High-Speed Electronics in Practice

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MM7. Impulse Propagation on Cables and Telegraph Equation

IMPULSE PROPAGATION ON CABLES AND TELEGRAPH EQUATION

Targets:

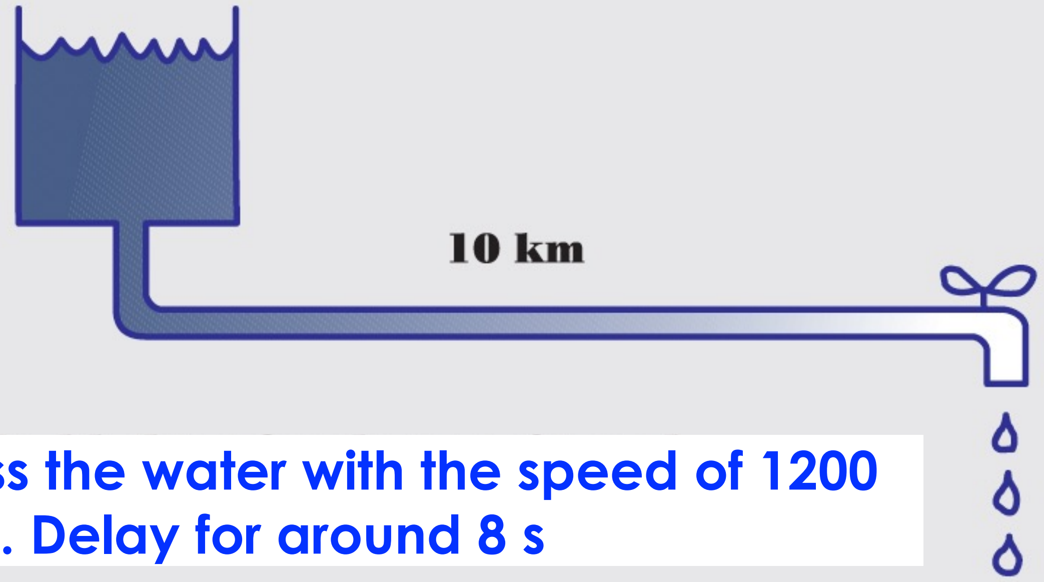
1. Read "Grundlmggende Transmissionsledningsteori" (Page 1-22) (before or after the lecture)

2. Be able to calculate with reflection map (lecture)

3. Finish the exercise (after the lecture)

Questions

Water container



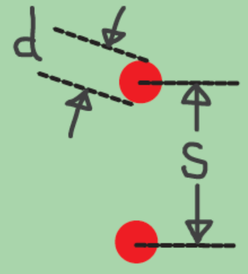
Press the water with the speed of 1200 m/s. Delay for around 8 s

How much current runs in the cable, right after turn on the switch?

Transmission Line

Blackboard (1)

Parallel cable



PARALLEL LEDER

$$L = \frac{\mu}{\pi} \cdot \ln\left(\frac{2S}{d}\right) \quad [H/m]$$

$$C = \pi \cdot \varepsilon \cdot \frac{1}{\ln\left(\frac{2S}{d}\right)} \quad [F/m]$$

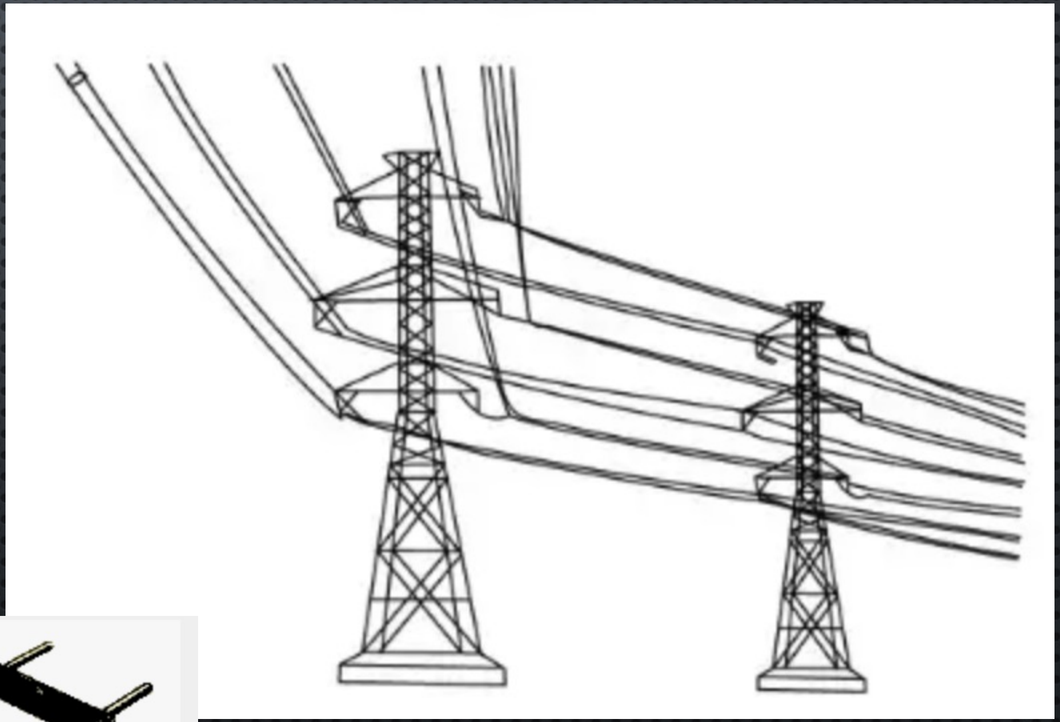
$$Z_0 = \frac{120}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{2S}{d}\right) \quad [\Omega]$$

