Question 4   Voltages x_1, x_2 [V], constant voltage load 1 [V]

x_1 upper bound by 0,5 [V], cost x_1^2

x_2 NOT upper bound, cost $2x_2^2$

Tjek Mohammed Note use jacobian

We use lagrangian

a) find the candidate solutions using the KKT condition

cost function

$$f(x_1, x_2) = x_1^2 + 2x_2^2$$

equality constraint

$$x_1 + x_2 = 1$$

e.g

$$x_1 + x_2 - 1 = 0$$

inequality constraints

$$x_1 \leq 0{,}5 \qquad x_1 \geq 0 \qquad x_2 \geq 0$$

e.g

$$x_1 - 0.5 \leq 0 \qquad -x_1 \leq 0 \qquad -x_2 \leq 0$$

Lagrangian

$$\mathcal{L}(x_1, x_2, \lambda, y_1, y_2, y_3) = x_1^2 + 2x_2^2 + \lambda(x_1 + x_2 - 1) + y_1(x_1 - 0{,}5) + y_2(-x_1) + y_3(-x_2)$$

can also be

$$\mathcal{L}(-\parallel-) = x_1^2 + 2x_2^2 + \lambda(x_1 + x_2 - 1) + y_1(-x_1 + 0{,}5) + y_2(x_1) + y_3(x_2)$$

Stationarity

$$\frac{\partial \mathcal{L}}{\partial x_1} = 2x_1 + \lambda + y_1 - y_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 4x_2 + \lambda - y_3 = 0$$

and

$$y_1 \geq 0, \; y_2 \geq 0, \; y_3 \geq 0$$

$$y_1(x_1 - 0{,}5) = 0$$

$$y_2(-x_1) = 0$$

$$y_3(-x_2) = 0$$

Try $x_1 = 0{,}5$

$y_1(0{,}5 - 0{,}5) = 0$

$0{,}5 + x_2 - 1 = 0, \; x_2 = 0{,}5$

$0{,}5 > 0$, for both x_1 and x_2

$y_2(-0{,}5) = 0, \; y_2 = 0$

$y_3(-0{,}5) = 0, \; y_3 = 0$

$2 \cdot 0{,}5 + \lambda + y_1 - 0 = 0 \rightarrow 1 + \lambda + y_1 = 0$

$4 \cdot 0{,}5 + \lambda - 0 = 0 \rightarrow 2 + \lambda = 0 \rightarrow \lambda = -2$

$1 + (-2) + y_1 = 0 \rightarrow y_1 = 1$

Try $x_1 = 0$

see MatLab

cost

for $x_1, x_2 = 0{,}5$

$f = 0{,}5^2 + 2 \cdot 0{,}5^2 = 0{,}75$

for $x_1 = 0, \; x_2 = 1$

$f = 0^2 + 2 \cdot 1^2 = 2$

Therefor $x_1, x_2 = 0{,}5$ better