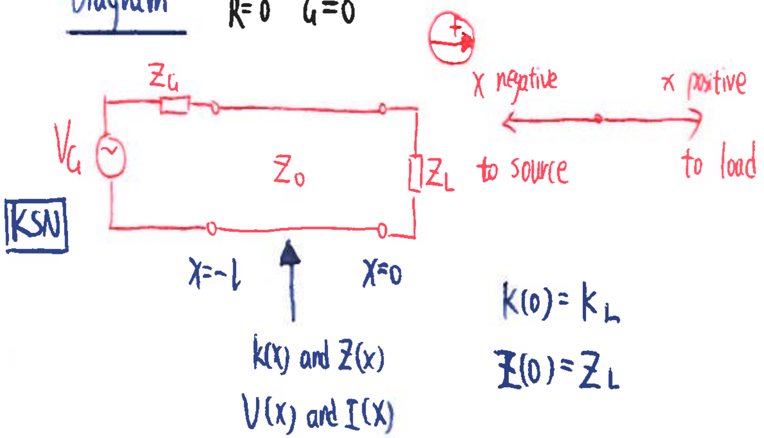
MM II

## (1) Transmission line model

**Diagram** R=0 G=0




$k(0)=k_L$

$Z(0)=Z_L$

**Important formulas**

$$Z(x) \triangleq \frac{V(x)}{I(x)} \; [\Omega] \qquad k(x) = \frac{V^-(x)}{V^+(x)} = -\frac{I^-(x)}{I^+(x)} \; [\cdot]$$

$$V(x) = V^+ e^{-j\beta x} + V^- e^{j\beta x} \; [V]$$

$$I(x) = I^+ e^{-j\beta x} + I^- e^{j\beta x} = \frac{1}{Z_0}\left[V^+ e^{-j\beta x} - V^- e^{j\beta x}\right] \; [A]$$

(Please note: $V^+$ is voltage amplitude, while $V^+(x)$ is voltage function)
(at x=0) (over different X)

$$I^+(x) = \frac{V^+(x)}{Z_0} \qquad I^-(x) = -\frac{V^-(x)}{Z_0} \; [A]$$

$$Z_0 = \frac{V^+(x)}{I^+(x)} = -\frac{V^-(x)}{I^-(x)} \; [\Omega]$$

$$k(x) = \frac{Z(x)-Z_0}{Z(x)+Z_0} \; [\cdot] \qquad Z(x) = Z_0 \cdot \frac{1+k(x)}{1-k(x)} \; [\Omega]$$

$$k(x) = k_L \cdot e^{j2\beta x} \; [\cdot]$$

Voltage on cable can be written as:

$$V(x) = V^+(x) + V^-(x) = V^+(x)\left(1+\frac{V^-(x)}{V^+(x)}\right)$$

$$= V^+(x)\,(1+k(x)) \quad [V]$$

For current:

$$I(x) = I^+(x)\,(1-k(x)) \quad [A]$$

**Amplitude of Voltage**

$$|V(x)| = |V^+(x)\,(1+k(x))|$$

$$= |V^+(x)| \cdot |1+k(x)|$$

$$= |V^+| \cdot |1+k(x)| \quad [V]$$

Normalized voltage:

$$\left|\frac{V(x)}{V^+}\right| = |1+k(x)| \quad [\cdot]$$

For reflection coefficient

$$|k(x)| = |k_L| \cdot |e^{j2\beta x}| = |k_L| \quad [\cdot]$$

$$\angle k(x) = \angle k_L + \angle e^{j2\beta x}$$

$$= \underbrace{\angle k_L}_{\text{reference}} + 2\beta x \quad [\text{rad}] \text{ or } [^\circ]$$

## (2) Vector figure

We investigate the function

$$\left| 1 + k(x) \right|$$

For the reflection coefficient

$$|k(x)| = |k_L|$$

$$\angle k(x) = \angle k_L + 2\beta x$$

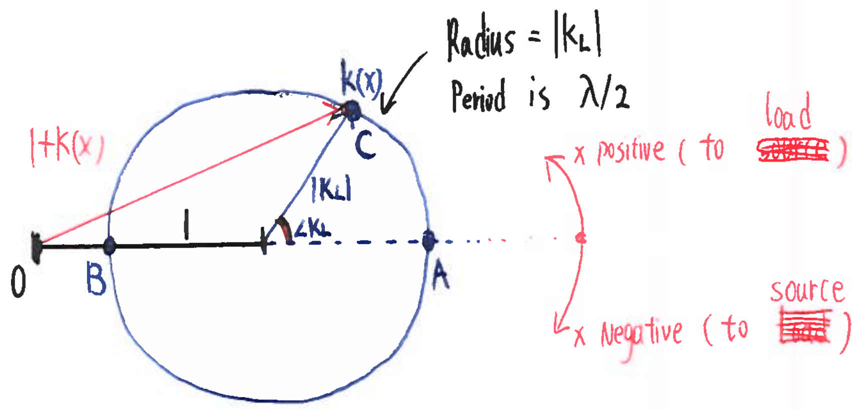
where:

$$2\beta x = 2\frac{\pi}{\lambda} \cdot x = 2\pi \cdot \frac{x}{\lambda/2} \ \text{[rad]}$$

(period is half $\lambda$)

