High-Speed Electronics in Practice

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Recall for MM1

Electric Field Equations

$$\nabla \times \bar{E} = \bar{0}$$

$$\nabla \cdot \bar{D} = \rho$$

$$\bar{D} = \varepsilon \bar{E}$$

Field around a point charge

$$\bar{E}(r,\theta,\varphi) = \frac{Q_p}{4\pi\varepsilon r^2}\cdot\hat{r} \quad \left[\frac{V}{m}\right]$$

$$V(r,\theta,\varphi) = \frac{Q_p}{4\pi\varepsilon r} \quad [V]$$

$$\nabla \times \bar{E} = \bar{0} \Rightarrow \bar{E} = -\nabla V \quad \& \quad V = -\int_C \bar{E}\cdot d\bar{l}$$

High-Speed Electronics in Practice

MM2. Current and Capacity

CURRENT AND CAPACITY

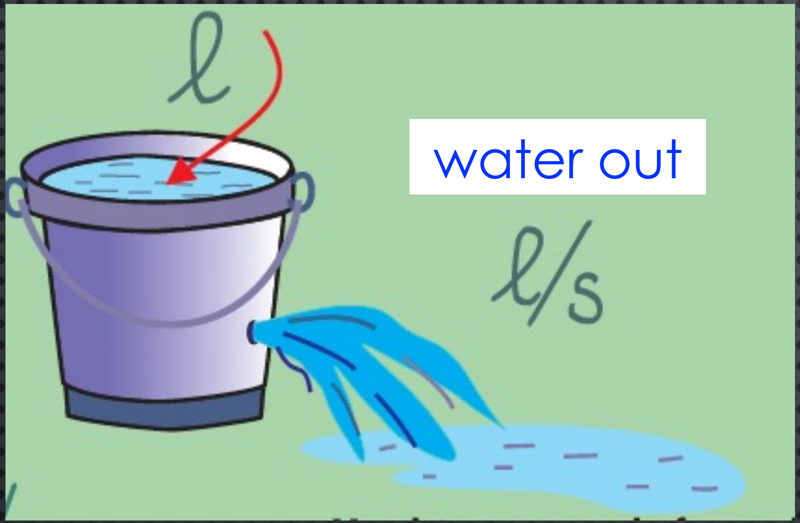
Targets:

1. Read "Quasistatiske elektriske og magnetiske felter" (Page 32 – 35, 48-56) (before or after the lecture)

2. Be able to calculate field and energy in a capacitor as well as interconnection of capacitors (lecture)

3. Finish the exercise (after the lecture)

Electric Currents



Liter/s

C/s

current out

Blackboard (1)

Quasi-static Field

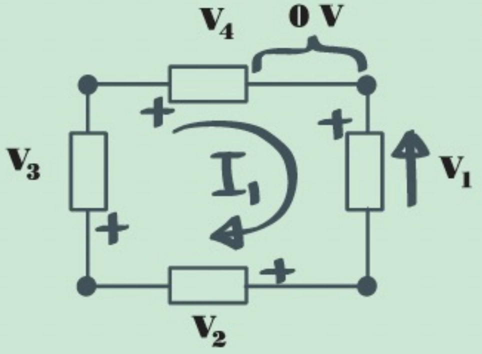
Quasi-static electric field is conservative so we have:

