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Modeling of urban road network traffic carrying capacity based on equivalent traffic flow

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ABSTRACT

The determination of the traffic carrying capacity of road network is of great significance to the scale control of urban vehicles and the control of transportation systems. Existing related works have proposed calculation models and solutions for different application environments. However, these studies have problems such as diversification of capacity definitions, incorrect analysis of factors, and excessive model constraints. In response to these problems, this paper aims to determine the connotation of traffic carrying capacity and put forward a novel calculation model for carrying capacity. Firstly, in-depth analysis found that the key factor affecting the number of vehicles is the service level of road network. Then, the connotation of traffic carrying capacity is clarified, that is, the maximum number of vehicles travelling simultaneously in the road network under a certain service level. Second, based on the definition of carrying capacity and the characterization relationship between average travel speed and service level, the calculation of carrying capacity is transformed into the establishment of relationship between average travel speed and maximum number of vehicles. The construction of this relationship includes five steps, which are proposal of equivalent lane, division of traffic basic units, construction of basic unit travel time model, calculation of basic unit carrying capacity, and calculation of road network carrying capacity. In particular, the proposal of equivalent lane achieves the conversion from intermittent flow to continuous flow, and the abstraction of traffic flow further unifies the operation process into a unified mode of free travel and stacked release. Finally, the VISSIM simulation software is employed to verify and evaluate the proposed models. The results show that the average relative error between the simulated data and the calculated data obtained from the model is -0. 46%. Findings from this study will provide an innovative idea for the evaluation of urban road network performance. It can also provide a basis for urban managers to improve service level of road network and optimize the design of transportation system.

1. Introduction

The traffic carrying capacity of urban road network reflects the road network's ability to deal with traffic demand. It plays an important role in alleviating the imbalance of traffic supply-demand and formulating urban control plans. Specifically, it cannot only be directly used for the estimation of urban car ownership and the control of road network perimeter, but also has guiding significance for the improvement of service level and the design of road infrastructure. However, the existing research related to carrying capacity

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has problems such as diversification of capacity definitions, incorrect analysis of influencing factors, excessive model constraints, and difficulty in model solving. Hence, the determination of the definition of traffic carrying capacity and the construction of its calculation model are issues worth exploring and challenging. This is of great significance for the sustainable development of modern cities.

1.1. Research background

In recent years, the research related to the traffic carrying capacity of urban road network is the vehicle accumulation in the Macroscopic Fundamental Diagram and road network capacity. Among them, the study of road network capacity is mainly divided into two categories: narrow road network capacity and generalized road network capacity.

The Macroscopic Fundamental Diagram (MFD) of urban traffic describes a relationship between network vehicle accumulation and network outflow under specific conditions (mainly homogeneity in the spatial distribution of congestion and the network topology) [1]. Vehicle accumulation in the MFD refers to the number of vehicles travelling in the road network involved in this diagram. Among them, the critical accumulation (the vehicle accumulation corresponding to the maximum flow) is the key point of road network control. It plays an important role in congestion control, signal planning, and multi-modal network analysis [2–4]. However, MFD is mainly concerned with maximizing traffic flow, and does not consider whether the adjusted number of vehicles matches the actual travel demand. In other words, when the travel demand is restricted and cannot be adjusted to the critical accumulation, the MFD is ineffective in this case.

Road network capacity refers to the maximum number of vehicles that the road network can handle or accommodate in a certain period [5–10]. Among them, this paper summarizes the concepts of studying the number of vehicles in the local area into the narrow road network capacity. What they have in common is to calculate the maximum number of vehicles passing through critical sections of road network. Correspondingly, the concepts of studying the overall carrying performance of network is summarized as the generalized road network capacity. The characteristic of this type is to broaden the research scope of the narrow road network capacity, and calculate the maximum number of vehicles that the entire road network can accommodate. Next, we introduce them separately.

The narrow road network capacity includes two classes: traditional class and derived class. The traditional class is to study the maximum number of vehicles passing through the road section under different traffic environments [5–6]. The derived class (also known as reserve capacity) is based on the traditional one, from the perspective of signal control, to explore how much traffic capacity is left in the road network [7–8]. The two classes of control ideas are the same, and both maximize the traffic capacity of the road network through the reasonable allocation of OD demand. It emphasizes the throughput of the road network. Relevant papers on the narrow road network capacity display the solving algorithms from different perspectives, but there are problems such as strict model constraints and inaccurate solutions.

Generalized road network capacity is classified as space-time consumption type [9] and supply analysis type [10]. The former is to explore the maximum capacity of the road network based on the utilization rate of time-space resources. This type is simple and easy to understand, but the results are affected by the survey period. Different traffic conditions during the survey period will result in different results. In turn, the results deviate from the true value, which has an impact on subsequent management and control (such as perimeter control). The latter is a variant of space-time consumption method, which is to determine the capacity from the perspective of road supply. Although this type can provide a macroscopic reference for supply and demand research, there are many parameters in the model that need to be calibrated by field investigation. So it is difficult to obtain the road network design plan with the optimal carrying efficiency before the road network is put into operation.

1.2. Proposal of traffic carrying capacity and research motivation

By summarizing the pros and cons of vehicle accumulation in the MFD and road network capacity, we can conclude: The function of vehicle accumulation in MFD is to determine the critical value corresponding to the maximum throughput, without considering the problem that the actual travel demand cannot match the adjusted flow. Road network capacity makes up for this shortcoming and realizes the maximization of the number of vehicles from different aspects. However, there are still problems such as incorrect analysis of influencing factors [9], inaccurate solutions of the model [6, 8], and difficulty in parameter calibration [9–10]. In particular, there has been no progress on the challenge of what is the key factor that affects the generalized road network capacity. Existing studies still think that the difference in the calculation period is the inducement.

This paper first conducts an in-depth analysis of the factors that affect the number of vehicles in the road network. It is found that the key factor is not the difference in the calculation/survey period, but the change of traffic condition during that period [9] (see Section 2. 2. 2 for details). However, the traffic condition is a relatively general concept, which cannot be directly used for quantitative description in the actual environment. So next, it is necessary to further analyze which characteristic indicators can reflect the traffic condition, and then the characteristic variables that affect the number of vehicles in the road network are clarified. In the urban road network, signal timing, road channelization, traffic control strategies, etc. will all affect the operation of traffic flow, which will eventually show up as the difference in the average travel speed of vehicle. The average travel speed directly influences the distance between vehicles, which in turn affects the number of vehicles. So the number of vehicles in the road network is not constant, but changes with speed. It can be seen that the average travel speed is the critical indicator that reflects the traffic condition or the situation of the number of vehicles in the road network. Moreover, the average travel speed essentially represents the service level of road network. In summary, the service level of road network is the key characteristic variable that affects the number of vehicles in the road network.

Based on the above analysis, this paper proposes a novel index for evaluating road network performance, which is called traffic carrying capacity. It refers to the maximum number of vehicles travelling simultaneously in the road network under a certain service level. It should be noted that:

(i) When the road network topology and infrastructure are provided, carrying capacity is uniquely determined by the traffic condition (service level);
(ii) Traffic carrying capacity is a variable independent of the time dimension, i. e. when the calculation period changes but the traffic condition does not change, it remains invariant;
(iii) Traffic carrying capacity is the intrinsic property of the network itself, emphasizing the ability of the road network to carry vehicles at the same time.

After determining the definition of traffic carrying capacity, a new model needs to be established to calculate it. This paper first considers the influence of the discontinuity of traffic flow on the calculation, and proposes the concept of equivalent lane. Next, we analyze the structure of the road network and divide the road network into multiple traffic basic units. Then, by abstracting the traveling process of traffic flow, the travel time model of basic unit is constructed and carrying capacity of basic unit is calculated. Finally, road network carrying capacity is obtained considering the relationship between basic unit and road network, as well as the influence of network homogeneity. To the best of our knowledge, this study makes the first attempt for the carrying capacity calculation utilizing equivalent-abstract idea.

