5. FIR filtre

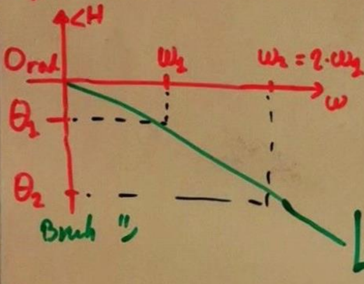
Can be seen from z^{M}

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{\sum_{k=0}^{M} b_k z^{M-k}}{z^{M}}$$

All poles located at 0 ⇒ always stable

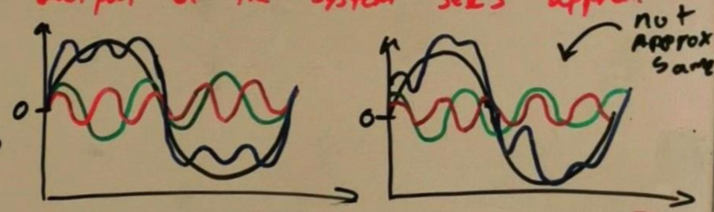
You can design with linear phase → constant group delay NO Feedback

System with linear phase



θ_2 is twice θ_1 which gives the same $\overbrace{\text{time delay}}^{\Delta t}$ to all frequencies. Therefor output of the system sees approx. same input waveform.

Approx Same



group delay $= \dfrac{d\,\angle H(e^{j\omega})}{d\omega}$ ← Phase response

Constant group delay = linear Phase = en konstant

from general linear Phase and general TF we can get following eq.

$\sum_{n=-\infty}^{\infty} h[n]\sin(\beta + (n-\alpha)\omega) = 0$ which should hold for all ω

without proof the following fulfills the conditions:

$h[n] = h[2\alpha - n] = h[M-n]$ where $\alpha = \frac{M}{2} \Rightarrow 2\alpha = M$ M = Integer even or odd

$\beta = \begin{cases} 0 \\ \pi \end{cases}$ or $\beta = \begin{cases} \pi/2 \\ 3\pi/2 \end{cases}$ Hold

We assume $h[n]$ is Causal and Symmetric impulse response

$\sum_{n=0}^{\infty}$ −ll− then: $h[n] = 0$ outside $0 \leq n \leq M$ where $n = \frac{M}{2} = \alpha$ is symmetry point

From this we can conclude the system $h[n]$ is FIR

Sym Impulse Response: $h[n] = h[M-n]$

Anti-Sym Impulse Response: $h[n] = -h[M-n]$

FIR Filter Types

M	even	odd
Sym $\beta = 0$ or π	I	II
Anti-Sym $\beta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$	III	IV