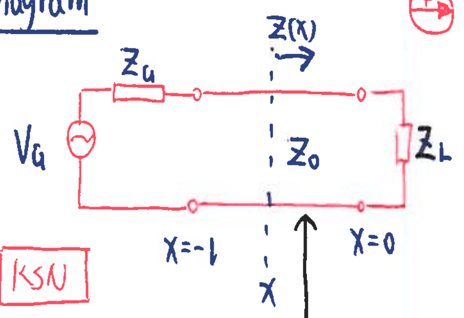
MN13

(1) Transmissionline cable models for calculations

Diagram



KSN

source ⟷ load

$K(0)=K_L$

$Z(0)=Z_L$

$K(x)$ and $Z(x)$

$V(x)$ and $I(x)$

$P_{trans}(x)$

$$\boxed{\begin{array}{l} \gamma=\sqrt{ZY}\ [m^{-1}] \\ Z_0=\sqrt{\frac{Z}{Y}}\ [\Omega] \end{array}}$$

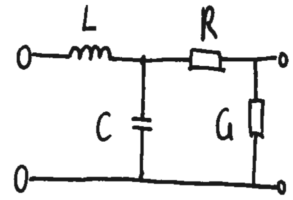
Propagation constant:

$\gamma = \alpha + j\beta \quad [m^{-1}]$

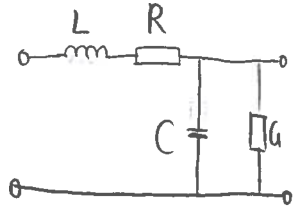
We replace $j\beta$ in formula with γ:

$V(x) = V^+ e^{-\gamma x} + V^- e^{+\gamma x} \quad [V]$

Cable models



or



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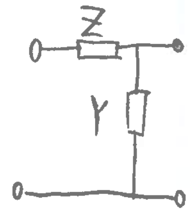


$Z=R+j\omega L\ [\Omega/m]$

$Y=G+j\omega C\ [S/m]$

Calculate to the secondary constant:

$L, C, R, G \longrightarrow Z_0, \gamma$

Four cable models:

1. General cable models
2. Lossless cable model
3. Distortionless cable model
4. Low loss cable model

(2) Four cable models

① General cable model

$$\gamma = \sqrt{ZY} = \sqrt{(R+jwL)(G+jwC)} \quad [m^{-1}]$$

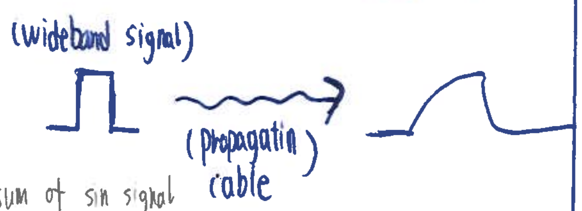
$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+jwL}{G+jwC}} \quad [\Omega]$$

α [Np/m] Attenuation constant

β [rad/m] Phase propagation constant (wave number k)

$$v = \frac{w}{\beta} = \frac{w}{Im(\gamma)} \quad [\frac{m}{s}]$$ Phase speed

Frequency Dependent



(sum of sin signal with different weight and frequency)

② Lossless cable model

If $wL \gg R$ and $wC \gg G$,
we have $R \approx 0$ and $G \approx 0$.

So

$$\gamma = \sqrt{jwL \cdot jwC} = jw\sqrt{LC} \quad [m^{-1}]$$

$$Z_0 = \sqrt{\frac{jwL}{jwC}} = \sqrt{\frac{L}{C}} \quad [\Omega]$$

$$\alpha = 0 \quad [Np/m]$$

$$V = \frac{w}{\beta} = \frac{1}{\sqrt{LC}} \quad [\frac{m}{s}]$$

Frequency Independent

③ Distortionless cable model

We have:

$$\frac{R}{L} = \frac{G}{C} \quad (\text{or } RC = LG)$$

So

$$\gamma = \alpha + j\beta \quad [m^{-1}]$$

$$\alpha = \frac{R}{Z_0} = GZ_0 \quad [Np/m]$$

$$\left(\text{or } \alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)\right)$$

$$\beta = w\sqrt{LC} \quad [rad/m]$$

$$V = \frac{1}{\sqrt{LC}} \quad [\frac{m}{s}]$$ Frequency Independent

$$Z_0 = \sqrt{\frac{L}{C}} \quad [\Omega]$$

④ Low loss cable model

We have

$wL \approx R$ and $wC \approx G$

So

$$\gamma = \alpha + j\beta \quad [m^{-1}]$$

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \quad [Np/m]$$

$$\beta = w\sqrt{LC} \quad [rad/m]$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad [\Omega]$$

$$V = \frac{1}{\sqrt{LC}}$$ Frequency Independent

Propagation speed

If $\frac{d\beta}{dw}$ is constant, v become constant.

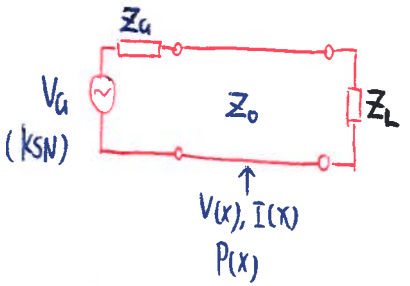
Phase speed: $V_f = \frac{1}{\beta/w} \quad [\frac{m}{s}]$

Group speed: $V_{gr} = \frac{1}{d\beta/dw} \quad [\frac{m}{s}]$

phase speed can be faster than light speed

(3) Calculations with lossy cables

Power transmission in KSN



To calculate the following two cases:

① Loss due to ohmic loss in cables

$\alpha \neq 0$, $Z_L = Z_0$, $Z_G = Z_0$

②. Loss due to reflection:

$\alpha = 0$, $Z_L \neq Z_0$, $Z_G = Z_0$

① Loss due to ohmic loss

We have only $V^+(x)$ and $I^+(x)$, when $k_L = 0$.

$$P_{trans}(x) = \frac{1}{2} Re[V^+(x) \cdot I^{+*}(x)]$$

$$\vdots$$

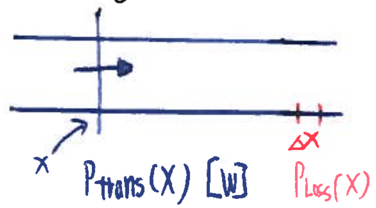
$$= \frac{1}{2} \left|\frac{V^+}{Z_0}\right|^2 \cdot Re[Z_0] \cdot e^{-2\alpha x}$$

If Z_0 is real, then:

$$P_{trans}(x) = \frac{1}{2} \frac{|V^+|^2}{Z_0} \cdot e^{-2\alpha x} \quad [W]$$

Loss per meter:

$$P_{Loss}(x) = -\frac{\partial}{\partial x} P_{trans}(x) = P_{trans}(x) \cdot 2\alpha \quad \left[\frac{W}{m}\right]$$



② Loss due to reflection

$$P_{trans} = \frac{1}{2} Re[V(x) \cdot I^*(x)]$$

$$\vdots$$

$$= \frac{1}{2} \frac{|V^+|^2}{Z_0} \cdot (1 - |k_L|^2)$$

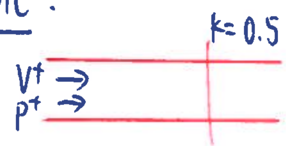
$$P_{trans} = P^+ - P^-$$

where:

$$P^+ = \frac{1}{2} \frac{|V^+|^2}{Z_0}$$

$$P^- = P^+ \cdot |k_L|^2$$

Example:



$P^+ = 10\ W \qquad P^- = 10 \cdot 0.5^2 = 2.5\ W$

$P_{trans} = P^+ - P^- = 7.5\ W$