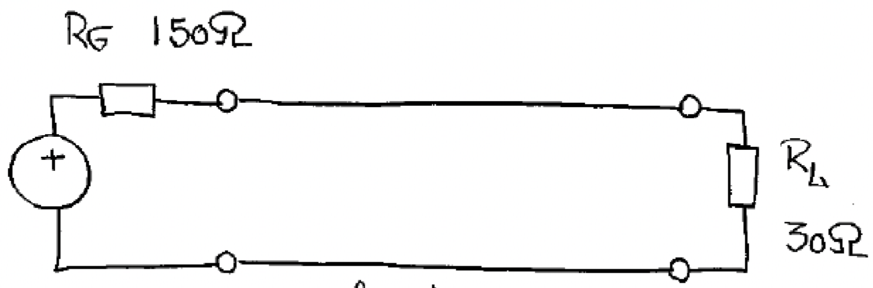
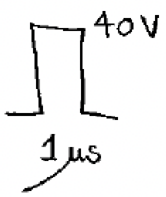
7.1





$\ell = 400\,m$

$v = 200\,m/\mu s$

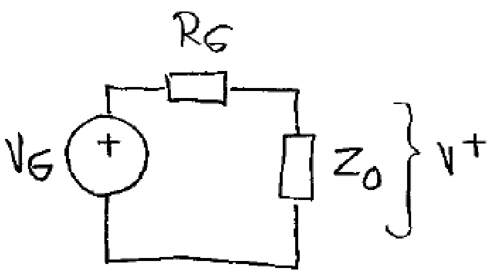
$Z_0 = 50\,\Omega$

a) Refleksionskoefficienter

$$K_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{30-50}{30+50} = \frac{-20}{+80} = -\frac{1}{4}$$

$$K_G = \frac{R_G - Z_0}{R_G + Z_0} = \frac{150-50}{150+50} = \frac{100}{200} = +\frac{1}{2}$$

Første indfaldende impuls:



