

Ex. 2.

$$f(x) = x^T A^T A x + b^T x$$

$$A = \begin{bmatrix} \frac{1}{5} & -\frac{1}{3} \\ \frac{1}{20} & \frac{1}{5} \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$Q = A^T A$$

a: gradient function

$$g(x) = \nabla f(x)$$

Hint:

$$\nabla f(x) = \frac{d}{dx} (x^T Q x + b^T x + c) = Q^T x + Q x + b$$

$$g(x) = \nabla f(x) = A A^T x + A^T A x + b$$

b: Hessian function

$$H_f(x) = \nabla g(x) = \nabla \nabla f(x) \text{ or Jacobian}(g(x)) = H_f(x)$$

$$\nabla g(x) = \frac{d}{dx} (A A^T x + A^T A x + b) = A A^T + A^T A$$

$$\text{eig}(\nabla g(x)) = \begin{matrix} 0.350 & \text{or} & 2.026 \\ 0.58 & -0.58 & \text{2nd result} \end{matrix} ; H(x) \text{ eig} \geq 0 \text{ (PSD)}$$

c: is it convex non convex??

Yes, because eig of  $H_f(x)$  is positive semidefinite

d: Jacobian of  $f(x) = A x$  ;  $A = \mathbb{R}^{n \times m}$

$$J_A(x) = A$$

$$\Rightarrow J_f(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \dots & \frac{\partial f_n}{\partial x_m} \end{bmatrix}$$