To sum up, the contributions of this study are five-fold:

- A novel calculation model for traffic carrying capacity of urban road network is put forward.
- The model utilizes the ideas of lane equivalence and traffic flow abstraction, while considering the comprehensive influence of signal timing and speed.
- The model overcomes the problems of complex model parameters and difficult calibration in the past, and proposes a practical and easy-to-operate calculation process.
- The model gives impressive results in the multi-perspective simulation experiments.
- The model has good application value in the improvement of road network service level and the optimization of urban traffic systems.

The remainder of this paper is organized as follows. Section 2 overviews the related work of traffic carrying capacity. Section 3 develops the model of traffic carrying capacity based on the idea of lane equivalence and traffic flow abstraction. Section 4 provides case studies and model comparisons from different perspectives to verify the great performance of proposed model. Section 5 concludes the presented work.

2. Literature review

The research related to the traffic carrying capacity is the vehicle accumulation in the MFD and road network capacity. Among them, the study of road network capacity is mainly divided into two categories: narrow road network capacity and generalized road network capacity. Next, we will introduce them respectively.

2.1. Vehicle accumulation in the MFD

There is a long research history concerning the MFD. The first idea came from Godfrey [11] and was further verified in empirical and simulation-based data. Ardekani and Herman [12] put forward the steady-state relations between the number of vehicles and vehicle speeds / flows. Later, Daganzo [13] re-initiated the concept of macroscopic traffic flow analysis. In particular, Daganzo and Geroliminis [1] employed variational theory to derive the MFD with network properties (link capacity, average link length, etc.) and found it to be a well-defined and reproducible curve. Subsequently, a number of studies have proved the existence of MFD [14–15]. Buisson and Ladier [16] utilized data from loop detectors in Toulouse (France), to investigate the influence of heterogeneous conditions on MFD. They found that heterogeneity factors contributing to hysteresis in the MFD curve included: the distance between the detector and the traffic signal, the location of onset and offset congestion, and the type of roads (urban and highway). Further works studied the influence of loop detectors on the shape of the MFD [17].

Since then, people have increasingly realized that MFD may have important value for the control and planning of urban road networks. Ji and Geroliminis [18] focused on relaxing the binding conjecture about the existence of global MFD at the network level, and solved the problem of decomposing the road network into sub-networks by the clustering algorithm, thereby reducing the heterogeneity in road congestion. Ramezani et al. [19] proposed two aggregated models, region- and subregion-based MFDs, to study the dynamics of heterogeneity. They determined how heterogeneity affects the accuracy scatter and hysteresis of the multi-subregion MFD model. In addition, there is also numerous applications regarding signal plan [3,20], multi-modal network analysis [4,21], congestion control [2,22].

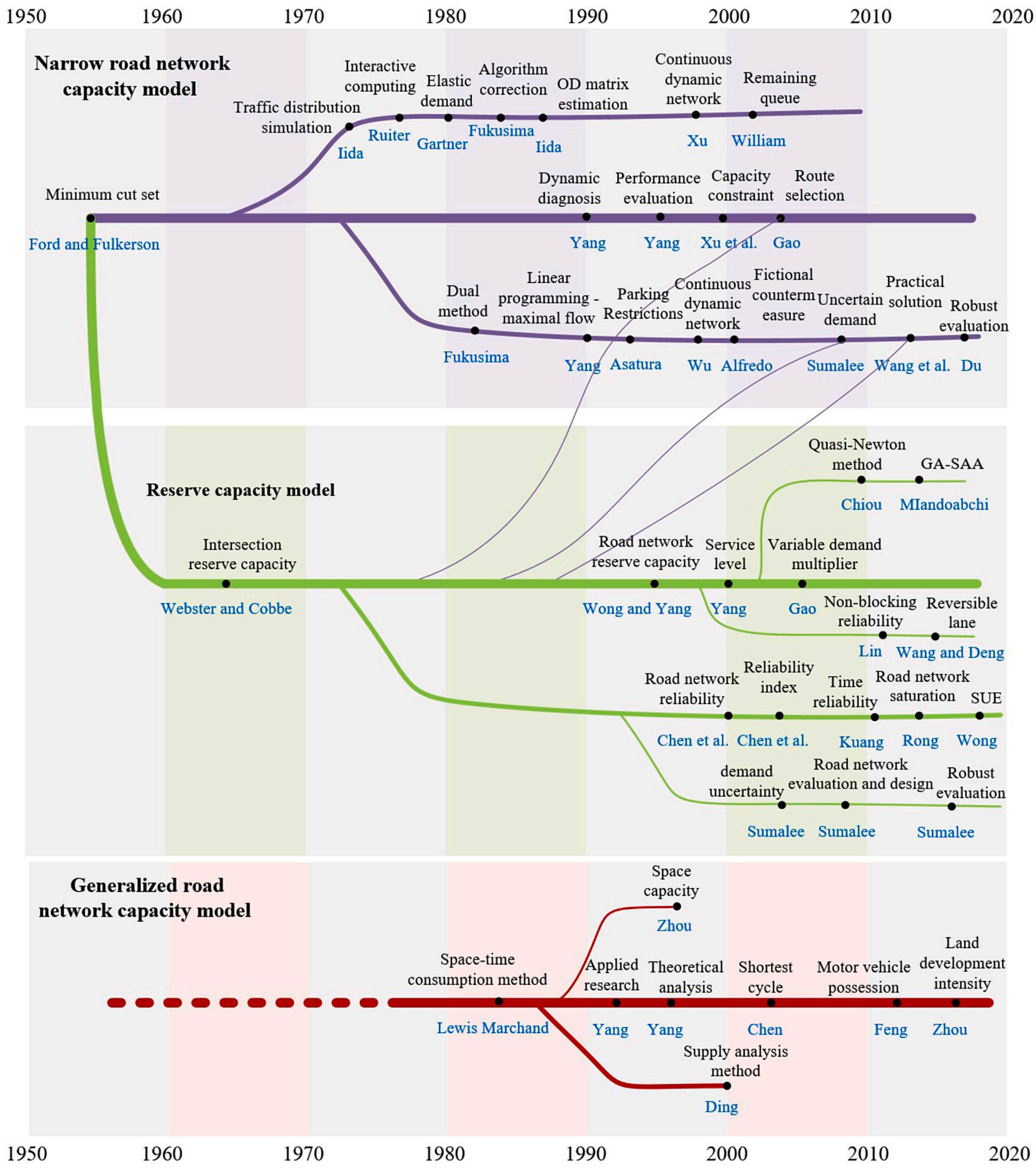


Fig. 1. Research history of road network capacity. .

2.2. Road network capacity

2.2.1. Models of narrow road network capacity

The earliest algorithm of narrow road network capacity is the minimum cut set method [5], which calculates the maximum traffic volume when the road network reaches saturation. It abstracts the road network into an ideal graph with the single starting point and ending point, but the maximum traffic flow solved is difficult to match the actual road network. Therefore, some scholars have improved the algorithm and proposed the method suitable for the real road network called traffic distribution simulation [6, 23]. The key of this method is also the process of finding the minimum cut set of road network. It utilizes a balanced network model for traffic distribution and the results are direct and reliable. The disadvantage is that the origins and destinations of traffic area are hard to obtain.

With the deepening of study, researchers put forward the linear programming method [24]. The linear programming method contains two types: single-level and bi-level. The former is essentially the problem of calculating the maximum flow of multiple origin-destination network [25]. The latter consists of upper and lower models and the rationality of algorithm is affected by the form of objective function and distribution model [26]. Meantime, non-linear mathematical programming method with balance constraint is also applied to determine signal settings and link capacity expansion [27]. Overall, linear/non-linear programming takes into account the balance between system and individual, with clear modeling objectives and constraints. However, it is based on the pre-selection of route, without considering the randomness of the individual route selection. The function is also a non-convex model and there is no exact solution.

In the wake of extension of road network capacity, reserve capacity appears as an important indicator in people's vision. The reserve capacity aims to study the road network capacity from the perspective of signal control and is calculated through the bi-level programming method in the form of the demand multiplier [7]. Existing papers have been carried out from two aspects: Deterministic User Equilibrium [28–29] and Stochastic User Equilibrium [30–31]. The detailed works are shown in Fig. 1.

2.2.2. Models of generalized road network capacity

The classic generalized road network capacity model is Space-time consumption method [9]. Specifically, road network capacity defined by this method refers to the maximum number of traffic individuals that can be served in the container of urban road network with time-space property. The calculated capacity is the ratio of total space-time resources to space-time resources occupied by traffic individuals. The formula is:

$$C_{ts} = \frac{C_{total}}{C_{indiv}} = \frac{\int_{t_0}^{t_1} L_{network} \cdot w_{network} dt}{\int_{t_2}^{t_3} L_{veh} \cdot w_{veh} dt} = \frac{S_{effective} \cdot T_{effective}}{d_{travel} \cdot w_{dynamic} \cdot \bar{t}} \tag{1}$$

where C_{ts} is road network capacity; C_{total} is total space-time resources of road network; C_{indiv} is average time-space consumption of traffic individuals in one trip. $L_{network}$ and $w_{network}$ are total length and width of road network, L_{veh} and w_{veh} are length and width of road network occupied by traffic individuals. $S_{effective}$ and $T_{effective}$ are effective area and operational time of road network; d_{travel} is space headway between traffic individuals during travel; $w_{dynamic}$ is dynamic width of vehicle while traveling; $\bar{t}$ is average travel time of traffic individuals in unit time.

This model is simple and the relevant parameters are measurable. Among them, the special variable $\bar{t}$ has a significant effect. During different travel periods, average travel time of traffic individuals will be affected by various factors (vehicle conflicts, traffic accidents, etc.). These factors all affect the calculation of road network capacity. Thus, slight differences in calculation/survey period will lead to changes in the road network capacity. That is to say, the road network capacity calculated by these variables is not the authentic maximum number of vehicles travelling in the road network, but the surveyed value in actual traffic environment during a specific period. In short, maximum number of vehicles travelling in the road network is independent of average travel time of traffic individual. The change in the number of vehicles is not due to the difference in calculation period, but the difference of traffic condition in the calculation period.

The supply analysis method [10] is a variant of space-time consumption method. It analyzes road network capacity from the perspective of road supply and calculates the maximum number of kilometers that the road network can provide within one hour in the case of unbalanced traffic volume. However, the results can only be used as a macroscopic reference for supply and demand research. The problems are that there are too many parameters to be calibrated and parameters are difficult to measure. With the deepening of research, scholars are committed to correcting the problems in the space-time consumption model [32]. In addition, some scholars have conducted research in the application areas of road network capacity, such as the prediction of motor vehicle possession [33] and the allocation of parking resources [34].

3. Methodology

Notations:

N- set of signalized intersections in the road network, $N = \{1, 2, 3\cdots, \mathrm{I}\}$;
R- set of links in the road network, $R = \{ij, \cdots\}$, ij represents the link from i to j,$ij \in R$,$j = i + 1$;
B- set of traffic basic units in the road network, including the upstream road travel area R_{ij} and the downstream intersection travel area R_j,$B = \{1, 2, 3\cdots, \mathrm{u}\}$, $B^u = \{R_{ij}^u, R_j^u\}$;
C_n- cycle of signalized intersection n, $n \in N$;
P_n- phase sequence of signalized intersection n, $n \in N$;
g_{pn}- green light time for phase$p$at signalized intersection n, $p \in P_n$, $n \in N$;
λ_{pn}- split for phase$p$at signalized intersection n, $p \in P_n$, $n \in N$;
λ- the vector of all splits, $\lambda = \{\lambda_{pn}, p \in P_n, n \in N\}$;
$m_{pn,ij}$- number of lanes allocated for g_{pn} on link ij, among them, $m_{pn,ij}^o$ represents the number of original lanes, $m_{pn,ij}^e$ represents the number of equivalent lanes, $m_{pn,ij}^{o,u}$ represents the number of original lanes of uth traffic basic unit, $m_{pn,ij}^{e,u}$ represents the number of equivalent lanes of uth traffic basic unit;

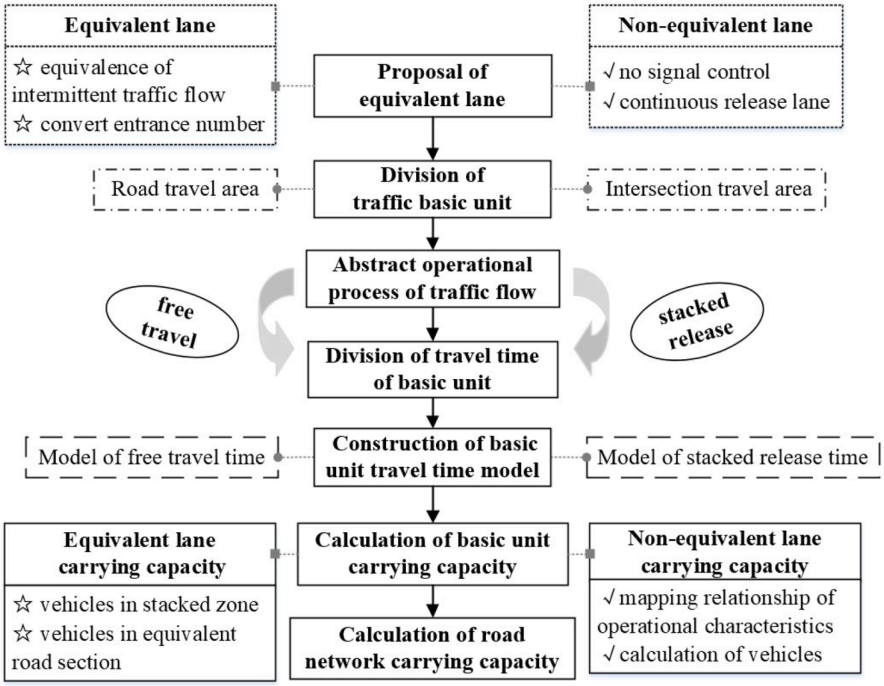


Fig. 2. Overall framework of modeling. .

$\mathbf{m}^o$- the vector of original lane numbers;
$\mathbf{m}^e$- the vector of equivalent lane numbers;
$s_{pn,ij}$- saturated flow rate corresponding to $m_{pn,ij}$;
T^u- the time required for the vehicle to pass through the uth traffic basic unit;
L^u- the length of uth traffic basic unit;
L^u_{free}- the distance traveled at the free-flow speed on the uth traffic basic unit;
$\overline{V}^u_{\text{expected}}$- expected average travel speed of uth traffic basic unit; among them, the average travel speed is the average value of the instantaneous speeds of all vehicles on the road at a certain moment;
$\overline{V}_{\text{expected}}$- expected average travel speed of road network;
V^u_{free}- free-flow speed of uth traffic basic unit;
A^u- traffic carrying capacity of uth traffic basic unit in the road network, $u \in B$;
A- traffic carrying capacity of road network, $A = \{A^u, u \in B\}$;
$\overline{h}^u_{pn,ij}$- average saturated time headway of uth basic unit, $p \in P_n$, $n \in N$;
$\xi_k(\varphi)$- traffic proportion of kth moving direction during period φ,$k = \{right, through, left\}$, $\xi_{right}$is right-turn direction, $\xi_{through}$ is through direction, ξ_{left} is left-turn direction.

After clarifying the definition of traffic carrying capacity, it is necessary to consider how to construct a carrying capacity calculation model. Due to the intermittent traveling of vehicles, it is difficult to establish the corresponding relationship between number of vehicles and level of service directly. Therefore, we specially put forward the concept of equivalent lane, which converts discontinuous flow in the road network into continuous flow. Then, by analyzing the composition of road network, the basic unit of road network is determined and the minimum change area of road network carrying capacity is cleared. However, we found that traffic blocks still occur in the equivalent basic unit. For tackling this problem, the vehicle traveling process is abstracted into a "free travel - stacked release" mode. Based on the travel time model of basic unit, the carrying capacity models of equivalent lane and non-equivalent lane are established, and then the carrying capacity of basic unit is got. Finally, road network carrying capacity is obtained considering the relationship between basic unit and road network, as well as the influence of network homogeneity (Fig. 2).

