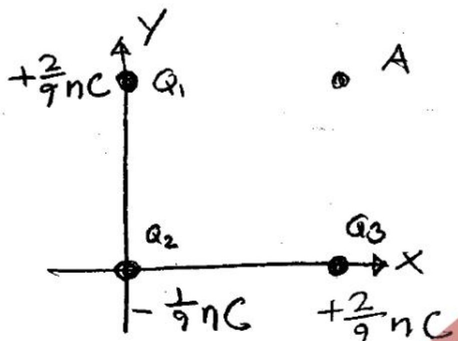
1.1



$Z=0$

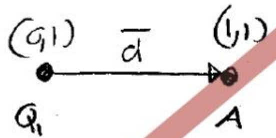
Q_1 i $(0,1)$

Q_2 i $(0,0)$

Q_3 i $(1,0)$

Punkt A i $(1,1)$

a) For Q_1:



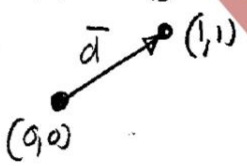
$\bar{d} = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$d = |\bar{d}| = 1$

$$\bar{E}_1 = \frac{Q_1}{4\pi\varepsilon_0 d^2}\cdot\hat{d} = \frac{\frac{2}{9}\cdot 1E-9}{\frac{1}{9}\cdot 1E-9\cdot d^2} = \frac{2}{1^2} = 2\cdot\begin{bmatrix}1\\0\end{bmatrix}\ \frac{V}{m}$$

$$4\pi\varepsilon_0 = \frac{4\pi}{36\pi}\cdot 10^{-9}\ F/m = \frac{1}{9}\cdot 1E-9$$

For Q_2:



$\bar{d} = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

$d = |\bar{d}| = \sqrt{1^2+1^2} = \sqrt{2}$

$$\bar{E}_2 = \frac{-1}{\sqrt{2}^2}\cdot\hat{d} = -\frac{1}{2}\cdot\begin{bmatrix}1\\1\end{bmatrix}\frac{1}{\sqrt{2}}\ \frac{V}{m}$$

$$\bar{E}_3 = \frac{2}{1^2}\cdot\hat{d} = 2\cdot\begin{bmatrix}0\\1\end{bmatrix}\ \frac{V}{m}$$

$$E_{TOT} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= 2 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{-1}{2\sqrt{2}} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} + 2 \end{bmatrix} = \begin{bmatrix} \frac{4\sqrt{2}-1}{2\sqrt{2}} \\ \frac{4\sqrt{2}-1}{2\sqrt{2}} \end{bmatrix} = \frac{\sqrt{32}-1}{\sqrt{8}} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= 1{,}6464 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{V}{m}$$

b) Potentialet

For Q_1:

$$d = |\vec{d}| = 1$$

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 \cdot d} = \frac{2}{1} = +2 \text{ V}$$

For Q_2:

$$d = |\vec{d}| = \sqrt{2}$$

$$V_2 = \frac{Q_2}{4\pi\varepsilon_0 \, d} = \frac{-1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \text{ V}$$

For Q_3:

$$V_3 = \frac{Q_3}{4\pi\varepsilon_0 d} = \frac{2}{1} = +2 \text{ V}$$

$$V_{TOT} = V_1 + V_2 + V_3$$

$$= 2 + 2 - \frac{1}{\sqrt{2}} = 4 - \frac{1}{\sqrt{2}} = 3{,}2929 \text{ V}$$

c) Vi definerer: $\overline{A} = \begin{bmatrix}1\\1\end{bmatrix}$

Kraften bliver:

$$\overline{F} = q \cdot \overline{E} = -1{,}602 \cdot 10^{-19} \cdot -1{,}6464 \cdot \overline{A}$$

$$= 0{,}2638 \cdot 10^{-18} \cdot \overline{A} \quad N$$

$$= 0{,}2638 \cdot \begin{bmatrix}1\\1\end{bmatrix} \quad aN$$

d) Accelerationen bliver:

$$\overline{a} = \frac{\overline{F}}{m} = \frac{0{,}2638E-18}{9{,}107E-31} \cdot \begin{bmatrix}1\\1\end{bmatrix} \quad \frac{m}{s^2}$$

$$= 289{,}62 \cdot 10^{9} \ \frac{m}{s^2}$$

$$= 289{,}62 \cdot 10^{6} \quad km/s^2$$

e) Kraften bliver:

$$\overline{F} = Q \cdot \overline{E}\Big|_{2,2,1C} = 1 \cdot 1{,}6464 \cdot 9 \cdot 10^{+9} \cdot \overline{A}$$

$$= 14{,}82\,E9 \cdot \overline{A}$$

$$|\overline{F}| = F = 14{,}82E9 \cdot |\overline{A}| = 14{,}82E9 \cdot \sqrt{2}$$

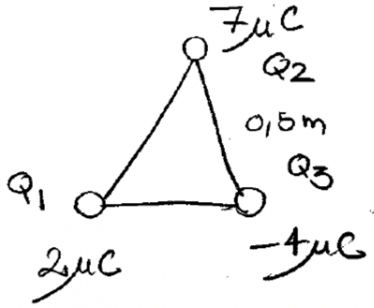
$$= 20{,}96E9 \quad N$$

Omregning til kp:

$$F = \frac{20{,}96E9}{9{,}82} = 2{,}13E9 \quad kp$$

Svarende til 2,13 millioner tons.

1.2

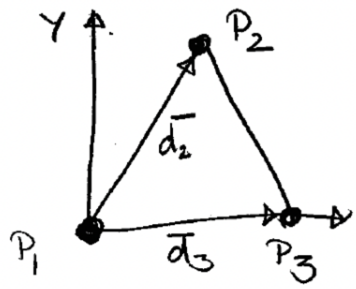


En ligesidet trekant.

$\cos 60° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

a) Vi placerer (0,0) ved de 2 µC og anvender et 2-dimensionelt koordinatsystem.



$P_1 = (0,0)$

$P_2 = \left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$

$P_3 = \left(\frac{1}{2}, 0\right)$

$|\bar{d}_2| = d_2 = 0,5$

$|\bar{d}_3| = d_3 = 0,5$

$$\hat{d}_2 = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\hat{d}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Feltet fra Q_2:

$$\bar{E}_2 = \frac{-Q_2}{4\pi\varepsilon_0 \cdot d_2^2} \cdot \hat{d}_2$$

$$= \frac{-7E\text{-}6}{\frac{1}{9}E\text{-}9 \cdot (\frac{1}{2})^2} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= -7 \cdot 9 \cdot 4 \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \cdot 1E3$$

$$= -252E3 \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \frac{V}{m}$$

$$4\pi\varepsilon_0 = \frac{4\pi}{36\pi} \cdot 1E\text{-}9$$

$$= \frac{1}{9} \cdot 1E\text{-}9$$

Feltet fra Q_3

$$\overline{E}_3 = \frac{-Q_3}{4\pi\varepsilon_0 \cdot d_3^2} \cdot \hat{d}_3$$

$$= \frac{+4E-6}{\frac{1}{9}E-9 \cdot (\frac{1}{2})^2} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= 4 \cdot 9 \cdot 4\,E3 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \quad 144\,E3 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \frac{V}{m}$$

Det totale felt:

$$\overline{E} = \overline{E}_2 + \overline{E}_3 = \begin{bmatrix} -252 \cdot 0{,}5 + 144 \\ -252 \cdot \sqrt{3} \cdot 0{,}5 \end{bmatrix} \quad kV/m$$

$$= \begin{bmatrix} 18 \\ -218{,}2 \end{bmatrix} \quad \frac{kV}{m}$$

b) Kraften bliver:

$$\overline{F} = Q \cdot \overline{E} = \quad 2E-6 \cdot \overline{E} = \begin{bmatrix} 36 \\ -436{,}5 \end{bmatrix} \; mN$$