UNIVERSITÄT BERN

CAS Applied Data Science - Module 2 – Day 2

Statistical Inference for Data Science

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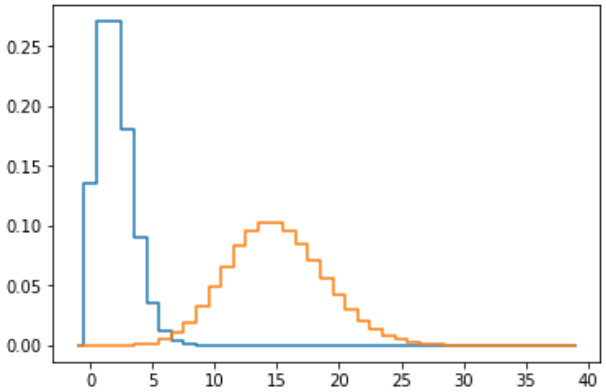
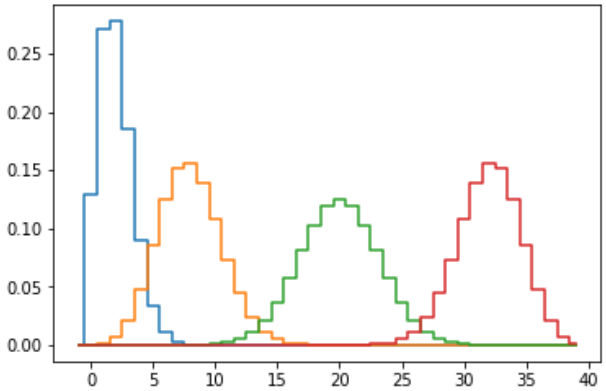
Questions from Notebook 2 (see also google form)

- What is the blue distribution ?
- Examples of use ?
- Difference with the other colored curve(s) ?





- Why are measured observables random variables ?
- Which probability distribution of a RV is the most important ?
- Thumb of rule, when is the normal distribution a good approximation ?
- Can you mention 5 descriptive statistical measures ?

Exercise 2.3.4

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It is important to obtain some routine with the computation of probabilities and quantiles.

Let X be binomially distributed with n = 60 and p = 0.4. Compute the following.

- P(X = 24) (PMF), P(X ≤ 24) (CDF)
- Compute the mean and standard deviation of X.

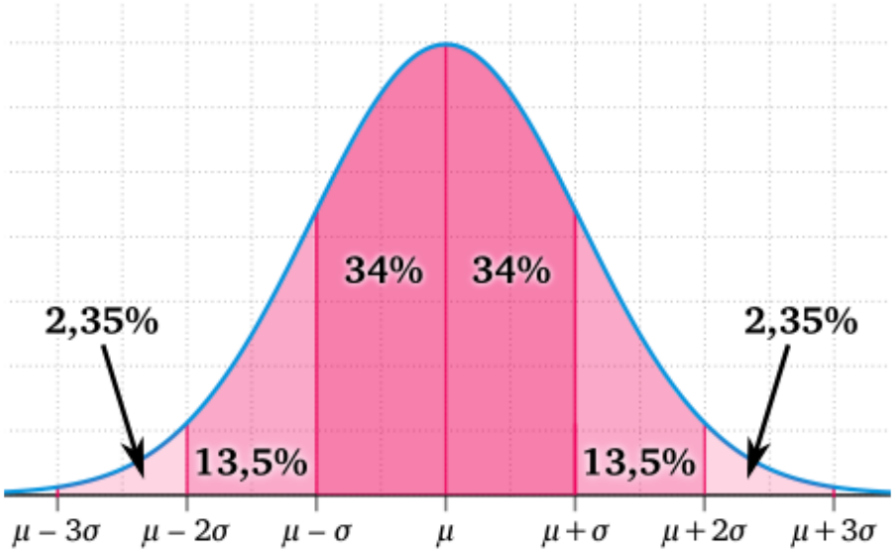
In [28]:
```
p24     = scipy.stats.binom.pmf(24,60,0.4) #
pLEQ24  = scipy.stats.binom.cdf(24,60,0.4) #

print(p24,pLEQ24)
mean, var, skew, kurt = scipy.stats.binom.stats(60, 0.4, moments='mvsk'
print(mean, var)
```

```
0.10466918336534053 0.5557755727497353
24.0 14.399999999999999
```

Questions from Notebook 2

- Difference between statistical and systematic uncertainties ?
- Interpretation of one standard deviation ?



