3. Frekvensresponse og geometrisk fortolkning ampresponse og faseresponse

Filter design: $H(z) = \dfrac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = \dfrac{B(z)}{A(z)}$ We want to find poles and zeros of $H(z)$

3 scenarios:

	extend:
$M = N$	z^M
$M < N$	z^N
$M > N$	z^M

Factorize $H(z)$: $H(z) = \dfrac{b_0}{a_0} \dfrac{\prod_{k=1}^{L} (z - c_k)}{\prod_{k=1}^{L} (z - d_k)}$, $L = \max\{N, M\}$ We want same degree of polynomium

frequency response i.e $z = e^{jw}$

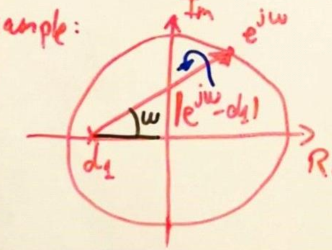
$H(e^{jw}) = \dfrac{b_0}{a_0} \dfrac{\prod_{k=1}^{L} (e^{jw} - c_k)}{\prod_{k=1}^{L} (e^{jw} - d_k)}$ → From here we can get amp and phase response

Amp resp: $|H(e^{jw})| = \left|\dfrac{b_0}{a_0}\right| \dfrac{\prod_{k=1}^{L} |e^{jw} - c_k|}{\prod_{k=1}^{L} |e^{jw} - d_k|}$; · Phase resp: $\angle H(e^{jw}) =$

$\arg\left\{\dfrac{b_0}{a_0}\right\} + \sum_{k=1}^{L} \arg\{e^{jw} - c_k\} - \sum_{k=1}^{L} \arg\{e^{jw} - d_k\}$

angles of the vectors

Example:



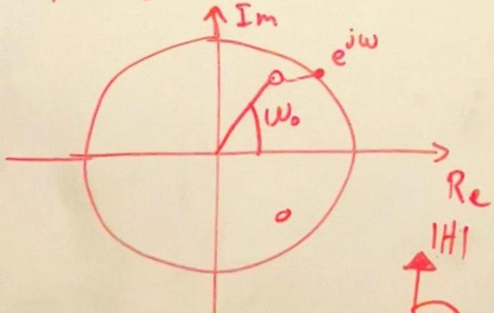
For each vector $|V_k|$ as a function of ω

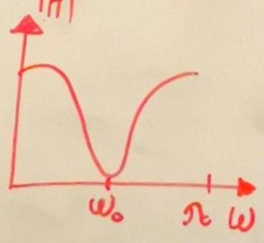
$|V_1| = \sqrt{((\cos(\omega) + 1)^2 + \sin(\omega))^2}$ eval $|H(e^{jw})|$ for $\omega \in [0; \pi]$

For each $\arg\{V_k\}$ —〃—

$\tan \varphi_k = \dfrac{\sin(\omega)}{\cos(\omega) + 1} \Rightarrow \varphi_k = \arctan\left\{\dfrac{\sin(\omega)}{\cos(\omega) + 1}\right\}$

Pole/zero location:





More Examples here: :cat: