EIT5 solutions to exam exercises 2021

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EXERCISE 1

We are given $m(t) = 0.1\cos(2\pi\, 50\, t) + 0.2\cos(2\pi\, 3400\, t)$, $f_c = 1.4$ MHz, $A_c = 0.5$ V, and $k_a = 0.3$. Therefore, the carrier wave is $c(t) = 0.5\cos(2\pi\, 1\,400\,000\, t)$.

Recall that the modulated wave in DSBAM is given as

$$\begin{aligned} s(t) &= c(t)\,[1 + k_a\, m(t)] && (1)\\ &= 0.5\cos(2\pi\, 1\,400\,000\, t)\,[1 + 0.3\,(0.1\cos(2\pi\, 50\, t) + 0.2\cos(2\pi\, 3400\, t))] && (2)\\ &= 0.5\,[1 + 0.03\cos(2\pi\, 50\, t) + 0.06\cos(2\pi\, 3400\, t)]\cos(2\pi\, 1\,400\,000\, t) && (3)\\ &= [0.5 + 0.015\cos(2\pi\, 50\, t) + 0.03\cos(2\pi\, 3400\, t)]\cos(2\pi\, 1\,400\,000\, t) && (4) \end{aligned}$$

To obtain the spectrum of the modulated wave, recall that

$$\mathcal{F}\left(A\cos(2\pi\, f_a\, t)\right) = \frac{A}{2}\left[\delta(f - f_a) + \delta(f + f_a)\right] \tag{5}$$

and, from the properties of convolution in the frequency domain

$$\mathcal{F}\left(A\cos(2\pi\, f_a\, t)\cos(2\pi\, f_b\, t)\right) = \frac{A}{4}\left[\delta(f - f_a + f_b) + \delta(f + f_a + f_b) + \delta(f - f_a - f_b) + \delta(f + f_a - f_b)\right]. \tag{6}$$

Thus, by using $f_c = 1.4\,$MHz for brevity, we get

$$\begin{aligned} S(f) &= \mathcal{F}(s(t)) && (7)\\ &= \frac{0.5}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]\\ &\quad + \frac{0.015}{4}\left[\delta(f - f_c + 50) + \delta(f + f_c + 50) + \delta(f - f_c - 50) + \delta(f + f_c - 50)\right]\\ &\quad + \frac{0.03}{4}\left[\delta(f - f_c + 3400) + \delta(f + f_c + 3400) + \delta(f - f_c - 3400) + \delta(f + f_c - 3400)\right] && (8)\\ &= 0.25\left[\delta(f - f_c) + \delta(f + f_c)\right]\\ &\quad + 0.00375\left[\delta(f - f_c + 50) + \delta(f + f_c + 50) + \delta(f - f_c - 50) + \delta(f + f_c - 50)\right]\\ &\quad + 0.0075\left[\delta(f - f_c + 3400) + \delta(f + f_c + 3400) + \delta(f - f_c - 3400) + \delta(f + f_c - 3400)\right]. && (9) \end{aligned}$$

Note that the formula we have in slide 24 of Lecture 1

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{k_a A_c}{2}\left[M(f - f_c) + M(f + f_c)\right], \tag{10}$$

where $M(f) = \mathcal{F}(m(t))$ can be used to reach the same result. The amplitude spectrum is given in Fig. 1.

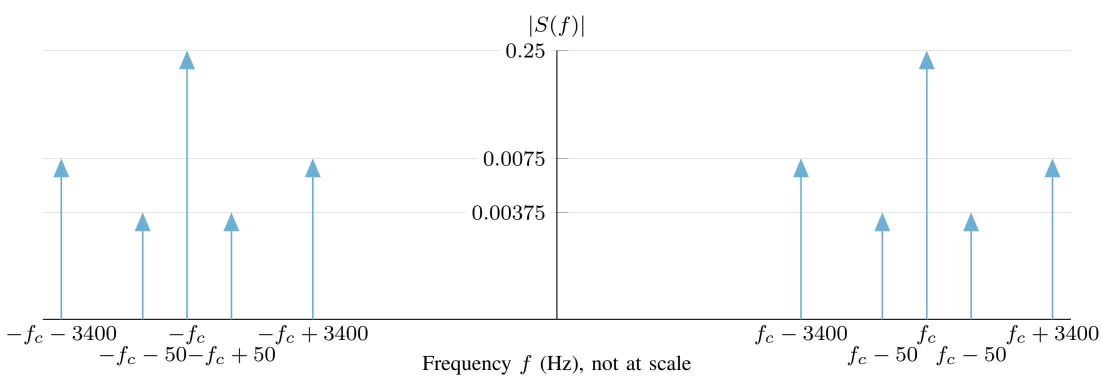

Fig. 1. Amplitude spectrum of the signal for exercise 1

Exercise 2

We know that

$$\Delta f = k_f A_m \tag{11}$$

and that

$$\beta = \frac{\Delta f}{f_m^*}. \tag{12}$$

a) The exercise gives $A_m = 0.1$ V, the maximum frequency for the baseband signal $f_m^* = 16\,000\,\text{Hz}$, and $k_f = 200\,000$ Hz/V, so we calculate $\Delta f = 200\,000 \times 0.1 = 20\,000\,\text{Hz}$. Then,

$$\beta = \frac{20\,000}{16\,000} = \frac{5}{4}. \tag{13}$$

b) With this, we can calculate the approximate bandwidth of the FM modulated wave as

$$B_{\text{FM}} \approx 2\Delta f + 2f_m^* = 40\,000 + 2(16\,000) = 72\,\text{kHz} \tag{14}$$

