

- L12

2)

$$\dot{x} = \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u$$

(1) Controllable Canonical Form

$$C = (B \ AB) = \begin{pmatrix} 4 & 1 \\ 3 & 0 \end{pmatrix} \Rightarrow C^{-1} = \frac{1}{-3} \begin{pmatrix} 0 & -1 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 1/3 \\ 1 & -4/3 \end{pmatrix}$$

$$s_2 = (0 \ 1) \begin{pmatrix} 0 & 1/3 \\ 1 & -4/3 \end{pmatrix} = (1 \ -4/3)$$

$$s_1 = s_2 A = (1 \ -4/3) \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} = (-1 \ \frac{5}{3})$$

$$T^{-1} = \begin{pmatrix} -1 & 5/3 \\ 1 & -4/3 \end{pmatrix} \Rightarrow T = \frac{1}{-1/3} \begin{pmatrix} -4/3 & -5/3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 3 & 3 \end{pmatrix}$$

$$A_c = T^{-1} A T = \begin{pmatrix} -1 & 5/3 \\ 1 & -4/3 \end{pmatrix} \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} \begin{pmatrix} 4 & 5 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5/3 \\ 1 & -4/3 \end{pmatrix} \begin{pmatrix} 1 & 8 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$$

$$B_c = T^{-1} B = \begin{pmatrix} -1 & 5/3 \\ 1 & -4/3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\dot{x} = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

(2) choose char poly: $\lambda^2 + 3\lambda + 2$; open loop: $\lambda^2 + \lambda - 2$

$$F_c = (1-3 \ -2-2) = (-2 \ -4)$$

$$F = F_c T^{-1} = (-2 \ -4) \begin{pmatrix} -1 & 5/3 \\ 1 & -4/3 \end{pmatrix} = (-2 \ 2)$$

$$(3) A + BF = \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} (k_1 \ k_2) = \begin{pmatrix} 7+4k_1 & -9+4k_2 \\ 6+3k_1 & -8+3k_2 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 0 & -2 \end{pmatrix}$$

Poles: $(-1, -2)$