$$V^+ = V_G \cdot \frac{Z_0}{Z_0 + R_G}$$

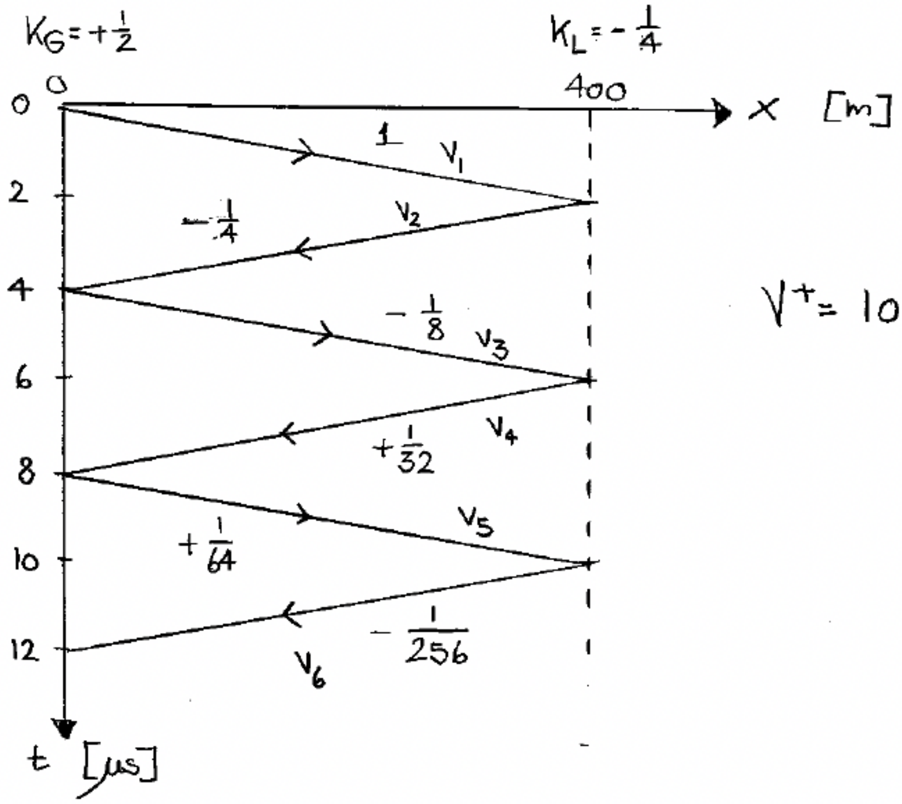
$$= 40 \cdot \frac{50}{50 + 150}$$

$$= 40 \cdot \frac{50}{200}$$

$$= 10\ V$$

b) Refleksionsdiagram

$$\Delta t = \frac{l}{v} = \frac{400}{200E6} = 2\,\mu s$$



$V^+ = 10\ V$

$V_1 = 1$

$V_2 = V_1 \cdot K_L = -\frac{1}{4}$

$V_3 = V_2 \cdot K_G = -\frac{1}{8}$

$V_4 = V_3 \cdot K_L = +\frac{1}{32}$

$V_5 = V_4 \cdot K_G = +\frac{1}{64}$

$V_6 = V_5 \cdot K_L = -\frac{1}{256}$

c) Spændingerne ved generatoren vil være

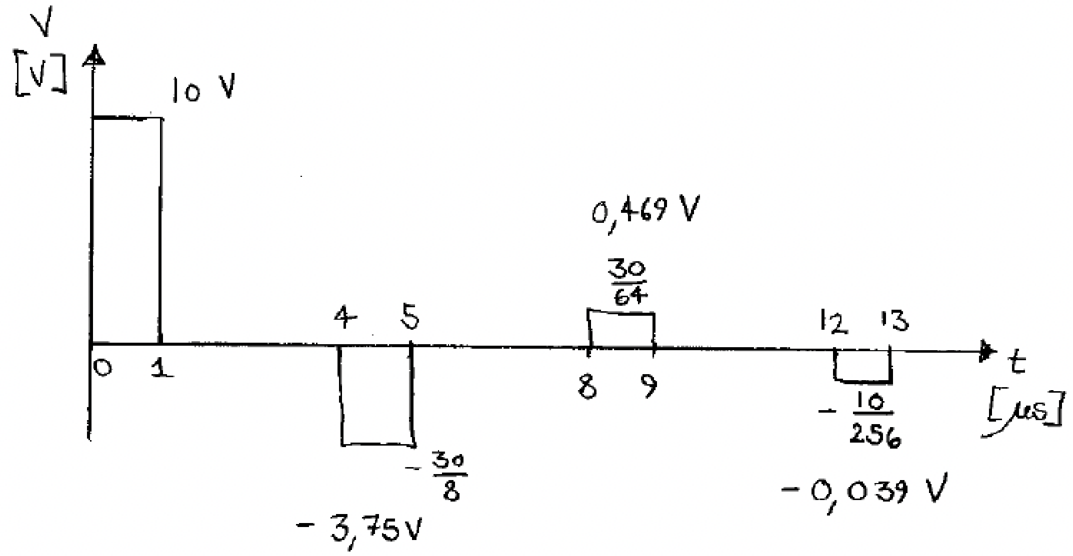
$t = 0\ \mu s \qquad V_1 \cdot V^+ = 1 \cdot 10 = 10\,V$

$t = 4\ \mu s \qquad (V_2 + V_3) \cdot V^+ = (-\frac{1}{4} - \frac{1}{8}) \cdot 10 = -\frac{30}{8}$

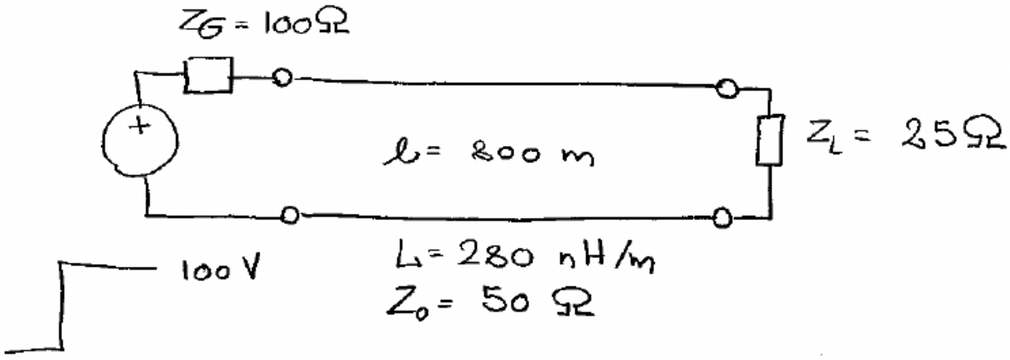
$t = 8\ \mu s \qquad (V_4 + V_5) \cdot V^+ = (\frac{1}{32} + \frac{1}{64}) \cdot 10 = \frac{30}{64}$