2nd day: Parameter Estimation

Two important estimation methods

- Least squares
- Maximum likelihood

Regression

- Linear and non-linear
- Non-parametric

More concepts

- p-values
- Confidence Intervals

Inferential statistics and parameter estimation

Situation

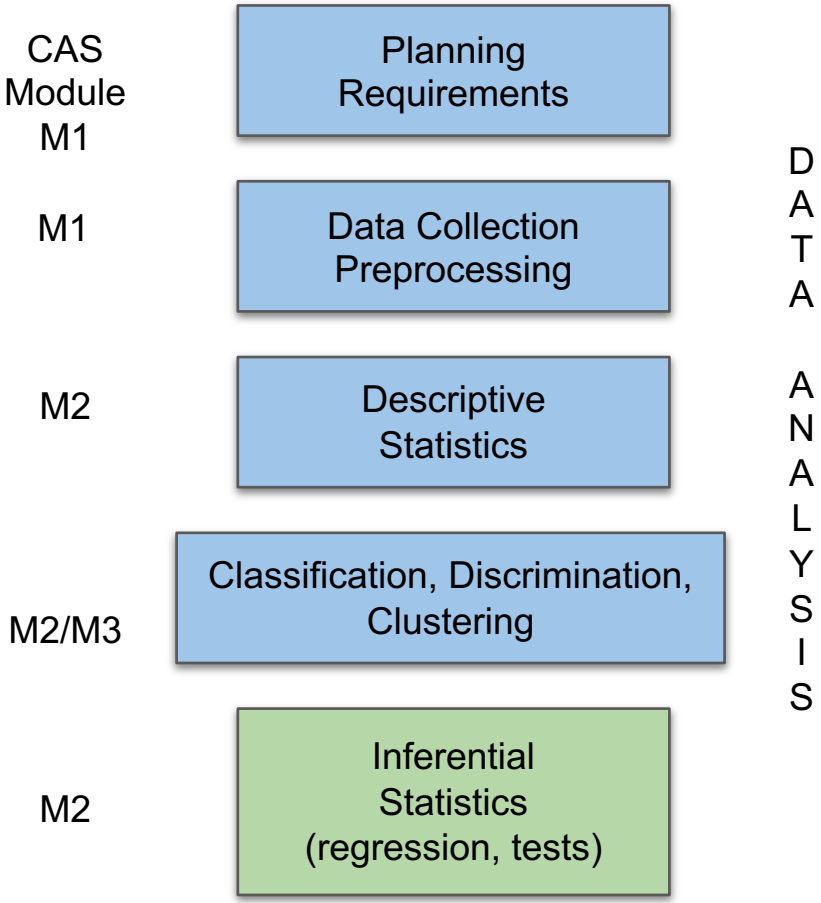
Topics in Statistics edit · view

General topics	Probability	Descriptive statistics	Inferential statistics	Specialized topics
• Levels of measurement • Sampling • Statistical survey • Design of experiments • Data analysis • Statistical graphics • History of statistics	• Probability theory • Random variable • Probability distribution • Independence • Expected value • Variance, covariance • Central limit theorem	• Averages • Statistical dispersion • Summary statistics • Skewness • Correlation • Frequency distribution • Contingency table	• Hypothesis testing • Estimator • Maximum likelihood • Bayesian inference • Non-parametric statistics • Analysis of variance • Regression models	• Computational statistics • Decision theory • Multilevel models • Multivariate statistics • Statistical process control • Survival analysis • Time series analysis

Inferential statistics

- Statistical inference is the process of using data analysis to deduce properties of an underlying probability distribution.[1]

- Inferential statistical analysis infers properties of a population, for example by testing hypotheses and deriving estimates. It is assumed that the observed data set is sampled from a larger population.

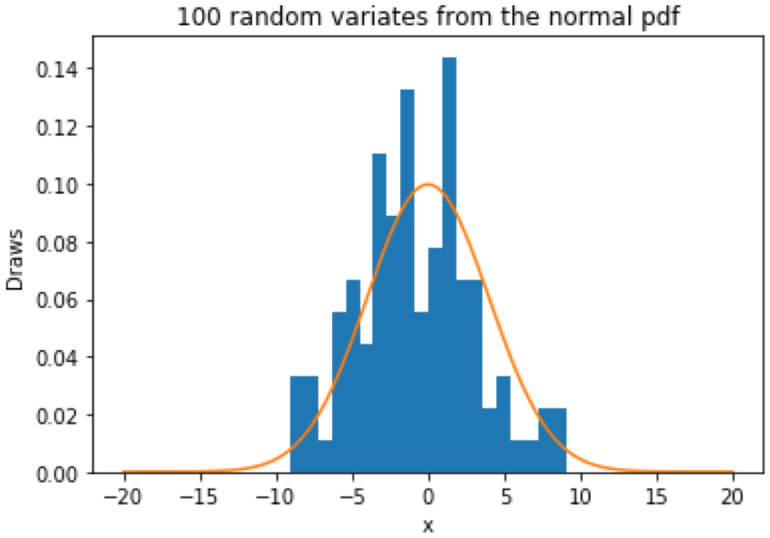


Parameter estimation

Situation

- We have data
- We have (chosen) a model describing the data (pdf or pmf)
- The model has parameters
- We want to estimate the parameters from the data

In this case the mean and the standard deviation of a normal distribution.



