

1.1 LMMSE

Let $X = [X_1, X_2, X_3, X_4]^T$ be a gaussian random vector with mean $[1, -3, 0, 2]^T$ and covariance matrix

$$C_X = \begin{bmatrix} 1 & -1 & 0.5 & -1 \\ -1 & 5 & 2.5 & 3 \\ 0.5 & 2.5 & 6.5 & 2 \\ -1 & 3 & 2 & 2.5 \end{bmatrix}$$

• Expression of the LMMSE of X_4 given X_1, X_2, X_3 .

$$\hat{\Theta} = x_4 \text{ given } \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \hat{\Theta} = \begin{bmatrix} x_4 \end{bmatrix} - C_{x_4, \underline{x}} C_{\underline{x}, \underline{x}}^{-1} \begin{bmatrix} \underline{x} \end{bmatrix} + C_{x_4, \underline{x}} C_{\underline{x}, \underline{x}}^{-1} \underline{x}$$

$$\hat{\Theta} = 2 + [-1, 3, 2] \begin{bmatrix} 1 & -1 & 0.5 \\ -1 & 5 & 2.5 \\ 0.5 & 2.5 & 6.5 \end{bmatrix}^{-1} \left(\underline{x} - \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} \right)$$

$$\hat{\Theta} = 2 + \begin{bmatrix} -1.64 & 1.45 & -0.63 \\ 0.48 & 1.17 & -0.38 \\ 0.31 & -0.56 & 0.5 \end{bmatrix} \left(\underline{x} - \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} \right)$$

$$\hat{\Theta} = 4.03 \text{ for } \underline{x} = \begin{bmatrix} 0.5 \\ -1 \\ 4 \end{bmatrix}$$