- Q7 Explain the general concept of PCA and how it can be computed using SVD. ; Covariance Matrix $C = \frac{1}{N} X^{(c)} (X^{(c)})^T$

-PCA: Dimension Reduction and find direction that captures most variance.

-Constrained problem $\max_{u} u^T \underset{\overset{\uparrow}{\text{covariance}}}{C} u$ s.t $u^T u = 1$ ← Unit vector

-Use Lagrangian $\mathcal{L}(u,\lambda) = u^T C u - \lambda(u^T u - 1)$. Take derivative $\frac{\partial \mathcal{L}}{\partial u} = Cu - \lambda u$

-Stationary point: $Cu = \lambda u$. This means that u is an eigenvector of C with eigenvalue λ. We want largest eigenvalue of C $\lambda = \lambda_1$ 90° !!

- Second eigenvector you add second constraint: $u_2^T u_2 = 1$, $u_2^T u_1 = 0$ $\max_{u_2} u_2^T C u_2$ s.t

Pick λ_2 as the nex largest eigenvalue

- How choose K : Individual Explained Var: $\frac{\lambda_i}{\sum_{i=1}^{d} \lambda_i}$, Cumulative explained Var: $\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{d} \lambda_i}$

-SVD: Directly on $X^{(c)}$. Avoids forming C and is more numerically stable cause C is expensive.

- $X = U \Sigma V^T$ ← Decomposition of X → Data Matrix $\mathbb{R}^{d \times N}$

XX^T → Singular left vectors $\mathbb{R}^{d \times d}$

Σ = Singular Values σ, $\mathbb{R}^{d \times N}$

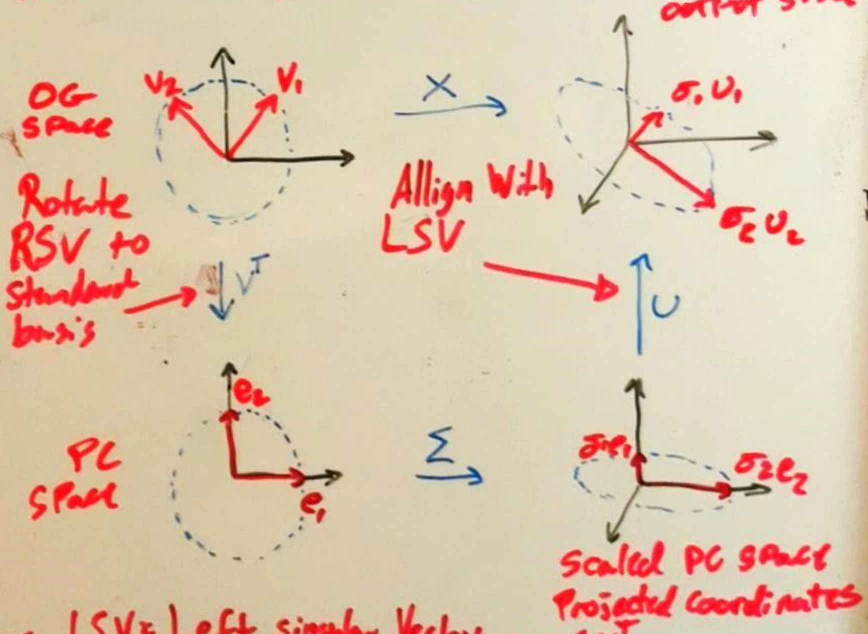
$V = X^T X$; Singular right vectors, $\mathbb{R}^{N \times N}$

Rank, $r = \text{rank}(X) \leq \min(d, N)$

Rows and coloumns with zeros Unsignificant.

-If $d > N$ then Dimension Eraser

If $d < N$ Dimension ADDER



RSV=Right singular Vectors, LSV= Left singular Vectors $z = \Sigma V^T$