

AMM4

# (1) Ampere's law and Faraday's law

## Ampere's law

magnetomotive force

$$\text{MMF} = F = \oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{a} = I_{\text{tot}} [A]$$

Differential - or point - form

$$\nabla \times \vec{H} = \vec{J}$$



## Faraday's law

electromotive force

~~emf~~ 
$$\text{emf} = -V = \oint_C \vec{E} \cdot d\vec{l} = \int_S -\frac{\partial}{\partial t} \vec{B} \cdot d\vec{a} = -\frac{\partial}{\partial t} \phi [V]$$

Differential - or point - form

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$



## Technical formula

A's law :  $H \cdot l = N \cdot I$   $[A \cdot m]$

F's law :  $V = -j\omega BAN$  ~~emf~~  $[wbv]$

Flux :  $\phi = BA$   $[wb]$

Technical unit:

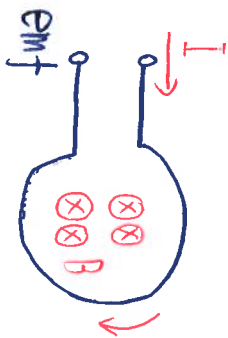
$[AV]$  Ampere turns

$[wbv]$  web turns

$[m^2]$  square ohm

$$A = BAN \quad [wbv]$$

## (2) Self induction



$$\phi = \int \vec{B} \cdot d\vec{a} \quad [\text{wb}]$$

$$\mathcal{E}_{mf} = -\frac{d}{dt} \phi = -\frac{d\phi}{dI} \cdot N \cdot \frac{dI}{dt}$$

$$= -\frac{dI}{dt} \cdot L \quad [\text{V}]$$

where

$$L \triangleq \frac{d\phi}{dI} \cdot N \left[ \frac{\text{wb}}{\text{A}} = \text{H} \right] \text{ is } \underline{\text{self induction}}$$

We see in  $kSN$

$$V = j\omega L \quad (\text{V})$$

$$\Rightarrow \frac{V}{I} = j\omega L$$

### (3) Magnetic materials

Many materials are HILS

according to  $\epsilon, \mu, \sigma$ :

- Homogeneous
- Isotropic
- Linear
- Stationary (time invariant)

For HILS materials,  $\epsilon, \mu, \sigma$  are constant

### Magnetic materials

Diamagnetic

$$\mu_r \leq 1$$

linear

Cu, Ag

Paramagnetic

$$\mu_r \geq 1$$

linear

air, Al

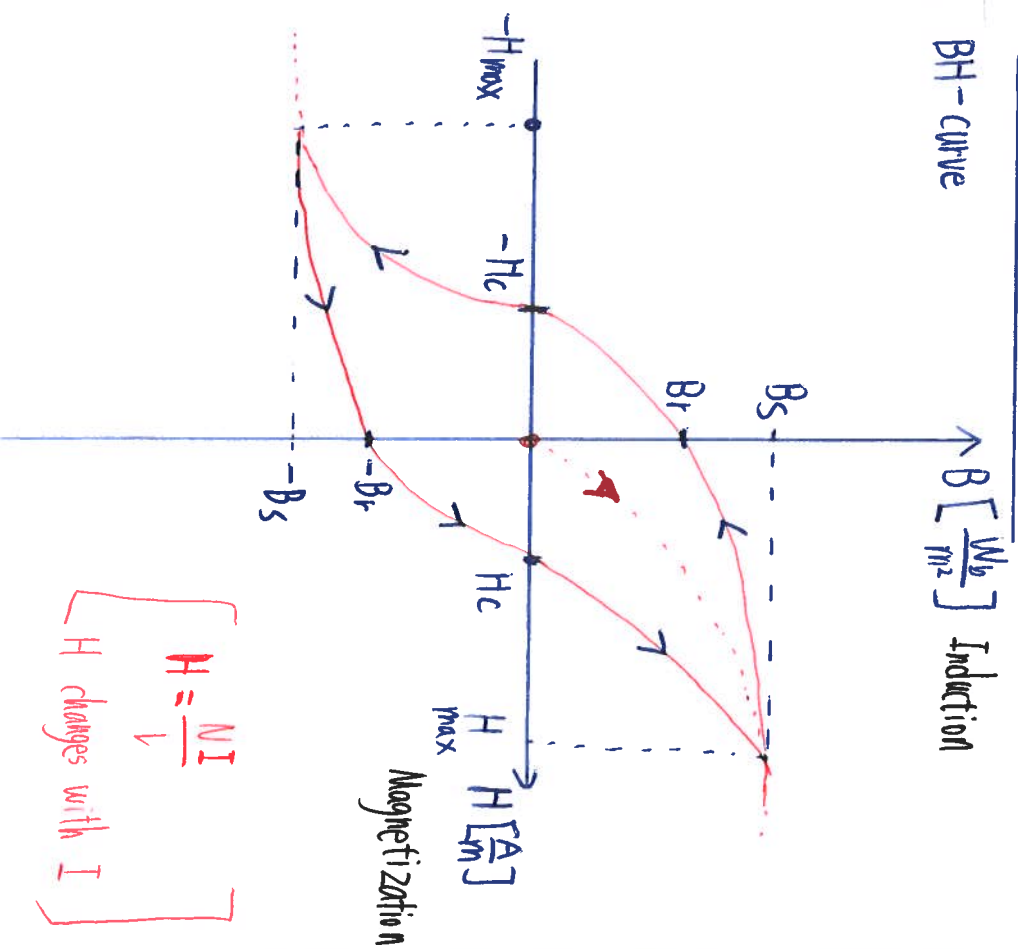
Ferromagnetic

$$\mu_r \gg 1$$

nonlinear

Iron

#### (4) Hysteresis loop



#### Typical values

$$B_s \left[ \frac{Wb}{m^2} \right]$$

Saturation flux

$$H_{max} \left[ \frac{A}{m} \right]$$

Corresponding magnetization

$$B_r \left[ \frac{Wb}{m^2} \right]$$

Residual magnetism

$$H_c \left[ \frac{A}{m} \right]$$

Coercive force

Energy per loop (Energy loss)

$$W_m = \oint B \, dH \left[ \frac{A}{m} \cdot \frac{Vs}{m^2} = \frac{J}{m^3} \right]$$

#### Permeability

$$\mu_{Av} = \frac{B_s}{H_{max}}$$

$$\mu_{Ac} = \frac{dB}{dH}$$