Lecture 2

Ex 1.

1)

Transfer function: $\frac{\mathrm{Kp}}{s^2 + s + \mathrm{Kp}}$

For at finde overshoot så kan man aflæse grafen på slide 12.

$\zeta \approx 0.45$ for 20% overshoot

$2\zeta\omega_n = 1$

$\omega_n^2 = \mathrm{Kp} \Leftrightarrow \omega_n = \sqrt{\mathrm{Kp}}$

$2 \cdot 0.45 \cdot \sqrt{\mathrm{Kp}} = 1 \Leftrightarrow \mathrm{Kp} = 1.23$

```
syms Kp
vpa(solve(2*0.45*sqrt(Kp)==1,Kp));
```

2)

Rise time and settle time:

$\omega_n = \sqrt{\mathrm{Kp}} = \sqrt{1.23} = 1.109$

$T_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{1.109} = 1.6\,\mathrm{sekunder}$

$T_s(x) = \frac{-\ln(x)}{\omega_n\zeta} = \frac{-\ln(0.02)}{1.109 \cdot 0.45} = 7.8\,\mathrm{sekunder}$ hvor x er settle time i procent.

```
-log(0.02)/(1.109*0.45);
```

3)

System type 1 se systemet skal betragtet som Open Loop!

$K_v = \lim_{s\to 0} s \cdot G(s) \ = \lim_{s\to 0} s \cdot \frac{K}{s+1} \cdot \frac{1}{s} = K = 1.23$

$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K}{s+1} \cdot \frac{1}{s} = \infty$

$K_a = s^2 \cdot \lim_{s\to 0} G(s) = \lim_{s\to 0} s^2 \frac{K}{s+1} \cdot \frac{1}{s} = 0$

Deraf ved man systemet er af type 1 se slides side:

2. Transient reponse, system type and steady state errors (two controllers).

Another position control of the DC-motoren is shown in the figure.

- Determine the closed loop transfer function $T(s) = \frac{\Theta(s)}{R(s)}$.
- Find the K_1 and K_2 that gives an overshoot (M_P) equal 20 % and a settling time (2%) (t_s) equal 1 [sec].
- Find the rise time (t_r) of the system?
- What is the system type? Determine the steady state errors for a step, a ramp and a parabola.

Ex 2.