It should be noted here that the average travel speed is a quantitative, continuous indicator that can reflect the service level. No matter how the service level is graded in different guidelines, as long as the one-to-one correspondence between number of vehicles and average travel speed is established, the mapping relationship between service level and number of vehicles can be obtained. Therefore, it is equivalent to study the functional relationship between average travel speed and number of vehicles when modeling carrying capacity.

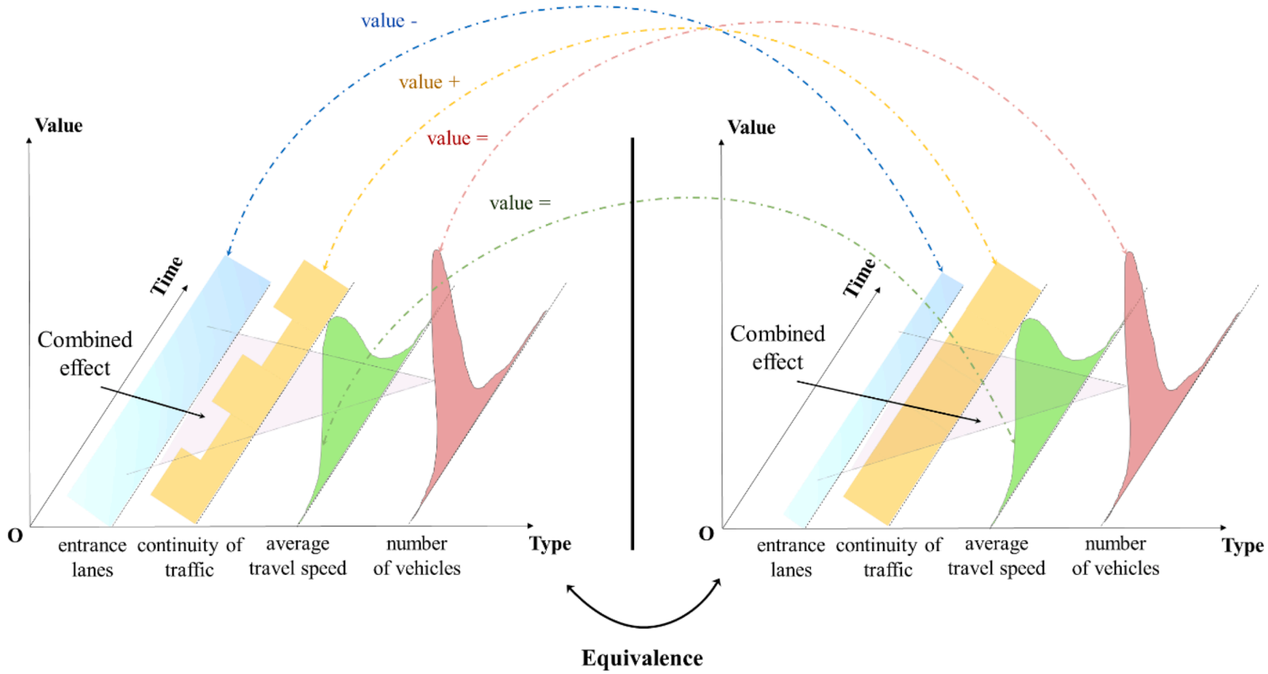


Fig. 3. Schematic diagram of equivalent lane. .

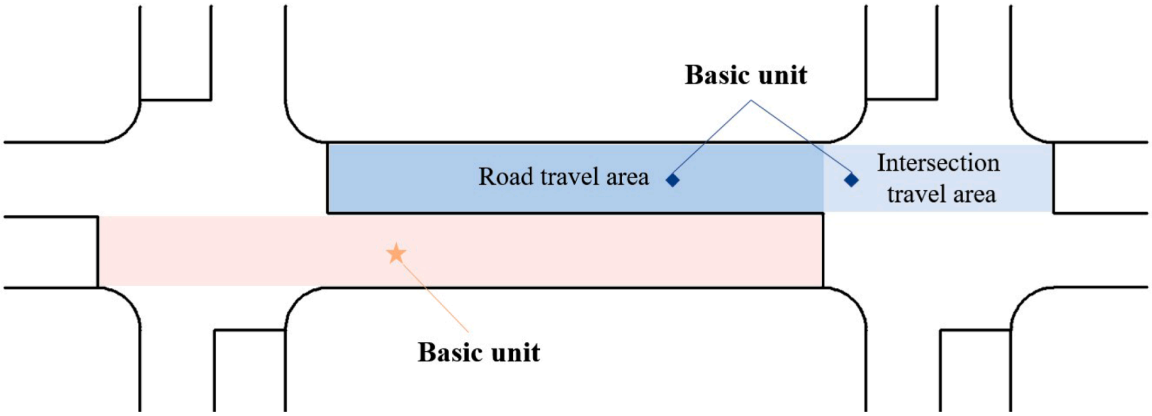


Fig. 4. Schematic diagram of traffic basic unit. .

3.1. Proposal of equivalent lane

Traffic flow in the urban link displays inherently stop-and-go behavior caused by traffic signals. As a consequence, the number of vehicles in the road network may dynamically change due to the influence of traffic flow and signal timing, making it challenging to directly calculate the carrying capacity. It is more difficult to establish the relationship between average travel speed and carrying capacity. To tackle with this problem, an innovative concept - equivalent lane - is proposed.

Equivalent lane is defined as a new type of entrance lane, on which the traffic flow is originally intermittent and then converted to continuous release flow. Its conversion is determined by the original number of entrances and green split at the intersection. The key idea of equivalent lane is to realize the transition from discontinuity to continuity of traffic flow under the premise of remaining the number of vehicles unchanged. Through this conversion process, on the basis of ensuring the theoretical correctness of equivalence, the complexity of calculation of carrying capacity is greatly simplified.

The specific idea of equivalent process is: When the traffic flow is unsaturated release at the intersection, there is still remaining space on the road that can be used by more vehicles while keeping the original travel speed of vehicle unchanged. That is, the maximum number of vehicles in the road network has not been reached under this condition. In other words, the road network carrying capacity can be obtained only when the vehicles are in the state of saturation release. Hence, based on the principle of conservation of traffic flow, the number of vehicles released at saturation condition during the green light is equal to the number of vehicles continuously saturated to release on the equivalent lane in that cycle. The formulas for the number of equivalent lanes can be expressed as Eq. (2) - (4). The change of related parameters before and after equivalence is shown in Fig. 3 [35]. In contrast, for the continuous release lane without signal control (such as, the right-turn lane in signalized intersection, or the lanes in non-signalized intersection), it is called non-equivalent lane.

$$s_{pn,ij} \cdot g_{pn} \cdot m^{o}_{pn,ij} = s_{pn,ij} \cdot C_n \cdot m^{e}_{pn,ij} \tag{2}$$

$$m^{e}_{pn,ij} = m^{o}_{pn,ij} \cdot \frac{g_{pn}}{C_n} = m^{o}_{pn,ij} \cdot \lambda_{pn} \tag{3}$$

$$\mathbf{m^e} = \mathbf{m^o} \cdot \boldsymbol{\lambda}. \tag{4}$$

It should be noted that equivalent lane only reduces the number of entrance lanes that remain open at the intersection, rather than reducing the number of lanes on the road section (see the left sub-graph in Fig. 5). Therefore, the original storage capacity of the road will not change, that is, it does not affect the maximum of queued vehicles that the road can accommodate. The validity and correctness of equivalent lane have been preliminarily proved in the authors' previous article [35].

3.2. Division of traffic basic unit

The purpose of this paper is to calculate the carrying capacity of entire road network, so it is necessary to analyze the composition of road network. Searching for the smallest component of the road network, we found that the road network consists of various links. Among them, each link is the section from the upstream intersection stop-line to the downstream intersection stop-line. We call this link the traffic basic unit, including road travel area and intersection travel area (Fig. 4).

Then, combined with the lane equivalence features provided in the previous section, the road network composed of continuous traffic flow is divided into multiple basic traffic units with the common attributes. Hence, the total number of vehicles in the road network is composed of the number of vehicles in each basic unit. In other words, the calculation of road network carrying capacity is transformed into two steps: first calculate the carrying capacity of each basic unit, and then calculate the relationship between the basic unit carrying capacity and the road network carrying capacity.