$t = 12\ \mu s \qquad V_6 \cdot V^+ = -\frac{1}{256} \cdot 10 = -\frac{10}{256}$

Impulserne forlænges til $1\ \mu s$



7.2



Beregning af udbredelseshastighed:

$$Z_0 = \sqrt{\frac{L}{C}} \Rightarrow Z_0^2 = \frac{L}{C} \Rightarrow C = \frac{L}{Z_0^2}$$

$$V = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L \cdot \frac{L}{Z_0^2}}} = \frac{Z_0}{L}$$

$$= \frac{50}{280E\text{-}9} = 178{,}571 \ m/\mu s$$

Beregning af tidsstep:

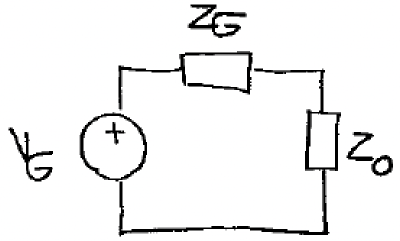
$$\Delta t = \frac{l}{V} = \frac{800}{178{,}6E6} = 4{,}48 \ \mu s$$

Beregning af refleksionskoefficienter:

$$K_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25-50}{25+50} = \frac{-25}{75} = -\frac{1}{3}$$

$$K_G = \frac{Z_G - Z_0}{Z_G + Z_0} = \frac{100-50}{100+50} = \frac{50}{150} = +\frac{1}{3}$$

Beregning af 1. indfaldende bølge:

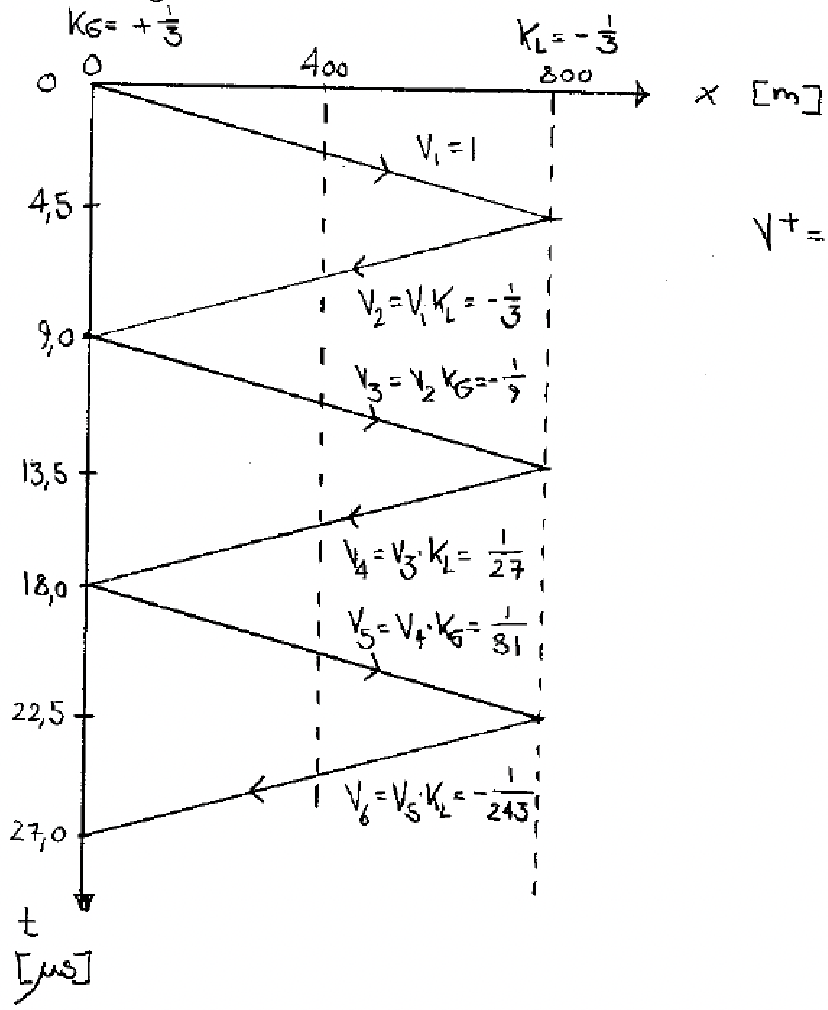


$$V^+ = V_G \cdot \frac{Z_0}{Z_0 + Z_G}$$

$$= 100 \cdot \frac{50}{50+100}$$

$$= \frac{100}{3} \quad V$$

Refleksionsdiagram



$$V^+ = \frac{100}{3} \ V$$

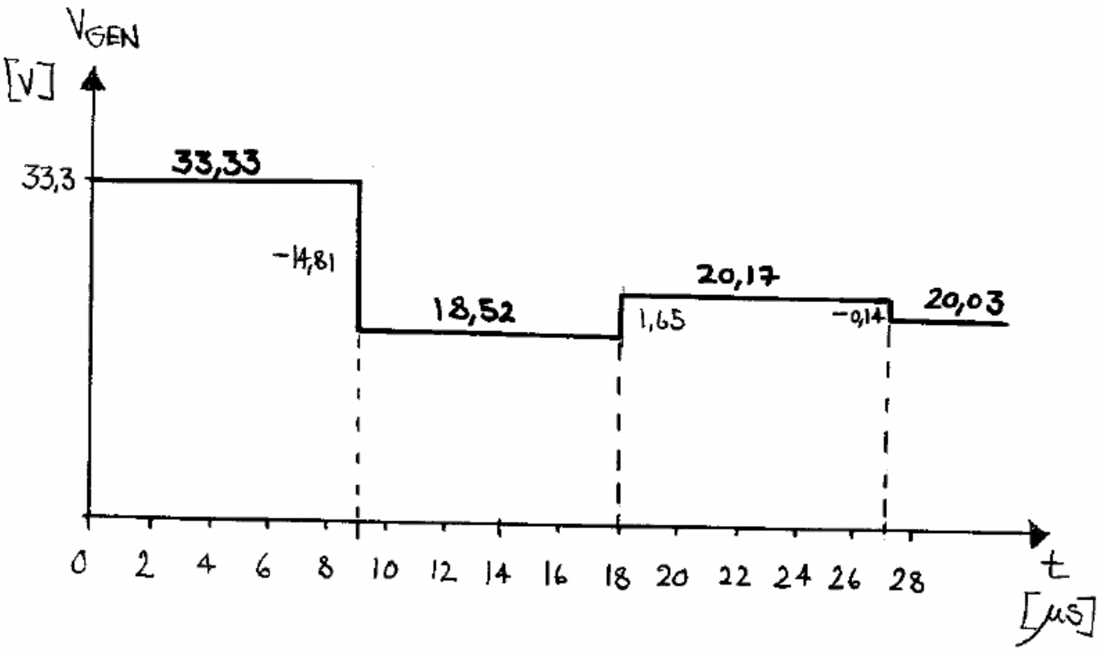
a) Spændingen ved generatoren.

$$t = 0\ \mu s \qquad V_1 \cdot V^+ = \frac{100}{3} = 33{,}33\ V$$

$$t = 9{,}0\ \mu s \qquad (V_2 + V_3) \cdot V^+ = \left(-\frac{1}{3} - \frac{1}{9}\right) \cdot \frac{100}{3} = -14{,}81\ V$$

$$t = 18{,}0\ \mu s \qquad (V_4 + V_5) \cdot V^+ = \left(\frac{1}{27} + \frac{1}{81}\right) \cdot \frac{100}{3} = 1{,}65\ V$$

$$t = 27{,}0\ \mu s \qquad V_6 \cdot V^+ = -\frac{1}{243} \cdot \frac{100}{3} = -0{,}14\ V$$

