MM1

(1) Electromagnetic field theory

1. Name, symbol, unit
2. Definition of the field
3. Rotation and divergence
4. Boundary condition

Electric field

$\bar{E}$ $[\frac{V}{m}]$ Electric field intensity (E field)

$\bar{D}$ $[\frac{C}{m^2}]$ Electric flux density (D field)

$$\bar{D} = \varepsilon \cdot \bar{E} \quad [\frac{C}{m^2}]$$

Permittivity or dielectric constant

$$\varepsilon = \varepsilon_r \cdot \varepsilon_0 \quad [\frac{F}{m}]$$

ε_r: relative permittivity ★

In vacuum:

$$\varepsilon_0 = \frac{1}{36\pi} \cdot 10^{-9} \frac{F}{m}$$

Example for relative permittivity

Air: 1, Glass: 5~9, ~~FR4: 4~5~~

FR4: 4~5, PE: 2.25, Nylon: 3.4

Metal: 1, Soil: 4,

Please notice that ε_r and its loss are frequency dependent.

Other fields

ρ $[\frac{C}{m^3}]$ charge density (Volum charge)

ρ_s $[\frac{C}{m^2}]$ surface charge density (area charge)

Q $[C]$ point charge (abstract model)

(2) Definition of electric field

Definition:

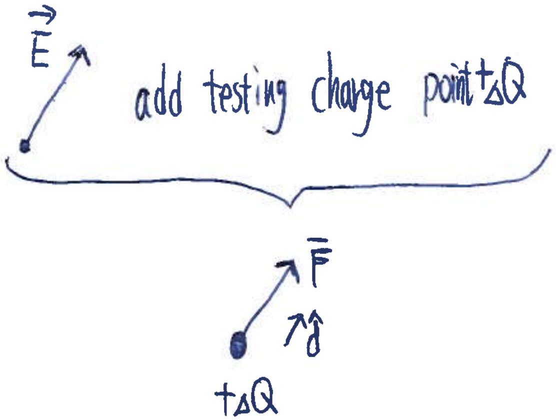
$$\bar{E} \triangleq \lim_{\Delta Q \to 0} \frac{|\bar{F}|}{\Delta Q} \cdot \hat{d} \quad \left[\frac{N}{C} = \frac{N\cdot m}{A\cdot s\cdot m} = \frac{J}{A\cdot s\cdot m} = \frac{W\cdot s}{A\cdot s\cdot m} = \frac{V\cdot A\cdot s}{A\cdot s\cdot m} = \frac{V}{m}\right]$$

★ use ISQ unit for calculation

where

$\bar{F}$ [N] is force

$\Delta\bar{Q}$ [C] is a positive testing charge



Vectors

$\bar{d}$ [m] direction vector

$$\hat{d} = \frac{\bar{d}}{|\bar{d}|} = \frac{\bar{d}}{d} \quad \left[\frac{m}{m} = \cdot\right] \quad \text{unit direction vector}$$

where $d = |\bar{d}| = \sqrt{d_x^2 + d_y^2 + d_z^2}$

others

$\bar{t}$, $\hat{t}$ Tangent vector, unit tangent vector

$\bar{n}$, $\hat{n}$ Normal vector, unit normal vector

(3) Coulomb's law

$$\bar{F} = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon \cdot d^2} \cdot \hat{d} \; [N]$$

where

Q_1, Q_2 [C] are charge

$\bar{F}$ [N] are force

d [m] is the distance between Q_1 and Q_2

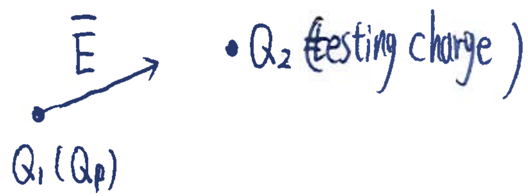
$\hat{d}$ [·] is unit direction vector

ε $[\frac{F}{m}]$ is permittivity

Electric field around a point charge

We let Q_1 a point charge and Q_2 a testing charge:

$$\bar{E}_{Q_p} = \frac{\bar{F}}{Q_2} = \frac{Q_1}{4\pi\varepsilon\, d^2} \cdot \hat{d} \; [\frac{V}{m}]$$



