MM 2

(1) Electric Currents

Currents out of one region

$$I = -\frac{dQ}{dt} = -\frac{\partial}{\partial t}\int_V \rho \, dV \quad [A]$$

$I \; [A = \frac{C}{s}]$

Currents out of one region

The out currents can also be expressed as:

$$I = \oint_S J \, d\bar{a} \qquad \bar{J} \left[\frac{A}{m^2}\right] \text{ Current density (area current)}$$

$$\left[A \quad \frac{A}{m^2} \quad m^2\right]$$

Apply Gauss (divergence) Theorem

$$\oint_S \bar{J} \cdot d\bar{a} = -\frac{\partial}{\partial t}\int_V \rho \, dv$$

$$\int_V \nabla \cdot \bar{J} \, dv = -\frac{\partial}{\partial t}\int_V \rho \, dv$$

$$\Rightarrow \nabla \cdot J = -\frac{\partial}{\partial t}\rho \qquad \text{(General)}$$

$$\nabla \cdot J = 0 \qquad \text{(for DC, or low frequency, or good conductor)}$$

(2) Quasi-Static field

$$\nabla \times \bar{E} = \bar{0} \quad \text{(KVL)}$$

$$\nabla \cdot \bar{J} = 0 \quad \text{(KCL)}$$

$$\bar{J} = \sigma \bar{E} \quad \text{(Ohm's law)}$$

Ohm's law

$$I = G \cdot V \quad \Rightarrow \quad \bar{J} = \sigma \bar{E}$$

$$[A = S \cdot V] \qquad \left[\frac{A}{m^2}\right] \left[\frac{S}{m}\right] \cdot \left[\frac{V}{m}\right]$$

$\sigma \left[\frac{S}{m}\right]$ conductivity

(specific conductor)

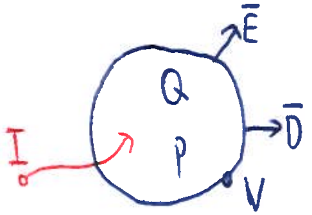
Example

air: $\sigma = 0 \ \frac{S}{m}$

copper: $\sigma_{Cu} = 58E6 \ \frac{S}{m}$

For the same σ, the ohm loss at high frequency is higher than low frequency (due to skin effect)

(3) Capacity



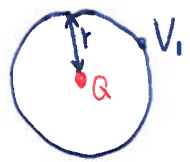
$$I = \frac{dQ}{dt} = \frac{dQ}{dV} \cdot \frac{dV}{dt} = C \cdot \frac{dV}{dt}$$

where $C \triangleq \frac{dQ}{dV}$ $\left[F = \frac{C}{V} = \frac{A \cdot s}{V}\right]$

is the capacity

$$\boxed{\begin{array}{l}\text{KSN} \\ I = C \cdot j\omega V \Rightarrow \frac{V}{I} = -j\frac{1}{\omega C} = Z\end{array}}$$

Capacity of a ball



Q is known and a point charge

So

$$V_1 = \frac{Q}{4\pi\varepsilon r} \; [V]$$

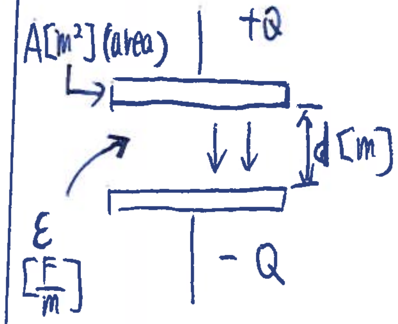
Capacity

$$C = \frac{Q}{V(Q)} = 4\pi\varepsilon r \; [F]$$

For air, we have:

$$C = 4\pi\varepsilon_0 r \; [F]$$

Capacity of parallel plates



The surface charge density over the ~~plane~~ plate.

$$\rho_s = \frac{Q}{A} \left[\frac{C}{m^2}\right]$$

D-field over the plate

$$D = \frac{Q}{A} = \rho_s \left[\frac{C}{m^2}\right]$$

E field

$$E = \frac{D}{\varepsilon} = \frac{Q}{\varepsilon A} \left[\frac{V}{m}\right]$$

Voltage between the plates:

$$V = \int E\,dl \approx E \cdot d = \frac{Qd}{\varepsilon \cdot A}$$

The capacity is

$$C = \frac{Q}{V(Q)} = \frac{\varepsilon \cdot A}{d} [F]$$