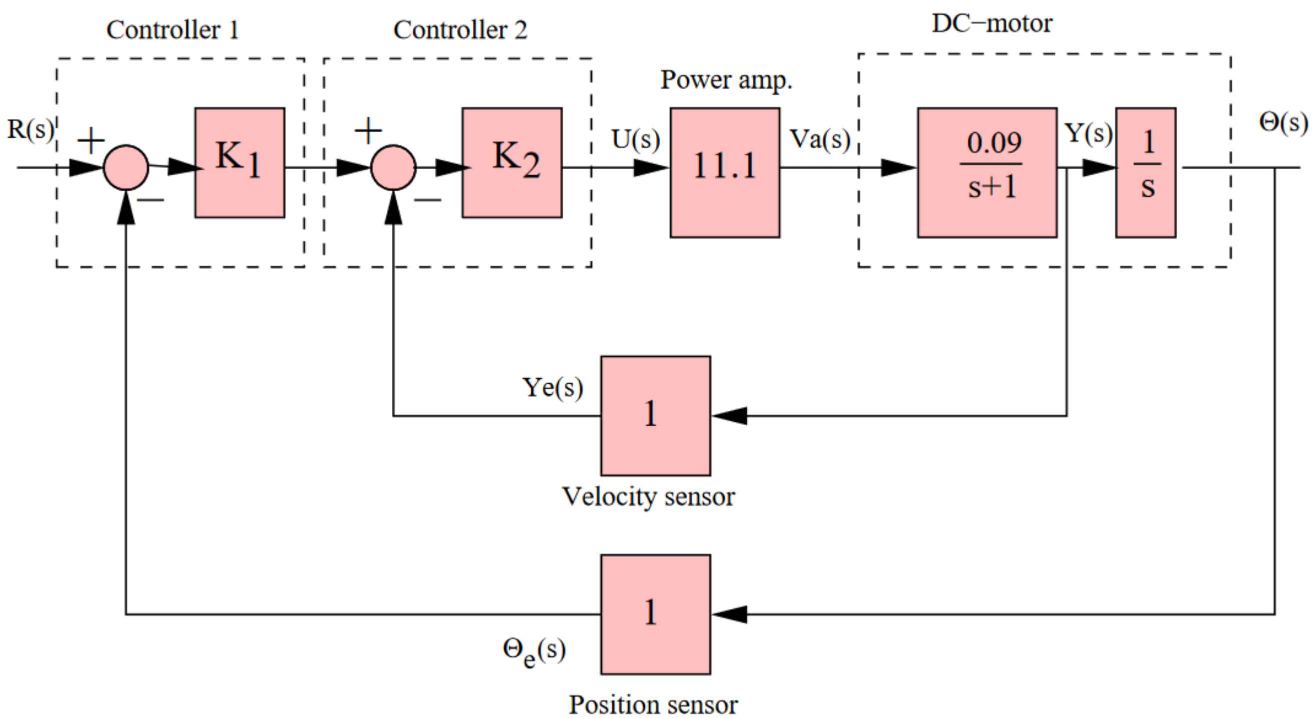

Figure 2: *Position controlled motor, two controllers*

1)

Transfer function:

Reducer det første med velocity sensor som close loop: $\dfrac{K_2\frac{1}{s+1}}{1+K_2\frac{1}{s+1}}$ reducér

$$\frac{\frac{K_2}{s+1}}{\frac{s+1+K_2}{s+1}} = \frac{K_2 s + K_2}{s^2+2s+K_2 s+K_2+1} = \frac{K_2(s+1)}{(s+1)(K_2+s+1)} = \frac{K_2}{(K_2+s+1)}$$

```
syms k1 k2 s
expr1 = (k2*11.1*(0.09/(s+1)))/(1+(k2*11.1*(0.09/(s+1))));
expr2 = (k1*expr1*(1/s))/(1+(k1*expr1*(1/s)));
```

```
collect(expr2,s); %fordi der rundes af så bliver der 1 i tæller og nævner i stedet for 999 og 1000 closed loop
```

ans =

$$\frac{999\,k_1\,k_2}{1000\,s^2 + (999\,k_2 + 1000)\,s + 999\,k_1\,k_2}$$

2)

Ved et overshoot på $\mathrm{Mp} = 20\%$ der er $\zeta \approx 0.45$ ses udfra fra en graf(slides), settle time % = 0.02% og $t_s = 1$ sekund

$$t_s(x) = \frac{-\ln(x)}{\zeta\omega_n} = \frac{-\ln(0.02)}{0.45 \cdot \omega_n} = 1 \Leftrightarrow \omega_n = 8.7 \text{ sekunder}$$

Herfra for at finde k1 og k2:

$$2\zeta\omega_n = K_2 + 1 \Leftrightarrow K_2 = 2\zeta\omega_n - 1 = 6.8$$

$$\omega_n^2 = K_1K_2 \Leftrightarrow K_1 = \frac{\omega^2}{K_2} = 11.1$$

3)

$t_r \approx \dfrac{1.8}{\omega_n} = \dfrac{1.8}{8.7} = 0.21$ sekunder se slide 14

4)

System type and system errors:

Open loop transfer function: $G(s) = \dfrac{K_1K_2}{K_2 + s + 1} \cdot \dfrac{1}{s}$ her ses der er en pole i 0 deraf en type 1 alt dette findes på slide 23

$$K_v = \lim_{s\to 0} \mathrm{sG}(s) = s \cdot \frac{K_1K_2}{K_2 + s + 1} \cdot \frac{1}{s} = \frac{K_1K_2}{K_2 + 1} = 9.67$$

Error = $\dfrac{1}{K_v} = 0.1$

Type 1	
Static error constant	Error
$K_p = \infty$	0
$K_v = \text{constant}$	$\frac{1}{K_v}$
$K_a = 0$	∞

```
clear all
close
syms s
k1 = 11.1;
k2 = 6.8;
x = limit((k1*k2)/(k2+s+1),s,0)
```

x =

$$\frac{629}{65}$$

var = 9.676923076923076923076923076923

```
ess = vpa(1/x) %ramp grundet tabel og type
```

ess = 0.10333863275039745627980922098569

Ex 3.