$$\nabla \times \overline{E} = \overline{0}$$

$$\overline{E} = -\nabla V \qquad \left[\frac{V}{m}\right]$$

$$\oint \overline{E} \bullet d\overline{r} = 0 \qquad [V]$$

Kirchhoffs maskelov



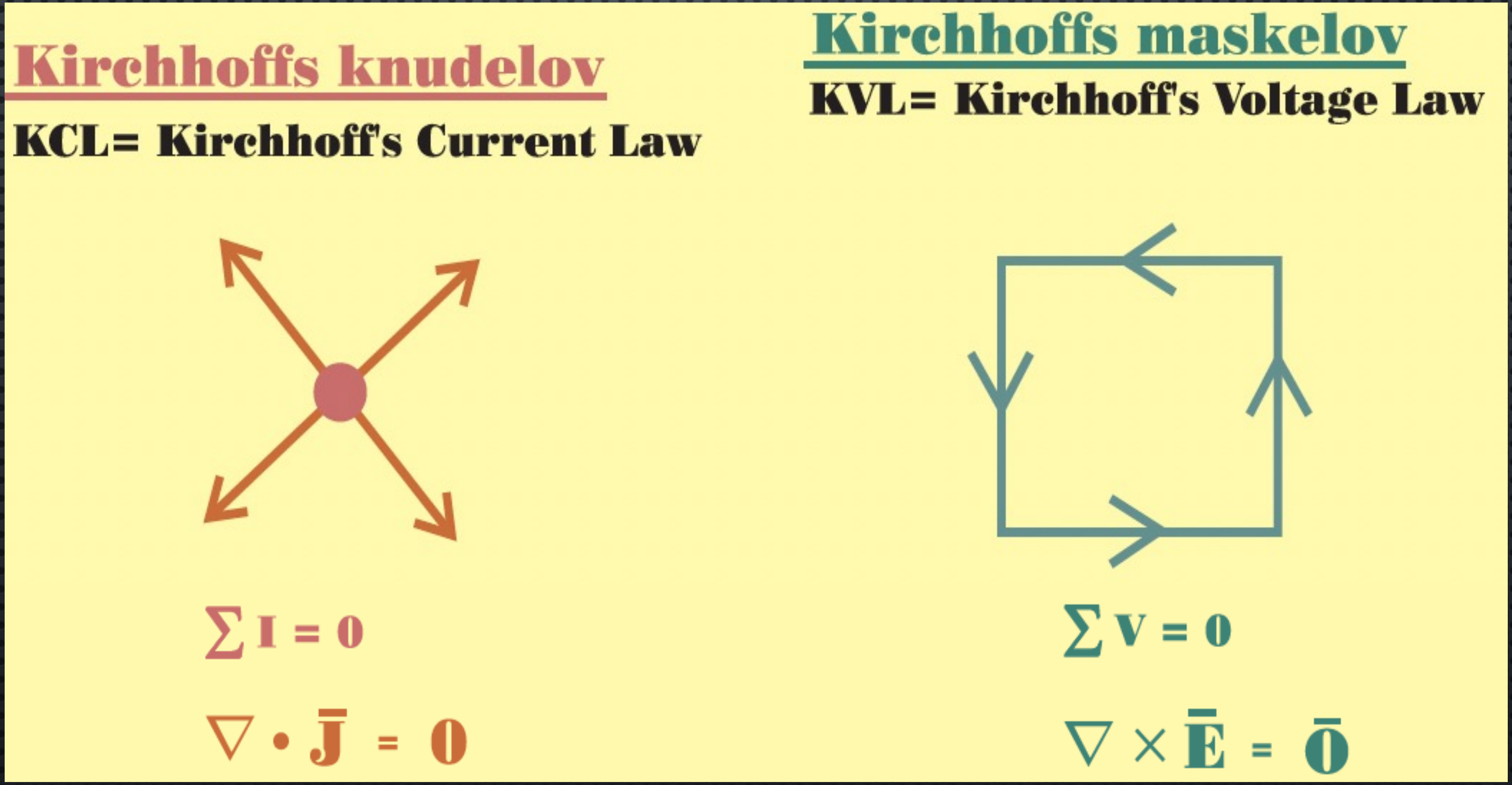
Idealiserede ledninger

$0\,\Omega$

$$\left[\frac{V}{m} \cdot m = V\right]$$

$$V_1 + V_2 + V_3 + V_4 = 0$$

Quasi-static Field



Blackboard (2)

Capacity

$$C = \frac{Q}{V} \quad [F]$$

V Method

1. **Assume a voltage (V) between the plates/bodies**

2. Find E and D by the gradient of V

3. Find charge density by the divergence of D and thus Q

4. Determine C= Q(V)/V

Q Method

1. **Assume +Q on one plate and -Q on the other**

2. Find E using Coulomb's law.

3. Find V between the plates by integrating E

4. Determine C= Q/V(Q)

Capacity

Ball capacitor

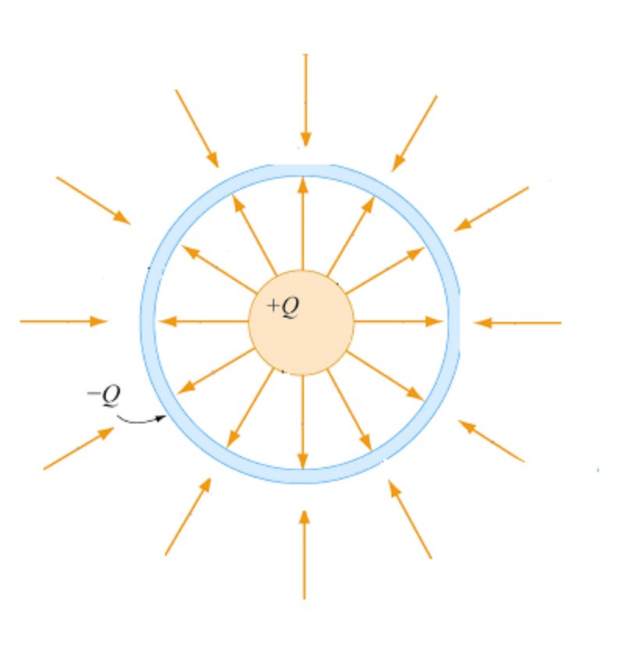
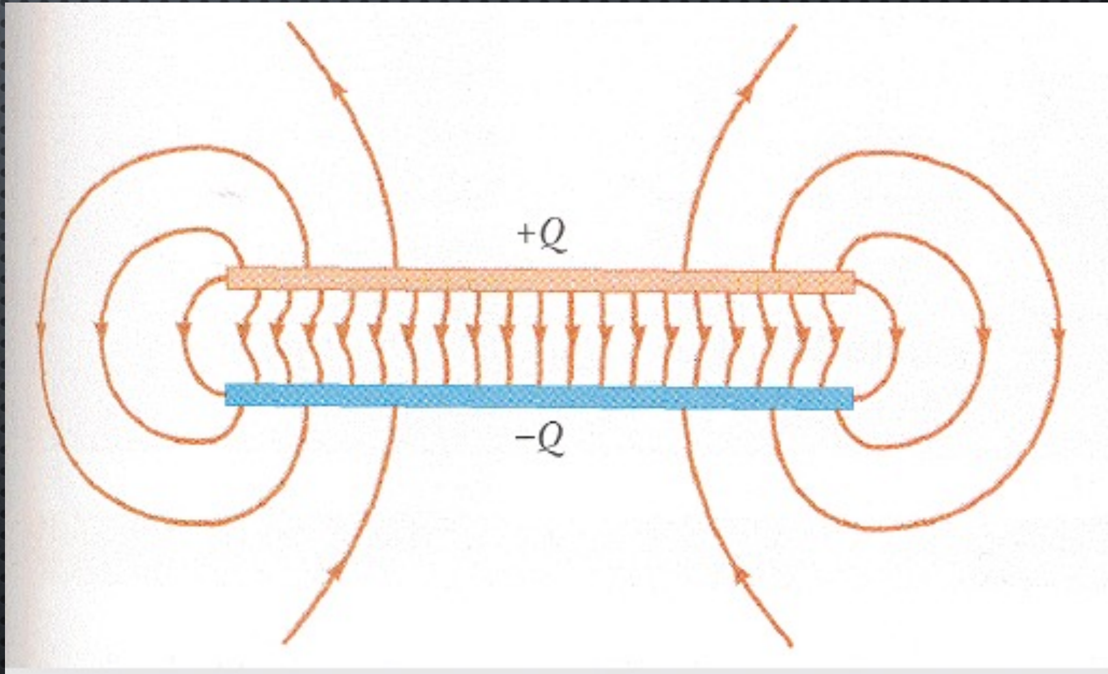


plate capacitor

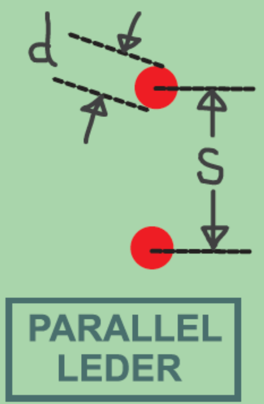


Blackboard (3)

Capacity

Capacity of commonly-used cables

Parallel cable



$$L = \frac{\mu}{\pi} \cdot \ln\left(\frac{2s}{d}\right) \quad [H/m]$$

$$C = \pi \cdot \varepsilon \cdot \frac{1}{\ln\left(\frac{2s}{d}\right)} \quad [F/m]$$

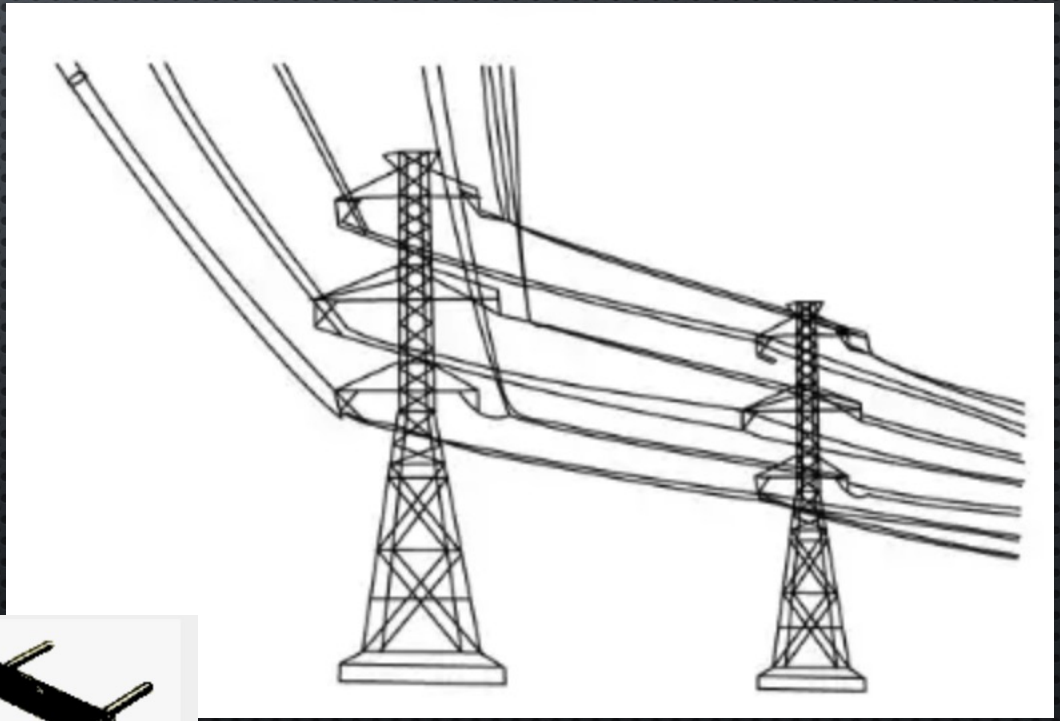
$$Z_0 = \frac{120}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{2s}{d}\right) \quad [\Omega]$$

