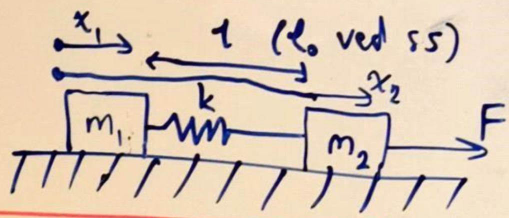
Control L11

1. Derive State Space Model

F: input

$y = \dot{x}_1$: output , $\dot{x}_2 = v_2$, V_1, V_2

$d = l - l_0$



$\longrightarrow t$

$$E_s = \frac{1}{2} k d^2 , \quad F_{s_1} = k d , \quad F_{s_2} = -k d$$

$$\dot{x}_1 = v_1 , \quad \dot{x}_2 = v_2$$

$$\dot{v}_2 = \frac{1}{m_2} F - \frac{k}{m_2} x_2 + \frac{k}{m_2} x_1$$

$$\dot{v}_1 = \frac{k}{m_1} x_2 - \frac{k}{m_1} x_1$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{V}_1 \\ \dot{V}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & 0 & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ V_1 \\ V_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{pmatrix}}_{B} (F)$$

$$V_1 = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}}_{C} \begin{pmatrix} x_1 \\ x_2 \\ V_1 \\ V_2 \end{pmatrix} + \underbrace{(0)}_{D} (F)$$

2) Eigenvalues (Proof by Matlab™)

$|A - \lambda i| = 0$;

$$\lambda_1 = 0 , \lambda_2 = 0 , \lambda_3 = \frac{\sqrt{-m_2 m_1 k (m_1 + m_2)}}{m_2 m_1} , \lambda_4 = -\lambda_3$$

if $m_1 = m_2 = k = 1$

then

$\lambda_1 = 0 , \lambda_2 = 0 , \lambda_3 = \sqrt{2} j , \lambda_4 = -\sqrt{2} j$

λ_1, λ_2 ss response ; λ_3, λ_4 swingy response no damping

3) Transfer Function:

$$G(s) = C(sI - A)^{-1} B + D$$

$$G(s) = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ \frac{k}{m_1} & \frac{-k}{m_1} & s & 0 \\ \frac{-k}{m_2} & \frac{k}{m_2} & 0 & s \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{pmatrix} + 0$$

MATLAB™ take the wheel!

$$G(s) = \frac{k}{m_1 m_2 s^3 + k m_1 s + k m_2 s}$$