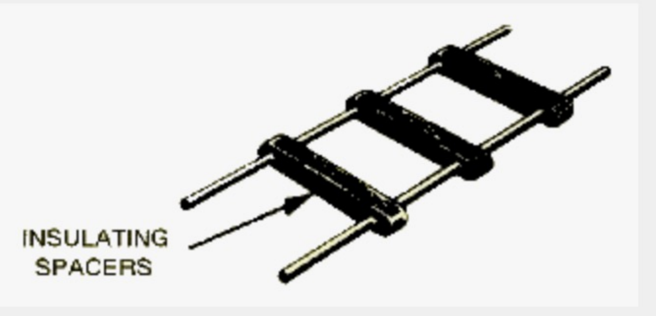
(for $\mu_r = 1$)

$$v = \frac{1}{\sqrt{\mu \cdot \varepsilon}} \quad [m/s]$$

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r \quad [F/m]$$

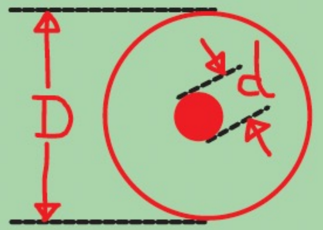
$$\mu = \mu_0 \cdot \mu_r \quad [H/m]$$





Transmission Line

Coaxial cable



KOAXIAL KABEL

$$L = \frac{\mu}{2\pi} \cdot \ln\left(\frac{D}{d}\right) \quad [H/m]$$

$$C = 2\pi \cdot \varepsilon \cdot \frac{1}{\ln\left(\frac{D}{d}\right)} \quad [F/m]$$

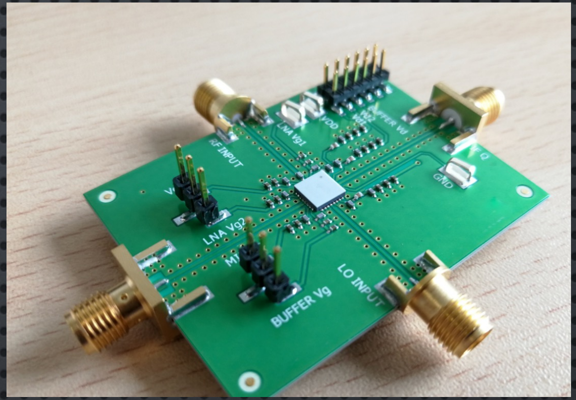
$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{D}{d}\right) \quad [\Omega]$$

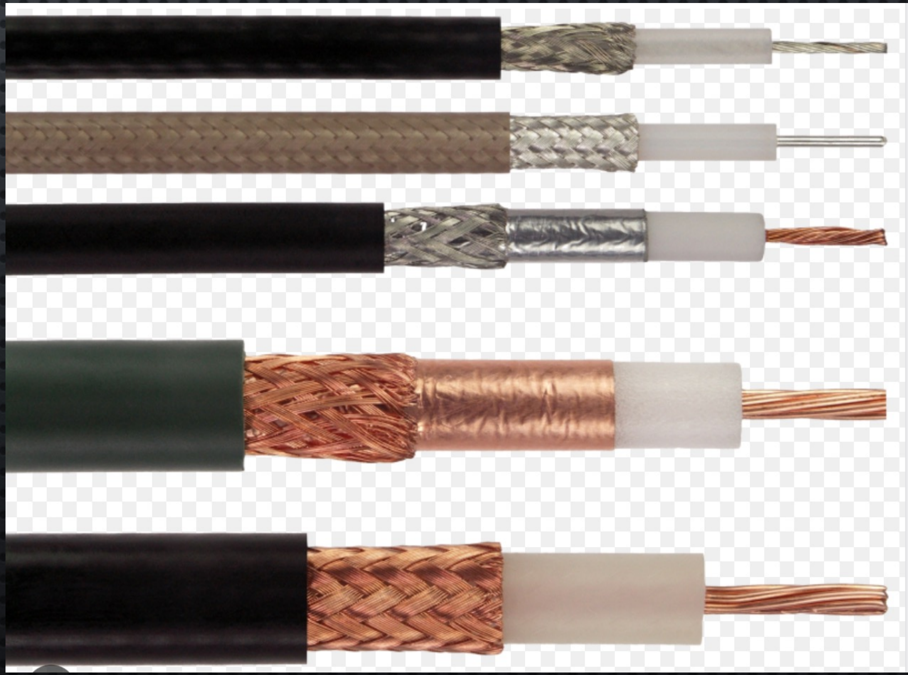
(for $\mu_r = 1$)

