Lec 5   Constrained Optimization 1

Ex. 1 X_1 [V], X_2 [V] provide constant voltage $U_L = 1$ [V]. Cost for first X_1^2

limit to 0.5 [V]. Cost of second is $2X_2^2$ and is not limit.

a) Formulat optimization problem. Convex? $f(x) = X_1^2 + 2X_2^2$, $H(f(x)) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

$$\min_{x \in \mathbb{R}} f(x) \quad \text{S.t} \quad \begin{cases} X_1 + X_2 = 1 \\ 0 \leq X_1 \leq 0.5 \\ X_2 \geq 0 \end{cases} \quad ; \quad \text{Since } Q = X_1 + 2X_2 \text{ is positive it is convex }$$

$\underset{\text{Hessian}}{\hookrightarrow}$

b) point $x = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ **[V] regular for the active constraints? Yes**

$X_1 + X_2 = 1$ Sat X ind: $0.5 + 0.5 = 1$ ✓   Tag ∇ af aktiv: $\nabla(X_1 + X_2) = [1\ 1]^T$

$X_1 = 0.5$ —''— : $0.5 = 0.5$ ✓ $\nabla(X_1) = [1\ 0]^T$

$X_1 = 0$ —''—: $0.5 = 0$ ✗

$X_2 = 0$ —''—: $0.5 = 0$ ✗

$$\text{rank}\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right) = \text{Full Rank}$$

c) Rewrite such that all constraints are equality constraints

         slack variable

$0 \leq X_1$; $-X_1 + 0 = S_1$ $S_1 \geq 0$ X_2 samme som X_1 med ny S

$X_1 \leq 0.5$; $X_1 - 0.5 + S_2 = 0$ $S_2 \geq 0$

d) Rewrite such the problem is inequality

$X_1 + X_2 = 1 \rightarrow X_1 + X_2 - 1 = 0 \rightarrow X_1 + X_2 - 1 \geq 0$

$\qquad\qquad\qquad \downarrow X_1 + X_2 - 1 \leq 0 \qquad = 0$

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for all $g(x) \leq 0$

↓

$g(x) + S = 0$, $S \geq 0$

$Y = S$, $X_1 \geq 0$

$X_1 \leq 0 \quad -X_1 + 0 = S_1 \quad S_1 \geq 0$

$X_1 - 0.5 + S_2 = 0 \quad S_2 \geq 0$

$h(x) = 0$

$h(x) \leq 0 \quad -h(x) \leq 0$