Spørgsmål 2 Bilinear

$-\infty < \Omega < \infty$ map to UC i.e. $-\pi < \omega < \pi$

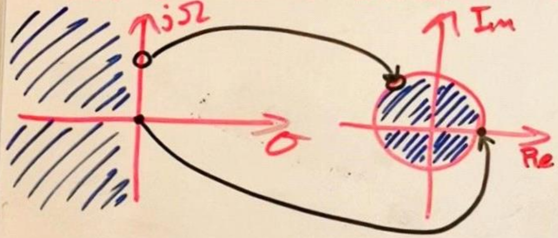
So we want $H(z) = H(s)|_{f(z)}$ udgangspunkt.

$z = e^{j\omega}$, $\omega = T\Omega$, $s = j\Omega$ ➡ $z = e^{jT\frac{s}{j}} = e^{sT}$ isolate sT => $\ln(z) = sT$

Solve for $s = \frac{1}{T}\ln(z) = \frac{1}{T}\cdot 2\left\{\frac{z-1}{z+1} + \frac{1}{3}\left(\frac{z-1}{z+1}\right)^3 + \dots\right\}$ then $\boxed{s = f(z) = \frac{2}{T}\cdot\frac{z-1}{z+1}}$ Bilinear transform

Taylor serie for ln

Mapping Bilinear Transform from S ⇒ Z plane



① $s=0 \Rightarrow z=1$; $z = \frac{1+\frac{T}{2}s}{1-\frac{T}{s}s}\Big|_{s=0} = 1$

② Left-handside of S-plane $Re(s) = \sigma < 0$

$$|z| = \frac{\sqrt{(1+\frac{T}{\sigma})^2 + (\frac{T}{2}\Omega)^2}}{\sqrt{(1-\frac{T}{\sigma})^2 + (\frac{T}{2}\Omega)^2}} < 1$$

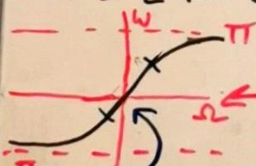
③ $j\Omega$-axis in s-plane $z = \frac{1+\frac{T}{2}(\sigma+j\Omega)}{1-\frac{T}{2}(\sigma+j\Omega)}$

$\sigma = 0 \Rightarrow |z| = 1 \ \forall \Omega$

Relation between Ω and ω ; Aliasing

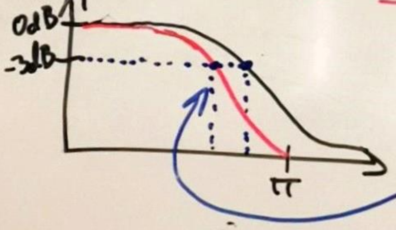
By using mit Armbøjningen and eulers formel

$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$; $]-\pi;\pi[$



this Avoids Aliasing

næsten lineær



$|H_c|$ – IIM

$|H(z)|$ – Bilinear

3dB WarPed!

Pre-WarPing

$f_c \cdot 2\pi$

$\Omega_{c,new} = \frac{2}{T}\cdot\tan\left(\frac{\omega_c}{2}\right)$

$\frac{1}{f_s}$

$\Rightarrow$ Sæt $\Omega_{c,new}$ ind i $H(s)$

Eks. 1. Butterworth HP

$H(s) = \frac{s}{s+\Omega_{c,new}}$

eks. 1. Butter LP

$H(s) = \frac{\Omega_{c,new}}{s+\Omega_{c,new}}$