3. Transient response, system type and steady state errors (PD-controller).

On the figure a PD-controlled DC-motor is shown.

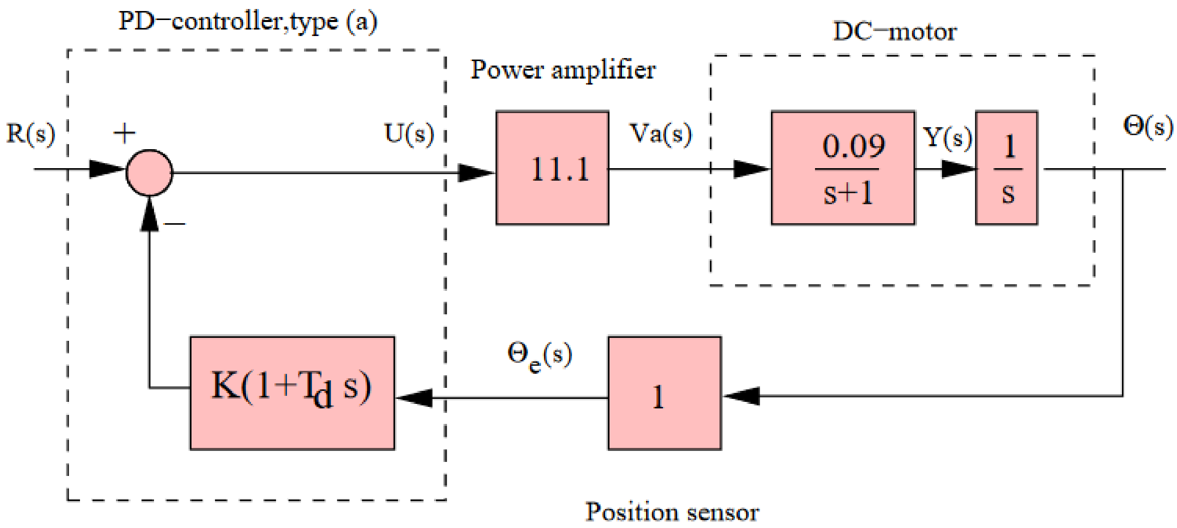

Figure 3: *Position controlled motor, PD-controller*

- Determine K and T_d giving an overshoot (M_P) of 20 % and a settling tir (t_s)(2%) equal to 1 [sec].
- Find the rise time (t_r).
- What is the system type? Determine the steady state errors for a step, a ran and a parabola.

1)

```
close all
clear
syms s K Td
expr1 = (1/(s+1))*1/s;
expr2 = K*(1+Td*s);
G = (expr1)/(1+(expr2*expr1)); %transfer function
collect(G,s)
```

ans =

$$\frac{1}{s^2 + (K\,\mathrm{Td} + 1)\,s + K}$$

For at finde overshoot så kan man aflæse grafen på slide 12.

$\zeta \approx 0.45$ for 20% overshoot

$$t_s(x) = \frac{-\ln(x)}{\zeta \omega_n} = \frac{-\ln(0.02)}{0.45 \cdot \omega_n} = 1 \Leftrightarrow \omega_n = 8.7 \text{ sekunder}$$

$$\omega_n^2 = K = 8.7^2 = 75.7$$

$$2\zeta\omega_n = T_d \cdot K + 1 \Leftrightarrow T_d = \frac{2\zeta\omega_n - 1}{K} = 0.09 \text{ sekunder}$$

2)

$$t_r \approx \frac{1.8}{\omega_n} = \frac{(1.8)}{\sqrt{75.7}} = 0.2 \text{ sekunder}$$

3)

$$\frac{1}{s^2 + (\mathrm{Td}\, K + 1)\, s + K}$$

Ingen unit feedback system derfor brug formel:

and the steady-state error is given again by the Final Value Theorem:

$$e_{ss} = \lim_{s \to 0} s \frac{1 - \mathcal{T}(s)}{s^{k+1}} = \lim_{s \to 0} \frac{1 - \mathcal{T}(s)}{s^k}. \tag{4.37}$$

