

Control L12

1) Determine Controllability

$$(1) \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$\det(B \mid AB) = \det \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} = 0$$

Not controllable

$$(2) \dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$\det(B \mid AB) = \det \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} = 0$$

! Controllable

$$(3) \dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$\det(B \mid AB) = \det \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = 1 \neq 0$$

Controllable

$$(4) \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

$$\det(B \mid AB) = \det \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = 0$$

! Controllable

$$(5) \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u$$

$$\det(B \mid AB) = \det \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} = 0$$

! Controllable

$$(6) \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

$$\det(B \mid AB) = \det \begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix} = -1 \neq 0$$

Controllable