Ex. 2.

$f(x) = x^T A^T A x + b^T x$

$A = \begin{bmatrix} \frac{1}{5} & -\frac{1}{3} \\ \frac{1}{20} & \frac{1}{5} \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ $Q = A^T A$

$A^T = \begin{bmatrix} \frac{1}{5} & \frac{1}{20} \\ -\frac{1}{3} & \frac{1}{5} \end{bmatrix}$, $b^T = [1 \ 4]$ $(AB)^T = B^T A^T$

a: gradient function

$g(x) = \nabla f(x)$

Hint:

$\nabla f(x) = \frac{d}{dx}\left(x^T Q x + b^T x + c\right) = Q^T x + Qx + b$

$g(x) = \nabla f(x) = A A^T x + A^T A x + b$

$Q^T = (A^T A)^T = A^T A$

b: Hessian function $2 A^T x$

$H_f(x) = \nabla g(x) = \nabla \nabla f(x)$ or $\text{Jacobian}(g(x)) = H_f(x)$

$\nabla g(x) = \frac{d}{dx}\left(A A^T x + A^T A x + b\right) = A A^T + A^T A$

$\text{eig}(\nabla g(x)) =$ 0,350 og 0,036 ; $H(x)$ eig ≥ 0 (PSD)

0.58 −0.58 2nd result

c: is it convex non convex??

Yes, because eig of $H_f(x)$ is positive semidefinite

d: Jacobian of $f(x) = Ax$; $A = \mathbb{R}^{n \times m}$

$J_f(x) = A \Rightarrow J_f(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial x_m} & \cdots & \frac{\partial f_N}{\partial x_m} \end{bmatrix}$