If spherical coordinates are applied:

$$\bar{E}_{Q_p}(r, \theta, \varphi) = \frac{Q_p}{4\pi\varepsilon \cdot r^2} \cdot \hat{r} \; [\frac{V}{m}]$$

Media independent field

$$\bar{D} = \varepsilon \cdot \bar{E}$$

$$= \frac{Q_p}{4\pi\, r^2} \cdot \hat{r} \; [\frac{C}{m^2}]$$

(4) Rotation and divergence

$\bar{E}$ field is spherical symmetry

So $\bar{E} = f(r)\cdot\hat{r}$

$$\nabla \times \bar{E} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\cdot\sin\theta\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ f(r) & 0 & 0 \end{vmatrix}$$

$$= \bar{0}$$

$\bar{E}$ field is conservative.

For divergence, we apply divergence theorem for electric charge

$$\oint_S \bar{D}\cdot d\bar{a} = \int_V \rho dV = Q_{TOT}$$

Since $\oint_S \bar{D}\cdot d\bar{a} = \int_V \nabla\cdot\bar{D} dv = \int_V \rho dV$

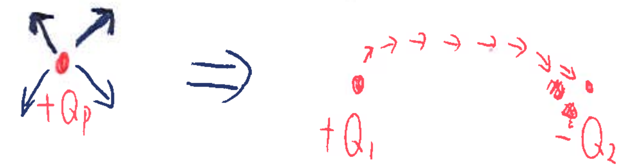
$$\Rightarrow \nabla\cdot\bar{D} = \rho$$

$$\Rightarrow \nabla\cdot\bar{E} = \frac{\rho}{\epsilon}$$

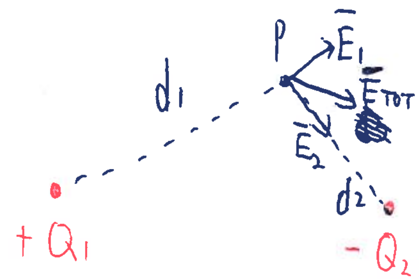
$\bar{E}$ field is a source field.

Illustration of electric field

We have one charge point $+Q_p$ that is placed in the electric field of $+Q_1$ and $-Q_2$



So :



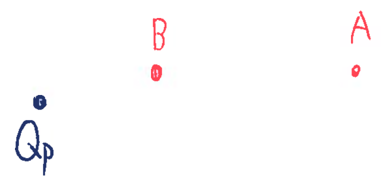
$$\left(\bar{E}_{TOT} = \bar{E}_1 + \bar{E}_2\right)$$

(5) Electric potential

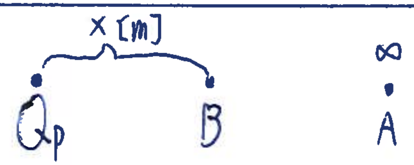
$$\nabla \times \bar{E} = \bar{0} \Rightarrow \begin{cases} \bar{E} = -\nabla V & [\frac{V}{m}] \\ -\int \bar{E}\cdot d\bar{l} = V & [V] \end{cases}$$

Potential between two points

$$V_{AB} = -\int_A^B \bar{E}\cdot d\bar{l} = \int_A^B \nabla V \cdot d\bar{l} = \int_A^B dV = V(B) - V(A)\ [V]$$



Potential around a point charge



$$V_{\infty B} = \int_\infty^x -\bar{E}\cdot d\bar{l} = -\int_\infty^x \frac{Q_p}{4\pi\epsilon r^2} dr$$

$$= -\frac{Q_p}{4\pi\epsilon}\int_\infty^x \frac{1}{r^2} dr = \frac{Q_p}{4\pi\epsilon}\left[\frac{1}{r}\right]_\infty^x$$

$$= \frac{Q_p}{4\pi\epsilon x}$$

$$= V(B) - V(\infty) = V(B) - 0 = V(B)$$

Potential of point B

Potential is a relative value. If the reference point is ∞, this potential is called absolute potential.