As before, the result of evaluating the limit in Eq. (4.37) can be zero, a nonzero constant, or infinite and if the solution to Eq. (4.37) is a nonzero constant, the system is referred to as *Type k*. Notice that a system of Type 1 or higher has a closed-loop DC gain of 1.0, which means that $\mathcal{T}(0) = 1$ in these cases.

$$e_{ss} = \lim_{s \to 0} s \frac{1 - T(s)}{s^{k+1}} = \lim_{s \to 0} \frac{1 - T(s)}{s^k}$$

Closed loop transfer function: $T(s) = \dfrac{1}{s^2 + (\mathrm{KT}_d + 1)s + K}$

$$e_{ss} = \lim_{s \to 0} \frac{1 - \dfrac{1}{s^2 + (\mathrm{KT}_d + 1)s + K}}{s^k} = \lim_{s \to 0} \frac{\dfrac{s^2 + (\mathrm{KT}_d + 1)s + K - 1}{s^2 + (\mathrm{KT}_d + 1)s + K}}{s^k} = \lim_{s \to 0} \frac{1}{s^k} \frac{K - 1}{K}$$ når s er gået imod 0

Det er non zero når k = 0 derfor er systemet type k eller type 0

The steady state error for step:

$$e_{ss} = \lim_{s \to 0} s(1 - T(s))R(s) = \lim_{s \to 0}(1 - T(s)) = \lim_{s \to 0} \left(\frac{s^2 + (\mathrm{KT}_d + 1)s + K - 1}{s^2 + (\mathrm{KT}_d + 1)s + K} \right) = \frac{K - 1}{K} = 0.9868$$

$$y_{ss} = 1 - 0.9868 = 0.0132$$

Ex. 4

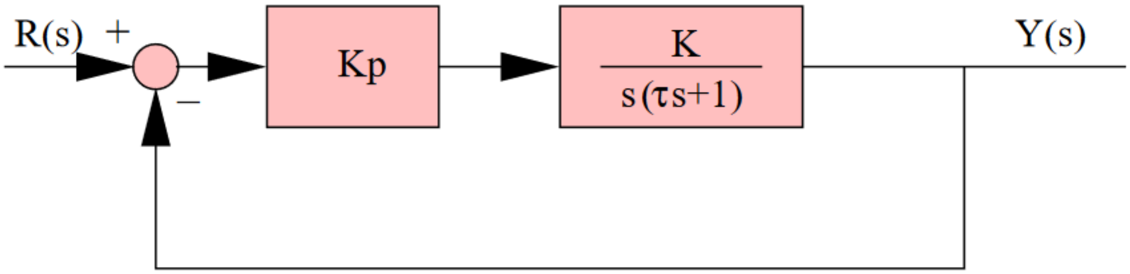

Figure 4: *Block diagram of a proportional-controlled system*

- With known $\tau > 0$ and $K > 0$ an expression for K_p giving 10 % overshoot must be determined. So find K_p given by τ and K.

Close loop transfer function:

$$T(s) = \frac{\mathrm{Kp} \cdot \frac{K}{s(\tau s + 1)}}{1 + \mathrm{Kp} \cdot \frac{K}{s(\tau s + 1)}} = \frac{\mathrm{Kp} \cdot K}{s^2 \tau + s + \mathrm{Kp} \cdot K} = \frac{\mathrm{Kp} \cdot K}{\tau} \cdot \frac{1}{s^2 + \frac{s}{\tau} + \frac{\mathrm{KpK}}{\tau}}$$

$$2\zeta\omega_n = \frac{1}{\tau} \text{ og } \omega_n^2 = \frac{\mathrm{KpK}}{\tau}$$

$$2\zeta\omega_n = \frac{1}{\tau} = 2\zeta\sqrt{\frac{\mathrm{KpK}}{\tau}} \Rightarrow \frac{1}{4\zeta^2\tau} = K_p K \Rightarrow K_p = \frac{1}{4\zeta^2\tau K}$$

sæt det her ind: Over shoot $10\% \approx 6.5$ for ζ værdi