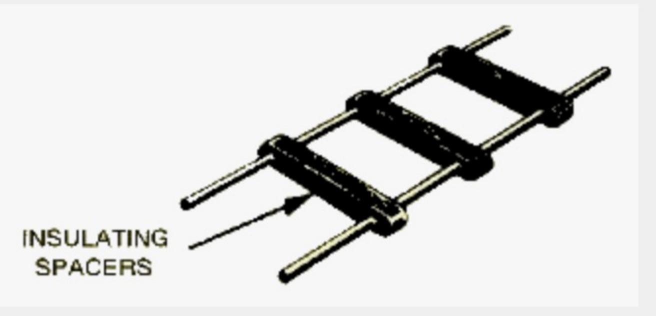
(for $\mu_r = 1$)

$$v = \frac{1}{\sqrt{\mu \cdot \varepsilon}} \quad [m/s]$$

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r \quad [F/m]$$

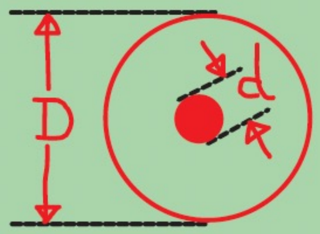
$$\mu = \mu_0 \cdot \mu_r \quad [H/m]$$





Capacity

Coaxial cable



KOAXIAL KABEL

$$L = \frac{\mu}{2\pi} \cdot \ln\left(\frac{D}{d}\right) \quad [H/m]$$

$$C = 2\pi \cdot \varepsilon \cdot \frac{1}{\ln\left(\frac{D}{d}\right)} \quad [F/m]$$

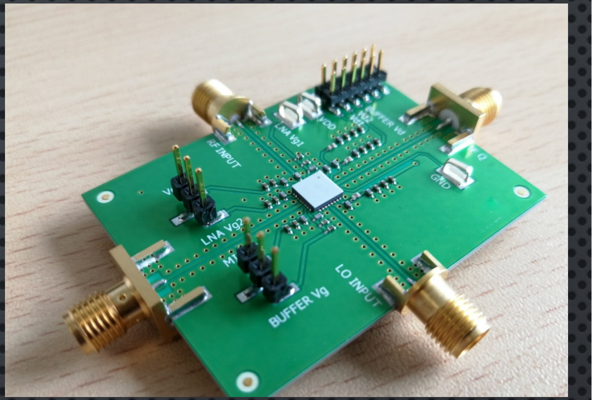
$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{D}{d}\right) \quad [\Omega]$$

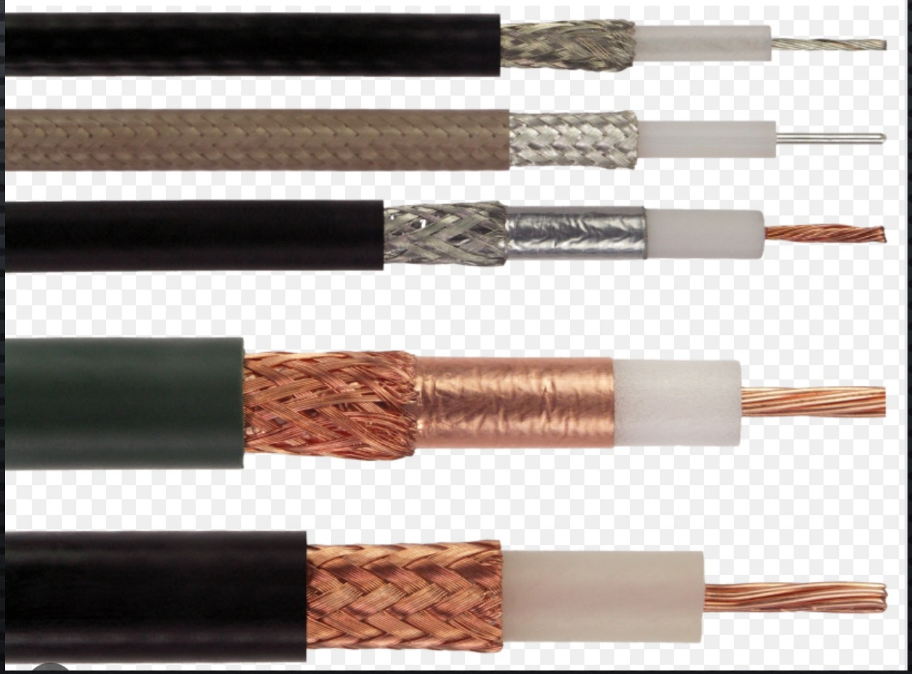
(for $\mu_r = 1$)

