Cascading Control

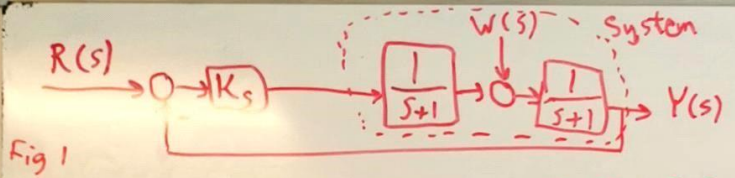

Fig 1

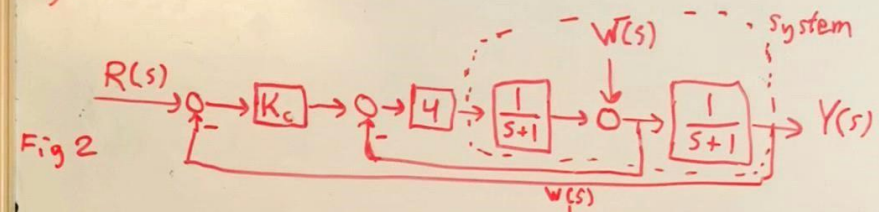

Fig 2

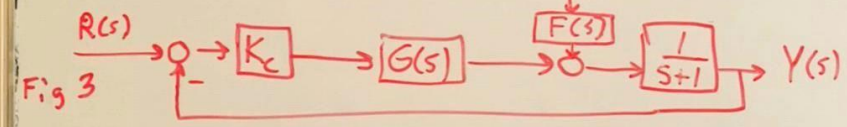

Fig 3

1) Find F(s) and G(s)

$$G(s) = \frac{4\frac{1}{s+1}}{1+4\frac{1}{s+1}} = \frac{4}{s+5}, \qquad F(s) = \frac{1}{1+4\frac{1}{s+1}} = \frac{s+1}{s+5}$$

2) Compare

Fig 2 is nice!

3) Determine series control K_s for %OS = 16% ($\zeta = 0.5$)

$$\frac{Y(s)}{R(s)} = \frac{K_s \frac{1}{s+1}\cdot\frac{1}{s+1}}{1+K_s\frac{1}{s+1}\cdot\frac{1}{s+1}} = \frac{K_s}{s^2+2s+1+K_s}$$

$2\zeta\omega_n = 2$

$\omega_n^2 = 1+K_s \Rightarrow \omega_n = \sqrt{1+K_s}$ $\quad 2\zeta\sqrt{1+K_s} = 2 \Rightarrow 2\cdot 0.5\cdot\sqrt{1+K_s} = 2 \Rightarrow \underline{K_s = 2^2-1 = 3}$

4) Determine cascade control K_c for %OS = 16% ($\zeta = 0.5$)

$$\frac{Y(s)}{R(s)} = \frac{K_c\frac{4}{s+5}\frac{1}{s+1}}{1+K_c\frac{4}{s+5}\cdot\frac{1}{s+1}} = \frac{4K_c}{s^2+6s+5+4K_c}, \qquad \omega_n = \sqrt{5+4K_c}$$

$6 = 2\zeta\omega_n = 2\zeta\sqrt{5+4K_c} \Rightarrow \underline{K_c = \frac{(3/\zeta)^2-5}{4} = 7.75}$ $\quad$ used $\zeta = 0.5$

5) Compare rise time

$T_r = \frac{1.8}{\omega_n}$, Series: $T_{r,s} = \frac{1.8}{\sqrt{1+3}} = \frac{1.8}{2} = 0.9$, Casc: $T_{r,c} = \frac{1.8}{\sqrt{5+4\cdot K_c}} = \frac{1.8}{6} = 0.3 \neq 0.9$