_\phi\sin\phi = \rho z\sin\phi\cos\phi - 3\rho\cos\phi\sin\phi$

$A_y = A_\rho\sin\phi + A_\phi\cos\phi = \rho z\sin\phi\sin\phi + 3\rho\cos\phi\cos\phi$

$$\vec{A} = \vec{a}_x(\rho z \sin\phi\cos\phi - 3\rho\cos\phi\sin\phi) + \vec{a}_y(\rho z\sin\phi\sin\phi + 3\rho\cos\phi\cos\phi) + \vec{a}_z\,\rho\cos\phi\sin\phi$$

(b). $\vec{B} = \vec{a}_r\, r^2 + \vec{a}_\phi \sin\theta$

$A_r = r^2, \quad A_\phi = \sin\theta, \quad A_\theta = 0$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ 0 & 1 & 0 \\ \cos\theta & 0 & -\sin\theta \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_\theta \end{bmatrix}$$

$$A_\rho = A_r\sin\theta + A_\theta\cos\theta = r^2\sin\theta$$

$$A_\phi = A_\phi = \sin\theta$$

$$A_z = A_r\cos\theta - A_\theta\sin\theta = r^2\cos\theta$$

$$\vec{B} = \vec{a}_\rho\, r^2\sin\theta + \vec{a}_\phi\sin\theta + \vec{a}_z\, r^2\cos\theta$$

(c). $\vec{C} = \vec{a}_x\, 3x + \vec{a}_y\, xy^2 + \vec{a}_z\, yz$

$A_x = 3x, \quad A_y = xy^2, \quad A_z = yz$

$$\begin{bmatrix} A_r \\ A_\phi \\ A_\theta \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$$

$$= 3x\sin\theta\cos\phi + xy^2\sin\theta\sin\phi + yz\cos\theta$$

$$A_\phi = -A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$$

$$= -3x\cos\theta\cos\phi + xy^2\cos\theta\sin\phi - yz\sin\theta$$

$$A_\theta = -A_x\sin\phi + A_y\cos\phi$$

$$= -3x\sin\phi + xy^2\cos\phi$$

$$\vec{c} = \vec{a}_r(3x\sin\theta\cos\phi + xy^2\sin\theta\sin\phi + yz\cos\theta) + \vec{a}_\phi(-3x\cos\theta\cos\phi + xy^2\cos\theta\sin\phi - yz\sin\theta)$$

$$+ \vec{a}_\theta(-3x\sin\phi + xy^2\cos\phi)$$