

Lec 13:

Ex1

Determine for systems which is observable:

Exercise 1

Determine for each of the following systems, whether it is observable:

$$(1) : \begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}, \quad (2) : \begin{cases} \dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}, \quad (3) : \begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}$$

$$(4) : \begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{cases}, \quad (5) : \begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 2 \end{pmatrix} x \end{cases}, \quad (6) : \begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{cases}$$

$$O = \begin{bmatrix} C \\ CA \end{bmatrix}, \det(O) \neq 0 \text{ så er den observable}$$

$$1) O = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \det(O) = 0 \text{ non observable}$$

Exercise 2

In the previous exercise sheet, we considered the following controllable system:

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{aligned} \tag{1}$$

which can be shown to be also observable with the output above. The state feedback

$$u = Fx = \begin{pmatrix} -2 & 2 \end{pmatrix} x$$

was shown to achieve a pole placement corresponding to the characteristic polynomial $s^2 + 3s + 2$.

1. Design a full order observer for (1) having characteristic polynomial $s^2 + 9s + 20$, i.e. find an observer gain L such that the eigenvalues of $A + LC$ become $\lambda_1 = -4$ og $\lambda_2 = -5$
2. Draw a diagram for an observer based compensator using F and L .
3. Verify, by computing the eigenvalues of the system matrix for the closed loop system:

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & BF \\ -LC & A+LC+BF \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

1) og 2)

Ex 2.

$$\dot{x} = \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u ; u = Fx = \begin{pmatrix} -2 & 2 \end{pmatrix} x$$

$$y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

1) Design full order observer

$$O = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 13 & -17 \end{pmatrix}$$

$$t_2 = \frac{1}{-30} \begin{bmatrix} -17 & -1 \\ -13 & 1 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{30} \\ -\frac{1}{30} \end{pmatrix}$$

$$t_1 = A t_2 = \begin{bmatrix} 7 & -9 \\ 6 & -8 \end{bmatrix} \cdot \begin{pmatrix} \frac{1}{30} \\ -\frac{1}{30} \end{pmatrix} = \begin{pmatrix} \frac{16}{30} \\ \frac{14}{30} \end{pmatrix}$$

$$T = \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 1 \\ 14 & -16 \end{bmatrix}$$

$$A_o = T^{-1} A T = \begin{bmatrix} 1 & 1 \\ 14 & -16 \end{bmatrix} \begin{bmatrix} 7 & -9 \\ 6 & -8 \end{bmatrix} \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} \Rightarrow \det(\lambda I - A) = \lambda^2 + \lambda - 2$$

$$C_o = (T = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix}) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$L_o = \begin{pmatrix} 1 & -9 \\ -2 & -28 \end{pmatrix} = \begin{pmatrix} -8 \\ -22 \end{pmatrix}$$

$$L = T L_o = \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix} \cdot \begin{pmatrix} -8 \\ -22 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

for closed loop

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & BF \\ -LC & A+LC+BF \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

eig $\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \lambda \begin{bmatrix} -5 \\ -7 \\ -2 \\ -1 \end{bmatrix}$ } the new poles
 } the same poles

eig(A+LC) = $\begin{bmatrix} -4 \\ -5 \end{bmatrix}$

```
close all
clear
```

```
xdot = [7 -9; 6 -8];
```

```
y = [1 1];
```

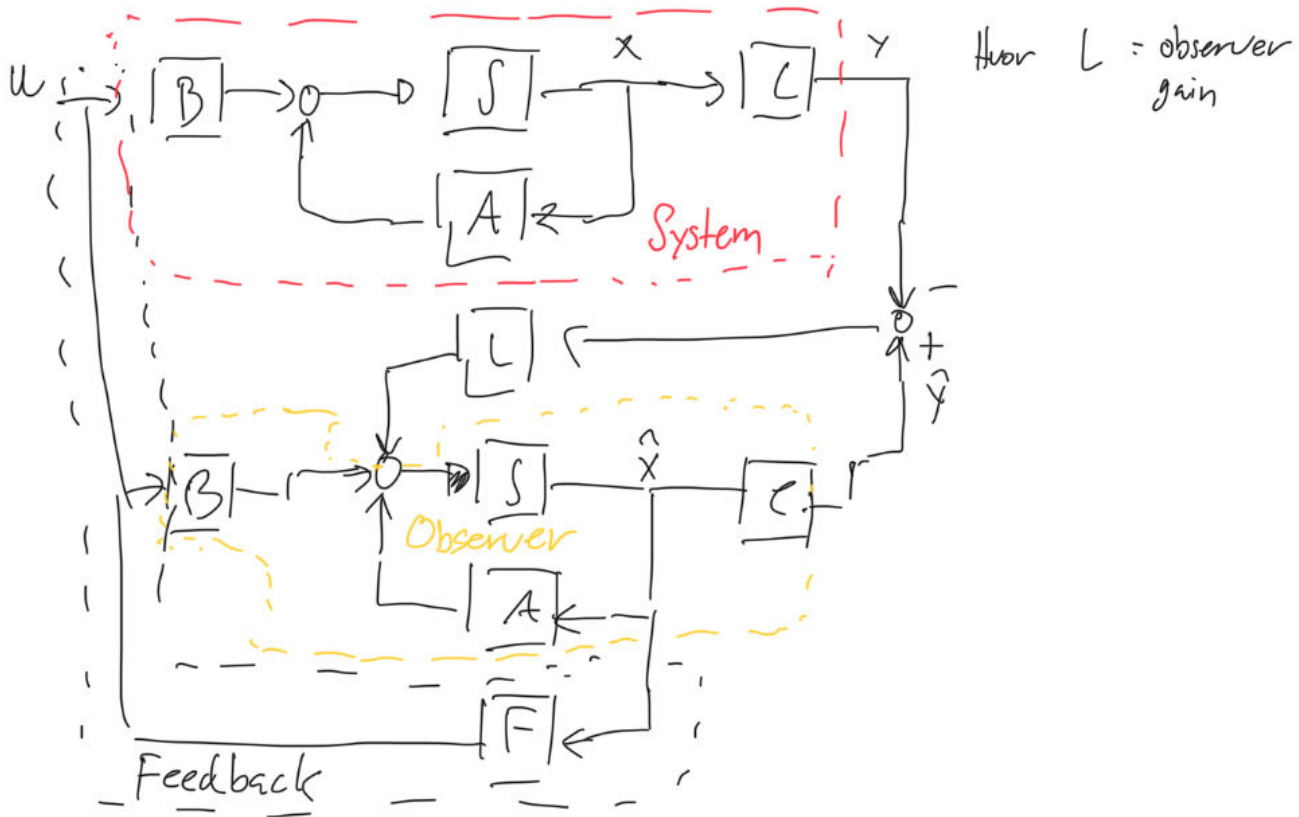
```
O = [y; y*xdot];
t2 = inv(O)*[0;1];
t1 = xdot*t2;
T = [t1(1) t2(1); t1(2) t2(2)];
Ao = inv(T)*xdot*T;
Co = y*T;
Lo = [-8; -22];
L = T*Lo;
eig(xdot + L*y);
```

```
ans = 2x1
```

ans = 2x1

-4
-5
-5
-4

2) Draw a diagram for an observer based compensator using F and L



3)

```
closedloop = [xdot [4;3]*[-2 2]; -L*y (xdot + L*y + [4;3]*[-2 2])];
eig(closedloop)
```

ans = 4x1

-5.000000000000002
-3.999999999999996
-2.000000000000001
-1.000000000000001