$$v = \frac{1}{\sqrt{\mu \cdot \varepsilon}} \quad [m/s]$$

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r \quad [F/m]$$

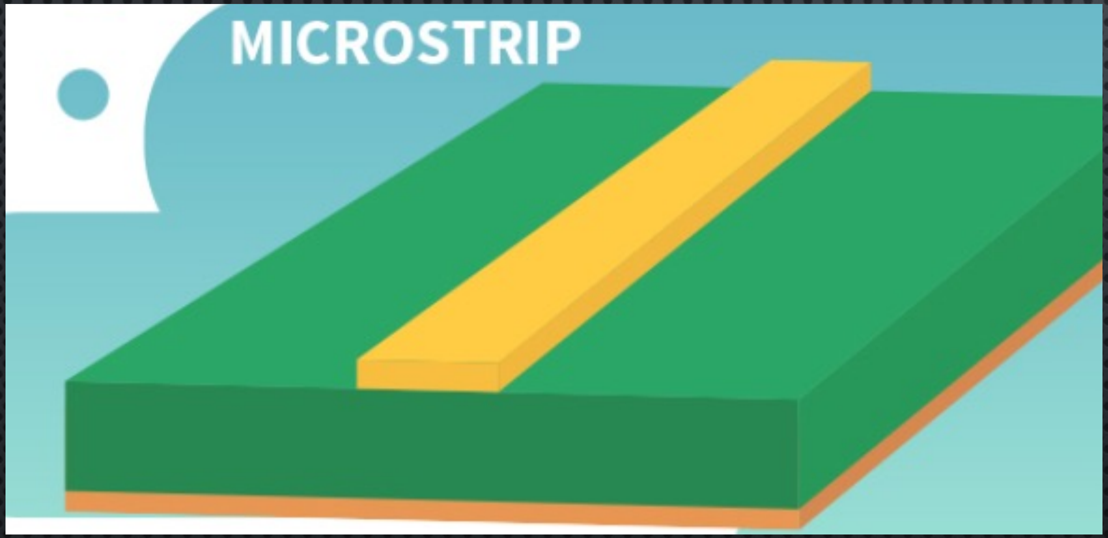
$$\mu = \mu_0 \cdot \mu_r \quad [H/m]$$

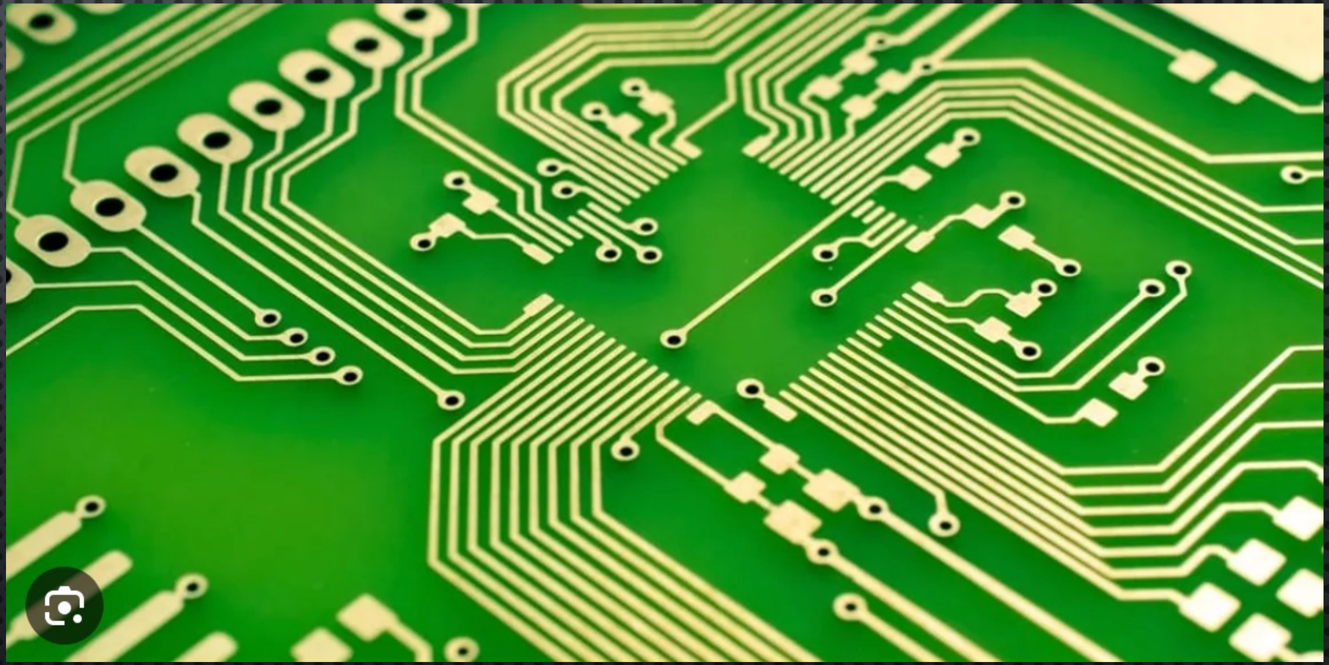




Transmission Line

Common transmission lines on PCB-Microstrip line

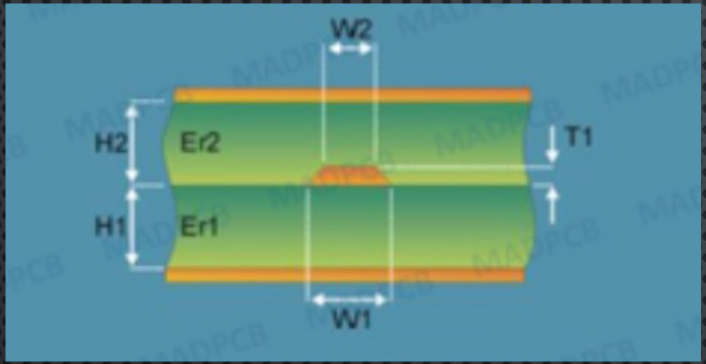


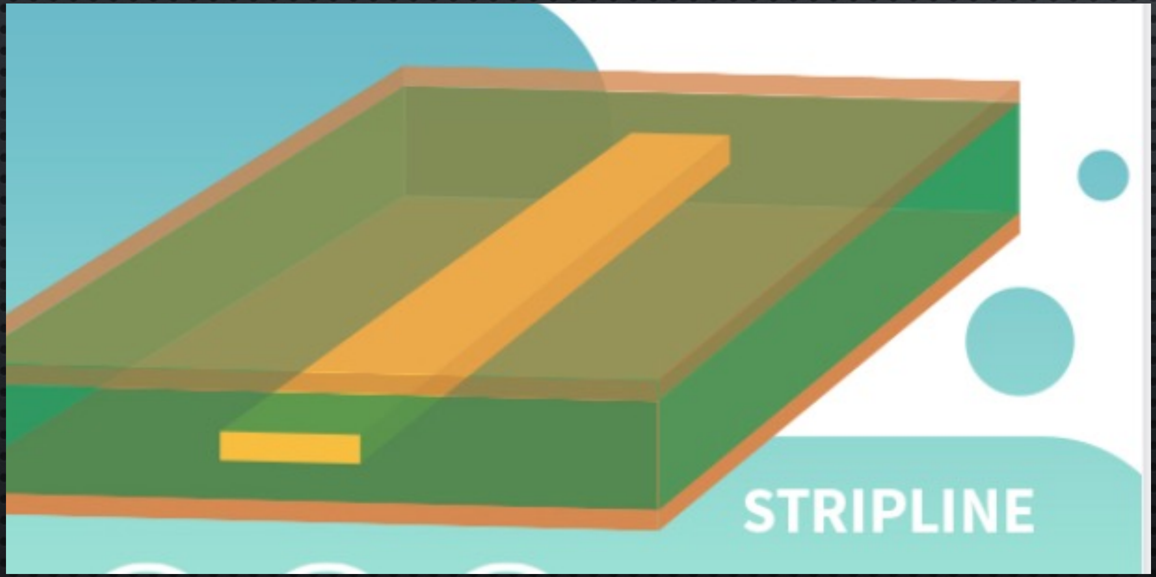


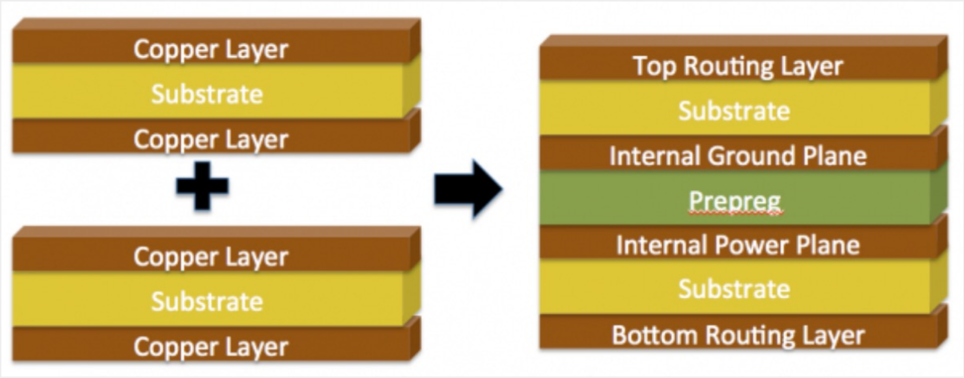
cheap solution