I. Exercise 3

a) In BPSK, a change in bits is indicated by a change in phase. According to slide 32 of Lecture 3, the symbols are represented as

$$\begin{cases} s_0(t) = A\cos(2\pi f_c t + 0) \\ s_1(t) = A\cos(2\pi f_c t + \pi). \end{cases} \tag{15}$$

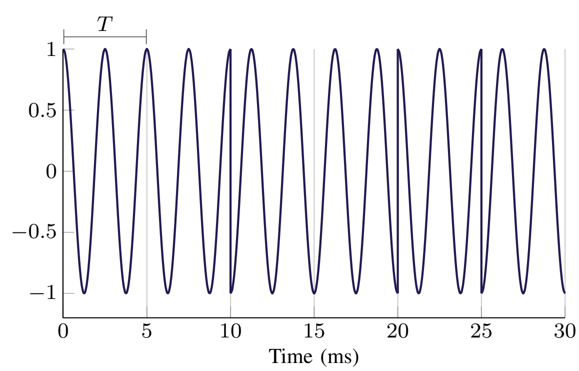

Fig. 2. BPSK signal given at the exam

In Fig. 2, the first symbol starts at $t = 0$ with a positive amplitude. Therefore, this first symbol must correspond to $s_0(t)$, as $\cos(0) = 1$. Building on this, the sequence of bits is

$$[0, 0, 1, 1, 0, 1]\,.$$

However, since the specific mapping of BPSK symbols to bits is not given in the exam, a shift in the carrier is acceptable. That is, a second solution to the exercise is to consider the representation

$$\text{Optional symbol representation:} \begin{cases} s_0(t) = A\cos(2\pi f_c t + \pi) \\ s_1(t) = A\cos(2\pi f_c t + 0). \end{cases} \tag{16}$$

This mapping gives the sequence $[1, 1, 0, 0, 1, 0]$.

b) The bit rate is determined by the number of bits per symbol $\log_2 M$ and the symbol rate R_{sym} as

$$R_b = R_{\text{sym}} \log_2 M. \tag{17}$$

Since the modulation is binary – only two symbols are possible – we have $M = 2$. Furthermore, we are given the symbol period $T = 5\,\text{ms}$. From here, we calculate the symbol rate as $R_{\text{sym}} = 1/T = 1/5 \times 10^{-3} = 200\,\text{Hz}$. As a consequence, the bit rate is

$$R_b = R_{\text{sym}} \log_2 M = 200 \log_2(2) = 200\,\text{bps} \tag{18}$$

c) The carrier frequency can be obtained from the number of cycles of the signal per unit time. In this case, the signal has two complete periods within each symbol. This means that, if the symbol period is $T = 5\,\text{ms}$, the period of the signal is half of that period, so $T_c = 2.5\,\text{ms}$. Since the frequency of a signal is the inverse of the period, we have that the carrier frequency is

$$f_c = 1/2.5 \times 10^{-3} = 400\,\text{Hz}. \tag{19}$$

d) The approximate bandwidth for BPSK is approximately twice the symbol rate. In b), we established that $R_{\text{sym}} = 200\,\text{Hz}$,so the bandwidth is $B_{\text{BPSK}} = 2(200) = 400\,\text{Hz}$.

e) In 4-ASK we have four different symbols, so $M = 4$. Each symbol represents $N = \log_2 M = 2$ bits. Therefore, for the signal obtained in a), namely

$$[0, 0, 1, 1, 0, 1]\,,$$

the first symbol must represent $[0, 0]$, the second symbol $[1, 1]$, and the third symbol $[0, 1]$.

From the 4-ASK image given at the exam we see that the amplitudes for these three symbols are $[3A, -3A, A]$. By using the same carrier frequency $f_c = 400\,\text{Hz}$ and symbol period $T = 5\,\text{ms}$, the signal is the one shown in Fig. 3

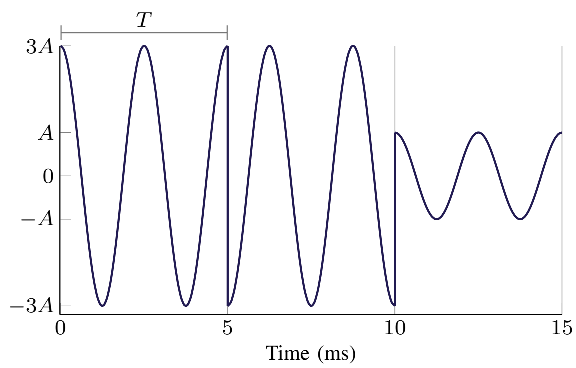

Fig. 3. 4-ASK signal to represent $[0, 0, 1, 1, 0, 1]$; only three symbols with amplitudes $[3A, -3A, A]$ are needed.