# EIT5 solutions to reexam exercises 2021

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## Exercise 1

We are given $m(t) = 0.5\cos(2\pi\, 50\, t) - 0.2\sin(2\pi\, 3400\, t)$, $f_c = 1.2$ MHz, $A_c = 0.5$ V, and $k_a = 0.4$. Therefore, the carrier wave is $c(t) = 0.5\cos(2\pi\, 1\,200\,000\, t)$.

Recall that the modulated wave in DSBAM is given as

$$
\begin{aligned}
s(t) &= c(t)\,[1 + k_a\, m(t)] && (1)\\
&= 0.5\cos(2\pi\, 1\,200\,000\, t)\,[1 + 0.4\,(0.5\cos(2\pi\, 50\, t) - 0.2\sin(2\pi\, 3400\, t))] && (2)\\
&= 0.5\,[1 + 0.2\cos(2\pi\, 50\, t) - 0.08\sin(2\pi\, 3400\, t)]\cos(2\pi\, 1\,200\,000\, t) && (3)\\
&= [0.5 + 0.1\cos(2\pi\, 50\, t) - 0.04\sin(2\pi\, 3400\, t)]\cos(2\pi\, 1\,200\,000\, t) && (4)
\end{aligned}
$$

To obtain the spectrum of the modulated wave, recall that

$$\mathcal{F}\left(A\cos(2\pi\, f_a\, t)\right) = \frac{A}{2}\left[\delta(f - f_a) + \delta(f + f_a)\right], \tag{5}$$

whereas

$$\mathcal{F}\left(A\sin(2j\pi\, f_a\, t)\right) = \frac{A}{2j}\left[\delta(f - f_a) - \delta(f + f_a)\right] \tag{6}$$

and, from the properties of convolution in the frequency domain

$$\mathcal{F}\left(A\cos(2\pi\, f_a\, t)\cos(2\pi\, f_b\, t\right) = \frac{A}{4}\left[\delta(f - f_a + f_b) + \delta(f + f_a + f_b) + \delta(f - f_a - f_b) + \delta(f + f_a - f_b)\right]. \tag{7}$$

and

$$\mathcal{F}\left(A\sin(2\pi\, f_a\, t)\cos(2\pi\, f_b\, t\right) = \frac{A}{4j}\left[\delta(f - f_a + f_b) - \delta(f + f_a + f_b) + \delta(f - f_a - f_b) - \delta(f + f_a - f_b)\right]. \tag{8}$$

Thus, by using $f_c = 1.2\,\text{MHz}$ for brevity, we get

$$
\begin{aligned}
S(f) &= \mathcal{F}(s(t)) && (9)\\
&= \frac{0.5}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]\\
&\quad + \frac{0.1}{4}\left[\delta(f - f_c + 50) + \delta(f + f_c + 50) + \delta(f - f_c - 50) + \delta(f + f_c - 50)\right]\\
&\quad - \frac{0.04}{4j}\left[\delta(f - f_c - 3400) + \delta(f + f_c - 3400) - \delta(f - f_c + 3400) - \delta(f + f_c + 3400)\right] && (10)\\
&= 0.25\left[\delta(f - f_c) + \delta(f + f_c)\right]\\
&\quad + 0.025\left[\delta(f - f_c + 50) + \delta(f + f_c + 50) + \delta(f - f_c - 50) + \delta(f + f_c - 50)\right]\\
&\quad - \frac{0.01}{j}\left[\delta(f - f_c - 3400) + \delta(f + f_c - 3400) - \delta(f - f_c + 3400) - \delta(f + f_c + 3400)\right]. && (11)
\end{aligned}
$$

To sketch the amplitude spectrum, given in Fig. 1, we need to take into account that $|j| = 1$.

Note that the formula we have in slide 24 of Lecture 1

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{k_a A_c}{2}\left[M(f - f_c) + M(f + f_c)\right], \tag{12}$$

where $M(f) = \mathcal{F}(m(t))$ can be used to reach the same result.

## Exercise 2

We know that

$$\Delta f = k_f A_m \tag{13}$$

and that

$$\beta = \frac{\Delta f}{f_m^*}. \tag{14}$$

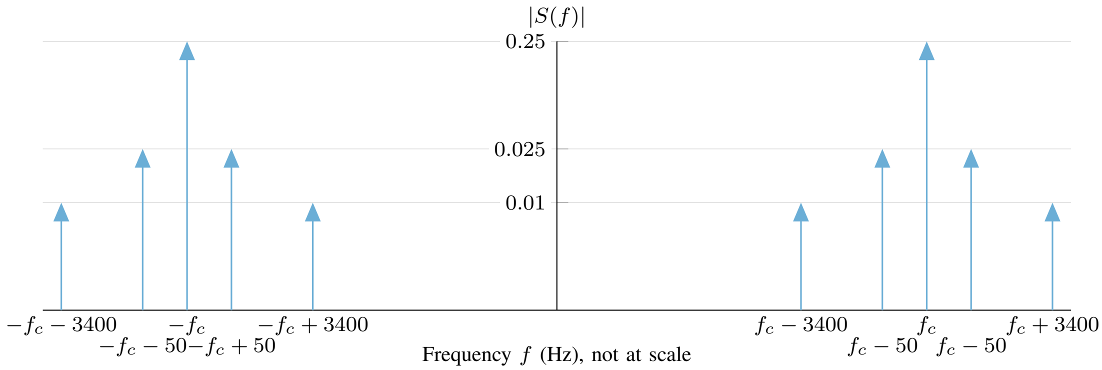


Fig. 1. Amplitude spectrum of the signal for exercise 1

a) The exercise gives $A_m = 0.1$ V, the maximum frequency for the baseband signal $f_m^* = 16\,000\,\text{Hz}$, and $k_f = 400\,000$ Hz/V, so we calculate $\Delta f = 400\,000 \times 0.1 = 40\,000\,\text{Hz}$. Then,