Transmission Line

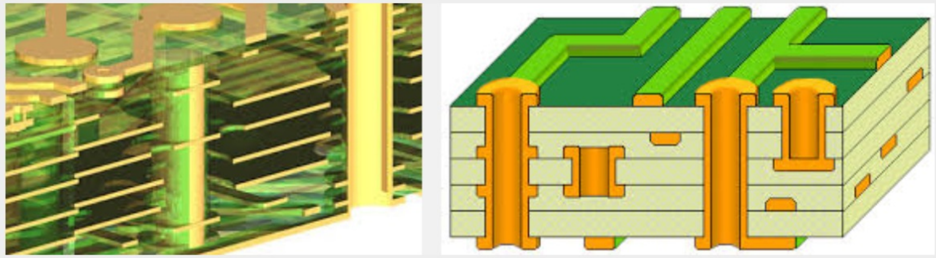
Common transmission lines on PCB PCB-Strip line







— Layers of copper+core are bonded together...



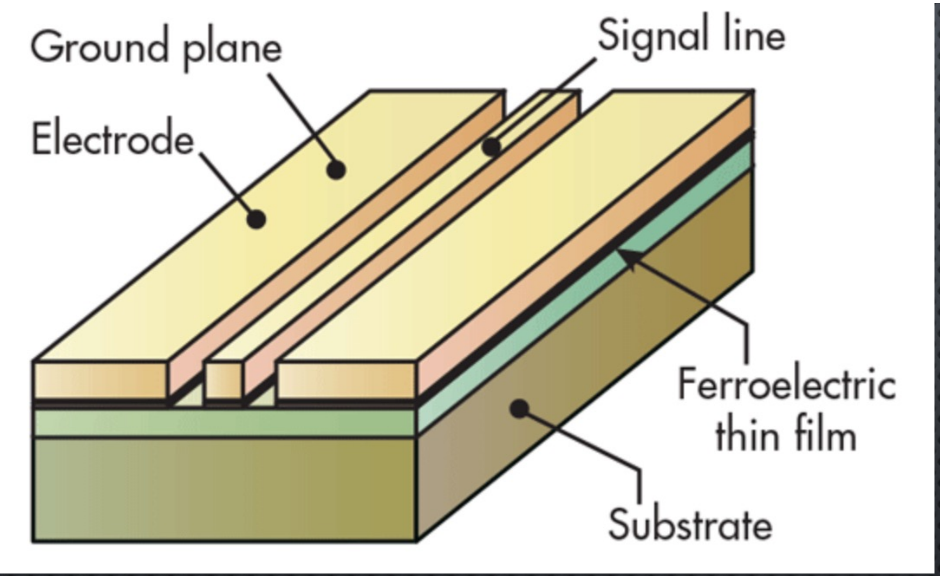
— Different types of vias

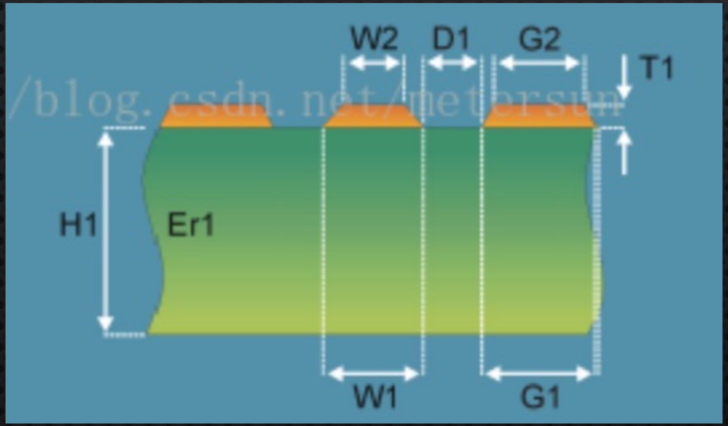
suitable for multi-layer PCB

Transmission Line

Common transmission lines on PCB-CPW and CPWG

Coplanar waveguide (CPW)





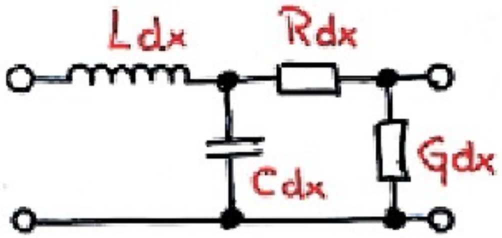
Coplanar waveguide grounded (CPWG)