Example: data in blue histogram, model in orange line graph. What is the mean and the sigma of the model?

Common point estimators (from data)

Mean

- Estimator for the mean of n measured x values

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Variance (and standard dev.)

- Another estimator

$$\widehat{\sigma^2} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \widehat{\mu})^2$$

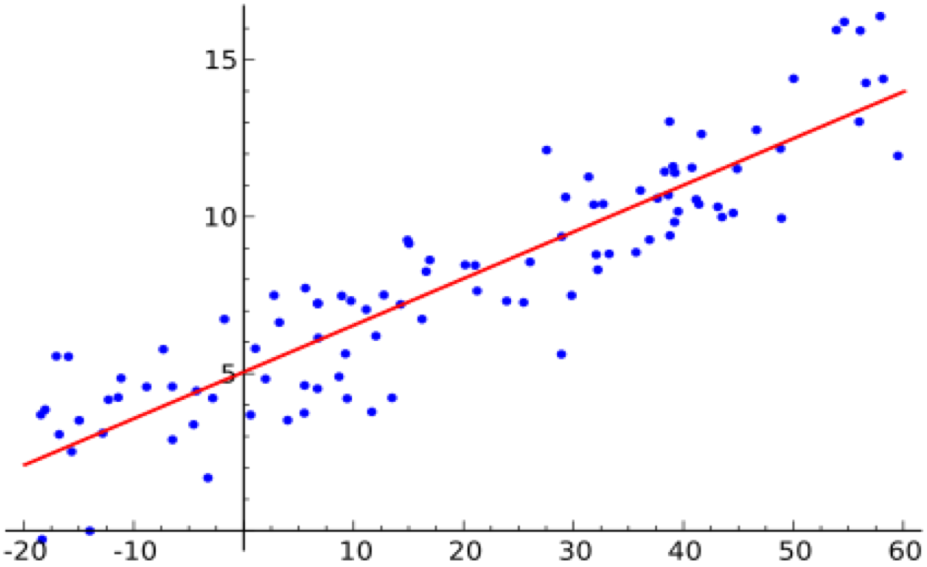
- Estimators often marked with *hat.* With Machine Learning techniques we fit many other model parameters from data.

Two important estimation methods

- Least Squares
- Maximum likelihood

Least Squares (LS) Method

- Minimise distance (residual) between data point and parameter
- Under certain conditions same as Maximum Likelihood Estimator



Maximum Likelihood (ML) Method

- The probability P(x|θ) is the probability to have gotten the data x actually obtained, given the theory (a set of parameters θ)
- The likelihood function L(θ|x) uses instead the data as input
  - Notation : L(θ) = L(θ|x)
  - It can be a simple function or something very complicated
- The ML estimator of θ maximises the likelihood function L
- For computational reasons, mostly the *–(log likelihood)* is used

$$-\frac{\partial \ln L}{\partial \theta_i} = 0\,, \qquad i = 1, \ldots, N$$

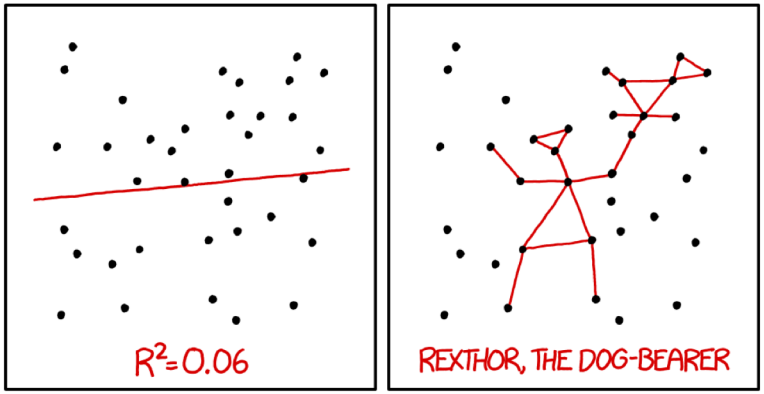
Example : Poisson distribution

- Probability Mass Function : $\mathrm{P}(k|\mu) = \exp(-\mu)\frac{\mu^k}{k!}$
- Example :
  - We fix $\mu = 20$: probability of getting 30 is

    P(30|20) = exp(-20) x 20^{30} /30! = 0.07
  - We measure k=20 : with ML, we compute $\mu = 20$!

