MM10

## (1) Cable constant

Primary : L , C

Secondary : v , Z

$$v = \frac{1}{\sqrt{LC}} \left[\frac{m}{s}\right]$$

$$Z_0 = \sqrt{\frac{L}{C}} \ [\Omega]$$

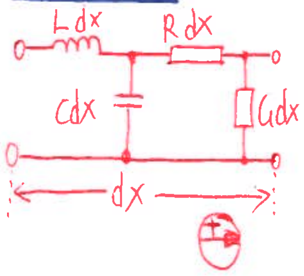
$$L = \frac{Z_0}{v} \left[\frac{H}{m}\right]$$

$$C = \frac{1}{vZ_0} \left[\frac{F}{m}\right]$$

(For loss free cable)

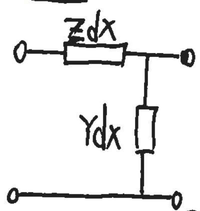
## Solution for harmonic (sine) signals

New model




$\Rightarrow$

$\boxed{KSN}$




$$Z = R + j\omega L \left[\frac{\Omega}{m}\right]$$

$$Y = G + j\omega C \left[\frac{S}{m}\right]$$

Voltage on cable:

$$\frac{dV}{dx} = -ZI \quad \left[\frac{V}{m}\right] \qquad (1)$$

and current on cable:

$$\frac{dI}{dx} = -YV \quad \left[\frac{A}{m}\right] \qquad (2)$$

V(x) and I(x) are KSN.

and Voltage and current on cable.

Add (1) and (2), we have:

$$\frac{d^2V}{dx^2} = ZYV \rightarrow \frac{d^2V}{dx^2} = \gamma^2 V$$

This is one dimension wave equation

We define.

$$\gamma \triangleq \sqrt{ZY} = \alpha + j\beta \ [m^{-1}]$$

Propagation constant

where:

$\alpha$ [Np/m] attenuation constant

$\beta$ [rad/m] phase propagation constant

## Neper ($N_p$)

Power $p = \frac{P_1}{P_2}$ [ · ]

Amplitude $A = \frac{A_1}{A_2}$ [ · ]

$P_{dB} = 10 \log P$

$A_{dB} = 20 \log A$

$A_{Np} = \ln A$ (i.e., $A = e^{A_{Np}}$)

$1[Np] = e^1 = 2.718\ldots = 20 \cdot \log e$ [dB]

$= 8.686$ [dB]

## (2) Lossless cable

We have $R = 0\,\frac{\Omega}{m}$, $G = 0\,\frac{S}{m}$

We get:

$$\gamma = \sqrt{ZY} = \sqrt{j\omega L \cdot j\omega C} = j\omega\sqrt{LC}$$

So

$$\alpha = 0 \quad Np/m$$

$$\beta = \omega\sqrt{LC} \quad [rad/m]$$

Solution:

$$V(x) = V^+(x) + V^-(x) = V^+e^{-j\beta x} + V^-e^{+j\beta x} \; [V]$$

$$I(x) = I^+(x) + I^-(x) = I^+e^{-j\beta x} + I^-e^{+j\beta x} \; [A]$$

$$= \frac{1}{Z_0}\left[V^+(x) - V^-(x)\right] \; [A]$$

We have:

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{L}{C}} \quad [\Omega]$$

$$v = \frac{1}{\sqrt{ZY}} \quad \left[\frac{m}{s}\right]$$

$$\beta = \omega\sqrt{ZY} = \frac{\omega}{v} \quad \left[\frac{rad}{m}\right] \leftarrow \bigstar$$

Note:

$V^+(x) = V^+e^{-j\beta x}$

$V^+(x)$ is complex function

$V^+ = V^+(0)$

complex constant

### Wave propagation

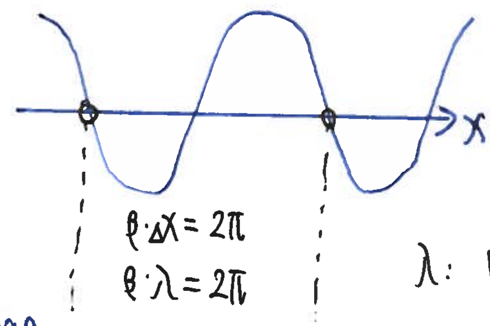
We see from the incident wave:

$$V^+(x) = V^+ \cdot e^{-j\beta x}$$

$$\xRightarrow{\boxed{KSN}} V^+ \cdot e^{-j\beta x} e^{j\omega t}$$

$$\rightarrow |V^+| \cdot \cos(\omega t - \beta x)$$

$\Rightarrow$ For growing time $t$




$\beta \cdot \Delta x = 2\pi$

$\beta \cdot \lambda = 2\pi$

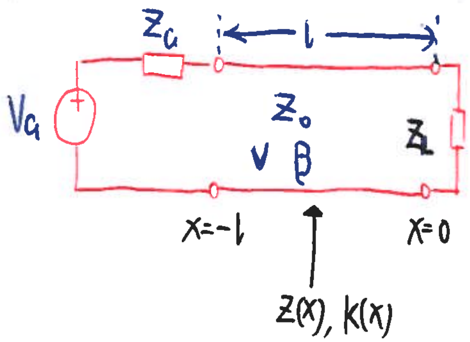
$\lambda$: wavelength [m]

where

$$\beta = \frac{\omega}{v} = \frac{2\pi}{\lambda} \Rightarrow \frac{2\pi f}{v} = \frac{2\pi}{\lambda} \Rightarrow \frac{f}{v} = \frac{1}{\lambda} \Rightarrow \lambda = \frac{v}{f}$$

$\bigstar$

## (3) Generalized $Z$ and $K$




It is defined:

$$Z(x) \triangleq \frac{V(x)}{I(x)} = \frac{V^+(x)+V^-(x)}{I^+(x)+I^-(x)} = Z_0 \cdot \frac{V^+(x)+V^-(x)}{V^+(x)-V^-(x)}$$

$$= Z_0 \cdot \frac{1+\frac{V^-}{V^+}}{1-\frac{V^-}{V^+}} = Z_0 \cdot \frac{1+K(x)}{1-K(x)} \; [\Omega] \quad (3)$$

Generalized Impedance

$$K(x) \triangleq \frac{V^-(x)}{V^+(x)} = \frac{V^- e^{+j\beta x}}{V^+ e^{-j\beta x}} = \frac{V^-(0)}{V^+(0)} \cdot e^{j2\beta x}$$

※· $\boxed{KSN}$

$$= K(0) \cdot e^{j2\beta x} = K(0) e^{-j2\beta l} \; [\cdot]$$

$\cancel{K(0) e^{j2\beta l}}$

When $K(0) = K_L$, we have: $K(x) = \underbrace{K_L \, e^{j2\beta x}}_{\text{amplitude is not changed with } x.}$

From (3), we also have:

$$K(x) = \frac{Z(x)-Z_0}{Z(x)+Z_0} \; [\;]$$

For $x=0$;

$$K_L = \frac{Z_L - Z_0}{Z_L + Z_0} \; [\cdot]$$

where, $Z(0) = Z_L$ and $K(0) = K_L$

We also have:

$$K(x) = \frac{V^-(x)}{V^+(x)} = -\frac{I^-(x)}{I^+(x)} \; [\cdot]$$

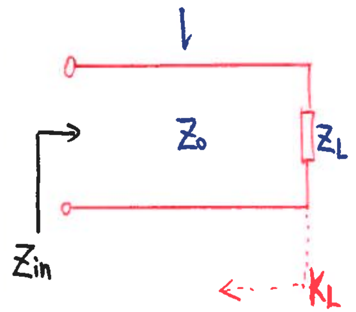
$$Z(x) = \frac{V(x)}{I(x)}$$

$$Z_0 = \frac{V^+(x)}{I^+(x)}$$

$$V(x) = V^+(x)\,(1+K(x))$$

$$I(x) = I^+(x)\,(1-K(x))$$

## Input impedance

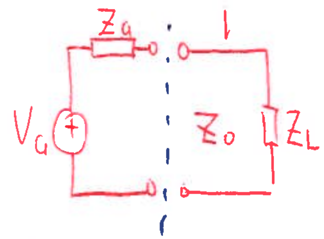



$$K_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$K(-l) = K_L \cdot e^{-j2\beta l}$$

$$Z_{in} = Z(-l) = Z_0 \cdot \frac{1+K(-l)}{1-K(-l)} \; [\Omega]$$

## Further calculations




$$V(-l) = V_G \cdot \frac{Z(-l)}{Z_G + Z(-l)}$$