$$v = \frac{1}{\sqrt{\mu \cdot \varepsilon}} \quad [m/s]$$

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r \quad [F/m]$$

$$\mu = \mu_0 \cdot \mu_r \quad [H/m]$$





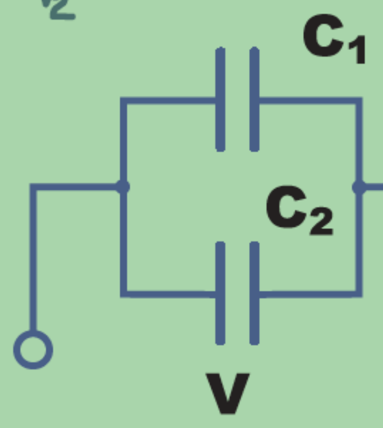
Capacitor Interconnection

Parallel connection

$V_1 = V_2$

$Q_1 = C_1 \cdot V$

$Q_2 = C_2 \cdot V$

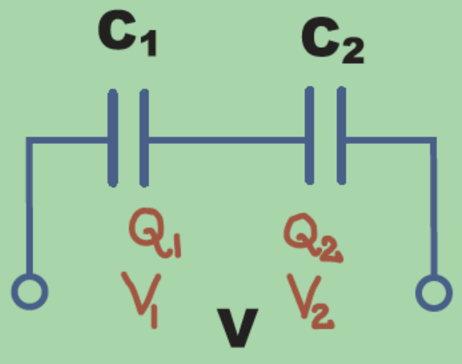


$$C_{RES} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$$

$$= C_1 + C_2$$

Series connection

$Q_1 = Q_2$



$$V_1 = \frac{Q_1}{C_1} \qquad V_2 = \frac{Q_2}{C_2}$$

$$V_{RES} = V_1 + V_2$$

$$= Q \cdot \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$C_{RES} = \frac{Q}{V_{RES}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Charging of Capacitor

