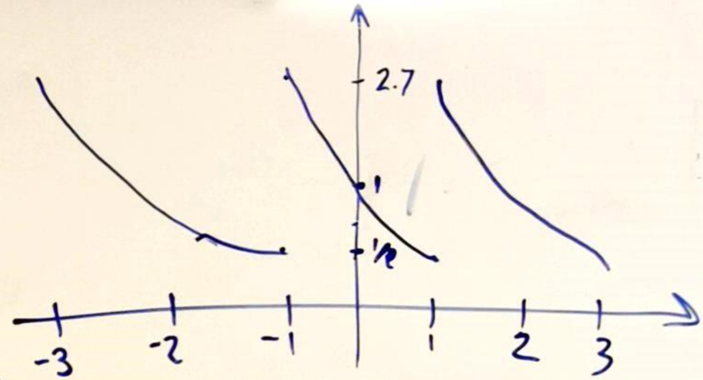
Lec 8

Periodic time signal $[-1,1]$ $x(t) = e^{-t}$

In seconds

a) Plot $x(t)$ in interval $t \in [-3, 3[$ sec



b) Derive $X[k]$

$$X[k] = \frac{1}{2}\int_{-1}^{1} e^{-t} e^{-j\frac{2\pi}{2}kt}\,dt = \frac{1}{2}\int_{-1}^{1} e^{-t(1+j\pi k)}\,dt$$

$$X[k] = \frac{1}{2}\cdot\frac{-1-j\pi k}{-1-j\pi k}\int_{-1}^{1} e^{-t(1+j\pi k)}\,dt = \frac{-1}{2}\cdot\frac{1}{1+j\pi k}\left[e^{-t(1+j\pi k)}\right]_{-1}^{1}$$

$$= \frac{-1}{2}\cdot\frac{1}{1+j\pi k}\left(e^{-1-j\pi k} - e^{1+j\pi k}\right) = \frac{e\cdot e^{j\pi k} - e^{-1}\cdot e^{-j\pi k}}{2(1+j\pi k)} =$$

$$= \frac{e(\cos(k\pi)+j\cdot\sin(k\pi)) - e^{-1}(\cos(k\pi) - j\cdot\sin(k\pi))}{2(1+jk\pi)} = \frac{e\cdot\cos(k\pi) - e^{-1}\cdot\cos(k\pi)}{2(1+jk\pi)}$$

$$= \boxed{\frac{(e-e^{-1})\cdot\cos(k\pi)}{2(1+jk\pi)}} = \frac{(e-e^{-1})\cdot(-1)^k}{2(1+jk\pi)} = \frac{(e-e^{-1})(-1)^k\cdot(1-jk\pi)}{2(1+jk\pi)(1-jk\pi)}$$

$$= \frac{(e-e^{-1})(-1)^k(1-jk\pi)}{1+k^2\pi^2}$$

$$\frac{A}{a+jb} = \frac{A(a-jb)}{a^2+b^2} = \frac{Aa-jAb}{a^2+b^2} = \frac{Aa}{a^2+b^2} - j\frac{Ab}{a^2+b^2}$$

$X[k] = Z = x+jy$

$|X[k]| = r = |Z| = \sqrt{x^2+y^2}$

$\angle X[k]$ $\theta = \angle Z = \arctan\left(\frac{y}{x}\right)$