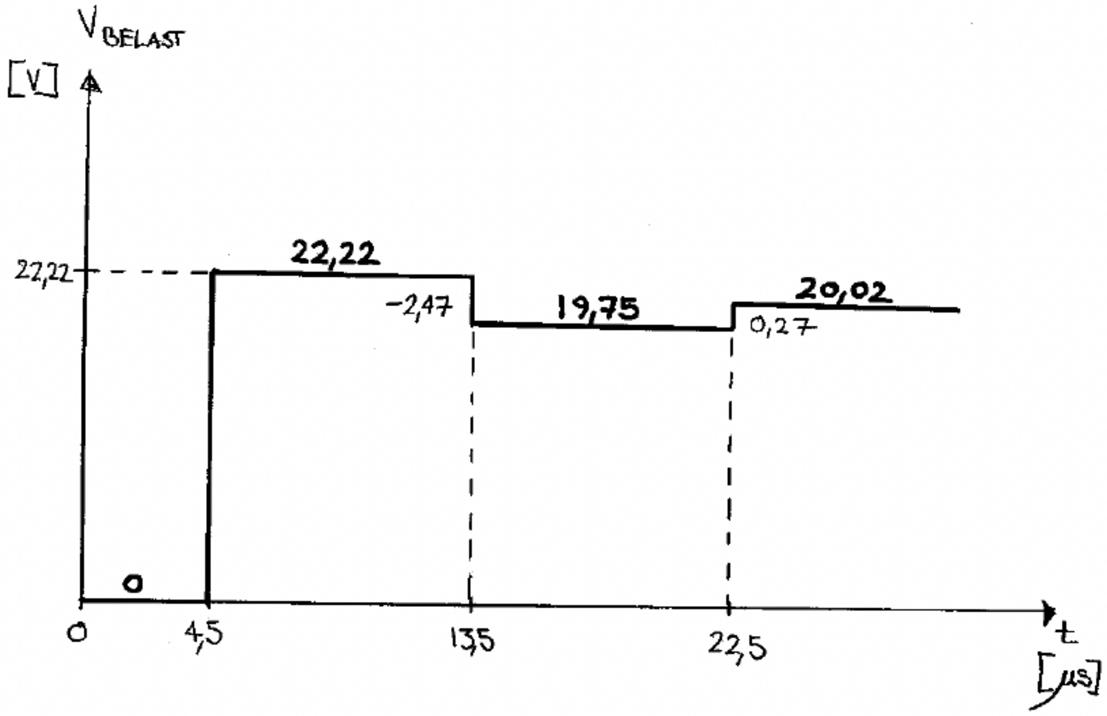


b) Spændingen ved belastningen.

$t = 4{,}5\ \mu s \qquad (V_1 + V_2)\,V^+ = \left(1 - \frac{1}{3}\right) \cdot \frac{100}{3} = 22{,}22\ V$

$t = 13{,}5\ \mu s \qquad (V_3 + V_4)\,V^+ = \left(-\frac{1}{9} + \frac{1}{27}\right) \cdot \frac{100}{3} = -2{,}47\ V$

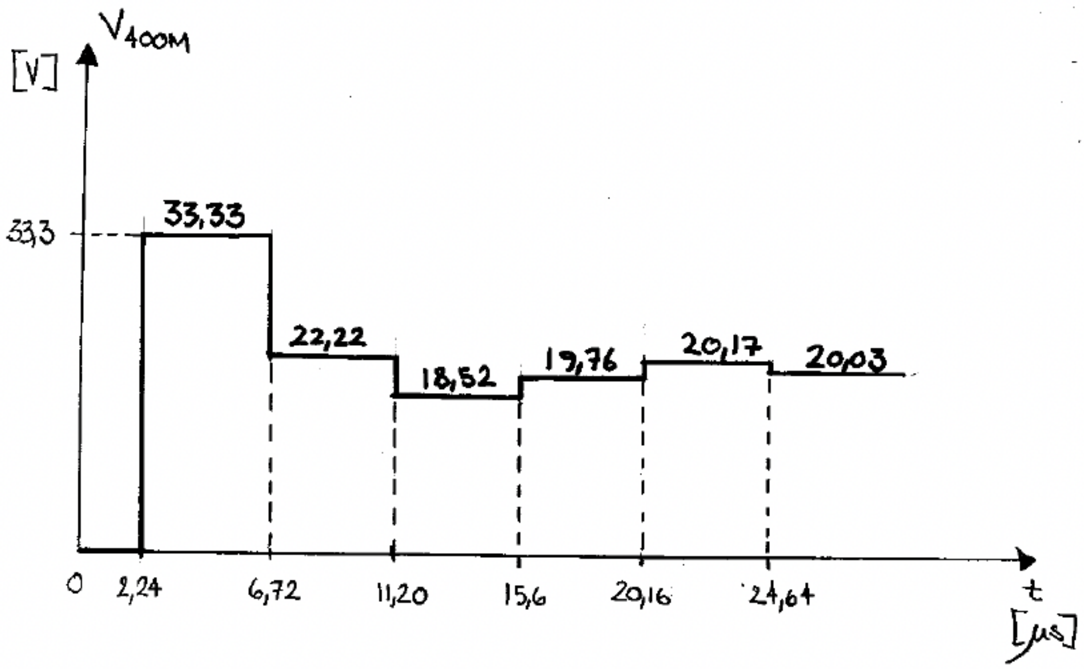
$t = 22{,}5\ \mu s \qquad (V_5 + V_6)\,V^+ = \left(\frac{1}{81} - \frac{1}{243}\right) \cdot \frac{100}{3} = 0{,}27\ V$



