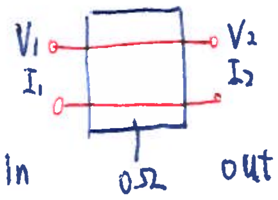
MM 7

(1) Transmission line

We investigate current and voltage on a transmission line:

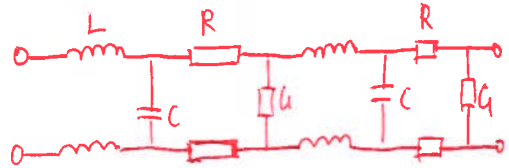


Often apply an ideal model where

$|V_1| = |V_2|$

$|I_1| = |I_2|$

Cabel model 1 (general)



A system with distributed parameters

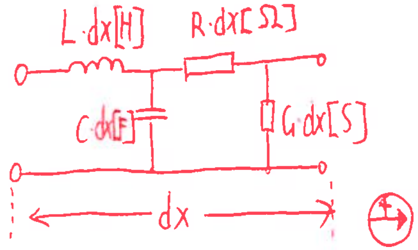
Primary cable constants:

$L\ [\frac{H}{m}]$ $\quad R\ [\frac{\Omega}{m}]$

$C\ [\frac{F}{m}]$ $\quad G\ [\frac{S}{m}]$

The same symboles L, C, R, G to the circuit theory

Cable model 2 (infinite small)

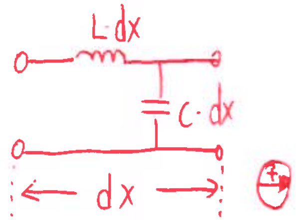


Positive x direction →

Symbol 

Cable model 3 (lossless)

lossless : $R = G = 0$



(2) Telegraph equation

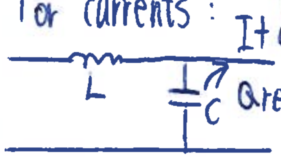
Voltage over dx [m]

(Lenz's law)

$$dV = -\frac{\partial \phi}{\partial t} = -L \cdot dx \frac{\partial I}{\partial t}$$

$$\frac{\partial V}{\partial x} = -L \frac{\partial I}{\partial t} \qquad (1)$$

For currents : I+dI



Q reduction leads to dI increase

$$dI = -\frac{\partial Q}{\partial t} = -C dx \frac{\partial V}{\partial t}$$

$$\frac{\partial I}{\partial x} = -C \frac{\partial V}{\partial t} \qquad (2)$$

(1) and (2) are telegraph equations.

[V(x,t), I(x,t) are functions]

Wave equations

Differentiate (1) with x

$$\frac{\partial^2 V}{\partial x^2} = -L \cdot \frac{\partial^2 I}{\partial x \partial t} \qquad (3)$$

and differentiate (2) with t

$$\frac{\partial^2 I}{\partial x \partial t} = -C \frac{\partial^2 V}{\partial t^2} \qquad (4)$$

Combine (3) and (4)

$$\frac{1}{LC} \frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial t^2} \qquad (5)$$

One dimension wave equation

Solutions

The solution of wave equations can be :

$V(x,t) = k_1 V_1(x,t) + k_2 V_2(x,t)$

V_1 and V_2 are solutions.

Any functions with parameters:

$(t \pm x\sqrt{LC})$

are solutions.

For example : (different wave forms)

$V(x,t) = 25 \cdot (t + x\sqrt{LC})$

$V(x,t) = 45 \cos[w(t - x\sqrt{CL})]$

$V(x,t) = U(t - x\sqrt{LC})$

Propagation speed

$V = \frac{1}{\sqrt{CL}} \left[\frac{m}{s}\right]$

are the propagation speed in cable

Light speed in vacuum

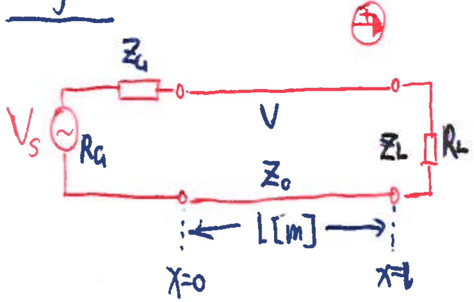
$c \approx 3E8 \frac{m}{s}$

This is the max speed of energy and information propagation

So $V \leq C$

(3) Paradigm model ★

Diagram



Current and voltage:

$$V(x,t) = V^+(t-\tfrac{x}{v}) + V^-(t+\tfrac{x}{v}) \quad [V]$$

$$I(x,t) = I^+(t-\tfrac{x}{v}) + I^-(t+\tfrac{x}{v}) \quad [A]$$

$$= \frac{1}{Z_0}\left[V^+(t-\tfrac{x}{v}) - V^-(t+\tfrac{x}{v})\right] [A]$$

It is defined:

$$Z_0 = \sqrt{\frac{L}{C}} \quad [\Omega]$$

Characteristic impedance

We can find (when wave equation is linear)

$$I^+ = \frac{V^+}{Z_0} \qquad I^- = -\frac{V^-}{Z_0} \quad [A]$$

$$\Rightarrow Z_0 = \frac{V^+}{I^+} = -\frac{V^-}{I^-} \quad [\Omega]$$

It is also defined:

$$k_L = \frac{V^-}{V^+} = -\frac{I^-}{I^+} \quad [\cdot]$$

Reflection coefficient

It can also be calculated by:

$$k_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad [\cdot]$$

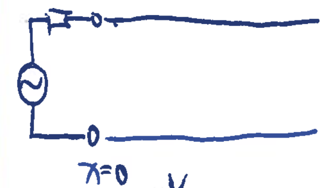
$V^+(x,t)$ and $I^+(x,t)$ are incident wave

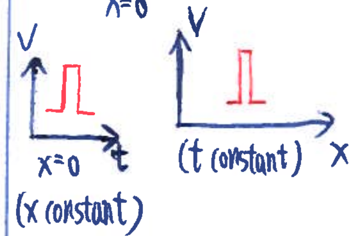
$V^-(x,t)$ and $I^-(x,t)$ are reflective wave

Propagation on cable

We see from incident wave:

$$V(x,t) = V^+(t-\tfrac{x}{v}) \; [V]$$





V^+ propagate to right

V^- propagate to left

Reflection

We have

$$V^- = k_L V^+$$

$$I^- = -k_L I^+$$

Summary with k_L and Z_L

Z_L	k_L
0	-1
∞	+1
Z_0	0

$$\left[k_L = \frac{Z_L - Z_0}{Z_L + Z_0} \right]$$