$$\beta = \frac{40\,000}{16\,000} = \frac{10}{4}. \tag{15}$$

b) With this, we can calculate the approximate bandwidth of the FM modulated wave as

$$B_{\text{FM}} \approx 2\Delta f + 2f_m^* = 80\,000 + 2(16\,000) = 112\,\text{kHz} \tag{16}$$

## I. Exercise 3

a) In OOK, a bit equal to 1 is signaled by a non-zero amplitude of the carrier, namely

$$\begin{cases} s_0(t) = 0 \\ s_1(t) = A\cos(2\pi f_c t). \end{cases} \tag{17}$$

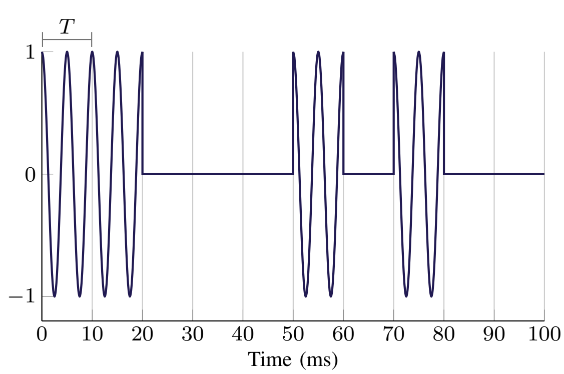


Fig. 2. OOK signal given at the reexam

In Fig. 2, the first symbol starts at $t = 0$ with a positive amplitude. Therefore, this first symbol must correspond to $s_1(t)$. Building on this, the sequence of bits is

$$[1, 1, 0, 0, 0, 1, 0, 1, 0, 0]\,.$$

b) The bit rate is determined by the number of bits per symbol $\log_2 M$ and the symbol rate $R_{\text{sym}}$ as

$$R_b = R_{\text{sym}} \log_2 M. \tag{18}$$

Since the modulation is binary – only two symbols are possible – we have $M = 2$. Furthermore, we are given the symbol period $T = 10\,\text{ms}$. From here, we calculate the symbol rate as $R_{\text{sym}} = 1/T = 1/10 \times 10^{-3} = 100\,\text{Hz}$. As a consequence, the bit rate is

$$R_b = R_{\text{sym}} \log_2 M = 100 \log_2(2) = 100 \,\text{bps} \tag{19}$$

c) The carrier frequency can be obtained from the number of cycles of the signal per unit time. In this case, the signal has two complete periods within each symbol. This means that, if the symbol period is $T = 10\,\text{ms}$, the period of the signal is half of that period, so $T_c = 5\,\text{ms}$. Since the frequency of a signal is the inverse of the period, we have that the carrier frequency is

$$f_c = 1/5 \times 10^{-3} = 200\,\text{Hz}. \tag{20}$$

d) The approximate bandwidth for BPSK is approximately twice the symbol rate. In b), we established that $R_{\text{sym}} = 100\,\text{Hz}$,so the bandwidth is $B_{\text{BPSK}} = 2(100) = 200\,\text{Hz}$.

e) In 4-ASK we have four different symbols, so $M = 4$. Each symbol represents $N = \log_2 M = 2$ bits. Therefore, for the signal obtained in a), namely

$$[1, 1, 0, 0, 0, 1, 0, 1, 0, 0]\,,$$

the first symbol must represent $[1, 1]$, the second symbol $[0, 0]$, the third symbol $[0, 1]$, the fourth symbol $[0, 1]$, and the fifth symbol $[0, 0]$.

From the 4-ASK image given at the reexam we see that the amplitudes for these three symbols are $[-3A, 3A, A, A, 3A]$. By using the same carrier frequency $f_c = 200\,\text{Hz}$ and symbol period $T = 10\,\text{ms}$, the signal is the one shown in Fig. 3

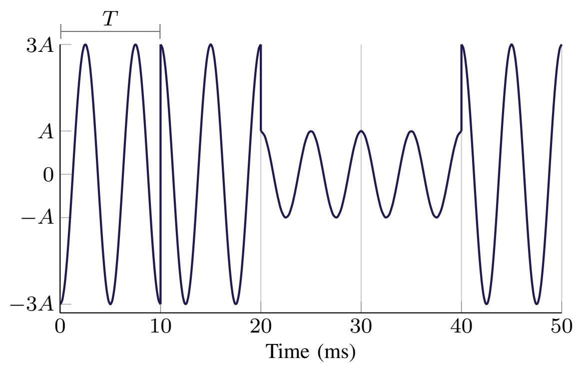


Fig. 3. 4-ASK signal to represent $[1, 1, 0, 0, 0, 1, 0, 1, 0, 0]$; only five symbols with amplitudes $[-3A, 3A, A, A, 3A]$ are needed.