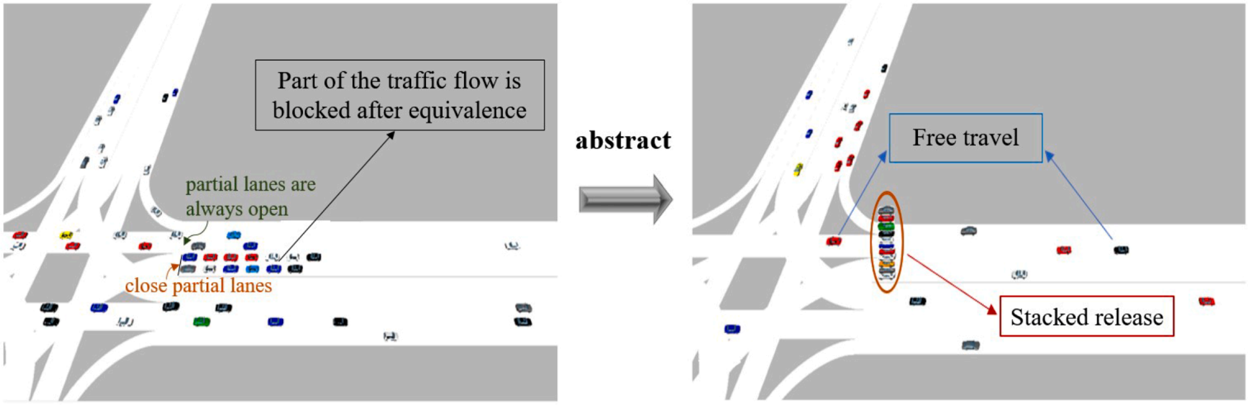


Fig. 5. Abstract process of traffic flow. .

3.3. Construction of basic unit travel time model

Before calculating the carrying capacity of basic unit, we found that when the traffic volume is large, some vehicles will be blocked at the intersection. This is because after the lane equivalence, the number of new entrance lanes is less than the number of lanes on the road section (Fig. 3). To calculate the total number of vehicles containing blocked traffic flow, we propose an idea of abstracting vehicle operation. Based on this abstraction, the traveling process of vehicle in the basic unit includes two parts, namely the free travel (free travel on road section and intersection internal road) and stacked release (saturated release of stacked vehicles in front of stop-line) (Fig. 5). The reason why this abstraction can be implemented is that no matter what speed the blocked vehicles move on the road or in what form they queue to pass through the intersection, as long as the total travel time remains unchanged, traffic flow can always be equivalent to the process of "free travel - stacked release". This ensures the reliability and accuracy of abstraction.

Corresponding to the abstract process, the total travel time of basic unit T^u includes free travel time T^u_{free} and stacked release time T^u_{stacked}. Based on the given average travel speed and free-flow speed, the total travel time and free travel time can be calculated respectively, and then the time relationship between different parts of basic unit is utilized to obtain the stacked release time. Then travel time calculation models of different parts are constructed. The formulas are shown in Eqs. (5)-(7). The division of basic unit simplifies the calculation of road network carrying capacity and the travel time models of different parts put the foundation for the calculation of carrying capacity.

$$T^u = \frac{L^u}{\overline{V}^u_{\text{expected}}} \tag{5}$$

$$T^u_{free} = \frac{L^u_{free}}{V^u_{free}} \tag{6}$$

$$T^u_{\text{stacked}} = T^u - T^u_{free} = \frac{L^u}{\overline{V}^u_{expected}} - \frac{L^u_{free}}{V^u_{free}} \tag{7}$$

3.4. Calculation of basic unit carrying capacity

Combining the above series of conversion processes, the lanes of basic unit include two types: equivalent lane and non-equivalent lane. Therefore, the carrying capacity of basic unit is the sum of the capacities of these two parts. The specific formula is:

$$A^u = A^u_{equivalent} + A^u_{non-equivalent} \tag{8}$$

where $A^u_{equivalent}$ represents the equivalent lane carrying capacity of the uth basic unit, and $A^u_{non-equivalent}$ represents the non-equivalent lane carrying capacity of the uth basic unit.

3.4.1. Calculation of equivalent lane carrying capacity

Regarding equivalent lane, based on the classification of basic unit and the abstraction of traffic flow, the carrying capacity of this part is the sum of maximum number of vehicles travelling on the stacked areas and equivalent road sections. The maximum number of vehicles in the stacked area is obtained according to stacked release time and average saturated time headway; the maximum number of vehicles on the equivalent road section is obtained according to lane length and desired space headway. Based on the above, the equivalent lane carrying capacity can be expressed as:

$$A^u_{equivalent} = A^{u,stacked}_{equivalent} + A^{u,free}_{equivalent} = \sum_{pn \in P_u} \left[\left(\frac{T^u_{\text{stacked}}}{\overline{h}^u_{pn,ij}} + \frac{L^u_{free}}{d^u_{free}} \right) \cdot m^{e,u}_{pn,ij} \right] \tag{9}$$

where $A^{u,stacked}_{equivalent}$ represents the carrying capacity of stacked area of the uth basic unit, and $A^{u,free}_{equivalent}$ represents the carrying capacity of equivalent road section of the uth basic unit. $\overline{h}^u_{pn,ij}$ is the average saturated time headway of uth basic unit and d^u_{free} is the desired space headway of uth basic unit. P_u represents the total number of phases contained in the uth basic unit.

Among them, the average saturated time headway and the desired space headway are calculated according to following research.

(1) Average saturated time headway

After the traffic flow abstraction and the lane equivalence, vehicles on the virtual road can travel continuously. Aiming at this operation mode of traffic flow, the bivariate distribution model of speed and headway proposed in [36] is suitable to determine time headway. The joint probability density function (PDF) of the distribution can be described as follows:

$$f\left(V_{expected}, h^u_{pn,ij} | \gamma\right) = f_{V_{expected}}\left(V_{expected}\right) f^u_{h_{pn,ij}}\left(h^u_{pn,ij}\right) \left[1 + \gamma\left(2F_{V_{expected}}\left(V_{expected}\right) - 1\right)\left(2F_{h^u_{pn,ij}}\left(h^u_{pn,ij}\right) - 1\right)\right] \tag{10}$$

where $f(V_{\text{expected}}, h^u_{\text{pn},ij}|\gamma)$ represents the PDF of a bivariate distribution of speed and headway, and $|\gamma| \leq 1$ is a scalar coefficient. $f_{V_{\text{expected}}}(V_{\text{expected}})$and $f_{h^u_{pn,ij}}(h^u_{pn,ij})$ are the marginal PDFs of speed and headway. $F_{V_{\text{expected}}}(V_{\text{expected}})$ and $F_{h^u_{pn,ij}}(h^u_{\text{pn},ij})$ are the marginal cumulative distribution functions of speed and headway.

The Pearson's product–moment correlation coefficient ρ is defined as:

$$\begin{aligned} \rho &= \frac{Cov\left(V_{\text{expected}}, h^u_{pn,ij}\right)}{\sigma_{V_{\text{expected}}} \sigma_{h^u_{pn,ij}}} \\ &= \frac{w}{\sigma_{V_{expected}} \sigma_{h^u_{pn,ij}}} \int V_{expected}\left[2F_{V_{expected}}\left(V_{expected}\right) - 1\right] f_{V_{expected}}\left(V_{expected}\right) dV_{expected} \int h^u_{pn,ij}\left[2F_{h^u_{pn,ij}}\left(h^u_{pn,ij}\right) - 1\right] f_{h^u_{pn,ij}}\left(h^u_{pn,ij}\right) dh^u_{pn,ij} \end{aligned} \tag{11}$$

where $Cov(V_{\text{expected}}, h^u_{\text{pn},ij})$ is covariance function, $\sigma_{V_{\text{expected}}}$ and $\sigma_{h^u_{pn,ij}}$ are the standard deviations of variables V_{expected} and $h^u_{\text{pn},ij}$.

