IIR-Filter ; Impulse Invariant metoden :

ω = Diskret
Ω = Kontinuert

$\omega = \Omega T = 2\pi f \frac{1}{f_s}$

$T = \frac{1}{f_s}$

Ex. 1 $R = 1\,k\Omega$ $C = 1\,\mu F$

Transfer function for lowpass

$$H(s) = \frac{1}{1 + sRC} \xrightarrow{\text{Invers laplace}} h(t) = \frac{1}{RC} e^{-\frac{t}{RC}}$$

Impouuuls respons

$$\Downarrow$$

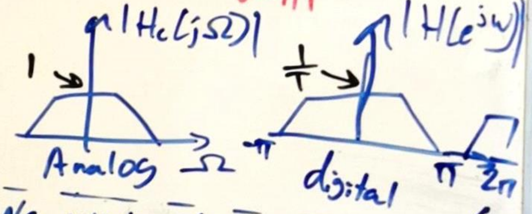
$$h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} \cdot u(t) \Longrightarrow h[nT] = h[n] = T \cdot h(t)\Big|_{t=nT}$$

for at gøre causal

To have normalized Amplitude

$$\Downarrow$$

$$h[n] = \frac{T}{RC} e^{-\frac{nT}{RC}} \cdot u[n] \longrightarrow$$ Express as a convolution sum



Convolution sum: $y[n] = h[n] * x[n]$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Sæt h[n] fra tidligere

$$= \frac{T}{RC} \sum_{k=0}^{\infty} e^{\frac{-kT}{RC}} \cdot x[n-k]$$

Note: Matematisk godt til at vise hvad filteret gør ved et input men svært at implementer

We want discrete time TF $H(z)$.

$$H(z) = Z\{h(n)\}$$

$$= \sum_{n=0}^{\infty} h(n) \cdot z^{-n}$$

causal

$$= \frac{T}{RC} \sum_{n=0}^{\infty} \underbrace{\left\{ e^{\frac{-T}{RC}} \cdot z^{-1} \right\}^n}$$ $\longrightarrow$ geo serie rewrite form

$$H(z) = \frac{b}{1 - a z^{-1}}, \quad b = \frac{T}{RC}, \quad a = e^{\frac{-T}{RC}}$$