Lec 2

1)

$$H_c(s) = \frac{2}{s^2 + 4s + 3}$$

A) Find $H(z)$

følg Nochilas' noter (eller snyd som John)

$$H_c(s) = \frac{2}{(s+1)(s+3)} = \frac{A_1}{s+1} + \frac{A_2}{s+3} = \frac{1}{s+1} - \frac{1}{s+3}$$

$$H_c(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k} \Rightarrow H(z) = \sum_{k=1}^{N} \frac{T A_k}{1 - e^{s_k T} z^{-1}}$$

$$H(z) = \frac{T}{1 - e^{-T} z^{-1}} - \frac{T}{1 - e^{-3T} z^{-1}} = \frac{Tz}{z - e^{-T}} - \frac{Tz}{z - e^{-3T}}$$

B) Find $H(z)\big|_{f_s = 1\,\mathrm{Hz}}$ og plot $|H(e^{j\omega})|$

$$H(z)\big|_{T=1} = \frac{z}{z - e^{-1}} - \frac{z}{z - e^{-3}}$$

C) Find filterets 3 dB cutoff frekvens (ved aflæsning)

$$|H(e^{j\omega})| = -3\,\mathrm{dB} = \frac{1}{\sqrt{2}} \Rightarrow \omega \approx 0.56 \cdot \pi \quad (?)$$

d) Find differensligning

$$H(z) = \frac{Y(z)}{X(z)} = \frac{T}{1 - e^{-T} z^{-1}} - \frac{T}{1 - e^{-3T} z^{-1}} \Rightarrow Y(z)(1 - e^{-T} z^{-1})(1 - e^{-3T} z^{-1}) = X(z)\,T\,(1 - e^{-3T} z^{-1} - 1 + e^{-T} z^{-1})$$

$$Y(z)\left(1 + e^{-T} e^{-3T} z^{-2} - e^{-T} z^{-1} - e^{-3T} z^{-1}\right) = X(z)\,T\,(e^{-T} - e^{-3T})\, z^{-1}$$

[redacted]

$$Y(z) = X(z)\,T\,(e^{-T} - e^{-3T}) z^{-1} + Y(z)(e^{-T} + e^{-3T}) z^{-1} - Y(z)\, e^{-T} e^{-3T} z^{-2}$$

$$Z^{-1} \Rightarrow y[n] = T(e^{-T} - e^{-3T})\, x[n-1] + (e^{-T} + e^{-3T})\, y[n-1] - e^{-T} e^{-3T}\, y[n-2]$$

Hermed IO-relation !!