Alle Opgaver Løst igen ark :)

Lecture 1

Problem 1.1.

A hypothetical isotropic antenna is radiating in freespace. At a distance of 100 meters from the antenna the total electrical field (E_Θ) is measured to be 5 V/m. Find the

a) Power density (W_{rad})

(a) Solution:

1.1

$$r = 100\,m$$

$$E_\theta = 5\ V/m$$

$$U = r^2 W_{RAD}$$

$$U = \frac{r^2}{2\eta}|E_\theta|^2$$

$$W_{RAD} = \frac{1}{2}[E \times H^*] = \frac{|E_\theta|^2}{2\eta} = \frac{\left(5\ \frac{V}{m}\right)^2}{2 \cdot 120\pi\,\Omega} = 0.03316\ \frac{W}{m^2}$$

```
close all
clear
r = 100;
Etheta = 5;
eta = 120*pi;
Wrad = abs(Etheta)^2/(2*eta) %answer in W/m^2
```

```
Wrad =
       0.0331572798108115
```

b) Power radiated (P_{rad})

Solution:

$$P_{RAD} = \int_0^{2\pi}\left[\int_0^{\pi} W_{RAD} \cdot r^2 \cdot \sin\theta\, d\theta\right] d\varphi = W_{RAD} \cdot r^2 \int_0^{2\pi}\left[\int_0^{\pi} \sin\theta\, d\theta\right.$$

$$= 0.03316\ \frac{W}{m^2} \cdot (100\ m)^2 \cdot 4\pi = 4166.9\ W$$

```
syms t g
```

```
fun1 = int(Wrad*r^2*sin(t),t,0,pi);
Prad = int(fun1,g,0,2*pi);
vpa(Prad) %answer in W;
```

ans = 4166.666666666676908934378419728

Problem 1.2

The maximum radiation intensity of a 90% efficient antenna is 200 mW/unit solid angle.
Find the directivity and gain (dimensionless and in dB) when the:
a) Input power is 125.66 mW

Solution:

$$D_0 = \frac{4\pi \cdot U_{max}}{P_{RAD}} = \frac{4\pi \cdot 200\ mW/U.S.A.}{0.9 \cdot 125.66\ mW} = 22.22 = 13.47\ dB$$
$$G_0 = e_t D_0 = 0.9 \cdot 22.22 = 20.00 = 13.01\ dB$$

```
close all
clear
Umax = 200e-3;
inputpower = 125.66e-3;
Prad = 0.9*inputpower;
D0 = (4*pi*Umax)/(Prad);%dimensionless
D0dB = 10*log10(D0);%dB
G0 = 0.9*D0;
G0dB=10*log10(G0)
```

```
G0dB =
        13.0104280430616
```

b) Radiated power is 125.66 mW

solution: Nu uden 0.9 fra tidligere af.

b) radiated power is 125.66 mW

$$D_0 = \frac{4\pi \cdot U_{max}}{P_{rad}} = \frac{4\pi \cdot 200\ mW/\text{Frihed}}{125.66\ mW} = 20 = 13.01\ dB$$
$$G_0 = e_t D_0 = 0.9 \cdot 20 = 18 = 12.55\ dB$$

Problem 1.3

In target-search ground-mapping radars it is desirable to have ecco power received from target, of constant cross section, to be independent of its range. For one such application, the desireble radiation intensity of the antenna is given by
$U(\Theta,\Phi) = 1$ for $0^\circ \leq \Theta < 20^\circ$
$U(\Theta,\Phi) = 0{,}342 \csc(\Theta)$ for $20^\circ \leq \Theta < 60^\circ$
$U(\Theta,\Phi) = 0$ for $60^\circ \leq \Theta \leq 180^\circ$

Find the directivity in dB using the exact formula.

Solution:

1.3

$$U(\theta,\phi) = 1 \quad \text{for} \quad 0^\circ \le \theta \le 20^\circ$$

$$U(\theta,\phi) = 0.342\cdot\csc(\theta) \quad \text{for} \quad 20^\circ \le \theta \le 60^\circ$$

$$U(\theta,\phi) = 0 \quad \text{for} \quad 60^\circ \le \theta \le 180^\circ$$

$$P_{RAD} = \int_0^{2\pi}\left[\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\,d\theta\right]d\phi$$

$$= 2\pi\cdot\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\,d\theta$$

$$= 2\pi\left[\int_{0^\circ}^{20^\circ}\sin(\theta)\,d\theta + \int_{20^\circ}^{60^\circ} 0.342\cdot\csc(\theta)\cdot\sin(\theta)\,d\theta + \int_{60^\circ}^{180^\circ} 0\cdot\sin(\theta)\,d\theta\right]$$

$$= 2\pi\left[-\cos(\theta)\right]_{0^\circ}^{20^\circ} + 2\pi\cdot 0.342\cdot\left[\theta\right]_{20^\circ=\pi/9}^{60^\circ=3\pi/9}$$

$$= -2\pi(0.94-1) + 2\pi\cdot 0.342\left(\frac{3\pi}{9} - \frac{\pi}{9}\right)$$

$$= 1.88 \text{ W}$$

$$D_0 = \frac{4\pi\cdot U_{max}}{P_{RAD}} = \frac{4\pi}{1.88} = 6.687 = 8.25 \text{ dB}$$

```
syms t
Umax = 1;
Prad = 2*pi*(int(sin(t),t,deg2rad(0),deg2rad(20))
+ int(0.342*csc(t)*sin(t),t,deg2rad(20),deg2rad(60)) +
int(0*sin(t),t,deg2rad(60),deg2rad(180)));
D0 = (4*pi*Umax)/(Prad);
vpa(D0);
toDbD0 = 10*log10(D0);
vpa(toDbD0) %dB
```

ans = 8.2525943806096608660124972406441

Problem 1.4

The radiation intensity of an antenna is given by

$U(\Theta,\Phi) = \cos^4(\Theta)\sin^2(\Phi)$,
for $0 \le \Theta \le \pi/2$ and $0 \le \Phi \le 2\pi$ (i.e. In the upper half-space)
It is 0 in the lower half-space.

Find the:
a) Exact directivity (dimensionless and in dB).

Solution: Husk at hvis Umax ikke er givet så er den normaliseret til 1!!

1.4

$$U(\theta,\phi) = \cos^4(\theta)\cdot\sin^2(\phi) \quad \text{for } 0 \le \theta \le \frac{\pi}{2},\ 0 \le \phi \le 2\pi$$

$$U(\theta,\phi) = 0 \quad \text{elsewise}$$

a)

$$P_{RAD} = \int_0^{2\pi}\left[\int_0^{\pi/2} \cos^4(\theta)\cdot\sin^2(\phi)\cdot\sin(\theta)\, d\theta\right] d\phi$$

$$= \int_0^{2\pi} \sin^2(\phi)\, d\phi \cdot \int_0^{\pi/2} \cos^4(\theta)\cdot\sin(\theta)\, d\theta$$

$$= \frac{\pi}{5}\ \text{W} \quad \text{(apparently)}$$

$$D_0 = \frac{4\pi\cdot U_{max}}{P_{RAD}} = \frac{4\pi}{\pi/5} = 20 = 13\ \text{dB}$$

```
close all
clear
syms t
Umax = 1;
```

```
Prad = int(sin(t)^2,t,0,2*pi) * int(cos(t)^4*sin(t)^2*sin(t),t,0,pi/2); % Andet
svar men jeg kan ikke se hvordan svaret kan være rigtigt?
vpa(Prad);
D0 = 4*pi*Umax/Prad;
vpa(10*log10(D0))
```

ans = $18.450980400142568307122162585926$

b) Elevation plane half-power beamwidth in degrees.

Solution:

b) $U(\theta, \phi = \frac{\pi}{2}) = \cos^4(\theta)$

$\cos^4\left(\frac{HPBW}{2}\right) = \frac{1}{2} \Rightarrow 2 \cdot \cos^{-1}(\sqrt{0.5}) = 65.5° = HPBW$

b) Elevation plane: theta varies, phi fixed. => choose $\phi = \dfrac{\pi}{2}$

$$U(\theta, \phi = \pi/2) = \cos^4(\theta) \qquad 0 \le \theta \le \pi/2$$

$$\cos^4\left[\frac{HPBW(elevation)}{2}\right] = \frac{1}{2}$$

$$HPBW(elevation) = 2 \cdot \cos^{-1}(\sqrt{0.5}) = 65.5°$$

```
close all
clear
syms x
HPBW = solve(cos(x/2)^4==1/2,x);
deg = rad2deg(HPBW);
vpa(deg) % Nummer 2!
```

ans =

$$\begin{pmatrix} -65.530199479297808348650789880557 \\ 65.530199479297808348650789880557 \\ -180.0 + 87.580662372692780339764902555967\,\mathrm{i} \\ 180.0 - 87.580662372692780339764902555967\,\mathrm{i} \end{pmatrix}$$

Lecture 2

2.1 For an X-band (8.2 – 12.4 GHz) rectangular horn, with aperture dimension of 5,5 cm and 7,4 cm , find its maximum effective aperture in cm^2 when its gain over isotropic is:

a) 14,8 dB

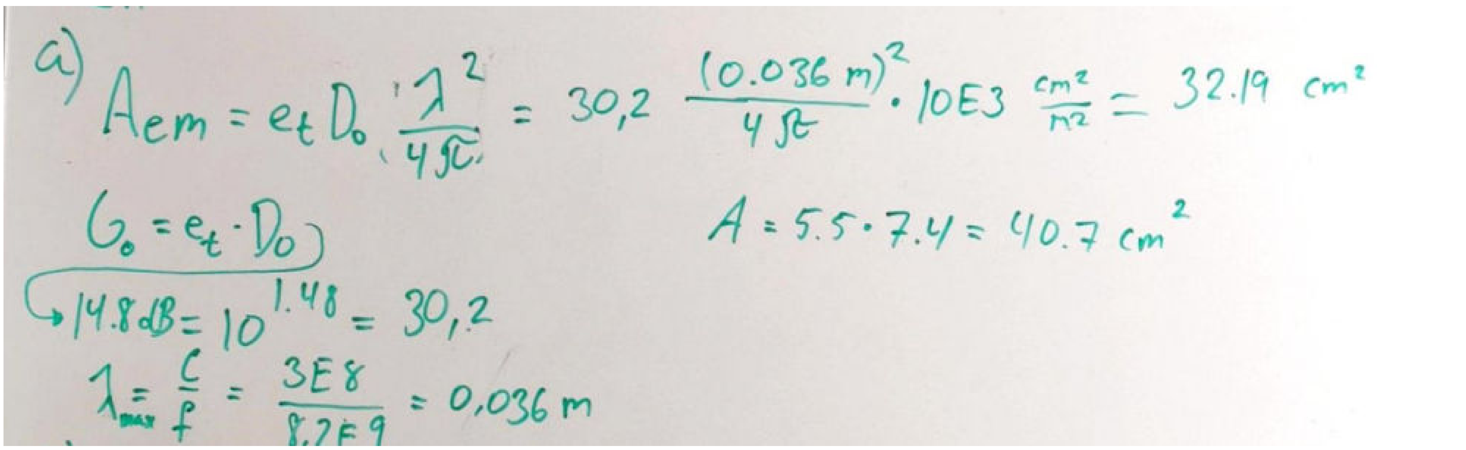
b) 16,5 dB

c) 18,0 dB

Solution to a) b) c)

Slide 5 lecture 2 for formel men den er isotropic og derfor kun dele af formlen bliver brugt.

a) b) og c)




```
close all
clear
syms;
% chose the small frequency instead apparently for the first, and middle
% for last and so on for the gainzz.
etDo = db2pow([14.8 16.5 18]);
lam = 3e8./[8.2e9 10.3e9 12.4e9];
Aem = etDo.*(lam.^2/(4*pi)) %HUSK ISQ :D
```

```
Aem = 1×3
    0.00321665838623415    0.00301549210670105    0.00293893022335922
```

```
A = 5.5*7.4;
```

Problems

2.2 A communication satellite is in the stationary orbit about the earth (22.300 statute miles ~ 36.000 km). Its transmitter generates 8 Watt. Assume the transmitting antenna is isotropic. Its signal is received by a 210 foot diameter tracking parabol antenna on the earth. Also assume no resistive losses in either antenna, perfect polarization match and perfect impedance matching at both antennas. At a frequency of 2 GHz, determinate the:

a) Power density in Watts/m^2 incident on the receiving antenna.

Solution:

2.2 Power Density Watts/m^2

a) $R = 36.000$ km, $f = 2$ GHz

$P_t = 8$ W

$$W_0 = \frac{P_t}{4\pi R^2} = \frac{8W}{4\pi (36000\,km)^2} = \underline{\underline{4.91E\text{-}16\ W/m^2}}$$

b)

$$P_r = A_{em} W_0 = G_r \frac{\lambda^2}{4\pi} \cdot W_0$$

$$= 10^6 \frac{(3E8/2E9)^2}{4\pi} \cdot 4.91E\text{-}16 = \underline{\underline{0.88\ pW}}$$

```
close all
clear
R = 36000e3;
```

```
f = 2e9;
Pt = 8;
Wo = Pt/(4*pi*R^2); %W/m^2 unit
```

b) Power received by the ground based antenna whose gain is 60 dBi

Solution:

$$(b)\quad A_{em} = \frac{\lambda^2}{4\pi} e_t D_0, \quad D_0 = 60dB = 10^6, \quad \lambda = 0.15m$$

$$A_{em} = \frac{(0.15)^2}{4\pi} 10^6 = 1790.493m^2$$

$$P_{received} = A_{em} P_i = 1790.493 \cdot 4.943 \cdot 10^{-16} = 8.85 \cdot 10^{-13}\, watts$$

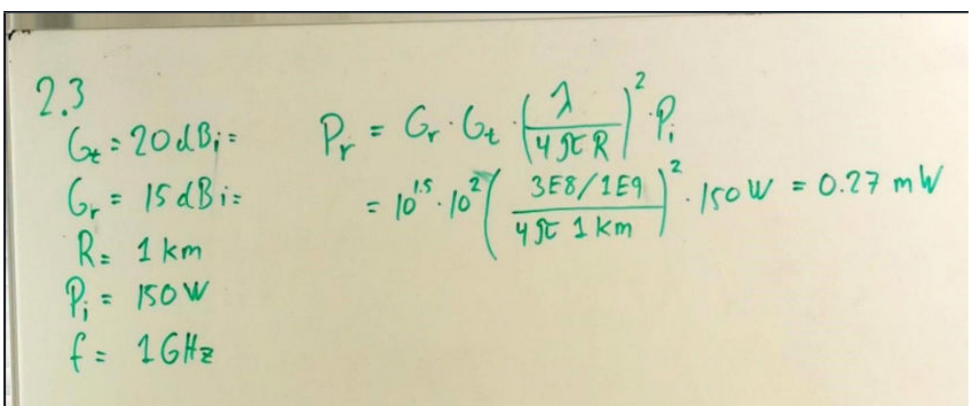
```
Do = db2pow(60);
Pr = 10^6*(3e8/2e9)^2/(4*pi)*Wo%Watts
```

```
Pr =
       8.7952416356196e-13
```

2.3 Transmitting and receiving antennas operating at 1 GHz with gains of 20 and 15 dBi respectively, are separated by a distance of 1 km.

Find the maximum power delivered to the load when the input power is 150 W. You can assume the antennas are polarization matched.

Solution:



$$\frac{P_r}{P_t} = \left|\hat{\rho}_t \bullet \hat{\rho}_r\right|^2 \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r}$$

$$G_{0t} = 20dB \Rightarrow G_{0t}\left(power\ ratio\right) = 10^2 = 100$$

$$G_{0r} = 15dB \Rightarrow G_{0r}\left(power\ ratio\right) = 10^{1.5} = 31.623$$

$$f = 1GHz \Rightarrow \lambda = 0.3m$$

$$R = 10^3 m$$

$$(a) \quad for \left|\hat{\rho}_t \bullet \hat{\rho}_r\right|^2 = 1$$

$$P_r = \left(\frac{0.3}{4\pi \cdot 10^3}\right)^2 (100)(31.623)\left(150 \cdot 10^3\right) = 270.344 \mu Watts$$

```
close all
clear
Gt = db2pow(20);
Gr = db2pow(15);
R = 1e3;
Pi = 150;
f = 1e9;
lam = 3e8/f;
Pr = Gr*Gt*(lam/(4*pi*R))^2*Pi;
num2sip(Pr) %Watts
```

```
ans =
'270.34 μ'
```

2.4 Repeat Problem 3 for the case of a reflecting ground and antenna height of both the receiver and transmitter of:

I. 3 meters

II. 5 meters

III. 10 meters

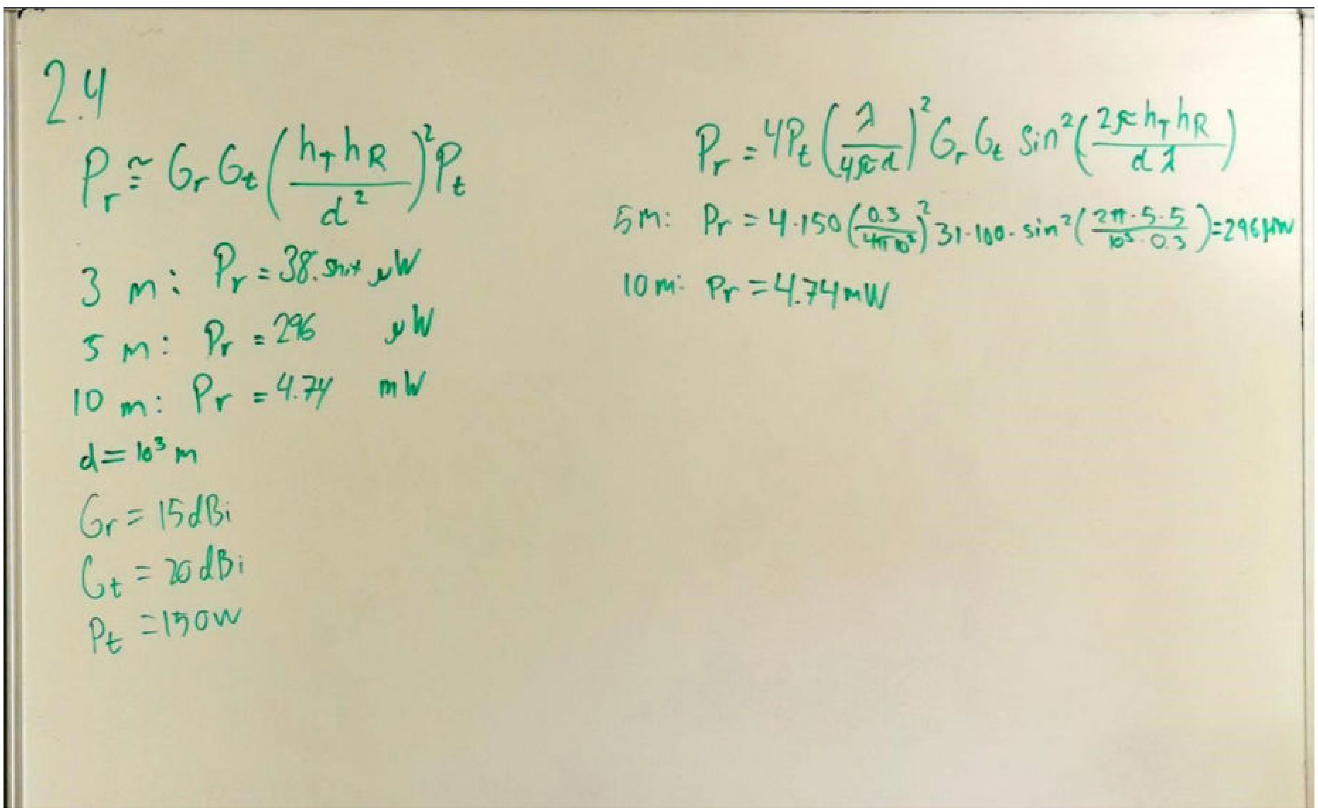
Problems

2.4 Repeat Problem 3 for the case of a reflecting ground and antenna height of both the receiver and transmitter of:

I. 3 meters

II. 5 meters

III. 10 meters



Problem 4.

Repeat Problem 3 for the case of a reflecting ground and antenna height of both th
receiver and transmitter of:

I. 3 meters

II. 5 meters

III. 10 meters

$$(a) \quad h_T = h_R = 3m$$

$$P_R = P_T \cdot G_T \cdot G_R \left(\frac{h_T h_R}{d^2} \right)^2 \quad eq\ 2.22,\ d \gg h_T, h_R$$

$$P_R = 150 \cdot 100 \cdot 31 \left(\frac{3 \cdot 3}{1000^2} \right) \sim 38 \mu W$$

$$(b) \quad h_T = h_R = 5m$$

$$P_R = P_T \cdot G_T \cdot G_R \left(\frac{h_T h_R}{d^2} \right)^2 \quad eq\ 2.22,\ d \overset{?}{\gg} h_T, h_R$$

$$P_R \sim 270 \mu W \Rightarrow same\ as\ free\ space$$

1)

```
close all
clear
Gr = db2pow(15);
Gt = db2pow(20);
Pt = 150;
d = 1e3;
f = 1e9;
lam = 3e8/f;
ht = 3;
hr = 3;
Pr = Gr*Gt*((ht*hr)/((d)^2))^2*Pt;
num2sip(Pr); % for 3 meter anden formel for den anden grundet en relation
```

2)

```
ht1 = [5 10];
```

```
hr1 = [5 10];
4*Pt*(lam/(4*pi*d))^2.*Gr.*Gt*sin((2*pi.*ht1.*hr1)/(d*lam)).^2;
```

Problem 4.

$$use\ equation\ 2.21\ as\ d\ is\ not \gg h_T, h_R$$

$$P_R = 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \cdot \sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$

$$P_R = 4\cdot 150\left(\frac{0.3}{4\pi\cdot 1000}\right)^2 \cdot 100\cdot 31\cdot \sin^2\left(\frac{2\pi\cdot 5\cdot 5}{0.3\cdot 1000}\right) \sim 296\mu W$$

explained by Fig 2.5, close to the breaking point!

i.e. Friis eq = flat reflecting surface

$eq\quad 2.119\ Balanis = 2.22\ Parson$

Problem 4.

$$(c)\quad h_T, h_R = 10m$$

$$P_R = 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \cdot \sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right) = 4.74mW$$

$\Rightarrow compare\ to\ free\ space, Wrong\ ?\ or$

$why?\quad No\ grazing\ angle,\ \rho \neq -1$

Compare to freespace, wrong or?

a) Max power will be 4x freespace due to doubel E-field. => Could be OK

b) Or reflection coefficient diffrent from -1 due to wrong assumption of grasing angle

Lecture 3

Problem 3.1

Design an ordinary end-fire uniform linear array with only maximum so that its directivity is 20 dBi. The spacing between the elements is λ/4, and its length is much greater that the spacing. Dertermine the:

a) Number of elements

Solution:

Lecture 5

Problem 1 – The Price of a Protocol

Consider that Alice wants to transmit 5,000 bytes of data to Bob. The communication protocol between Alice and Bob consists of the following elements: an error detection mechanism based on CRC, a feedback mechanism based on ARQ with sequence number, and a two-way connection with piggybacking in order to enable transmission/feedback from/to either Alice or Bob. The accordingly frame structure is shown in Fig. 1.

(a) What is the minimum number of frames required to be transmitted by Alice in order to convey the 5,000 bytes of data to Bob?

Solution:

```
number_frames = ceil(5000/64)% frames
```

```
number_frames =
    79
```

(b) Consider that the communication channel that connects Alice and Bob supports a data rate of 1 kbit/s. What would be the minimum time required for Alice's data to reach Bob if the ARQ procedure was not present and communication was error-free?

Solution:

```
totalbits = (24+(64*8))*number_frames; % Hele framen bruges.
min_time = totalbits/1e3; % 1e3 = 1kbits
```

(c) What is the efficiency of the protocol adopted? Then, what is the goodput in bit/s following the considerations made in (b)?

Solution:

```
effeciency = (64*8)/((64*8)+24) %mængde data i tæller, totalmængde bits i nævner
```

```
effeciency =
```

```
0.955223880597015
```

```
goodput = 1e3*effeciency %bits/s
```

```
goodput =
    955.223880597015
```

(d) Considering a very specific scenario where only Alice transmits and Bob receives with a fixed data packet length of 64 bytes. How would you change the frame to make the protocol more efficient? What is the efficiency gain and the new goodput when comparing the new protocol to the old one?

Solution:

- Fjern pkt length nu hvor size er fixed.

```
new_effeciency = 512/530; % 530 = uden packet length
new_goodput = new_effeciency*1e3;
```

```
new_goodput =
    966.037735849057
```

Problem 2 – Feedback Latency

Consider the same scenario introduced in Problem 1 and the frame depicted in Fig. 1. Now, assume that a frame sent by Alice takes from 5 to 20 ms to be received by Bob, which is also valid from Bob to Alice. On average, this delay in receiving is 10 ms. Recall that Alice wants to transmit 5,000 bytes of data to Bob.

(a) What is the minimum time required by Alice to know that a frame has been lost? And the maximum?

Solution: 5ms+5ms (min) 20ms + 20ms(max)

(b) Assuming that all frames are received successfully, what is the average throughput in bit/s of Alice's transmission by considering now the ACKs from Bob?

Solution: $\frac{536\text{bits} \cdot 79 \text{ frames}}{20\text{ms} \cdot 79 \text{ frames}} = 26800 \frac{\text{bits}}{s}$

```
average_throughput = (536*79)/(20e-3*79);
```

Consider now that Alice transmits 10,000 frames to Bob and assume that every 100-th frame is lost in transmission. Consider only that frames from Alice can be in error, while ACKs/NACKs are perfectly sent from Bob.

(c) How many retransmissions would occur?

Solution:

```
retransmission = (10e3/100) + 1 % Husk at retransmission som man regner fra
10e3/100 også fejler derfor + 1 :)
```

```
retransmission =
   101
```

(d) How long would it take to transmit all frames on average? Assume that all retrans-mission is successful

Solution:

$$\frac{10100 \text{ frames} \cdot 536 \text{ bits}}{26.8\text{E3 averagethroughput}} = 202\text{sekunder}$$

```
average_transmit = (10100*536)/26.8e3;
```

Alternative answer:

```
(10e3+100)*20e-3; %Total frame gange average delaye for receive
```

```
ans =
   202
```

Problem 3 – RS-232

For this problem, assume that Alice and Bob are connected via a serial link and their communication follows the recommended standard 232 (RS-232).

(a) Draw a diagram showing the evolution over time of the voltage levels within an RS-232 frame to transmit the lowercase ASCII character "**h**" from Alice to Bob.[1]

Solution: Billede på github

(b) Consider a baud rate of 9,600 bit/s. How long would it take for to Alice send the sentence "hello world" to Bob assuming error-free communication? Assume that each RS-232 frame is comprised of a start bit and a stop bit and that there are no parity bits.

Solution:

```
x = (11*10)/(9600); % i sekunder for totalt mængde af karaktere(11) som fylder 10
bit
num2sip(x);
```

```
ans =
'11.458 m'
```

(c) Assume the use of the parity bit now and consider the even parity bit scheme[2]. The transmitter wants to send the following ASCII character: 0 1 1 0 0 0 0 1. Which character is it? What is the parity check bit that needs to be appended to this data? Consider the following reception situations for the data sent:

(i) 0 1 1 1 0 0 0 1. Which character was received? Can the error be detected?

(ii) 0 1 1 1 0 1 0 1. Which character was received? Can the error be detected?

Solution:

ACII = 'a' og paritiy bit = 1 grundet even parity hvilket medfører at man tæller antal '1' og da dette er ulige = 1 og modsat for hvis det var lige. For ulige parity så ville ulige antal 1 give 0 og omvendt.

(i) q and yes.

(ii) u and no, fordi det er lige parity og der er lige antal 1 så skulle det sidste bit være 0

Problem 4 – ARQ with Limited Retransmissions

In Example 3 of Lecture 5, we show the average delay under the assumption of infinite retransmissions. This assumption means that, if we have an unbounded time to receive data, then the transmission success probability over the *Binary Symmetric Channel* (BSC) is always 1. These are the theoretical insights obtained by our analysis. However, infinite retransmission attempts might not be practical, considering the communication system's stringent latency requirement. So, in this problem, we investigate the performance when we set the constraint for the number of retransmissions. Let K be the maximum number of retransmissions for a single packet. We consider a scenario where Alice wants to deliver M packets to Bob in a BSC where the *Packet Error Rate* (PER) is $p_{\text{PER}} = 0.1$.

Solution:

$$1-(p_{\text{PER}})^{\text{Total transmissions}(K+M)} = \text{proability of succesful transmission}$$

(a) For $K = 1$, $P_s = \sum_{i=1}^{2} p_{\text{PER}}^{(i-1)}(1-p_{\text{PER}}) = 1 - p_{\text{PER}}^2 = 1 - 0.1^2 = 0.99$

For $K = 2$, $P_s = \sum_{i=1}^{3} p_{\text{PER}}^{(i-1)}(1-p_{\text{PER}}) = 1 - p_{\text{PER}}^3 = 1 - 0.1^3 = 0.999$

(b) Calculate the throughput for K = 1, and then for K = 2

$$\text{Number of packet}(M) \cdot \frac{\text{proability of succes transmissions}}{\text{number of transmissiomns}(M+K)} = 1 \cdot \frac{0.99}{2} = 0.495 \text{ packet per slot}$$

$$1 \cdot \frac{0.999}{3} = 0.333 \text{ Packet per slot}$$

(c) Characterize the relationship between the throughput, the probability of successful packet transmission, and the value of K.

Solution: Increasing the number of retransmissions leads to a higher probability of successful packet transmission for a single packet, while it deteriorates the throughput.

TLDR: Betal ved kasse 1

(d) Let us consider the case where $M = 2$, meaning that Alice wants to send 2 packets to Bob. In this case, we are interested in the probability of the 2 data packets being transmitted by a deadline of 5 slots. This probability is known as the *reliability* of the communication system. Does the number of retransmissions that Alice can send for each packet affect the reliability? Explain it.

Solution:

If the set number of retransmissions makes the total number of transmissions for the 2 packets higher than 5 slots, then the reliability is zero.

Fx: M=2 og K=1 så hvis hver message fejler og en retransmission sker så det totalt 4 hvilket er under 5 slots hvilket er godt. Mere end det så er reliability zero.

Notice that the reliability also depends on Bob successfully receiving the data packets. Hence, if the total number of transmissions for the 2 packets is below 5 slots, the communication reliability is defined by the packet error rate.

Lecture 6

Problem 1 – TDMA-Based Scheduling

In this problem, we are going to develop a functional medium access control (MAC) scheme in the context of a cellular network. Consider the system depicted in Fig. 1, where a base station (Basil) wants to communicate with three terminals: Zoya, Yoshi, and Xia. The communication occurs according to a *Time Division Multiple Access* (TDMA) scheme. This means that the time domain is sliced into time slots and that at some point each slot is allocated to a single terminal to which Basil wants to transmit or receive data. Note that Basil owns a powerful role since she can control who is able to speak/hear. To develop the most simple functional MAC scheme, we will develop 4 different frame types, where each executes a different function. Throughout the problem, we will consider that Basil has the capacity of serving 3 users at most.

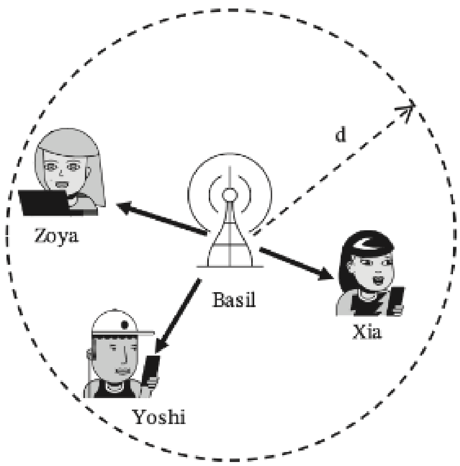


Figure 1: A cellular network that is operating according to a TDMA scheme.

(a) Basil needs to specify the frame that is meant to be sent in the frame header so that the users can understand what is happening at the moment. How many bits are necessary to differentiate 4 frame types?

Solution: log2(4) Hvor 4 er de forskellige different frame types.

Frame 1: Initial Access

In the beginning, Zoya, Yoshi, and Xia have not established a link with Basil and they are all in a receive state, that is, waiting for someone to speak to them so they can figure out what to do. Therefore, Basil needs to create a frame to enable the users to connect to her. Let's call this the link establishment frame. In this frame, there is no payload data being sent, just an invite packet from Basil asking the users: "does anyone want to connect?". After this invite packet, Basil changes to the receive mode in order to listen for some answers. If a user wants to connect, it should send an ACK to Basil. Basil then sends an ACK back to ensure that the link was established and with this ACK also a number between 1 and 3 in order to associate each user with a time slot that is going to be useful when performing data transmission. After this process, Zoya, Yoshi, and Xia can start to communicate with Basil.

(b) Enlist what can go wrong with this process. What do Basil and the users need to do when each one of the problems occurs?

Solution:

- Det kan gå galt hvis de alle gerne vil snak med Basil på samme tid og der sker ingen ting.
- ACK can be lost in transmission for both parties and nothing happens then try again to fix it :)

Frame 2: Link Termination

Consider that Zoya, Yoshi, and Xia have links established to Basil. Assume that we now have a new user, Walt, that wants to communicate with Basil. In view that Basil can serve 3 users at most, the engineering question here is: "how can Walt be served by Basil?"

(c) Design a link termination frame that enables Basil to free resources when suitable.

Solution:

Based on the initial access frame, a link termination frame would be comprised of a reference signal sent by Basil to establish that this frame has begun. Then, Basil sends the address of the user that she wants to be terminated. The user listens to it and withdraws its connection. The user no longer has access to the network.

Frames 3 and 4: Downlink/Uplink Transmissions

In wireless communication terminology, the transmitting direction of having Basil send data to the users is called *downlink*, while the other way around is called *uplink*. Basil needs to design a frame in order to differentiate the directions and specify who needs to listen/hear. Both downlink and uplink transmission frames contain 3 slots, which are assumed to agree to the size of a data packet.

(d) Let us consider a downlink transmission frame where Zoya, Yoshi, and Xia already have established links, and each one is associated with a single slot. Assume that the data rate of each data packet sent by Basil is 1 kbit/s. However, Xia needs to hear all the slots in order to receive the packet meant for her. What is the equivalent data rate of Xia?

Solution:

1kbit /s bliver delt i mellem 3 som skal høre det hele

```
1e3/3;
```

```
ans =
         333.333333333333
```

A Very Simple Functional MAC Scheme

Now that we have defined all 4 frames, we can evaluate how this MAC scheme actually works.

(e) Draw a timing diagram showing the following steps:

- Zoya tries to establish a link.
- Basil transmits to Zoya.
- Zoya transmits to the Basil.
- Yoshi and Xia try to establish a link simultaneously.
- Basil transmits to Zoya.
- Xia tries to establish a link.
- Zoya and Xia transmit to Basil.

Make sure to specify the frame type used in each step.

Solution:

Sæt billede ind fra Github.

Problem 2 – ALOHA Protocol

According to Fig. 2, consider a slotted ALOHA scheme involving five users competing to send their packets over 6 time slots. We assume that users are sending multiple copies of the same packet.

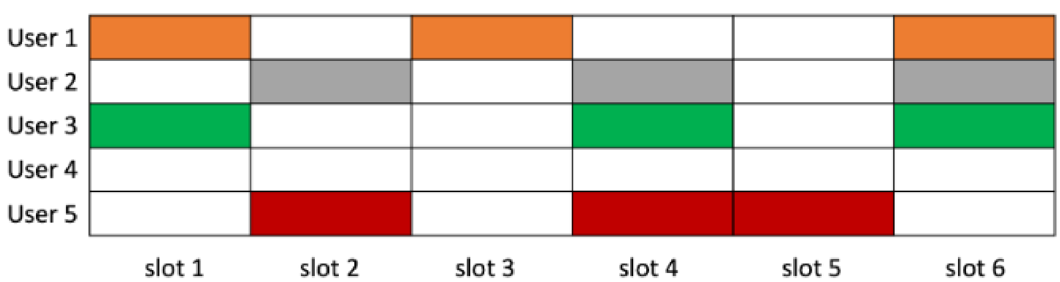


Figure 2: Example of slotted ALOHA. Each user sends multiple copies of the same packet.

(a) How many users have successfully sent their packets without colliding with one another? Who are those users?

Solution:

User 1, Slot 3 and User 5 Slot 5

(b) Consider the decoding technique of *Successive Interference Cancellation* (SIC), which can be applied at the receiver to improve the throughput of the communication. In principle, the SIC algorithm works by removing the replicas of the already resolved packets. For example in slot 1, assuming User 1's packet has been resolved, SIC can be used to exclude User 1's packet. From there, User 2's packet can be easily decoded without collision. So by assuming the use of the SIC technique, how many users' packets can be successfully decoded? What is the decoding order to achieve it?

Solution:

U have to pick either slot 3 or slot 5 since there are only one package which means the sequence will either start with user 1 or user 5.

For slotted ALOHA of N nodes, the throughput is $Np(1-p)^{N-1}$, where p is the transmission probability. The optimal p is $1/N$, resulting in a throughput of $(1-\frac{1}{N})^{N-1}$. Now, consider the following multiple access scheme that combines TDMA and slotted ALOHA. There are 20 users, separated into two groups, one of 4 users and the other of 16 users. Even time slots (*i.e.*, $0, 2, 4, \dots$) are reserved for the 4-user group. Odd time slots (*i.e.*, $1, 3, 5, \dots$) are reserved for the 16-user group. Contention within each group is resolved by the slotted ALOHA protocol (*e.g.*, when a user in the 16-user group wants to send, it waits for an odd slot and then transmits with a probability p).

(c) Determine the *average* throughput (in packets/slot) of the system, assuming that every user always has something to send and, in each group, the users use the optimal transmission probability.

Solution:

$\frac{\left(1-\frac{1}{4}\right)^{4-1}+\left(1-\frac{1}{16}\right)^{(16-1)}}{2} = 0.4$ Brug formlen som er i givet i opgave beskrivelsen.

Problem 3 – Token Ring and Round-Robin

Consider the system in Fig. 3, where we have 8 communicating nodes positioned in a ring architecture. Here we have a *token ring system*, where Node 0 starts with the token and hence is allowed to communicate first. Then, after a node finishes its transmission, the node successively passes the token to the next one following the counterclockwise direction. Table 1 reports the time each node takes to execute its transmission.

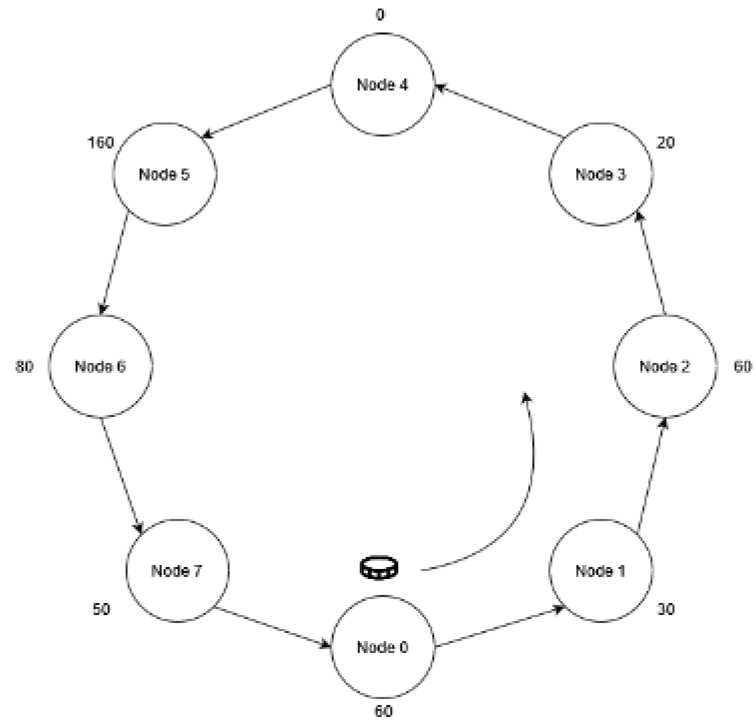


Figure 3: Architecture of the token ring system.

Time is set to 0 when the original token holder (Node 0) starts its first transmission. Considering that the token handover is instantaneous, answer the following:

Node ID	0	1	2	3	4	5	6	7
Time to execute (ms)	60	30	60	20	0	160	80	50

Table 1: Execution times.

(a) How long does it take before Node 6 has sent its message?

Solution:

$60 + 30 + 60 + 20 + 0 + 160 + 80 = 410$

(b) What are the consequences of having a system where we do not limit communication time? How does this affect the performance of the various nodes?

Solution: Det betyder at en node kan bruge lang tid på at sende så rimelig unfair :)

Ingen garanti på latens for at kommunikere.

To make sharing of the communication medium fairer we add a *round-robin* approach to the system. Now our nodes can only transmit for 50 ms at a time before they have to pass the token on to the next node.

(c) How long time before Node 6 has finalized transmitting its message?

Solution: 50+30+50+20+50+50+50+10+10+50+30=400 (Så man skal to gange rund eftersom at 50 er maksimal det der kan sendes så der er en rest for nogen)

(d) Which nodes have had the time before they finish their communication increased, which has it decreased, and which are unchanged?

Solution:

- Increased: Node; 2,0,5
- Decreased: Node; 1,3,6,7
- Unchanged: Node; 4

(e) Nodes in this architecture would normally only know the existence of their neighbors and nothing more. From the perspective of Node 0, can you make any assumptions on the minimum or the maximum number of nodes, from when Node 0 gets the token back?

- Is it the same if we do not apply the round-robin approach?
- If the original token holder changed, would the same amount of information on the number of nodes in the system be available, and which nodes would know more?

Solution:

If we think of time when doing a full circle of the ring, Node 0 may communicate again after 300 ms. If we consider the maximum time we may use the medium (50 ms), Node 0 knows at least 6 nodes are in the system since 300. If we consider the maximum time we may use the medium (50ms), Node 0 knows atleast 6 nodes are in the system since $\frac{300}{50} = 6$

- No, single nodes like Node 5 can take long making it look like many nodes, while nodes like 4 are hidden away, and would not be noticed. The missing guarantee on communication time makes it impossible.
- The nodes know 2 things: how long before they get the token back again and how much they need to communicate. If the token were to start at Node 1, 3, or 4 they would know that there are at least 7 nodes in the system. Exemplifying, for Node 1, $\text{num\_nodes} = 1 + \lceil\frac{300-30}{50}\rceil = 7$, while, for Node 0, it would be $\text{num\_nodes} = 1 + \lceil\frac{300-50}{50}\rceil = 6$.

It wouldnt know, but with round robin it would since max delay time is implemented?

Lecture 7

Wireshark opgave sæt billede ind fra github.

Payload er 40 bytes istedet for 48 bytes som der står i github til løsning 2.e

Lecture 8

Problem 1 – Bellman-Ford Algorithm

The *Bellman-Ford algorithm* is an algorithm for finding and computing the shortest paths in a graph.[1] In this exercise, we are going to see an example of how this algorithm can be used when considering a weighted graph. But first, why are we interested in graphs and algorithms over them? A weighted graph can be used as a mathematical structure to represent a communication network, where nodes are devices such as computers and edges are links (cables). The weights of each edge can correspond to the communication cost/efficiency of the corresponding link. Therefore, to improve a practical network's performance, we can use graph algorithms to perform routing or other functionalities we would like our network to perform.

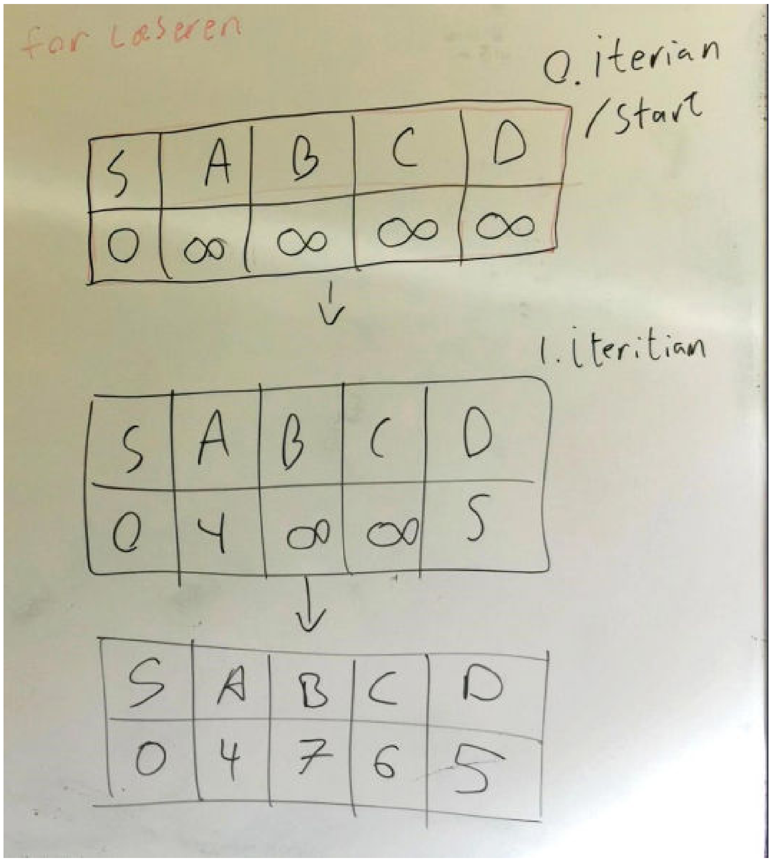
Consider the graph and weights illustrated in Fig. 1. Consider that the weight of each edge corresponds to the distance between the nodes. We want to find the shortest distance paths between the source node S and all the other nodes. To do this, we will apply the Bellman-Ford algorithm to find the shortest network paths. Answer the following:

(a) What is the maximum number of iterations required to complete the Bellman algorithm?

Solution: Number of vertices - 1 so it is 4

(b) Describe the Bellman algorithm through each iteration and construct the shortest path tree.

Solution:



Then, the shortest path tree: S-D-A-B-C

Path	Shortest Path	Distance
S-A	S-A	4
S-B	S-A-B	7
S-C	S-D-C	6
S-D	S-D	5

Problem 2 – Routing

The purpose of *routing* in a network is to find the best path to transmit a packet from a node X to another node Y. In Problem 1, we have learned how graphs can represent networks and how to use the Bellman-Ford algorithm. In this part, we will apply what we have learned to perform routing!

The *distance-vector routing protocol* is a routing protocol based on finding the shortest distances. Consider the network of Fig. 2 and that distance-vector routing protocol is used. To be familiarized with the nomenclature used by the protocol, please read `https://en.wikipedia.org/wiki/Distance-vector_routing_protocol`. Now, consider that a message is sent through the network in the figure and nodes B, D, and E have the following routing tables: B: (5, 0, 8, 12, 6, 2); D: (16, 12, 6, 0, 9, 10); E: (7, 6, 3, 9, 0, 4). Note that each routing table entry (A, B, C, D, E, F) corresponds to the *distance between the current node and all other nodes.* For example, B: (5, 0, 8, 12, 6, 2) means that the distance to transmit from B to A is 5, B to B is 0, B to C is 8, and so on. Consider that the distance between the C to B, D, and E links are 6, 3, and 5, respectively. Based on these three routing tables, *what is C's new routing table? Give the routing table and the corresponding best nodes.*

Hint: For example, to obtain the value for the first element of C's routing table, you have to determine the path that will cost the less from C to A based on the possible ways to reach A from C: through B, D, and E.

Solution:

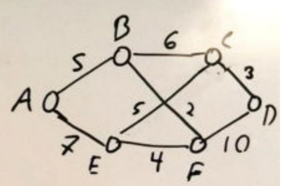
Problem 2

Routing table:

B (5, 0, 8, 12, 6, 2)

D(16, 12, 6, 0, 9, 10)

E(7, 6, 3, 9, 0, 4)



Answers:

C(11, 6, 0, 3, 5, 8)

Routing tables are "old" costs but can still be used so you can pick any cost from the tables giving the shortest route. But network costs cannot be changed.

Problem 3 – Flow control

A CPU executes instructions at the rate of 1000 MIPS (million instructions per second). A transmitter wants to send a packet of 64 bits to a receiver that uses this CPU. Consider that the processing of each packet costs 10 instructions. Consider that the CPU must `copy` the packet 4 times, where the `copy` operation over a packet can occur simultaneously. Can the receiver's CPU handle it if the transmitter and receiver are connected using a link to transmit data with 1 Gbps? For simplicity, assume that all instructions, even those that read or write memory, run at the total 1000-MIPS rate.

Motivation: This exercise represents an example of flow control, where the data rate of the link between the transmitter and the receiver should adapt to the processing capability of the receiver! There is no point in transmitting faster than the receiver can handle it.

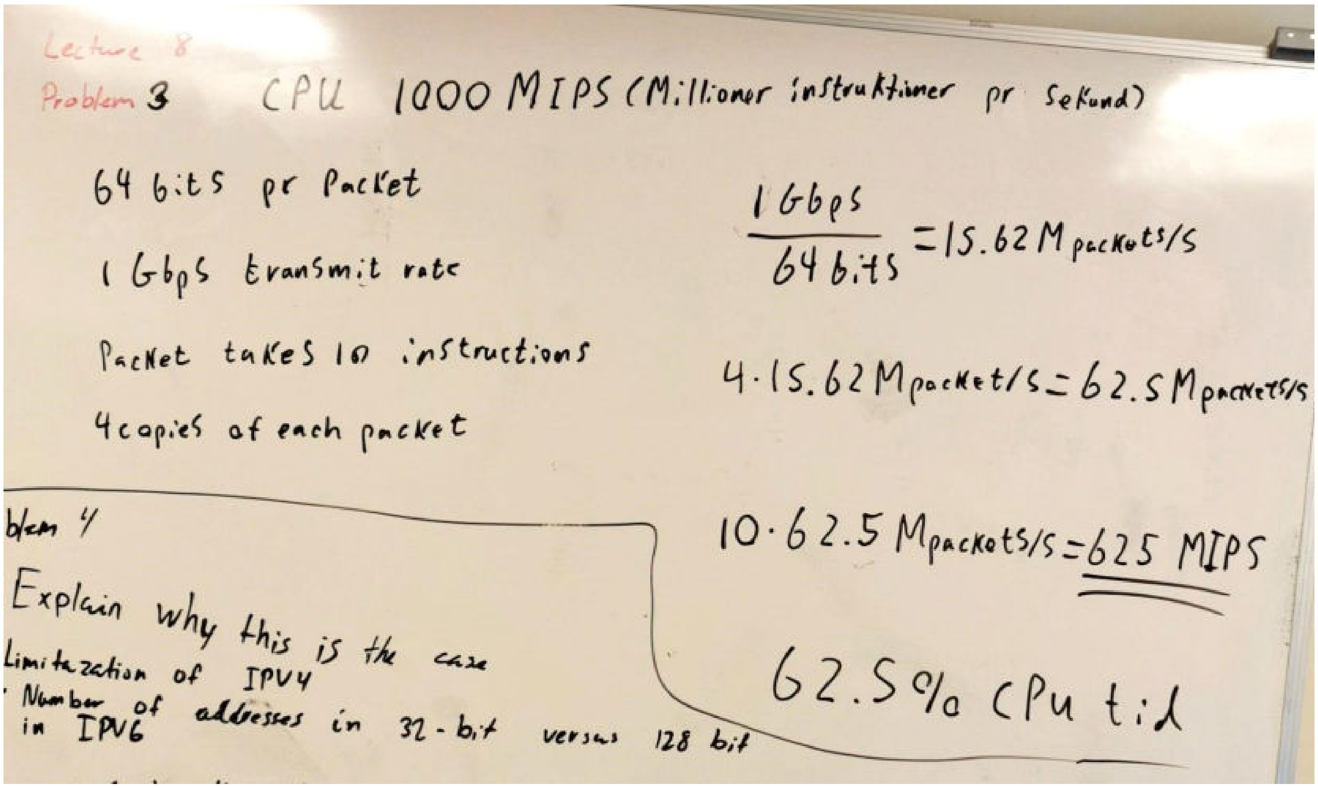
Solution:

Which is 40 instructions in 40 nanoseconds.

64 bits i pakken = 8 bytes:

$\frac{8}{40e-9} = 5\text{nano sekund}$ for enkelt byte men vi har gigabitpersekund så derfor: $\frac{8\text{bit}}{5e-9\text{sekund}} = 1600\frac{\text{gigabit}}{\text{sekund}}$ hvilket vil sige at den kan håndtere det eftersom det er større end 1 gigabit.

Egen solution fra github:



Problem 4 – TCP/IP

This exercise suggests that you read and familiarize yourself with the TCP-/IP, the protocol that enabled the internet boom.[2] Without this protocol, you would not be able to have Netflix, YouTube, and social media.

(a) To address the limitations of IP version 4, a major effort had to be undertaken via IETF that resulted in the design of IP version 6. There is still a significant reluctance to adopt this new version. However, no such major effort is needed to address the limitations of TCP. Explain why this is the case.

Solution: At der var for få addreser i IPV4 og derfor blev IPV6 lavet

Eftersom at TCP kan snakke sammen på forskellige versioner af sig selv men det kan IP (fordi det er fysisk) og derfor er der reluctance.

(b) In the figure of the TCP segment header as in Fig. 3, we saw that in addition to the 32-bit acknowledgment field, there is an ACK bit in the fourth word. Does this add

Solution:

Det tilføjer at man kan se om man bruger det 32bit acknowledge field.

Det spild hvis man nu ikke skal bruge feltet.

TCP segment header

Offsets	Octet	0								1								2								3							
Octet	Bit	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0
0	0	Source port																Destination port															
4	32	Sequence number																															
8	64	Acknowledgment number (if ACK set)																															
12	96	Data offset				Reserved 0 0 0			NS	CWR	ECE	URG	ACK	PSH	RST	SYN	FIN	Window Size															
16	128	Checksum																Urgent pointer (if URG set)															
20	160	Options (if *data offset* > 5. Padded at the end with "0" bits if necessary.)																															
⋮	⋮																																
60	480																																

Figure 3: TCP segment header.

Lecture 9

Problem 1

In encryption using block-cipher, what potential problem can occur when using Electronic Code Book? Using Cipher Block Chaining?

Solution:

- ECB; Large data must not have patterns cause it can be decrypted then.
- CBC; Packet cannot be lost since decryption is then impossible for subsequent packages.

Problem 2

Assume that two parties know each other's public keys. If one message is sent from A to B, what can be verified? If two messages are exchanged, what can be verified?

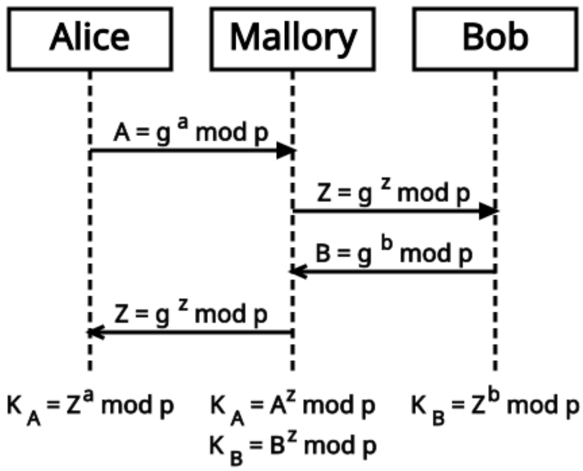
Solution: The receiver can only verify the message contents and the sender but not the transmitter can not verify anything.

But if they both send a message the receivers can decrypt the same thing iguees?.

Problem 3

With Diffie-Hellman key exchange, is it a man-in-the-middle attack possible? If possible, draw how this could be achieved.

Solution: YES :)



Problem 4

Assume the following scenario: A and B both have the knowledge of a secret key K (e.g. pre-shared). They communicate over an insecure channel. Define a protocol (by writing down a message sequence chart) in which A and B use the pre-shared key K to mutually authenticate each other and to agree on a common session key (different from the long-term pre-shared key K). Try to keep the number of exchanged messages as low as possible.

Solution:

- This is just a proposal to a solution, as long as the authentication is ensured and a new session key can be generated then the principles are correct.
- **Step 1.** A and B send starts a connection.

  Step 2. Authentication via a "challenge"; A sends a random string, and B hashes it with the key and sends it back to A. A can validate B by hashing the challenge and the key together and comparing the hash with what was received. A key point is that the challenge should be random, otherwise, it may be vulnerable to a replay attack. While the initial communication could be encrypted with a key, it is not necessary to authenticate as the one way function property of a hash makes reversing the operation computationally infeasible.

  Step 3. When authenticated, any key generation algorithm could be used e.g. RSA/DH to create the session key.

Problem 5

Asymmetric encryption and symmetric encryption are both actively used to ensure security. While Asymmetric encryption leads to more security it comes with computational expensive algorithms for encryption and decryption. Meanwhile, symmetric keys provide less security but are less computationally expensive. Assuming A has a public/private key pair, where B can ask what the public key is through an unsecured channel, how could a middle ground between the computational cost and security be met?

(a) To further enhance security enhancing the authentication is critical. What kind of technology could A possess, and how would this make a Man-in-the-middle attack more difficult?

Solution:

Man stoler på de certificates der kommer fra en authority

We use asymmetric encryption to create a secure channel, through which B can define a symmetric key that both can use for a session. By redefining the session key "often" the security may be further enhanced, of course at the cost of more computational complexity.

(a) By adding certificates to the mix, B can be sure that A is who A says he is. This is a critical part of e.g. HTTPS, where websites authenticate their identity through a trusted CA, hence establishing a server-client connection is asymmetricly encrypted. The user can then assume that the website is who they say and that the public key is not false, and data transmissions be done through a symmetrically encrypted channel.

Comparison Table

Feature	Symmetric Encryption	Asymmetric Encryption
Keys Used	One key (same for encryption and decryption).	Two keys (public and private).
Speed	Faster	Slower
Key Sharing	Requires secure channel for key sharing.	Public key can be shared freely.
Security	Vulnerable if the key is compromised.	More secure for key distribution.
Use Case	Encrypting large amounts of data.	Secure key exchange, authentication.
Examples	AES, DES, 3DES	RSA, ECC, DSA

Lecture 11

problem 1

In this exercise, you will learn how encoding can influence the rate of a channel.

(a) Consider the simple encoding scheme where we send an 8-bit message using the channel six times and decode it by taking the majority vote. What is the rate of this encoding scheme?

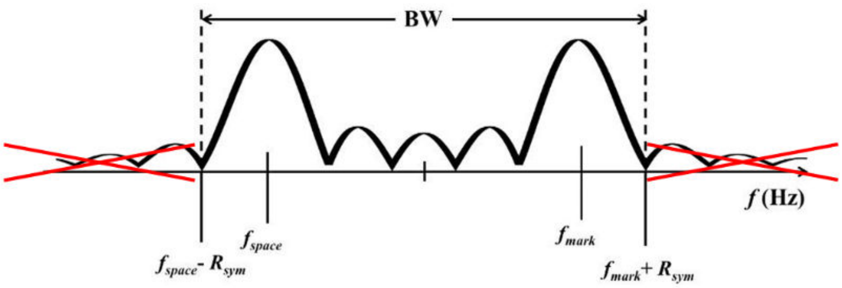
Solution:

use formula $R(\text{code rate}) = \dfrac{\log_2(\text{bits})}{\text{channel use}(n)} = \dfrac{3}{6} = 0.5$

(b) Consider a BFSK transmitter that sends information at a rate of 10 kbps using a space frequency of 500 kHz and a mark frequency of 700 kHz. How much bandwidth is needed for the transmission?

Solution:

forskellen i frekvens + 2*R



$$\mathrm{BW} = f_{\mathrm{mark}} - f_{\mathrm{space}} + 2 \cdot R_{\mathrm{sym}} \Rightarrow 700\mathrm{kHz} - 500\mathrm{kHz} + 2 \cdot 10\mathrm{kbps} = 220\mathrm{kHz}$$

Problem 2 – Digital Modulation

In this exercise, you will learn a digital modulation scheme and define its main properties. Consider the signal constellation shown in Fig. 1. Answer the following.

Solution:

(a) What type of modulation is represented?

16-PSK

(b) How many symbols are represented?

16 symbols

(c) How many bits does each symbol represent?

log2(16)=4bits

(d) What is the bit rate if the baud rate is 10000 symbols/second?

10000*4=40000

Problem 3 – Yet Another Digital Modulation Technique

Consider a communication system that transmits a bit stream using 16-QAM.

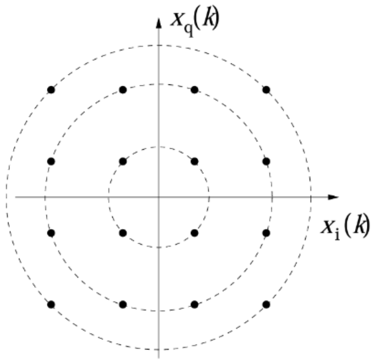
Solution:

(a) How many bits are transmitted per symbol?

log2(16)=4bits

(b) Suppose four different phases and four different amplitudes are used. Sketch a constellation diagram for the 16-QAM modulation. Label/Indicate the symbols. Is the order important to label them? Are there other ways to label them?

Solution:



Yes, the order in which the symbols are labeled matters. We can use for example Gray labeling. The idea is to reduce the quantity of errors and make them easier to be detected.

Problem 4 – An Error Detection Scheme

In this exercise, you will learn an error detection mechanism. In particular, we consider Error detection with Cyclic Redundancy Check (CRC). The message 11011 will be transmitted using CRC polynomial $x^3 + x + 1$ to protect it from errors.

Solution:

(a) How many bits are required to apply the CRC?

kigger på største eksponent hvilket er 3 ergo 3 bits requried

(b) Describe the CRC generation process and compute the message that should be transmitted.

```
padmessage = [0 0 0 1 1 0 1 1];%husk lsb først
crcpoly = [1 1 0 1];%det er fra polynomiet som giver 1 0 1 1 som kommer fra
exponenterne(husk igen lsb først)
[q,r] = gfdeconv(padmessage,crcpoly)%
```

```
q = 1×5
     1     1     1     1     1
r =
1
```

derfor er crc 001 og hele message er 11011001

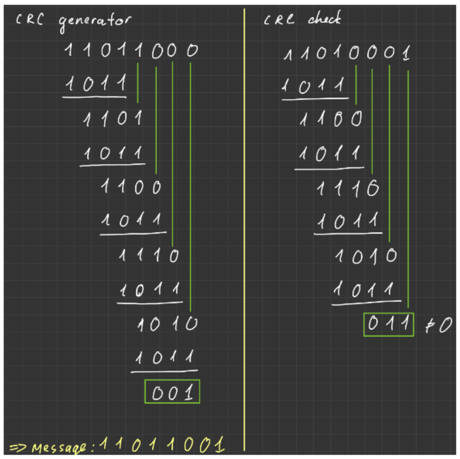
(c) Consider that the transmission is damaged, so the receiver receives 11010001. Will this error be detected?

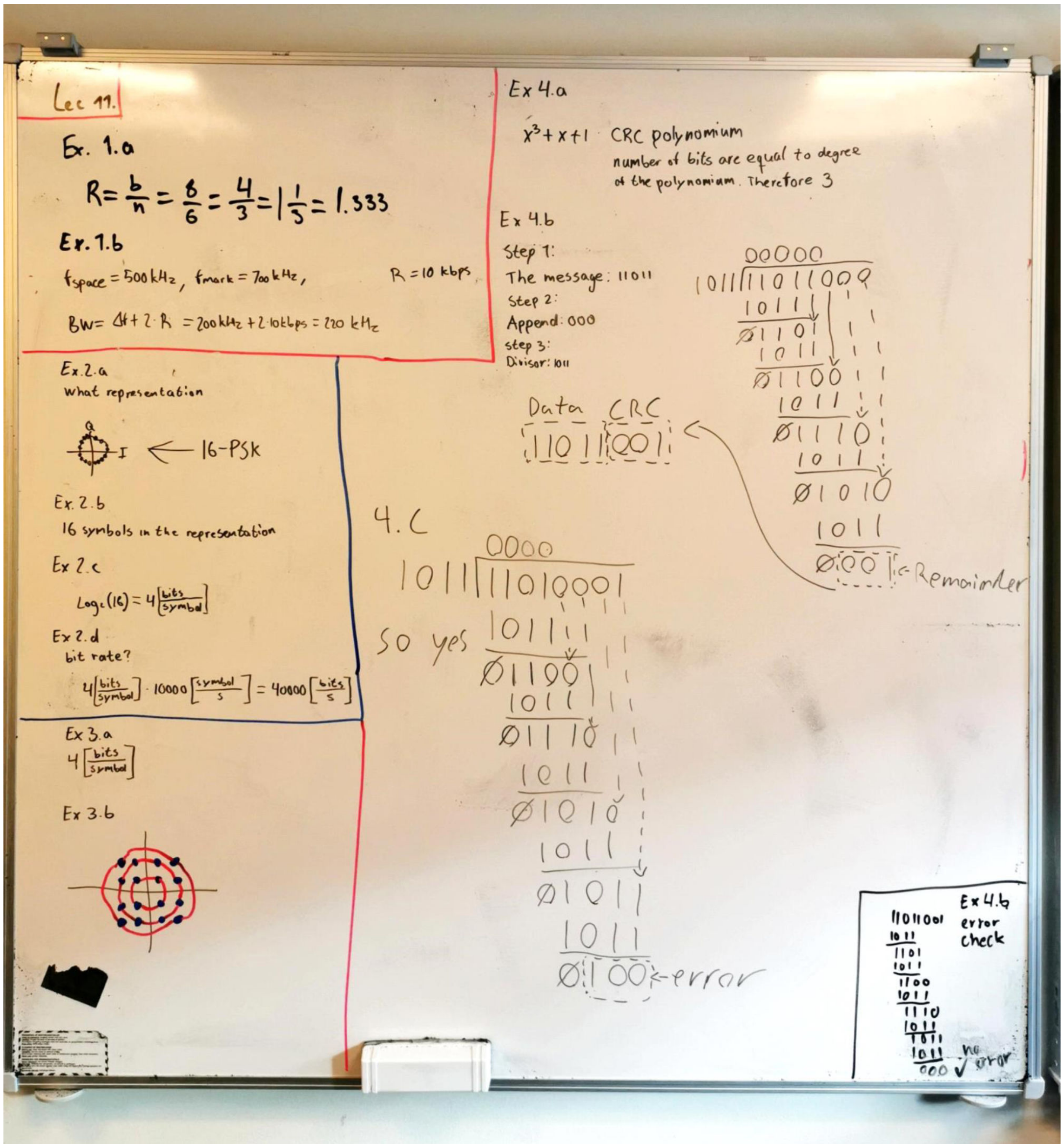
```
padmessage = [1 0 0 0 1 0 1 1];
crcpoly = [1 1 0 1];
[q,r] = gfdeconv(padmessage,crcpoly)% r er forskellige fra 0 ergo har den fundet
fejlen
```

```
q = 1×5
     0     1     1     1     1
r = 1×2
     1     1
```

hele message 11010011 fordi crc er forskellig fra 0 kan den detektere fejlen

Yes, the receiver can detect the error.





Problem 5 – Maximum Ratio Combinining and Incremental Retransmission

In this exercise, you will see how we can combine baseband bits to improve our communication. Then, you grasp the role of retransmission and how we can make it more efficiency through a thought exercise about incremental redundancy.

Solution:

(a) Let $\mathbf{s} = (s_1, s_2, \ldots s_u)$ represent the packet sent by Xia to Yoshi through the respective baseband symbols. Yoshi does not receive the first packet transmission correctly, Xia does not receive an ACK, and she retransmits the same $\mathbf{s}$. Let us look at the same received symbol from both packet transmissions:

$$y_{i,1} = hs_1 + n_{i,1}, \quad (1)$$
$$y_{i,2} = hs_2 + n_{i,2}, \quad (2)$$

where the index j stands for the jth packet transmission; $y_{i,j}$ is the ith symbol received by Yoshi and $n_{i,j}$ is the ith noise sample. It is assumed that the channel h stays constant during the transmission and the retransmission.

1. Using Chase combining or maximum ratio combining (MRC), what is the message that Yoshi creates? How do the signal-to-noise ratio (SNR) and nominal throughput change in this case?

Solution:

1. Yoshi creates:

$$y_i = y_{i,1} + y_{i,2} = 2hs_i + n_{i,2} + n_{i,2}. \quad (1)$$

If the noise samples are independent, then MRC makes the SNR of y_i double the original SNR under which the data is attempted to be decoded from the individual $y_{i,j}$. Assuming that the feedback from Yoshi is ideal and instantaneous, then double retransmissions will increase the overall transmission time of the packet two times, which decreases the nominal throughput two times.

det er en fejl at $n_{i,2}$ står dobbelt der skulle stå $n_{i,1}$ da noisen ikke er den samme for begge.

per retranmission skaleres SNR linært

2. The idea of partial retransmission is that instead of retransmitting the same $\mathbf{S}_{1,1} = \mathbf{S}_{1,2} = \mathbf{S}_{1,3} = \mathbf{s}$, Xia can retransmit another set of symbols $\mathbf{R}_{1,2}$ for the first retransmission, $\mathbf{R}_{1,3}$ for the second retransmission, etc. The redundancy of $\mathbf{R}_{1,2}, \mathbf{R}_{1,3}, \ldots$ is not introduced in the first transmission but upon feedback from the receiver. This is why this retransmission method is called *incremental redundancy.* Describe a simple protocol that uses incremental redundancy to show that this approach can improve the throughput without decreasing the reliability.

Solution:

2. This could be achieved by rich feedback instead of a simple NACK. After failing to decode the packet, Yoshi sends feedback to Xia, which contains information "I am missing b' bits of information to be able to decode". The key phrase is "in principle," as there is a major difficulty in finding coding/decoding methods that enable Yoshi to measure how much information he is missing to decode the packet correctly.

   The simplest way to use NACK for incremental redundancy is to retransmit only a subset of the symbols $\mathbf{S}_{1,1} = \mathbf{s}$ sent in the original transmission. For example, Xia retransmits s_1, s_2, s_3 only and during the retransmission, Yoshi receives $y_{i,2}$ for $i = 1, 2, 3$, as given by [eq. (1, 2), Prob. 1]. Then one option is that Yoshi replaces $y_{i,1}$ from the previously received packet with the respective $y_{i,2}$ for $i = 1, 2, 3$, while he reuses the remaining $y_{i,1}$ for $i > 3$ and attempts to decode the packet again.

Problem 1 – From the Analog to the Discrete World

Consider the signals:

$$x(t) = A_x \left(\sin(2\pi 4000t) + \cos(2\pi a_x t)\right) \tag{1}$$

$$y(t) = A_y \left(\sin(2\pi a_x t) + \cos(2\pi b_y t)\right), \tag{2}$$

where $0 \leq a_x \leq 8$ kHz and $0 \leq b_x \leq 16$ kHz. What are the Nyquist rate (sampling frequency) and sampling period for:

(a) $x(t)$?

(b) $y(t)$?

(c) $x(t) + y(t)$?

(d) $x(t)y(t)$?

Solution:

a) x(t) => 8000 * 2 = 16kHz , (den højeste frekvens i signalet)

b) y(t) => 16000*2=32kHz

c) x(t)+y(t)=> tager du den højeste derfor 32 kHz

d) x(t)y(t) => der addere du de to, 16+32= 48kHz

Let us focus now on $x(t)$. Assume $A_x = 1$ and $a_x = 8$ kHz. Imagine that we observe the signal for 1 ms and we start to observe the signal at $T = 0$. Answer the following:

(e) Using the result from (a), get the number of samples required to reconstruct $x(t)$ for $t \in [0, 1 \cdot 10^{-3}]$ according to the Nyquist rate.

(f) Using Matlab or Python, plot the signal $x(t)$ over this 1 ms and the respective sampled points. What are the values of $x(t)$ in the sampled points?

e)

Solution:

$$T = \frac{1}{F} = \frac{1}{16\text{kHz}} = 62.5\text{us}$$

$$\frac{1\text{ms}}{62.5\text{us}} = 16 \text{ samples}$$

f) Solution code for plotting graph.

```
close all
clear
% Parameters
Ax = 1;           % Amplitude, you can adjust this value
a_x = 8000;       % Frequency in Hz, you can adjust this value as well

% Continuous Time Vector for Smooth Plot
fs_continuous = 100000;           % High sampling frequency for smooth plot
t_continuous = 0:1/fs_continuous:0.001; % Time vector for continuous plot

% Discrete Time Vector for 16 samples over 1 ms
num_samples = 16;                 % Number of samples within 1 ms
t_samples = linspace(0, 0.001, num_samples); % 16 equally spaced points from 0 to 1
ms

% Continuous Signal Definition
x_t_continuous = Ax * (sin(2 * pi * 4000 * t_continuous) + cos(2 * pi * a_x *
t_continuous));

% Sampled Signal Definition
x_t_samples = Ax * (sin(2 * pi * 4000 * t_samples) + cos(2 * pi * a_x * t_samples));

% Plotting
figure;
plot(t_continuous * 1000, x_t_continuous, 'b-', 'LineWidth', 1.5); % Continuous
signal plot
hold on;
stem(t_samples * 1000, x_t_samples, 'r', 'LineWidth', 1.5); % Sampled signal plot
xlabel('Time (ms)');
ylabel('x(t)');
title('Plot of x(t) over 1 ms with 16 samples');
legend('Continuous Signal', 'Sampled Signal (16 samples)');
grid on;
hold off;
```

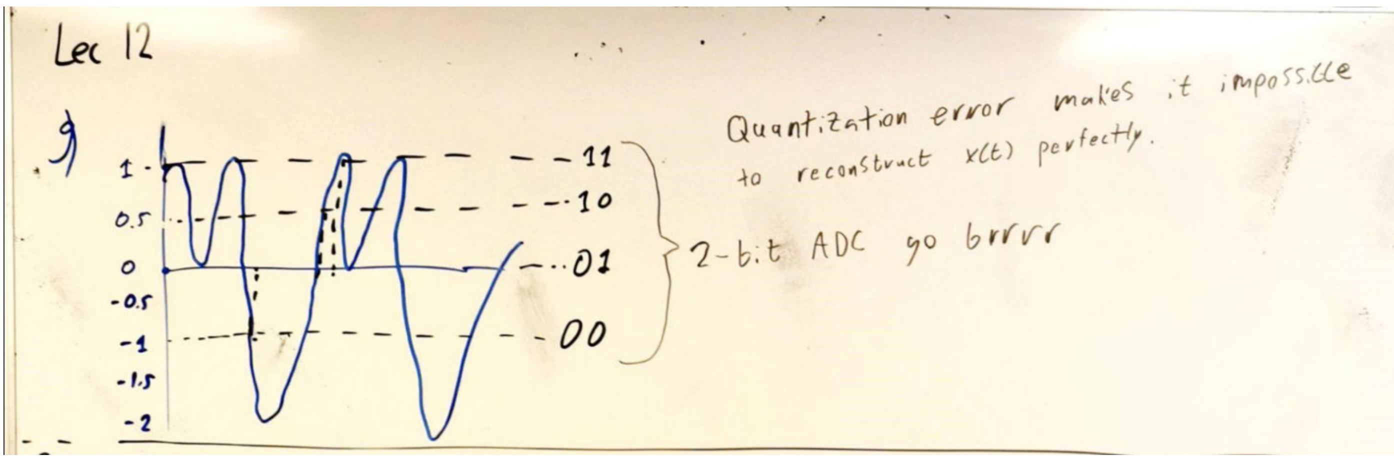
(g) The last step to the analog signal $x(t)$ become a completely discrete signal is to discretize its amplitude value. Read more about quantization on https://en.wikipedia.org/wiki/Quantization_(signal_processing)[1]. If you had access to a 2-bit resolution quantizer, how would you do the quantization of $x(t)$? After quantizing, can you get $x(t)$ back perfectly from the quantized version?

(optional) This optional exercise is about estimating the quantization error. Define the quantization error as $e_n = x[nT_s] - \hat{x}[nT_s]$, where $n = 1, 2, \ldots, N$ with N obtained in (e), T_s being the sampling rate, and $\hat{x}$ being the quantized version. Compute the average quantization error according to the:

$$\text{MSE} = \frac{1}{N}\sum_{n=1}^{N} e_n^2, \tag{3}$$

where MSE is the mean-squared error.

Solution: Impossible since the gap between bits are to large to reconstruct the signal or simply look at graph :).

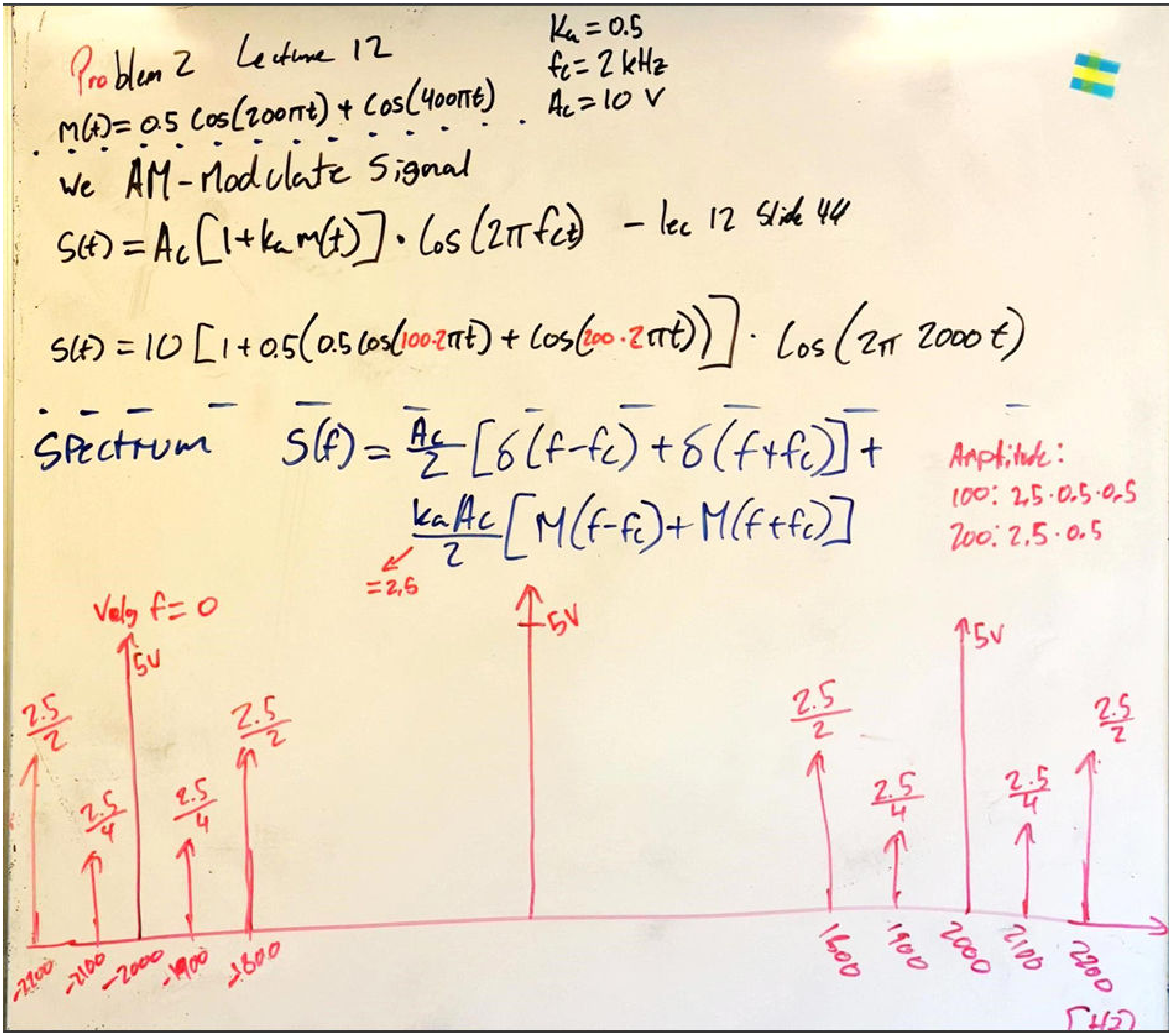


Problem 2 – Conventional AM modulation

Let $m(t) = 0.5\cos(200\pi t) + \cos(400\pi t)$ be the AM baseband modulating signal. Calculate the AM modulated signal and plot its spectrum given amplitude sensitivity $k_a = 0.5$, carrier frequency of $f_c = 2$ kHz, and carrier amplitude of $A_c = 10$ V.

Solution: Indsæt løsning for expansion af spectrum ligning og billede fra tavle.

Tavle løsning:



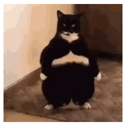
Problem 3 – 4-PAM

Consider an $M = 4$ PAM modulation. Answer the following:

(a) Enumerate the binary symbols that this modulation can represent.

Solution: 00 01 10 11 since 4-PAM has 4 in it. $\log 2(4) = 2$ therefor 2 bits represents it.

(b) Assume $A_1 = -2, A_2 = -1, A_3 = 1, A_4 = 2$ and $g_T(t) = \cos(2\pi t)$. Draw the signal waveforms and associate them with a symbol enumerated in (a). Please associate the according A_x to the symbol that represents x in the binary numeral system.

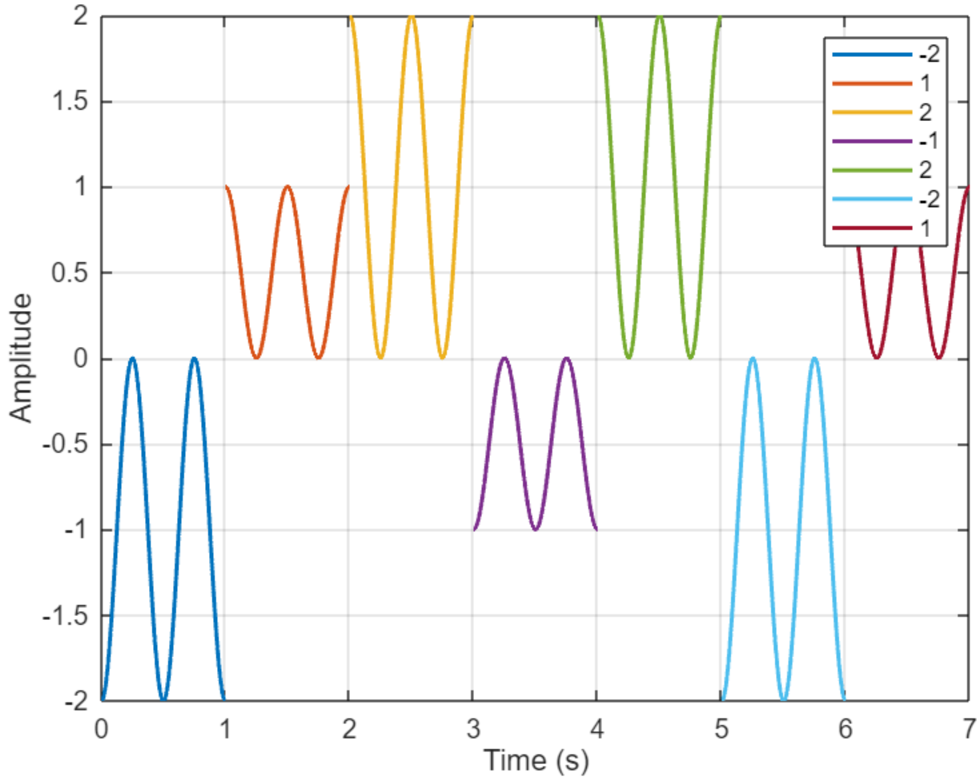
b) Solution: 

```
% Parameters
close all
clear
fs = 1000; % Sampling frequency
t = 0:1/fs:1;% Time vector (one period of the pulse shape)
g_t = cos(2*pi*t); % Pulse shape g(t)

% Amplitudes for 4-PAM
A = [-2, -1, 1, 2]; % Corresponding to binary symbols 00, 01, 10, 11
symbols = ["00", "01", "10", "11"]; % Binary symbols

% Plot the signal waveforms
figure;
for i = 1:length(A)
    plot(t+(i-1), A(i) * g_t, 'LineWidth', 1.5); % Scaled pulse shape
    hold on
    grid on;
end
title(['4-PAM Symbol: ', symbols(i), ' (Amplitude: ', num2str(A(i)), ')']);
    xlabel('Time (s)');
    ylabel('Amplitude');
    legend(symbols)
```

(c) Solution: Her er vi interesseret i C istedet for den nye besked. Hernæst er der tilføjet $\cos(2\pi \cdot 1\text{Hz} \cdot t)$i for loopet for at tilføje carrieren. Ellers så er værdierne bare trykket ind i C arrayet.

```
    % Parameters
close all
clear
fs = 1000; % Sampling frequency
t = 0:1/fs:1;% Time vector (one period of the pulse shape)
g_t = cos(2*pi*t); % Pulse shape g(t)

% Amplitudes for 4-PAM
A = [-2, -1, 1, 2]; % Corresponding to binary symbols 00, 01, 10, 11
symbols = ["00", "01", "10", "11"]; % Binary symbols
C = [-2, 1, 2, -1, 2, -2, 1];
symbolsC = string(C);
% Plot the signal waveforms
figure;
for i = 1:length(C)
    plot(t+(i-1), C(i) * g_t.*cos(2*pi*t), 'LineWidth', 1.5); % Scaled pulse shape
    hold on
    grid on;
end
```

```
% title(['4-PAM Symbol: ', symbols(i), ' (Amplitude: ', num2str(A(i)), ')']);
    xlabel('Time (s)');
    ylabel('Amplitude');
    % legend(symbols)
    legend(symbolsC)
```



(d) Assume that at the receiver side, the symbol "00" transmitted by the transmitter suffered from a noise component of $n = 0.5$. What is the value of r after the signal demodulator?

Solution: Sæt svararket ind!

(e) Based on the maximum-a-posterior principle, the received demodulated signals would be: $[-1.20, -2.40, 1.49, 2.10] \rightarrow [01, 00, 10, 11]$ (just find the closest symbol using the TRUE mapping $[-2, -1, 1, 2] \rightarrow [00, 01, 10, 11]$). Assuming that the sequence of true transmitted symbols was: $[00, 00, 11, 11]$, the percentage of error was 50%.

Solution:

Based on the maximum-a-posterior principle, the received demodulated signals would be: [−1.20, −2.40, 1.49, 2.10]→ [01, 00, 10, 11] (just find the closest symbol using the TRUE mapping [−2, −1, 1, 2]→ [00, 01, 10, 11]). Assuming that the sequence of true transmitted symbols was: [00, 00, 11, 11], the percentage of error was 50%.