

Ex 1 transmitter $x_1 = (-2, -4)$, distance between t_x and r_x
 $P(x) \propto \frac{1}{d^2}$

Following constraints in the euclidean plane:

$$C_1(x) = -x_1^2 - (x_2 + 4)^2 + 16 \geq 0, \quad C_2(x) = x_1 - x_2 - 6 \geq 0$$

a) Formulate the optimization problem. Is the optimization problem convex?

$$f_0(x) = d^2 = |x_r - x_t|^2 = (x_{r1} - (-2))^2 + (x_{r2} - (-4))^2 \quad \text{Cost function}$$

$$\min f_0(x)$$

$$\text{s.t. } x_r \in R$$

$$R = \{x_r \in \mathbb{R}^2 \mid C_1(x_r), C_2(x_r)\}$$

Optimization Problem

Note: PSD = positive Semi Definite

$$H(f_0(x)) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad H(C_1(x)) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad H(C_2(x)) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

↑
PSD

ND →

↑
PSD

However the constraints are $C_1 \geq 0$ meaning it needs to be concave to be convex
 general rule for constraint to be convex

| | |
|---------------|-------------------|
| $f(x) \leq 0$ | $f(x)$ is convex |
| $f(x) \geq 0$ | $f(x)$ is concave |

ex showing this, rewrite $C_1(x) = x_1^2 + (x_2 + 4)^2 \leq 16$

$$H(C_1(x)) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

b) use rule $C_3(x) = x_1^2 + (x_2 + 6)^2 \geq 2$

$$C_3(x) = x_1^2 + (x_2 + 6)^2 \geq 2 \rightarrow H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

however the function is convex but constraint

concave because $f(x) \geq 2$

Not convex

given by

$$C_3(x) = -x_1^2 - (x_2 + 6)^2 - 2 \rightarrow H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

↑
PSD

(look at sum of squares)
RAC