## Figure

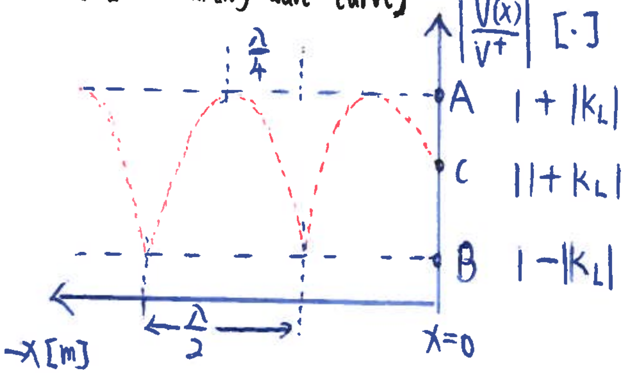
Vector in the complex plane




| point | name | $\angle k(x)$ | amplitude |
|---|---|---|---|
| A | Max. position | $0°$ | $1 + \|k_L\|$ |
| B | Min. position | $180°$ | $1 - \|k_L\|$ |
| C | Start position ($x=0$) | $\angle k_L$ | $\|1 + k_L\|$ |

## (3) Standing wave curve

[$Z_L \rightarrow$ Standing wave curve]



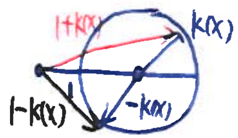

## Standing wave for currents

For current we have:

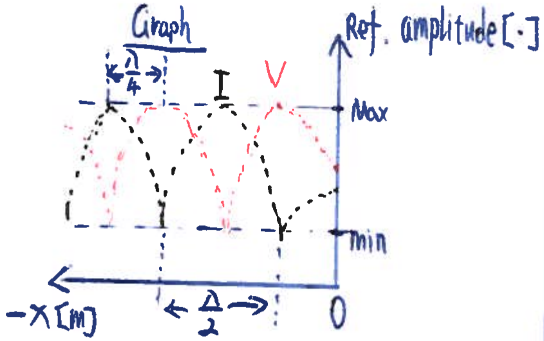
$$I(x) = I^+ (1-k(x))$$

$$\left|\frac{I(x)}{I^+}\right| = |1-k(x)|$$

### Vector figure




### Graph




## Standing wave condition

SWR standing wave ratio

VSWR voltage standing wave ratio

$$SWR \triangleq \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|k_L|}{1-|k_L|}$$

and the reverse

$$|k_L| = \frac{SWR-1}{SWR+1} \quad [\,]$$

Let:

$$Z_L = \begin{Bmatrix} \infty \\ 0 \\ jx \end{Bmatrix} \Rightarrow |k_L| = 1 \Rightarrow SWR\ \infty$$

$$Z_L = Z_0 \Rightarrow |k_L| = 0 \Rightarrow SWR\ 1$$

## (4) Reverse standing wave curve

[Standing wave curve → $Z_L$]

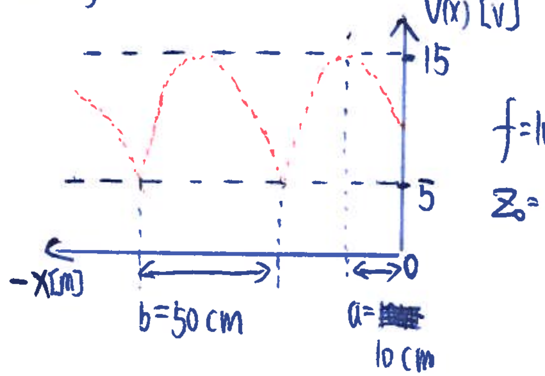
We have measured one standing wave curve, and require to find $Z_L$

Steps:

1. determine $|k_L|$
2. determine $\angle k_L$
3. calculate $Z_L = f(Z_0, k_L)$

## Example

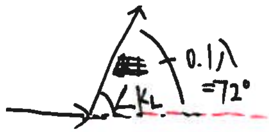
Standing wave curve




$f = 100\ MHz$

$Z_0 = 50\ \Omega$

Calculation:




$\lambda = 2b = 1\ m$

$a = 10cm = \frac{0.1m}{\lambda} \cdot \lambda = \frac{0.1m}{1m} \cdot \lambda = 0.1\lambda$

$1\lambda$ is corresponding to $720°$

$0.1\lambda$ is correpending to $72°$

$a$ is at $72°$, so $\angle k_L = 72°$

$SWR = \frac{V_{max}}{V_{min}} = \frac{15}{5} = 3$

$|k_L| = \frac{SWR-1}{SWR+1} = \frac{1}{2}$

Therefore $\left.\begin{array}{l} |k_L| = 0.5 \\ \angle k_L = 72° \end{array}\right\} \Rightarrow k_L = 0.5\angle 72°$

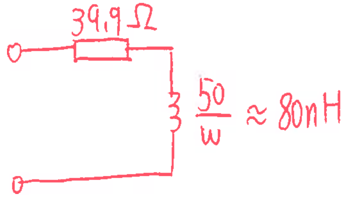
$= 0.5(\cos 72° + j \sin 72°)$

We have

$Z_L = Z_0 \frac{1+k_L}{1-k_L}$

$= 50 \cdot \frac{1 + 0.5\angle 72°}{1 - 0.5\angle 72°} \approx 39.9 + j50\ \Omega$

$\Downarrow$ component realization




$\boxed{w = 2\pi f}$