From Eq. (11) and the fact that$|w| \leq 1$, a bivariate FGM distribution is hence limited to the following.

$$|\rho| \leq \frac{1}{\sigma_{V_{expected}} \sigma_{h^u_{pn,ij}}} \int V_{expected}\left[2F_{V_{expected}}\left(V_{expected}\right) - 1\right] f_{V_{expected}}\left(V_{expected}\right) dV_{expected} \int h^u_{pn,ij}\left[2F_{h^u_{pn,ij}}\left(h^u_{pn,ij}\right) - 1\right] f_{h^u_{pn,ij}}\left(h^u_{pn,ij}\right) dh^u_{pn,ij} \tag{12}$$

After the bivariate distribution is constructed, the distribution of time headway can be determined when the speed value is given. Then the average saturated time headway can be obtained by using the expected value of time headway distribution, $\overline{h}^u_{\text{pn},ij}(V_{\text{expected}}) = E[h^u_{\text{pn},ij}(V_{\text{expected}})]$. The specific process is that if the marginal speed distributions are normal, skew-normal and skew-t mixture models, respectively, then the corresponding marginal headway distributions will be gamma, log-normal and log-logistic models. The three headway models are briefly expressed as Eq. (13), (15) and (17), and mathematical expectations are Eq. (14), (16) and (18).

$$f\left(h^u_{pn,ij} | \alpha, \beta\right) = \frac{{h^u_{pn,ij}}^{(\alpha-1)} e^{(-x/\beta)}}{\Gamma(\alpha)\beta^\alpha} \tag{13}$$

$$E\left(h^u_{pn,ij}\right) = \alpha\beta \tag{14}$$

$$f\left(h^u_{pn,ij} | \mu, \sigma^2\right) = \frac{1}{h^u_{pn,ij}\sqrt{2\pi\sigma^2}} e^{-\frac{\left[\ln\left(h^u_{pn,ij}\right) - \mu\right]^2}{2\sigma^2}} \tag{15}$$

$$E\left(h^u_{pn,ij}\right) = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{16}$$

$$f\left(h^u_{pn,ij} \middle| \alpha, \beta\right) = \frac{(\alpha/\beta)\left(h^u_{pn,ij}/\beta\right)^{\alpha-1}}{\left[1 + \left(h^u_{pn,ij}/\beta\right)^\alpha\right]^2} \tag{17}$$

$$E\left(h^u_{pn,ij}\right) = \frac{\beta\pi/\alpha}{\sin(\pi/\alpha)} \tag{18}$$

Where α is the shape parameter and β is the scale parameter, $\Gamma(\alpha)$ is the gamma function; μ and σ are the mean and standard deviations, respectively.

(1) Desired space headway

The desired space headway can be calculated directly using the intelligent driver model (IDM) [37]. The IDM assumed that each driver had a desired space headway, travel speed, and time headway, and used this as the goal to drive on the road. In this paper, the formula of space headway in accordance with the free-flow speed is as follows:

$$d^u_{free}\left(V^u_{free}, \Delta v\right) = d_0 + d_1\sqrt{\frac{V^u_{free}}{V_o}} + T_{safe}V^u_{free} + \frac{V^u_{free}\Delta v}{2\sqrt{ab}} + \bar{l}_v. \tag{19}$$

In the equilibrium state, the desired space headway is:

$$d^u_{free}\left(V^u_{free}, \Delta v\right) = d^u_{free}\left(V^u_{free}, 0\right)\left[1 - \left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} = \left(d_0 + V^u_{free}T_{safe}\right)\left[1 - \left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} + \bar{l}_v \tag{20}$$

where minimum safety distance d_0 is 2 m, jam distanced_1 is 0 m, safety time headway T_{safe} is 1. 6 s, limited maximum speed V_o is 90 km/h, acceleration exponent δ is 4, average vehicle length $\bar{l}_v$ is 5 m. a is maximum acceleration and $b$is desired deceleration.

Based on the analysis of $\bar{h}_{pn,ij}$ and d^u_{free}, the formula (9) can be described as:

$$\begin{aligned}
A^u_{equivalent} &= A^{u,stacked}_{equivalent} + A^{u,free}_{equivalent} = \sum_{pn\in P_u}\left[\left(\frac{T^u_{\text{stacked}}}{\bar{h}^u_{pn,ij}} + \frac{L^u_{free}}{d^u_{free}}\right)\cdot m^{e,u}_{pn,ij}\right] \\
&= \sum_{pn\in P_u}\left\{\left[\frac{\frac{L^u}{\overline{V}^u_{\text{expected}}} - \frac{L^u_{free}}{V^u_{free}}}{E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]} + \frac{L^u_{free}}{\left(}d_0 + V^u_{free}T_{safe}\right)\left[1 - \left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} + \bar{l}_v\right]\cdot m^{e,u}_{pn,ij}\right\} \\
&= \sum_{pn\in P_u}\left\{\left[\frac{L^uV^u_{free} - L^u_{free}\overline{V}^u_{\text{expected}}}{V^u_{free}\cdot\overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]} + \frac{L^u_{free}}{\left(d_0 + V^u_{free}T_{safe}\right)\left[1 - \left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} + \bar{l}_v}\right]\cdot m^{e,u}_{pn,ij}\right\}
\end{aligned} \tag{21}$$

3.4.2. Calculation of non-equivalent lane carrying capacity

For the non-equivalent lane, the carrying capacity is obtained by calculating the proportional relationship between the traffic in equivalent lane and the traffic in non-equivalent lane.

There are different methods for solving the proportion of traffic in different directions. In the actual road network, it can be calculated by historical data, or obtained by real-time data using the Extended Kalman Filter method. In the simulation road network, relative parameter values can be set. In this paper, we utilize the simulation software to simulate the road network, so the proportion of traffic in different directions can be directly set.

After obtaining the proportion in different directions, the carrying capacity of non-equivalent lane is:

$$A^u_{non-equivalent} = \frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}A^u_{equivalent} \tag{22}$$

where $\xi^u_{non-equivalent}$ represents the proportion of traffic in the non-equivalent lanes of the uth basic unit, $\xi^u_{equivalent}$ represents the proportion of traffic in the equivalent lanes of the uth basic unit.

3.4.3. Calculation of basic unit carrying capacity

Through the above series of derivation, the Eq. (8) of basic unit carrying capacity can be described as:

$$A^u = A^u_{equivalent} + A^u_{non-equivalent}$$

$$
\begin{aligned}
&= \sum_{pn\in P_u}\left\{\left[\frac{L^u V^u_{free} - L^u_{free}\overline{V}^u_{\text{expected}}}{V^u_{free}\cdot \overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]} + \frac{L^u_{free}}{\left(d_0 + V^u_{free}T_{safe}\right)\left[1-\left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} + \bar{l}_v}\right]\cdot m^{e,u}_{pn,ij}\right\} + \frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}A^u_{equivalent} \\
&= \left(1+\frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}\right)\cdot\sum_{pn\in P_u}\left\{\left[\frac{L^u V^u_{free} - L^u_{free}\overline{V}^u_{\text{expected}}}{V^u_{free}\cdot \overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]} + \frac{L^u_{free}}{\left(d_0 + V^u_{free}T_{safe}\right)\left[1-\left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} + \bar{l}_v}\right]\cdot m^{e,u}_{pn,ij}\right\}.
\end{aligned} \tag{23}
$$

Further analyzing the connection between the free travel vehicles and the stacked release vehicles, we found that the vehicles on equivalent road section and the vehicles in stacked area should maintain corresponding supplements and stability, which could not make the stacked zone unbalanced. In other words, in order to ensure that the average travel speed of basic unit remains unchanged, one vehicle on the road section is needed to supplement the reduced vehicle in the stacked area for each interval of headway. Otherwise, the basic unit will be out of balance and will not reach the original average travel speed. That is, the average time headway in the two conditions should be equal,$\overline{h}^u_{pn,ij}(V_{\text{expected}}) = \overline{h}_d(d^u_{free})$. Combined with the relationship between time headway and space headway, $d^u_{free} = \overline{h}_d\cdot V^u_{free}$ [38], the Eq. (23) can be derived as:

$$
\begin{aligned}
A^u &= A^u_{equivalent} + A^u_{non-equivalent} = \left(1+\frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}\right)\cdot\sum_{pn\in P_u}\left\{\left[\frac{L^u V^u_{free} - L^u_{free}\overline{V}^u_{\text{expected}}}{V^u_{free}\cdot \overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]} + \frac{L^u_{free}}{d^u_{free}}\right]\cdot m^{e,u}_{pn,ij}\right\} \\
&= \left(1+\frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}\right)\cdot\sum_{pn\in P_u}\left\{\left[\frac{L^u}{\overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]}\right]\cdot m^{e,u}_{pn,ij}\right\}.
\end{aligned} \tag{24}
$$

