MM3

(1) Magnetic field

$\bar{H}$ $[\frac{A}{m}]$ Magnetic field intensity

$\bar{B}$ $[\frac{Wb}{m^2} = \frac{V\cdot s}{m^2}]$ Magnetic flux density

$$\bar{B} = \mu \cdot \bar{H} \quad [\frac{Wb}{m^2}]$$

$$\mu = \mu_r \cdot \mu_0 \quad [\frac{H}{m}] \quad \text{permeability}$$

$$\mu_0 \triangleq 4\pi \cdot 10^{-7} \frac{H}{m}$$

Othe fields

$\bar{J}$ $[\frac{A}{m^2}]$ Current density (area current)

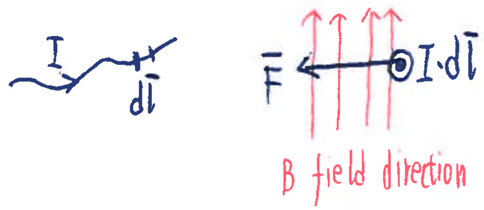
$\bar{J}_s$ $[\frac{A}{m}]$ surface current density (linear current)

Φ $[Wb = V\cdot s]$ Magnetic flux

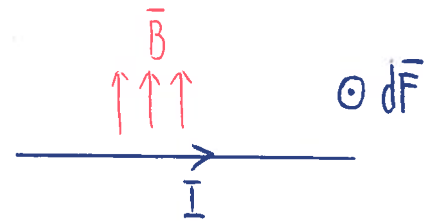
Definition of magnetic field

B-field definition

$$\bar{B} \triangleq \lim_{|I\cdot d\bar{l}| \to 0} \frac{|\bar{F}|}{|I d\bar{l}|} \cdot \hat{d} \quad [\frac{N}{A\cdot m} = \frac{Wb}{m^2} = T]$$



Laplace's law



Laplace's law

$$d\bar{F} = I d\bar{l} \times \bar{B} \quad [N]$$

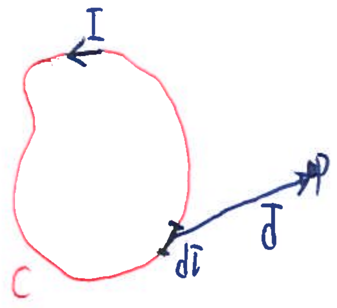
$$\Phi = \int_S \bar{B}\, d\bar{a} \quad [Wb]$$

$$Q = \int_S \bar{D}\, d\bar{a} \quad [C]$$

(2) Biot - Savart's law

Magnetic field : point P

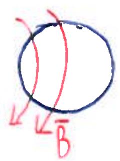
$$\bar{B}(P) = \frac{I\mu}{4\pi} \oint_C \frac{d\bar{l} \times \hat{d}}{d^2} \left[\frac{Wb}{m^2}\right]$$

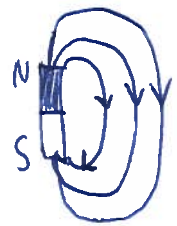


Divergence of B-field

Apply divergence theorem

$$\int_V \nabla \cdot \bar{B}\, dv = \oint_S \bar{B} \cdot d\bar{a} = 0$$





So $\nabla \cdot \bar{B} = 0$

Magnetic field is source free.

$(\nabla \cdot \bar{D} = \rho)$

(3) Ampere's law

$$mmf = \mathcal{F} = \oint_C \bar{H} \cdot d\bar{l} = \int_S \bar{J}\, d\bar{a} = I_{TOT}\ [A]$$

mmf is magnetomotive force
or magnetic voltage [A]

We apply rotation theorem

$$\int_S \nabla \times \bar{H} \cdot d\bar{a} = \int_S \bar{J} \cdot d\bar{a}$$

$$\Rightarrow \nabla \times \bar{H} = \bar{J}$$

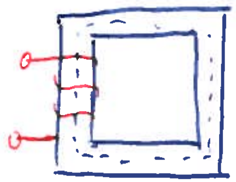
Magnetic field is not conservative

Ampere's law technical formula

$$H \cdot l = N \cdot I \quad (A)$$

$$\left[\frac{A}{m} \cdot m = \cdot A\right]$$

Example :



$I = 1A$

$N = 100$ turns

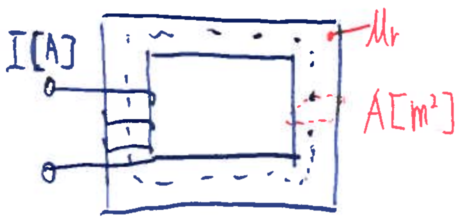
$l = 0.25\,m$

$$H \cdot l = N \cdot I$$

$$H = \frac{N \cdot I}{l}$$

$$= \frac{100 \cdot 1}{0.25} = 400 \frac{A}{m}$$

(4) Magnetic circuit



N turns

l [m] (magnetic middle way length)

Formula

$\phi = B \cdot A$ [wb]

$F = NI = H \cdot l$

Magnetic flux in Iron

$\phi = BA = \mu \frac{N \cdot I}{l} \cdot A$

So

$N \cdot I = \phi \frac{l}{\mu A}$

$\Rightarrow F = \phi \cdot \mathcal{R}$ [A]

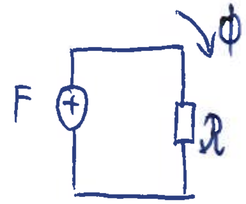
where

$\mathcal{R}$ $\left[\frac{A}{Wb} = H^{-1}\right]$

Magnetic ~~resistance~~ resistor

$\Lambda = \frac{1}{\mathcal{R}}$ $\left[\frac{Wb}{A} = H\right]$

Magnetic circuit

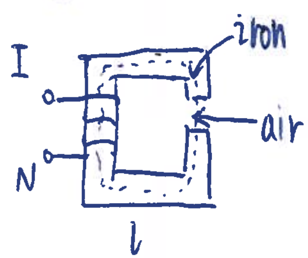


$F = NI$ [A] Magnetic voltage

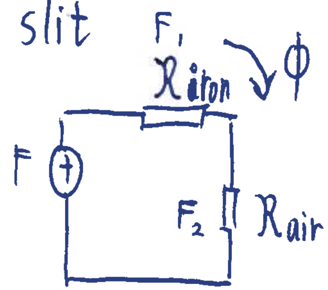
$\phi = \frac{F}{\mathcal{R}}$ [wb] Magnetic current

$\mathcal{R} = \frac{l}{\mu A}$ [H^{-1}] Magnetic ~~resistance~~ resistor

Magnetic circuit with air slit



$\Rightarrow$



$\phi = \frac{F}{\mathcal{R}_{air} + \mathcal{R}_{iron}}$ [wb]

Circuit equation:

$F = F_1 + F_2 = \phi \mathcal{R}_{iron} + \phi \mathcal{R}_{air}$

$= H_{iron} l_{iron} + H_{air} \cdot l_{air}$

$\phi = \phi_1 = \phi_2 = B_{air} A_{air} \equiv B_{iron} \cdot A_{iron}$