Lec 5 Constrained Optimization 1

Ex. 1 $X_1$ [V], $X_2$ [V] provide constant voltage $U_L = 1$ [V]. Cost for first $x_1^2$ limit to 0.5[V]. Cost of second is $2x_2^2$ and is not limit.

a) Formulat optimization problem. Convex? $f(x) = x_1^2 + 2x_2^2$, $H(f(x)) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

$$\min_{x \in R} f(x) \quad \text{S.t} \begin{cases} x_1 + x_2 = 1 \\ 0 \le x_1 \le 0.5 \\ x_2 \ge 0 \end{cases}$$

: Since $Q = x_1 + 2x_2$ is positive it is convex :)
↳ Hessian

↳ Sets and equalities are convex !!

b) point $x = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ [V] regular for the active constraints? Yes

$x_1 + x_2 = 1$ set x ind: $0.5 + 0.5 = 1$ ✓ Tag ∇ af aktiv: $\nabla(x_1 + x_2) = [1\ 1]^T$

$x_1 = 0.5$ —11— : $0.5 = 0.5$ ✓ $\nabla(x_1) = [1\ 0]^T$

$x_1 = 0$ —11— : $0.5 = 0$ ✗

$x_2 = 0$ —11— : $0.5 = 0$ ✗

$\text{rank}\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right) = \text{Full Rank}$

c) Rewrite such that all constraints are equality constraints
slack variable

$0 \le x_1$ ; $-x_1 + 0 = s_1$ $s_1 \ge 0$ $x_2$ samme som $x_1$ med ny $s$

$x_1 \le 0.5$; $x_1 - 0.5 + s_2 = 0$ $s_2 \ge 0$

d) Rewrite such the problem is inequality

$x_1 + x_2 = 1 \rightarrow x_1 + x_2 - 1 = 0 \rightarrow x_1 + x_2 - 1 \ge 0$ $= 0$
$\searrow\ x_1 + x_2 - 1 \le 0$

↳ Mohammads lesning

$x_1 \ge 0 \Rightarrow x_1 = y_1^2$

$x_2 \ge 0 \Rightarrow x_2 = y_2^2$

$-x_1 + 0.5 \ge 0 \Rightarrow -x_1 + 0.5 = y_3^2$

For all $g(x) \le 0$

↓

$g(x) + s = 0$, $s \ge 0$

$x_1 = s_1$ $x_1 \ge 0$

$x_1 \le 0$ $-x_1 + 0 = s_1$ $s_1 \ge 0$

$x_1 - 0.5 + s_2 = 0$ $s_2 \ge 0$

$h(x) = 0$

$h(x) \le 0$ $-h(x) \le 0$