Very popular in practical applicaitons

Telegraph Equations and Wave Equations

Blackboard (2)

Model for cable

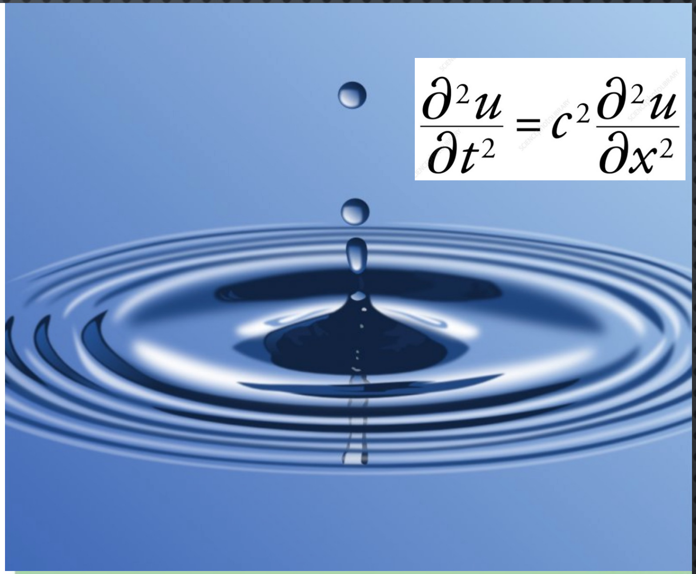


Telegraph equation

Lossless R=G=0

$$\frac{\partial V}{\partial x} = -L \cdot \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = -C \cdot \frac{\partial V}{\partial t}$$



$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Wave equation

Lossless R=G=0

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 V}{\partial t^2}$$

$$\frac{\partial^2 I}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 I}{\partial t^2}$$

Solutions

$$V = V^+\left(t - \frac{x}{v}\right) + V^-\left(t + \frac{x}{v}\right)$$

$$I = I^+\left(t - \frac{x}{v}\right) + I^-\left(t + \frac{x}{v}\right)$$

$$I = \frac{1}{Z_0}\left(V^+\left(t - \frac{x}{v}\right) - V^-\left(t + \frac{x}{v}\right)\right)$$

$$v = \frac{1}{\sqrt{LC}} \quad \left[\frac{m}{s}\right]$$

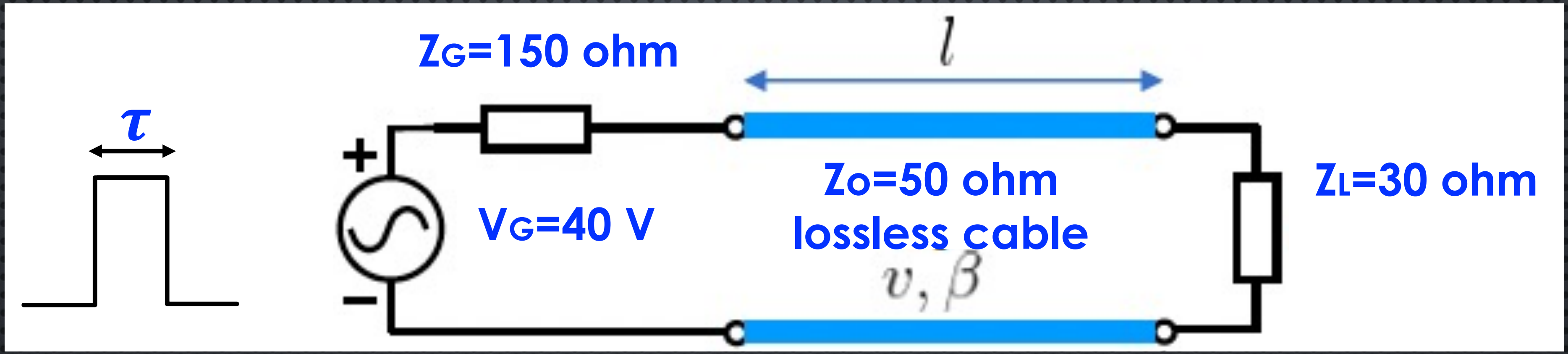
$$Z_0 = \sqrt{\frac{L}{C}} \quad [\Omega]$$

$$K_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V^-}{V^+} = -\frac{I^-}{I^+}$$

Paradigm Model

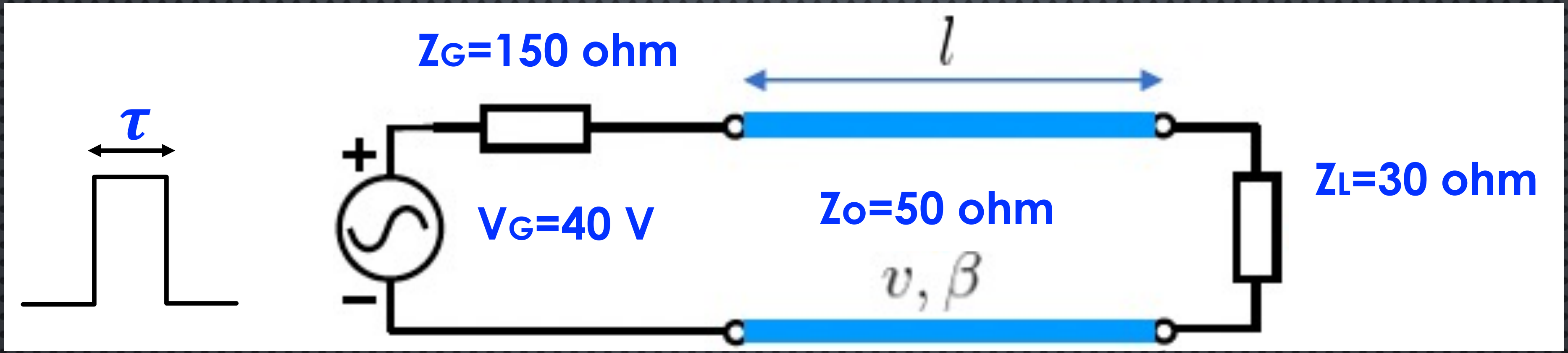
Blackboard (3)

