Q_3 Explain the Gauss-Newton method.

* optimized for least squares * for vector valued functions

* we wish to find a point x^* such that $F(x) = 0$ (i.e., $f_i(x^*) = 0$ for all i)

$F: \mathbb{R}^n \to \mathbb{R}^m$; $x \mapsto F(x) = [f_1(x), \dots, f_m(x)]^T$

IO Relation

Map to

Hint
$\|x\|_2 = \sqrt{x_1^2 + \dots + x_m^2}$

* we need real valued function (sum of squares)

$$f: \mathbb{R}^n \to \mathbb{R} \; ; \; x \mapsto f(x) = F(x)^T F(x) = \|F(x)\|_2^2 = \sum f_i(x)^2$$

* Minimize $f(x)$ (this is Least squares)

$$\min_{x \in \mathbb{R}^n} f(x)$$

Same as finding zeros of F

* we use NR

update: $x_{k+1} = x_k + \alpha_k d_k$

direction: $-H_f(x_k)^{-1} \nabla f(x_k)$

in Gauss Newton we APProx H and ∇

$$\nabla f(x) = 2\, J_F(x)^T F(x) \; ; \; H_f(x) \approx 2\, J_F(x)^T J_F(x)$$

Maybe look at Why ??? NoT on Slides

We can remove Second term of Hessian when residuals are small

meaning SSE Small

(Good)