c) Spændingen midt på kablet.

Løbetid for 400 m: $t_{400} = \frac{400}{V} = 2{,}24\ \mu s$

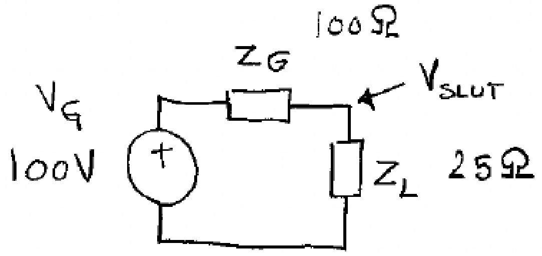
$t = 2{,}24\ \mu s$	$V_1 \cdot V^+ = 1 \cdot \frac{100}{3}$	$= 33{,}33\ V$
$t = 6{,}72\ \mu s$	$V_2 \cdot V^+ = -\frac{1}{3} \cdot \frac{100}{3}$	$= -11{,}11\ V$
$t = 11{,}20\ \mu s$	$V_3 \cdot V^+ = -\frac{1}{9} \cdot \frac{100}{3}$	$= -3{,}70\ V$
$t = 15{,}68\ \mu s$	$V_4 \cdot V^+ = \frac{1}{27} \cdot \frac{100}{3}$	$= 1{,}24\ V$
$t = 20{,}16\ \mu s$	$V_5 \cdot V^+ = \frac{1}{81} \cdot \frac{100}{3}$	$= 0{,}41\ V$
$t = 24{,}64\ \mu s$	$V_6 \cdot V^+ = -\frac{1}{243} \cdot \frac{100}{3}$	$= -0{,}14\ V$



d) Beregning af slutværdien.

Slutværdien, som bliver den samme for alle 3 steder, kan beregnes v.h.a. kredsløbsteorien.

Ækvivalentdiagram



$$V_{SLUT} = V_G \cdot \frac{Z_L}{Z_G + Z_L} = 100 \cdot \frac{25}{100+25} = 100 \cdot \frac{25}{125}$$

$$= 100 \cdot \frac{1}{5} = 20 \text{ V}$$

Slutværdien kan også beregnes ved at addere alle indfaldende og reflekterede bølger, som set ud fra de foregående spørgsmål.

Sommen af alle indfaldende spændinger:

$$\sum V^+ = 1 + K_L K_G + K_L^2 K_G^2 + K_L^3 K_G^3 + \ldots$$

$$= \sum_{n=0}^{\infty} (K_L K_G)^n = \sum_{n=0}^{\infty} \left(-\frac{1}{9}\right)^n$$

$$= \frac{1}{1 - \left(-\frac{1}{9}\right)} = \frac{1}{\frac{10}{9}} = \frac{9}{10}$$

Dette er de normerede spændingen.

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Sommen af alle reflekterede spændinger:

$$\sum V^- = K_L + K_L^2 K_G + K_L^3 K_G^2 + K_L^4 K_G^3 + \dots$$

$$= K_L \cdot (1 + K_L K_G + K_L^2 K_G^2 + K_L^3 K_G^3 + \dots)$$

$$= K_L \cdot \sum V^+ = -\frac{1}{3} \cdot \frac{9}{10} = -\frac{9}{30}$$

Totalt fås:

$$\sum V^+ + \sum V^- = \frac{9}{10} - \frac{9}{30} = \frac{27-9}{30}$$

$$= \frac{18}{30}$$

Denormering med $V^+ = \frac{100}{3}$ V giver:

$$V_{SLUT} = \frac{18}{30} \cdot \frac{100}{3} = \frac{1800}{90} = 20 \text{ V}$$

Beregning af geometrisk rækkesum:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Gælder for $x < 1$.