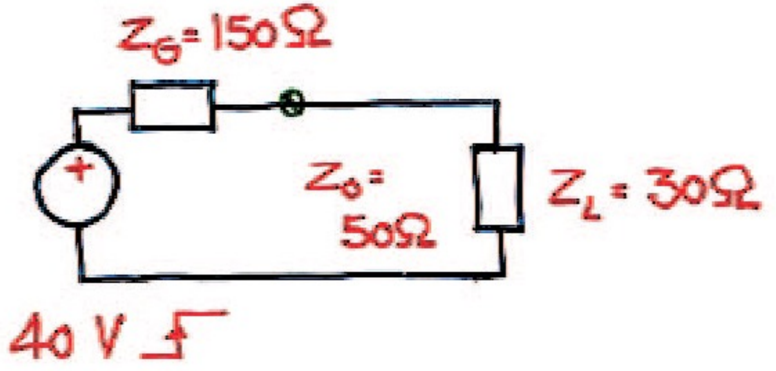
Reflection Diagram



Determine the current and voltage on the Z_L

Reflection Diagram



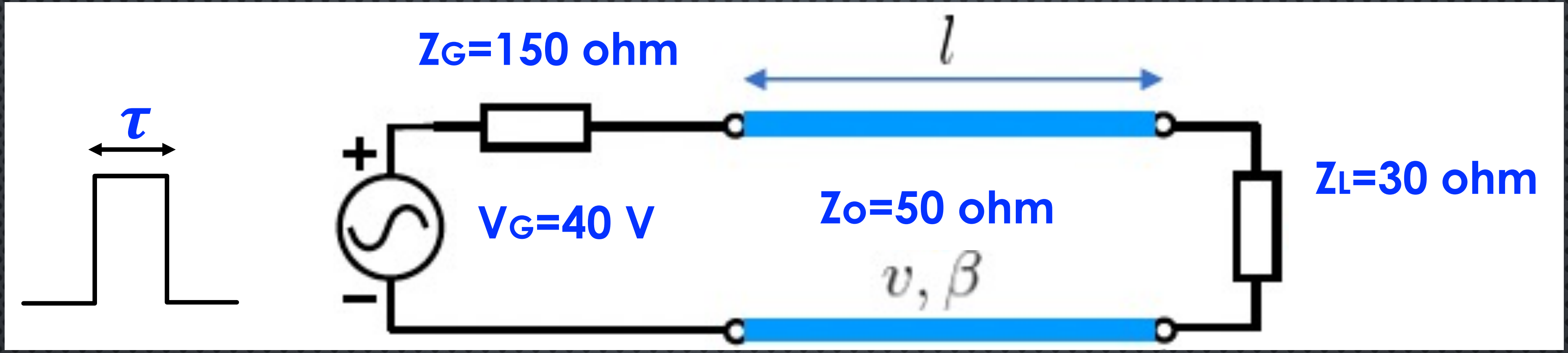


$$V_0 = 40\cdot\frac{30}{150+30} = \frac{1200}{180} = \frac{20}{3}\ \mathrm{V} \qquad (6{,}67\ \mathrm{V})$$

$$I_0 = 40\cdot\frac{1}{150+30} = \frac{40}{180} = \frac{4}{18}\ \mathrm{A} \qquad (0{,}22\ \mathrm{A})$$

There are many ways to calculate it.
First, we can utilize circuit theory.

Reflection Diagram



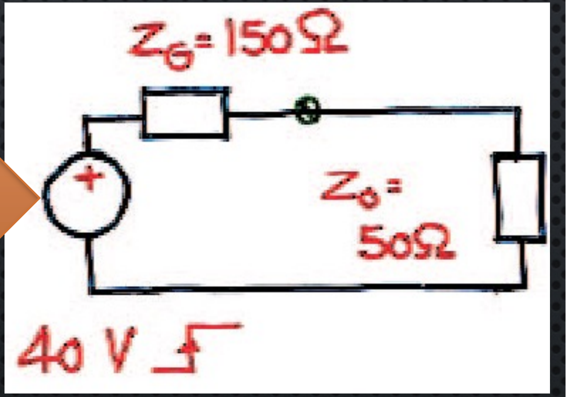
There are many ways to calculate it. We can also apply **transmission line theory** that gives more details.

Start:

