Lecture 13

Ex 1.

(1): $\begin{cases} \dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{cases}$; $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$\det(\mathcal{O}) = 0 \Rightarrow$ non observable

(6): $\begin{cases} \dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x \\ y = \begin{bmatrix} 1 & 1 \end{bmatrix} x \end{cases}$; $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

$\det(\mathcal{O}) = -1 \Rightarrow$ observable !!

Ex 2.

$\dot{x} = \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u$; $u = Fx = (-2 \;\; 2) x$

$y = (1 \;\; 1) x$

1) Design full order observer

$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 13 & -17 \end{bmatrix}$

$t_2 = \frac{1}{-30} \begin{bmatrix} -17 & -1 \\ -13 & 1 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{30} \\ -\frac{1}{30} \end{pmatrix}$

$t_1 = At_2 = \begin{bmatrix} 7 & -9 \\ 6 & -8 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{30} \\ -\frac{1}{30} \end{bmatrix} = \begin{bmatrix} \frac{16}{30} \\ \frac{14}{30} \end{bmatrix}$

$T = \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix}$ $\quad T^{-1} = \begin{bmatrix} 1 & 1 \\ 14 & -16 \end{bmatrix}$

$A_o = T^{-1} A T = \begin{bmatrix} 1 & 1 \\ 14 & -16 \end{bmatrix} \begin{bmatrix} 7 & -9 \\ 6 & -8 \end{bmatrix} \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} \Rightarrow \det(\lambda I - A) = \lambda^2 + \lambda - 2$

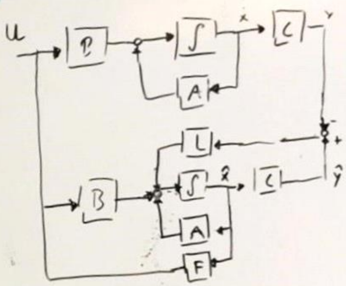
$C_o = CT = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$L_o = \begin{pmatrix} 1 - 9 \\ -2 - 20 \end{pmatrix} = \begin{pmatrix} -8 \\ -22 \end{pmatrix}$

[redacted] $L = TL_o = \begin{bmatrix} \frac{16}{30} & \frac{1}{30} \\ \frac{14}{30} & -\frac{1}{30} \end{bmatrix} \cdot \begin{bmatrix} -8 \\ -22 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$

$\text{eig}(A + LC) = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$

2) Draw diagram



3) Verify by computing eigenvalues for closed loop

$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & BF \\ -LC & A + LC + BF \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$

$\text{eig}\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \lambda \begin{bmatrix} -5 \\ -4 \\ -2 \\ -1 \end{bmatrix}$ $\begin{matrix} \} \text{ De nye poler} \\ \} \text{ De samme poler} \end{matrix}$