Q4 Explain how to solve convex OP using the KKT

* Used to relate constrained OP to unconstrained OP

* Let $x' \in \mathbb{R}^n$ be regular minimizer for

min $f(x)$ s.t $a(x) = 0$, $c(x) \geq_e 0$ (x if regular point the gradient of active constraints are linearly independent)

* Then there exists a vector λ' in $\mathbb{R}^p$ and a vector $\mu' \geq_e 0$ in $\mathbb{R}^q$ such that (1) $\nabla f(x') = J_a(x')^T \lambda' + J_u(x')^T \mu' = \sum \lambda_i' \nabla a_i(x') + \sum \mu_i' \nabla c_i(x')$.

(2) $0 = c(x')^T \mu' = \sum \mu_i' c_i(x')$ (Inequalities are active or non-influential for the solution.)

λ_i' and μ_i' are Lagrange multipliers, where $\mu' \geq_e 0$ are the KKT condition

* So the only terms (1) which contributes are those connected to the active constraints

if OP is convex then this conditions also apply for a Global minimizer (that is if f and $-c$ are convex and $a(x) = Ax + b$ is affine linear)

Slide 14

These first order conditions yields minimizer candidates by solving:

$\nabla f(x) - J_a(x)^T \lambda - J_c(x)^T \mu = 0$ ← from (1)

$a(x) = 0$

$c(x)^T \mu = 0$ ← from (2)

and the system

$c(x) \geq_e 0$,

$\mu \geq_e 0$

of $2q$ inequalities in the $n+p+q$ unknowns (x, λ, μ)

Second order: You take the Hessian and see if it is PD or PSD

$H_f(x) \succeq 0$ reg minimizer

$H_f(x) \succ 0$ Strong local minimizer

KKT = Karush Kuhn Tucker

Optimization Probelm = OP