3.5. Calculation of road network carrying capacity

Road network carrying capacity refers to the maximum number of vehicles travelling simultaneously in the road network, and has a one-to-one correspondence with the average travel speed. When the physical conditions of road network are unchanged, once the average travel speed is determined, the maximum number of vehicles in the road network won't change. In this case, it is equivalent to taking a photo of road network from the high altitude, then the number of vehicles included in the photo is the road network carrying capacity in that state. Therefore, the carrying capacity of road network is equal to the direct addition of the carrying capacity of each basic unit:

$$
\begin{aligned}
A &= \sum_{u\in B}A^u = \sum_{u\in B}\left(A^u_{equivalent} + A^u_{non-equivalent}\right) \\
&= \sum_{u\in B}\left\{\left(1+\frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}\right)\cdot\sum_{pn\in P_u}\left\{\left[\frac{L^u V^u_{free} - L^u_{free}\overline{V}_{\text{expected}}}{V^u_{free}\cdot \overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]} + \frac{L^u_{free}}{\left(d_0 + V^u_{free}T_{safe}\right)\left[1-\left(\frac{V^u_{free}}{V_o}\right)^{\delta}\right]^{-1/2} + \bar{l}_v}\right]\cdot m^{e,u}_{pn,ij}\right\}\right\} \\
&= \sum_{u\in B}\left\{\left(1+\frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}\right)\cdot\sum_{pn\in P_u}\left\{\left[\frac{L^u}{\overline{V}^u_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]}\right]\cdot m^{o,u}_{pn,ij}\cdot\lambda_{pn}\right\}\right\}.
\end{aligned} \tag{25}
$$

During the operation of traffic flow, there may be speed differences between basic units. The identical average travel speed of road network may correspond to two situations. (i) The average travel speed of each basic unit is the same as the average travel speed of road network. This state is named as homogeneous traffic state. (ii) The average travel speed of each basic unit is different from the average travel speed of road network. The speeds of some basic units are higher than the road network speed, and others are lower than it. This state is named as non-homogeneous traffic state. Thus, the influence of homogeneity of road network should be considered before calculating the carrying capacity of road network. Put another way, under non-homogeneous traffic conditions, whether the carrying capacity of road network is equal to the sum of carrying capacity of basic units calculated by directly using the average travel speed of road network. Next, we try to prove it.

Assumption 1. The non-homogeneity of the road network will not affect the calculation of road network carrying capacity.

Proof

This paper constructs a simple continuous road network consisting of only two basic units. In the case of the same physical conditions, only the expected average travel speed of each basic unit is changed to verify the influence of non-homogeneity. This road network includes two situations. (i) The average travel speed of each basic unit is the same as the road network, both of which are $\overline{V}$. The corresponding road network carrying capacity is A_{homo}. (ii) The average travel speeds of two subnetworks are $\overline{V}_1$ and $\overline{V}_2$, respectively, but the average speed of road network is also $\overline{V}$. The corresponding road network carrying capacity is A_{nonhomo}.

The carrying capacities in the two cases are expressed as:

$$\begin{aligned} A_{\text{homo}} &= \left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}\cdot\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}\cdot\overline{h}_d\left(d^2_{free}\right)} \\ &= \frac{1}{\overline{V}}\cdot\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{h}_d\left(d^2_{free}\right)}\right] \end{aligned} \tag{26}$$

$$A_{\text{nonhomo}} = \left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}_1\cdot\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}_2\cdot\overline{h}_d\left(d^2_{free}\right)} \tag{27}$$

Where $L^1, m^{o,1}, \lambda^1, d^1_{free}, \xi^1_{non-equivalent}, \xi^1_{equivalent}$ respectively represent the road length, number of original lanes, green split, desired space headway, traffic proportion of non-equivalent lane and traffic proportion of equivalent lane of basic unit 1. $L^2, m^{o,2}, \lambda^2, d^2_{free}, \xi^2_{non-equivalent}, \xi^2_{equivalent}$ respectively represent the road length, number of original lanes, green split, desired space headway, traffic proportion of non-equivalent lane and traffic proportion of equivalent lane of basic unit 2.

The travel speed of vehicle is the ratio of the road length traveled by the vehicle to the total travel time. The average travel speed of vehicles is the mean of the travel speeds of all vehicles. So the formula of the average travel speed of the road network is:

$$\begin{aligned} \overline{V} &= \frac{A^{\text{unit},1}_{\text{nonhomo}}\cdot\overline{V}_1+A^{\text{unit},2}_{\text{nonhomo}}\cdot\overline{V}_2}{A^{\text{unit},1}_{\text{nonhomo}}+A^{\text{unit},2}_{\text{nonhomo}}} \\ &= \frac{\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}_1\cdot\overline{h}_d\left(d^1_{free}\right)}\right]\cdot\overline{V}_1+\left[\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}_2\cdot\overline{h}_d\left(d^2_{free}\right)}\right]\cdot\overline{V}_2}{\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}_1\cdot\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}_2\cdot\overline{h}_d\left(d^2_{free}\right)}} \\ &= \frac{\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{h}_d\left(d^1_{free}\right)}\right]+\left[\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{h}_d\left(d^2_{free}\right)}\right]}{\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}_1\cdot\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}_2\cdot\overline{h}_d\left(d^2_{free}\right)}} \end{aligned} \tag{28}$$

Substituting formula (28) into formula (26), we can get:

$$\begin{aligned} A^{\text{unit}}_{\text{homo}} &= \frac{1}{\overline{V}}\cdot\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{h}_d\left(d^2_{free}\right)}\right] \\ &= \frac{\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{h}_d\left(d^2_{free}\right)}\right]\cdot\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}_1\cdot\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}_2\cdot\overline{h}_d\left(d^2_{free}\right)}\right]}{\left[\left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{h}_d\left(d^1_{free}\right)}\right]+\left[\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{h}_d\left(d^2_{free}\right)}\right]} \\ &= \left(1+\frac{\xi^1_{non-equivalent}}{\xi^1_{equivalent}}\right)\frac{L^1\cdot m^{o,1}\cdot\lambda^1}{\overline{V}_1\cdot\overline{h}_d\left(d^1_{free}\right)}+\left(1+\frac{\xi^2_{non-equivalent}}{\xi^2_{equivalent}}\right)\frac{L^2\cdot m^{o,2}\cdot\lambda^2}{\overline{V}_2\cdot\overline{h}_d\left(d^2_{free}\right)} \end{aligned} \tag{29}$$

Comparing Eq. (27) and (29), we can see that the Assumption 1 is correct. That is to say, the homogeneity of road network does not affect the calculation of carrying capacity. On the other hand, it further illustrates the self-consistency and effectiveness of the carrying capacity definition.

Therefore, the carrying capacity of basic unit can be directly calculated using the average travel speed of road network. Alter-

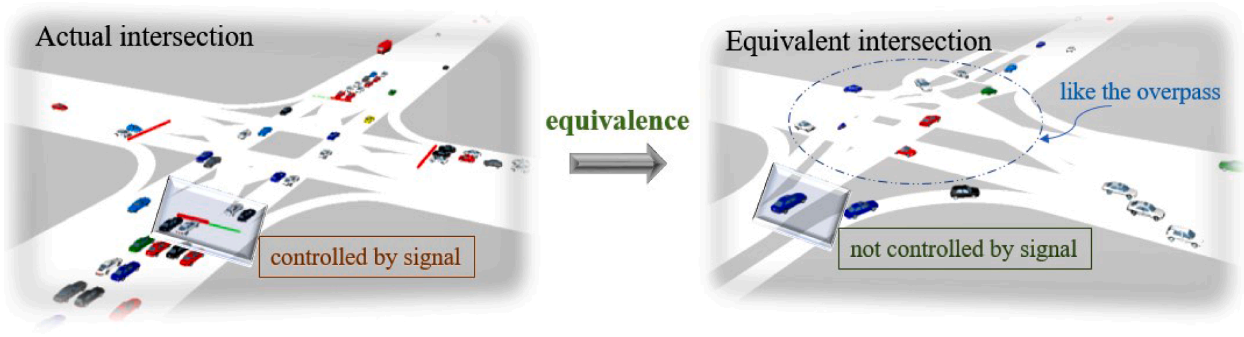


Fig. 6. Schematic diagram of traffic flow before and after equivalent. .

