Lec 13:

Ex1

Determine for systems which is observable:

Exercise 1

Determine for each of the following systems, whether it is observable:

$$(1):\begin{cases}\dot{x} &= \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}x\\ y &= \begin{pmatrix}1 & 0\end{pmatrix}x\end{cases},\quad (2):\begin{cases}\dot{x} &= \begin{pmatrix}-1 & 1\\ 0 & -1\end{pmatrix}x\\ y &= \begin{pmatrix}1 & 0\end{pmatrix}x\end{cases},\quad (3):\begin{cases}\dot{x} &= \begin{pmatrix}-1 & 0\\ 1 & -1\end{pmatrix}x\\ y &= \begin{pmatrix}1 & 0\end{pmatrix}x\end{cases}$$

$$(4):\begin{cases}\dot{x} &= \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}x\\ y &= \begin{pmatrix}1 & 1\end{pmatrix}x\end{cases},\quad (5):\begin{cases}\dot{x} &= \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}x\\ y &= \begin{pmatrix}1 & 2\end{pmatrix}x\end{cases},\quad (6):\begin{cases}\dot{x} &= \begin{pmatrix}-1 & 0\\ 0 & -2\end{pmatrix}x\\ y &= \begin{pmatrix}1 & 1\end{pmatrix}x\end{cases}$$

$O = \begin{bmatrix} C \\ \mathrm{CA} \end{bmatrix}$, $\det(O) \neq 0$ *så* er den observable

1) $O = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, $\det(O) = 0$ non observable

Exercise 2

In the previous exercise sheet, we considered the following controllable system:

$$\begin{aligned}\dot{x} &= \begin{pmatrix}7 & -9\\ 6 & -8\end{pmatrix}x + \begin{pmatrix}4\\ 3\end{pmatrix}u\\ y &= \begin{pmatrix}1 & 1\end{pmatrix}x\end{aligned} \tag{1}$$

which can be shown to be also observable with the output above. The state feedback

$$u = Fx = \begin{pmatrix} -2 & 2 \end{pmatrix} x$$

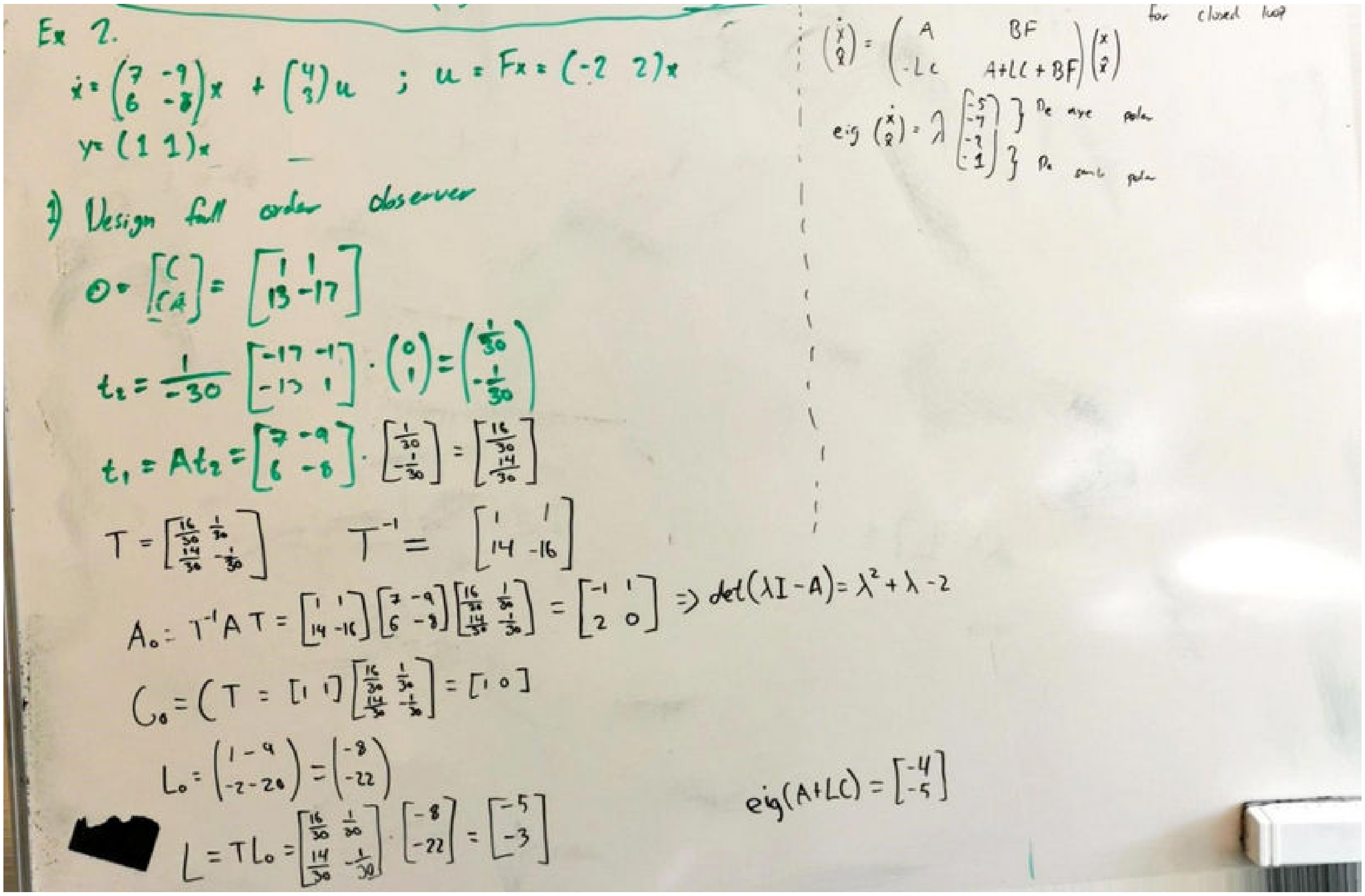
was shown to achieve a pole placement corresponding to the characteristic polynomial $s^2 + 3s + 2$.