$$V^+ = 40 \cdot \frac{50}{150+50} = \frac{2000}{200} = 10 \text{ V}$$

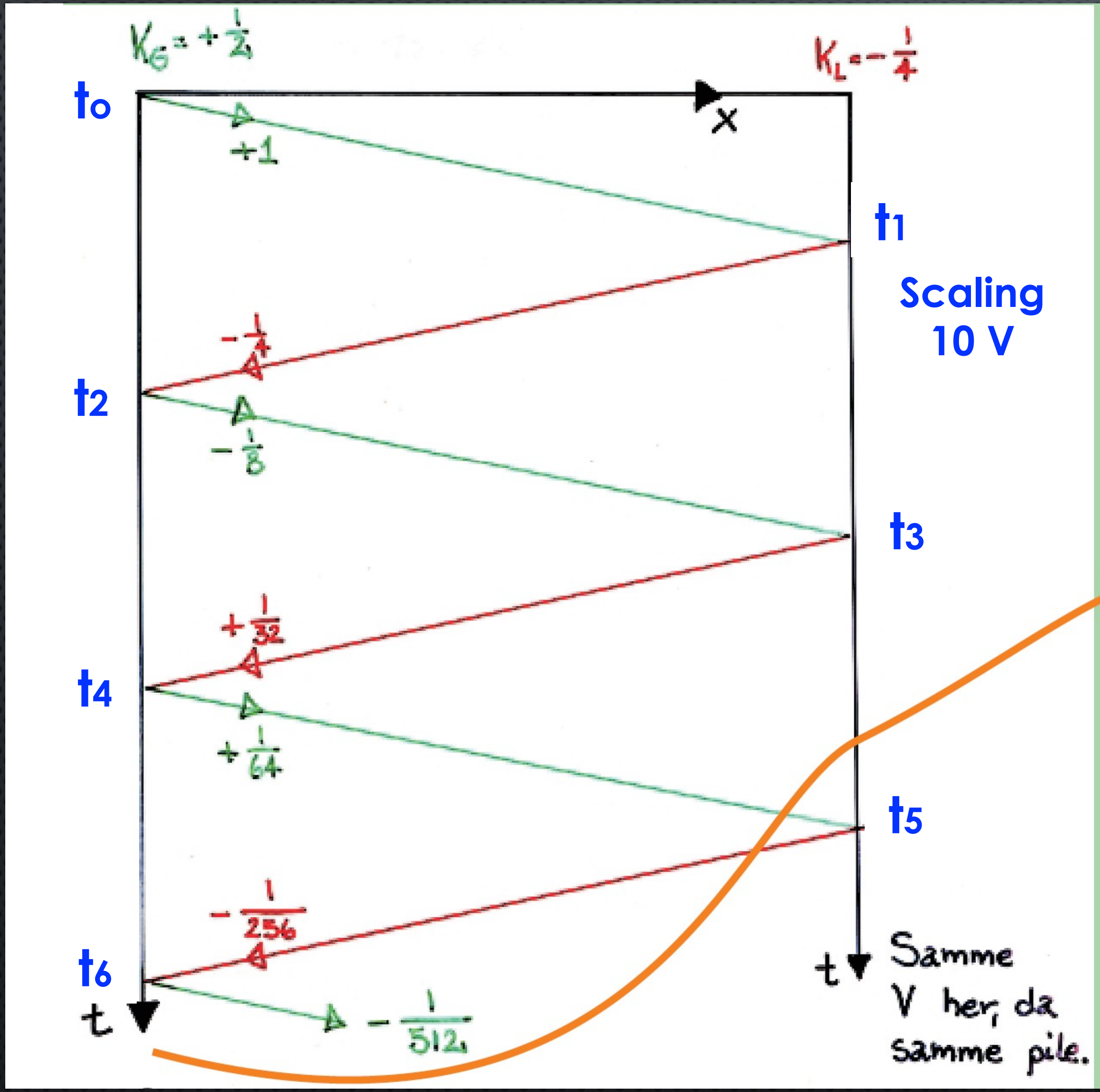
$$I^+ = 40 \cdot \frac{1}{150+50} = \frac{40}{200} = 0{,}2 \text{ A}$$

$$K_G = \frac{150-50}{150+50} = +\frac{1}{2} \qquad K_L = \frac{30-50}{30+50} = -\frac{1}{4}$$



Reflection Diagram

Reflection map



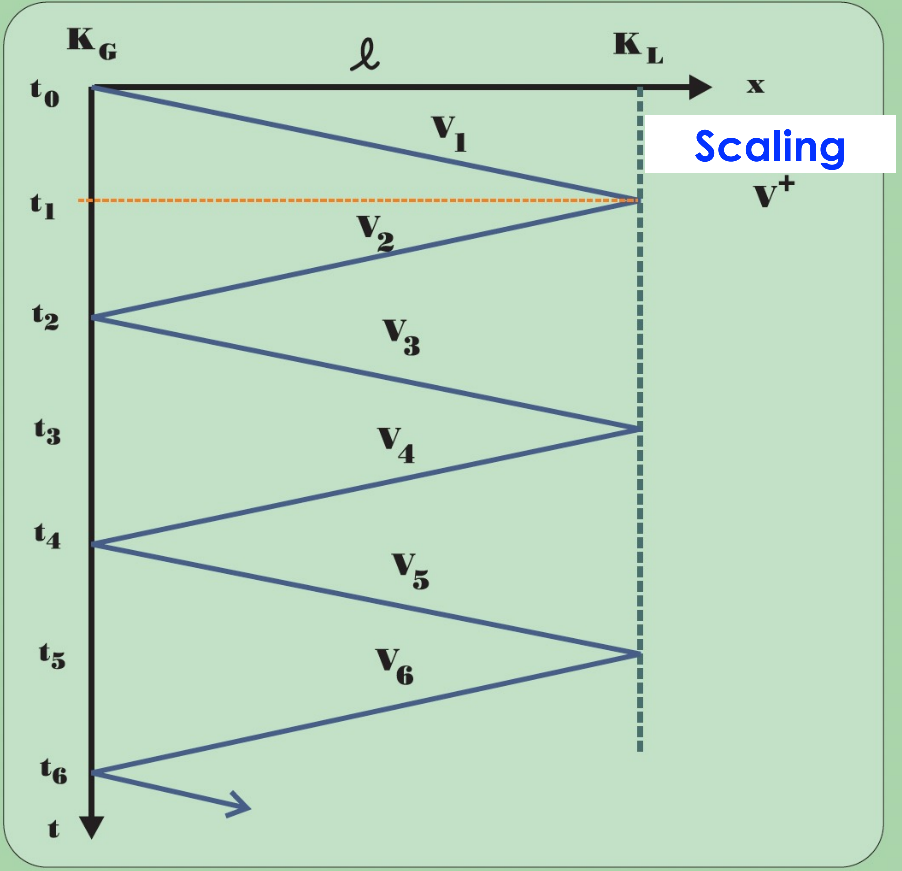
$$V_{TOTAL\ @\ \otimes} = 10 \cdot$$

$$1 + \left(-\tfrac{1}{4}\right) + \tfrac{1}{2}\left(-\tfrac{1}{4}\right) + \tfrac{1}{2}\left(-\tfrac{1}{4}\right)^2 + \left(\tfrac{1}{2}\right)^2\left(-\tfrac{1}{4}\right)^2 + \dots$$

$$= \left(\sum_{n=0}^{\infty} \left(-\tfrac{1}{8}\right)^n + \left(-\tfrac{1}{4}\right) \sum_{n=0}^{\infty} \left(-\tfrac{1}{8}\right)^n \right) \cdot 10 = \tfrac{3}{4} \cdot \tfrac{8}{9} \cdot 10 = \tfrac{20}{3}\ V$$

