Spm 6 mm 13 Observability, Observers and orbservability !!.

1) ①: $\begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}$; $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ $\det(\mathcal{O}) = 0 \Rightarrow$ Nonobservable

⑥ $\begin{cases} \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \\ y = \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{cases}$; $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ $\det(\mathcal{O}) = -1 \neq 0 \Rightarrow$ observable

- $\dot{x}$ Skal være fuld Rank
- Hvis uafhængig begge "output" skal observeres med ratio for at disting...
- hvis afhængig kan man nøjes med at kigge på det output der afhænger af den anden

2) $\dot{x} = \underbrace{\begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix}}_{A} x + \underbrace{\begin{pmatrix} 4 \\ 3 \end{pmatrix}}_{B} u$ Closed loop poly: $s^2 + 9s + 20$

$y = \underbrace{(1 \;\; 1)}_{C} x$. State feedback: $u = Fx = (-2 \;\; 2)x$

Shown to give pole placement acc. to $s^2 + 3s + 2$

1. $\begin{bmatrix} C \\ CA \end{bmatrix} = \mathcal{O} = \begin{bmatrix} 1 & 1 \\ 13 & -17 \end{bmatrix}$; Byg vores observable canonical form (A_o) som kræver T

$T = (t_1 \; t_2)$: $t_2 = \mathcal{O}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{30} \\ -\frac{1}{30} \end{bmatrix}$, $t_1 = At_2 = \begin{bmatrix} 16/30 \\ 14/30 \end{bmatrix}$. $A_o = T^{-1}AT = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} \Rightarrow \det(\lambda I - A) =$

$\lambda^2 + \lambda - 2$ } open loop

$C_o = CT = [1 \;\; 0]$, $L_o = \begin{bmatrix} \overbrace{1}^{\text{closed loop}} - 9 \\ \underbrace{-2}_{\text{Open loop charakthristic poly}} - 20 \end{bmatrix} = \begin{bmatrix} -8 \\ -22 \end{bmatrix}$, $L = TL_o = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$

$\mathrm{eig}(A + LC) = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$

$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & BF \\ -LC & A + LC + BF \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$ closed loop

$\mathrm{eig}\left(\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix}\right) = \lambda \begin{bmatrix} -5 \\ -4 \\ -2 \\ -1 \end{bmatrix}$ $\begin{matrix} \text{Nye poler} \\ \text{Gamle poler} \end{matrix}$ Observer based controller tilføjer nye poler til closed loop system