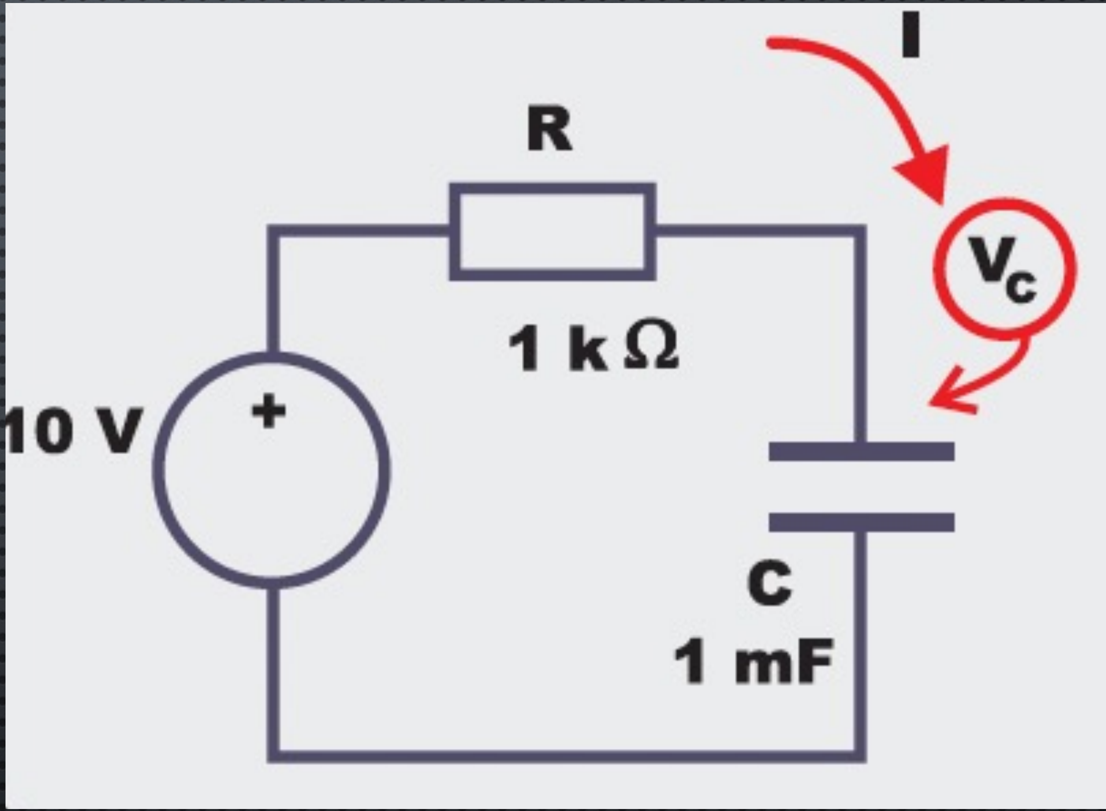
$$V_G = V_R + V_C$$

$V_C = 0$ til start

$$I = \frac{V_R}{R} = \frac{V_G - V_C}{R}$$

$$\frac{1}{R} V_G - \frac{1}{R} V_C = I = C \cdot \frac{d}{dt} V_c$$

$$\frac{d}{dt} V_c + \frac{1}{\tau} V_c - \frac{1}{\tau} V_G = 0$$



Solution of differential equation

$$V_C(t) = V_G\left(1 - e^{-\frac{t}{\tau}}\right)$$

Time constant

$\tau = R \cdot C$ [s]