1. Design a full order observer for (1) having characteristic polynomial $s^2 + 9s + 20$, i.e. find an observer gain L such that the eigenvalues of $A + LC$ become $\lambda_1 = -4$ og $\lambda_2 = -5$
2. Draw a diagram for an observer based compensator using F and L.
3. Verify, by computing the eigenvalues of the system matrix for the closed loop system:

;

$$\dot{\begin{pmatrix} x \\ \hat{x} \end{pmatrix}} = \begin{pmatrix} A & BF \\ -LC & A + LC + BF \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

1) og 2)




```
close all
clear

xdot = [7 -9; 6 -8];

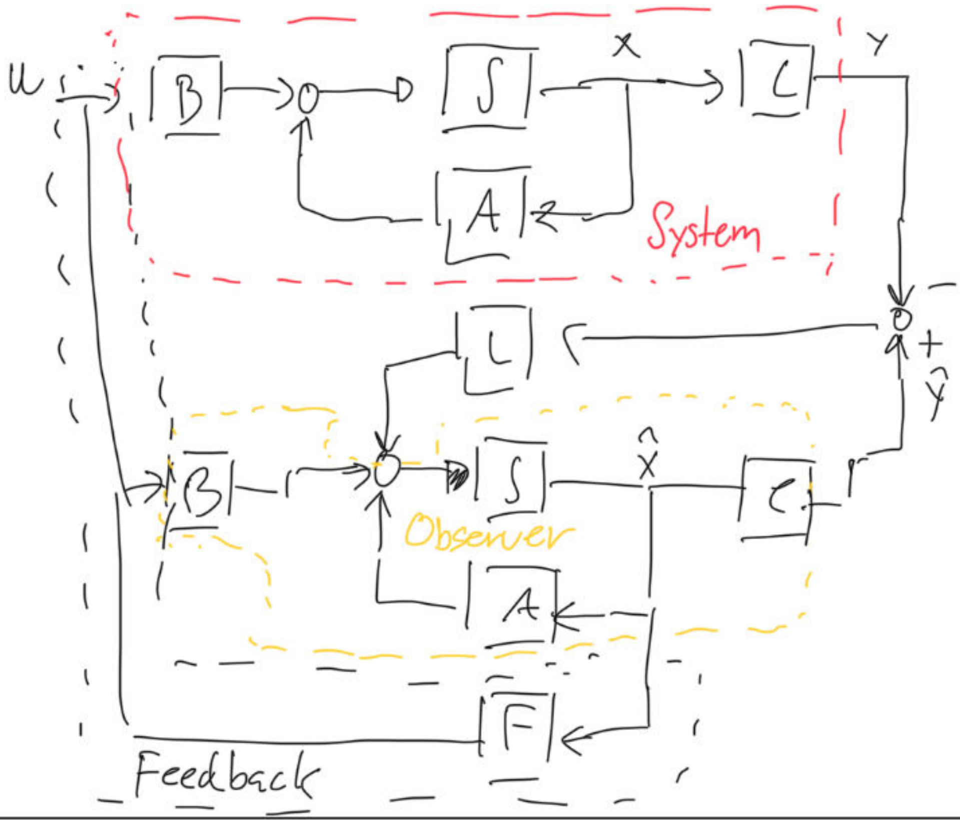
y = [1 1];

O = [y; y*xdot];
t2 = inv(O)*[0;1];
t1 = xdot*t2;
T = [t1(1) t2(1); t1(2) t2(2)];
Ao = inv(T)*xdot*T;
Co = y*T;
Lo = [-8; -22];
L = T*Lo;
eig(xdot + L*y);
```

ans = 2×1

```
                  -4
                  -5
ans = 2×1
                  -5
                  -4
```

2) Draw a diagram for an observer based compensator using F and L



Hvor L = observer gain

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3)

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closedloop = [xdot [4;3]*[-2 2]; -L*y (xdot + L*y + [4;3]*[-2 2])];
eig(closedloop)
```

```
ans = 4×1
         -5.00000000000002
         -3.99999999999996
         -2.00000000000001
         -1.00000000000001
```