Reflection Diagram

Reflection map



Diagram

Voltage calculation

$$V_1 = 1$$
$$V_2 = V_1 \cdot K_L$$
$$V_3 = V_2 \cdot K_G$$
$$V_4 = V_3 \cdot K_L$$
$$V_5 = V_4 \cdot K_G$$
$$V_6 = V_5 \cdot K_L$$

If it is current, it should apply:

$$K'_L = -K_L$$
$$K'_G = -K_G$$

Voltage

Time calculation

$$t_0 = 0$$
$$t_1 = t_0 + \Delta$$
$$t_2 = t_1 + \Delta$$
$$t_3 = t_2 + \Delta$$
$$t_4 = t_3 + \Delta$$
$$t_5 = t_4 + \Delta$$
$$t_6 = t_5 + \Delta$$

where

$$\Delta = \frac{\ell}{v} \quad [s]$$

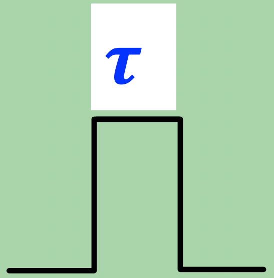
Time

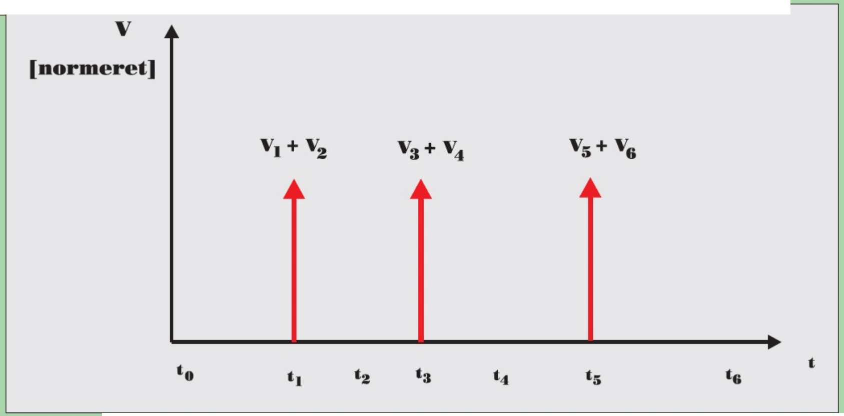
It is smart and clear to calculate the voltage (current) and time in same tables.

Reflection Diagram

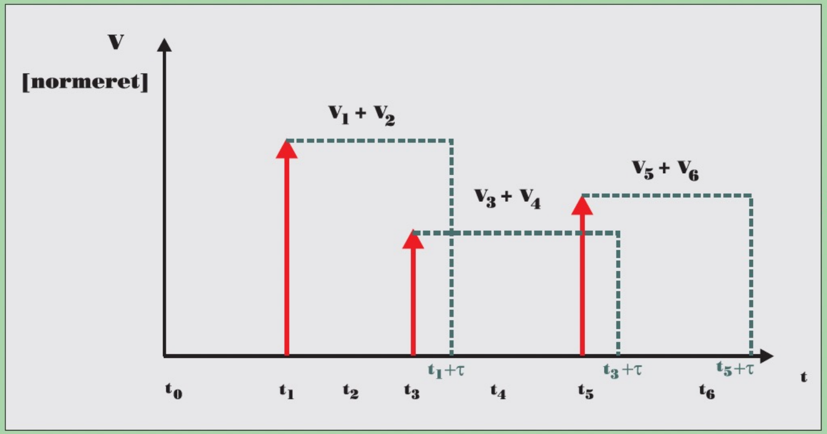
Stick diagram

Calculate the curve form with stick diagram

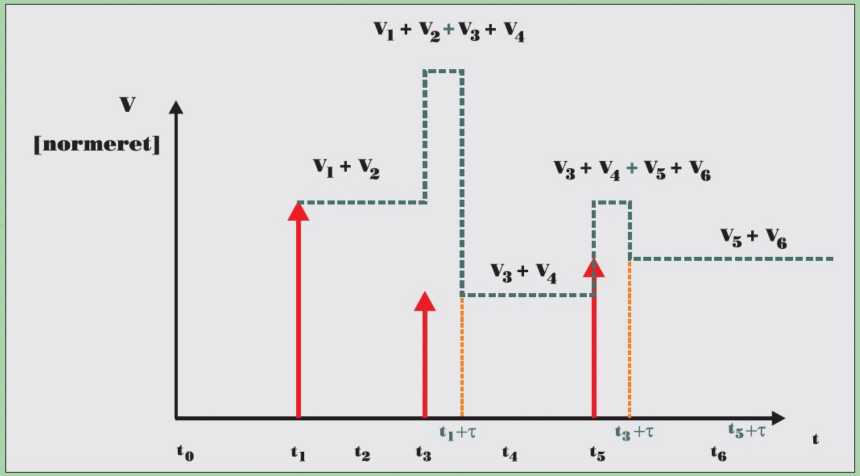




Make the wave leading edge with stick



Extend stick to full pulse width



Sum the impulse overlap part

Reflection Diagram

1. Calculate cable parameters. We should have speed v and characteristic impedance Z_0. Identify generator voltage, generator impedance and load impedance.

2. Determine the first incident signal. Calculate K_L and K_G. If it is current, remember to reverse the sign of the reflection coefficients.

3. Calculate the important time points. This is the time of the wave propagation on the cable, as well as the times of the leading edge of an impulse signal.

4. Draw the reflection map for the leading edge of the signals.

5. Make a "stick" graph for each signal. Calculate the voltage or currents.

6. Extend the sticks to the entire pulse and sum the signals if they overlap.

7. Multiply the entire graph by the scaling factor (the first incident V or I).