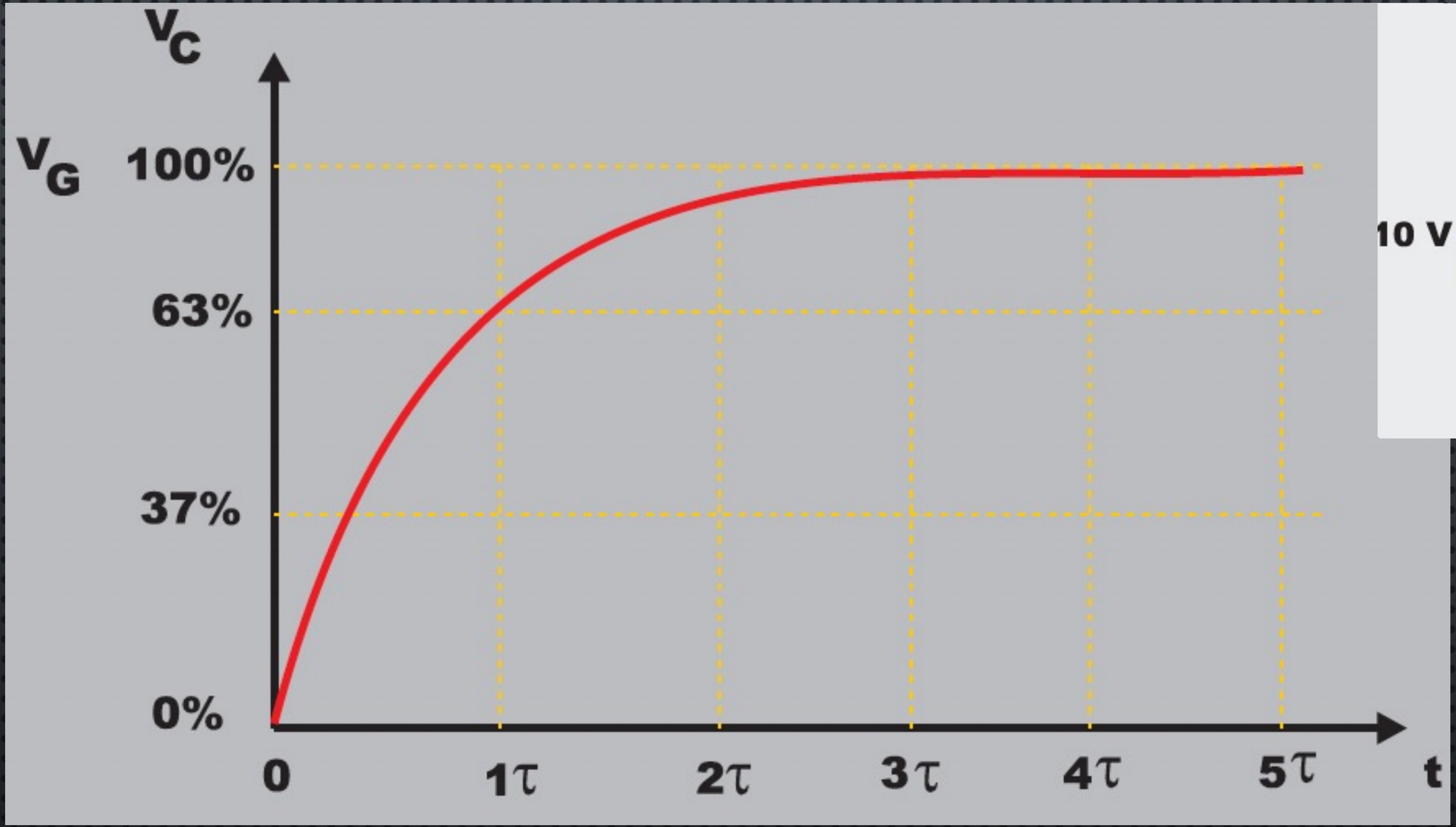
Example for the value

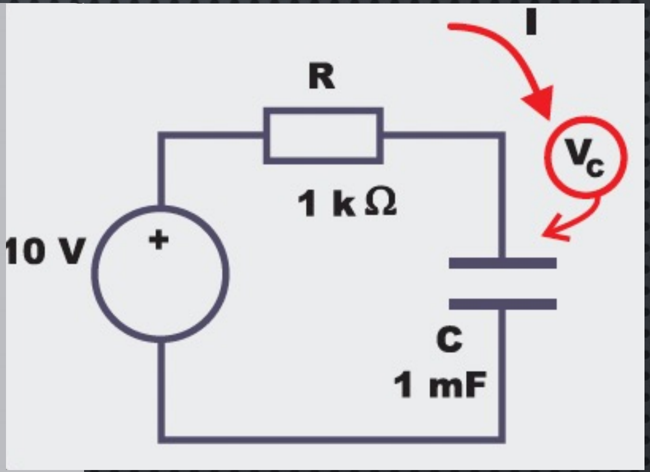
$e^{-1} = 37\%$

$1 - e^{-1} = 63\%$

$1 - e^{-5} = 99\%$

Charging of Capacitor



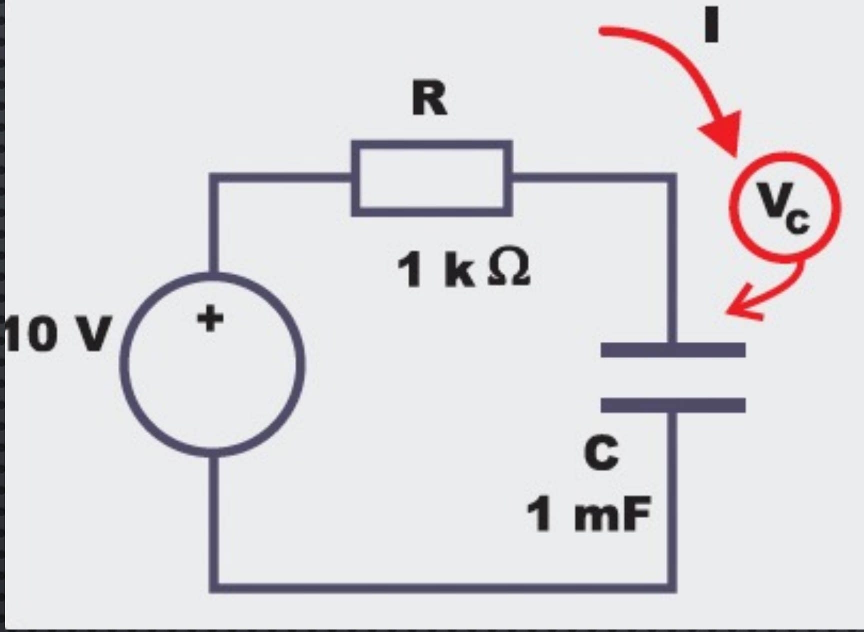


$$V_C(t) = V_G\left(1 - e^{-\frac{t}{\tau}}\right)$$

Time constant

$\tau = R \cdot C \quad [s]$

Charging of Capacitor



Time constant

$\tau = R \cdot C \quad [s]$

$1\,k\Omega \cdot 1\,mF = 1\,s$

V_c	I	V_c efter 0,1 s
0 V	$\frac{10\,V}{1\,k\Omega} = 10\,mA$	$V_c = \frac{1}{C}\int_0^t I\,dt = \frac{I \cdot t}{C} = \frac{10\,mA \cdot 0{,}1\,s}{1\,mF} = 1\,V$
1 V	$\frac{10 - 1}{1E3} = 9\,mA$	$V_c = \frac{I.t}{C} + V_{før} = 0{,}9 + 1 = 1{,}9\,V$

current I is a constant within a very short time

Charging of Capacitor

Energy is stored in capacitor by charging!!
Capacitor gives high output power in a short time than conventional battery

