Lecture 9

6) Consider the complex sequence:

$$x[n] \begin{cases} e^{jw_0 n}, & 0 \le n \le N-1 \\ \text{otherwise}, & 0 \end{cases}$$

Geometric Series !!

$$\sum_{n=0}^{N-1} r^n = \frac{1-r^N}{1-r}$$

(a) Find fourier transform $X(e^{jw})$ of $x[n]$

$$X(e^{jw}) = \sum_{n=0}^{N-1} x[n]\, e^{-jwn} = \sum_{n=0}^{N-1} e^{jw_0 n} e^{-jwn} = \sum_{n=0}^{N-1} e^{-jn(w-w_0)}$$

$$= \frac{1 - e^{(-jn(w-w_0))N}}{1 - e^{-jn(w-w_0)}}$$

$$= e^{-j(w-w_0)((N-1)/2)} \left(\frac{\sin((w-w_0)(N/2))}{\sin((w-w_0)/2)} \right)$$

(b) Find the N-point DFT $X[k]$ of the finite length sequence $x[n]$

Twiddle !!

$$W_N^{kn} = e^{-j\frac{2\pi}{N}}$$

$$X(k) = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}, \quad 0 \le k \le N-1$$

$$= \sum_{n=0}^{N-1} e^{jw_0 n} e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1 - e^{-jn(\frac{2\pi}{N}k - w_0)N}}{1 - e^{-jn(\frac{2\pi}{N}k - w_0)}}$$

$$= e^{-j(\frac{2\pi}{N}k - w_0)((N-1)/2)} \left(\frac{\sin((\frac{2\pi}{N}k - w_0)(N/2))}{\sin((\frac{2\pi}{N}k - w_0)/2)} \right)$$

c) $w_0 = \frac{2\pi k_0}{N} \downarrow$

$$X(k) = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}, \quad 0 \le k \le N-1$$

$$X(k) = \sum_{n=0}^{N-1} e^{j\frac{2\pi k_0}{N}n} \cdot e^{-j\frac{2\pi k}{N}n} = \frac{1 - e^{-jn(\frac{2\pi k}{N} - \frac{2\pi k_0}{N})N}}{1 - e^{-jn(\frac{2\pi k}{N} - \frac{2\pi k_0}{N})}} = \frac{1 - e^{-jn2\pi(k-k_0)}}{1 - e^{-jn\frac{2\pi}{N}(k-k_0)}}$$

1 page algebra

$$e^{-j(\frac{2\pi}{N})(k-k_0)(\frac{N-1}{2})} \cdot \frac{\sin \pi(k-k_0)}{\sin(\pi(k-k_0)/N)}$$