In inferential statistics we take the measures and try to compute the mean. The mean is not fixed!

Regression



first-order, very easy, often not enough

- Linear and non-linear
- Non-parametric

Normal distributions, linear models, poisson distributions, etc. (even neural networks) are parametric models --> they have distinct parameters (mean, standard deviation, etc.).
Non-parametric models are used when you have no idea of the "shape" of the data.

parameters from data - not vice versa

Different types

Linear

- Example straight line y = ax + b
  - Fit a and b with LS or ML
- You can then predict the future (extrapolation and interpolation)
- *Linear* refers to the parameters

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$$

Non-linear

- When dependent variable (y_i) not linear in the parameters, it is called non-linear regression (obviously)

Non-parametric

- Parametric models have a shape assumed
- If we have no clue about the shape, we may use non-parametric estimations
- These normally need more data, because also the shape must be somehow estimated

Typical Machine Learning Methods 1

When using data to fit the model (pdf) parameters, we call it machine learning. We can choose among an infinite number of models/methods. Popular ones are

- Linear regression (with logistic regression for classification)
- (Boosted) Decision trees (and random forest)
- Principal Component Analysis (dimension reduction)
- Nearest neighbor methods (k-means)
- Neural Networks
- In this CAS we will practice linear regression and neural networks (Module 3)

In machine learning the main factor is that you feed the machine with data and the machine then looks for the optimal parameters. You don't tell the parameters to the machine.
An often seen problem in machine learning ist overfitting.
In machine learning we often use multiple data samples (training sets and test sets) in order to control overfitting --> so the parameters does not become to specific to just one sample

Typical Machine Learning Methods 2

Most (all?) methods typically use either Least Squares or Maximum Likelihood for finding the optimal parameters.

When the model is fitted, it can be used for hypothesis testing, i.e. future predictions or classification.

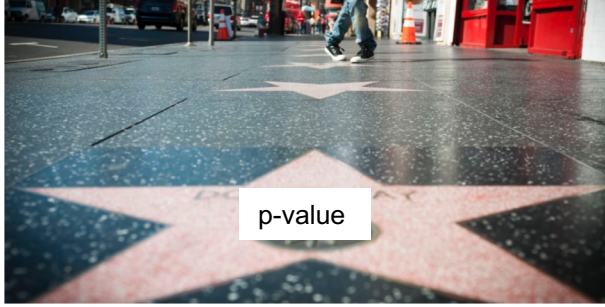
The simplest ML is fitting a straight line to some data points. The model has 2 parameters. linear regression

GPT-3 is a neural network with the capacity of 175 billion parameters (the bigges ever?) can learn languages and write "human-like" sentences. Very hard to distinguish from actual human.

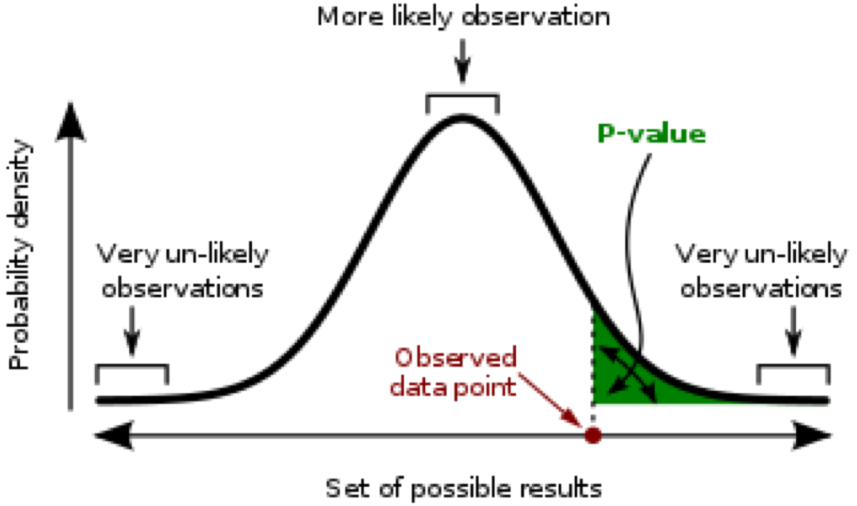
More concepts

- p-value
- Confidence Interval

p-value



- p-value is fraction of the surface above a certain data value
- Exercise (in pairs)
  - Assume a normal distribution of the age in the class.
  - Use the participants' ages to calculate the p-value of your age



A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

Confidence Intervals (CI)

- One sided and two sided

- In the normal case we have the nice relation between standard deviations and confidence levels: 2SD = CL of 95 %