Tid 0.1 s:	Strøm= 10.0000 mA,	Spænding= 1.0000 V
Tid 0.2 s:	Strøm= 9.0000 mA,	Spænding= 1.9000 V
Tid 0.3 s:	Strøm= 8.1000 mA,	Spænding= 2.7100 V
Tid 0.4 s:	Strøm= 7.2900 mA,	Spænding= 3.4390 V
Tid 0.5 s:	Strøm= 6.5610 mA,	Spænding= 4.0951 V
Tid 0.6 s:	Strøm= 5.9049 mA,	Spænding= 4.6856 V
Tid 0.7 s:	Strøm= 5.3144 mA,	Spænding= 5.2170 V
Tid 0.8 s:	Strøm= 4.7830 mA,	Spænding= 5.6953 V
Tid 0.9 s:	Strøm= 4.3047 mA,	Spænding= 6.1258 V
Tid 1.0 s:	Strøm= 3.8742 mA,	Spænding= 6.5132 V
Tid 1.1 s:	Strøm= 3.4868 mA,	Spænding= 6.8619 V
Tid 1.2 s:	Strøm= 3.1381 mA,	Spænding= 7.1757 V
Tid 1.3 s:	Strøm= 2.8243 mA,	Spænding= 7.4581 V
Tid 1.4 s:	Strøm= 2.5419 mA,	Spænding= 7.7123 V

Time constant

$\tau = R \cdot C \quad [s]$

$1\ k\Omega \cdot 1\ mF = 1\ s$

Energy Examples

Car battery

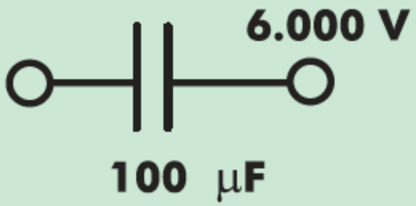
Car battery — **12 V / 100 Ah**

Energy of a fully charged battery

$$U = 12 \cdot 100 \cdot 3600 \quad Ws$$
$$= 12 \cdot 36 \cdot 10 \quad Ws = 4.320.000 \ Ws = 4{,}43 \ MJ$$

Kondensator

Energiindhold



$$U = \frac{1}{2} \cdot C \cdot V^2 = \frac{1}{2} \cdot 100E\text{-}6 \cdot 6.000^2$$
$$= 1.800 \quad Ws$$

2400 capacitors in parallel gives similar energy to car battery 4.3 MJ

Energy Examples

Heart starter

Energy

$$U = 400\ J = I \cdot V \cdot t = I^2 \cdot R \cdot t$$

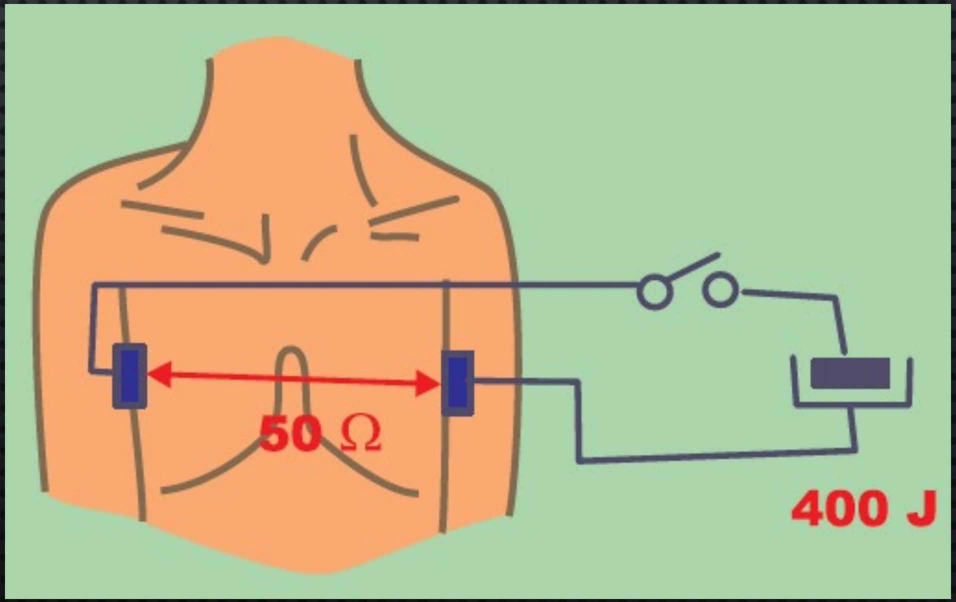
Impulse time

$$t = 3\ ms$$

Impulse current and voltage

$$I = \sqrt{\frac{U}{R \cdot t}} = 52\ A$$

$$V = \frac{U}{I \cdot t} = 2.600\ V$$



Energy Examples

Battery less IoT devices by sending impulse signal