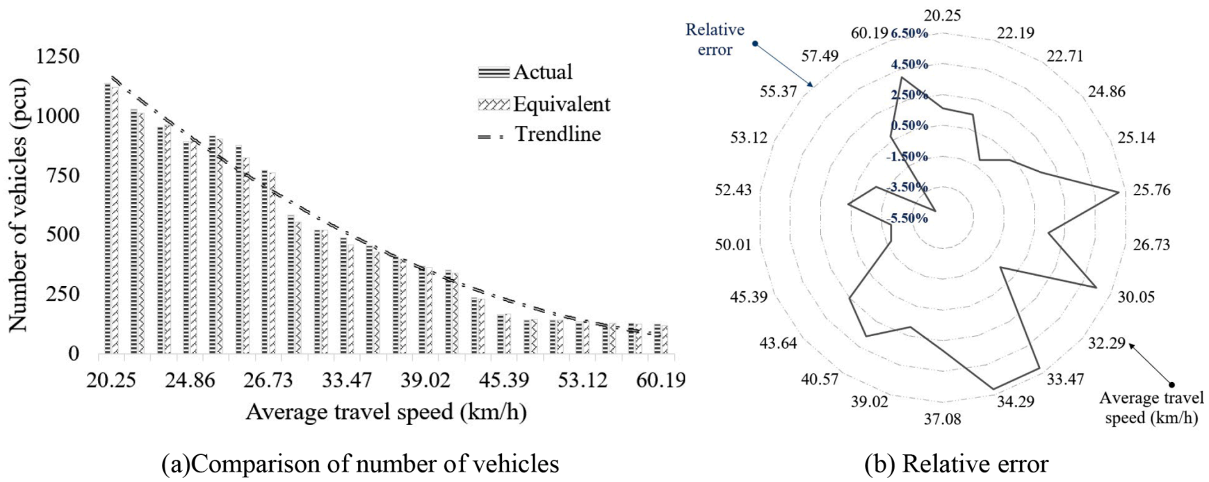


Fig. 7. Verification results of intersection. (a) Comparison of number of vehicles. (b) Relative error.

natively, each basic unit can first use its own speed to calculate respective carrying capacity, and then add up to get the road network capacity. All in all, the formula for calculating the carrying capacity of the road network at an expected average travel speed, that is, Eq. (25) can be transformed into:

$$
\begin{aligned}
A &= \sum_{u\in B} A^u = \sum_{u\in B}\left(A^u_{equivalent} + A^u_{non-equivalent}\right) \\
&= \sum_{u\in B}\left\{\left(1+\frac{\xi^u_{non-equivalent}}{\xi^u_{equivalent}}\right)\cdot\sum_{pn\in P_u}\left\{\left[\frac{L^u}{\overline{V}_{\text{expected}}\cdot E\left[h^u_{pn,ij}\left(V_{\text{expected}}\right)\right]}\right]\cdot m^{o,u}_{pn,ij}\cdot\lambda_{pn}\right\}\right\}.
\end{aligned} \tag{30}
$$

subject to:

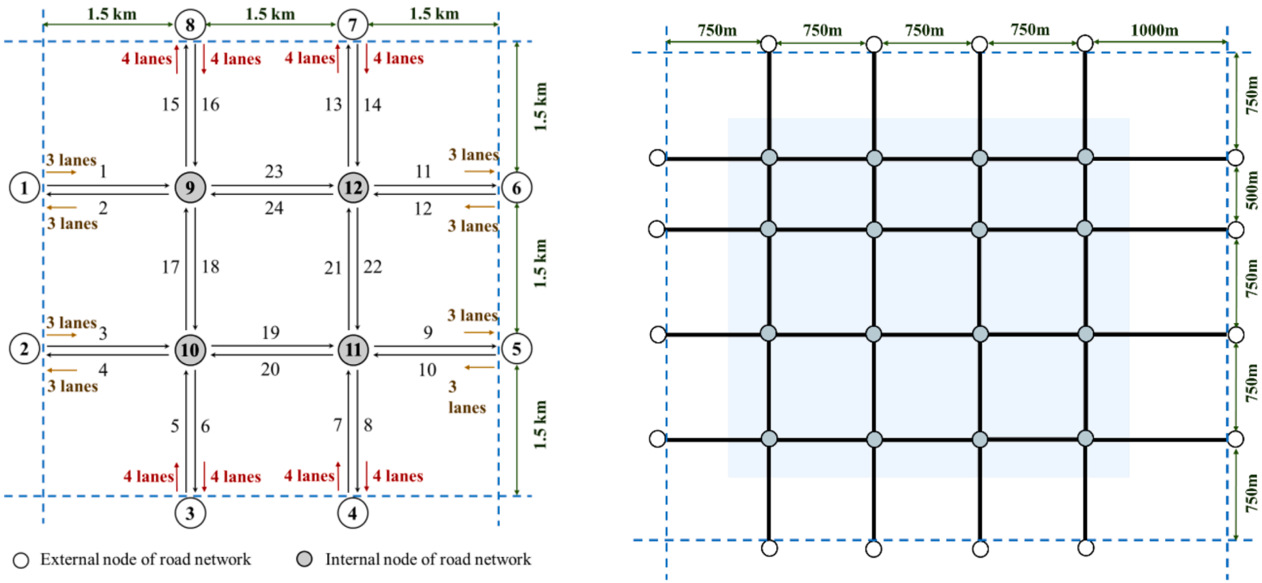
$$
\overline{V}_{\text{expected}} = \frac{\sum\limits_{u\in B}\overline{V}^u_{\text{expected}}\cdot A^u}{\sum\limits_{u\in B} A^u} \tag{31}
$$

$$
\sum_{k=\{right,through,left\}} \xi^u_k = \xi^u_{equivalent} + \xi^u_{non-equivalent} = 1, u\in B \tag{32}
$$

Solving algorithm: when the parameters $\overline{V}_{\text{expected}}$, $\xi^u_{equivalent}$, λ_{pn} in the model are alterable and the ranges are given, genetic algorithm can be utilized to solve the model. It includes initialization, evaluation of fitness, selection, crossover, mutation, etc. Conversely, if the relevant parameters in the model are determined, the results can be computed directly.

4. Simulation and result analysis

In this part, the correctness and validity of carrying capacity model are first verified. After proving that the model is feasible, we



(a) Road network 1

(b) Road network 2

Fig. 8. The example road networks. (a) Road network 1. (b) Road network 2.

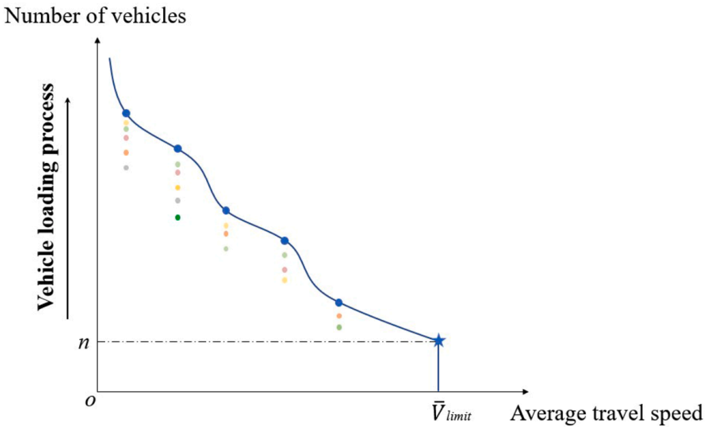


Fig. 9. The changing process of road network traffic state. .

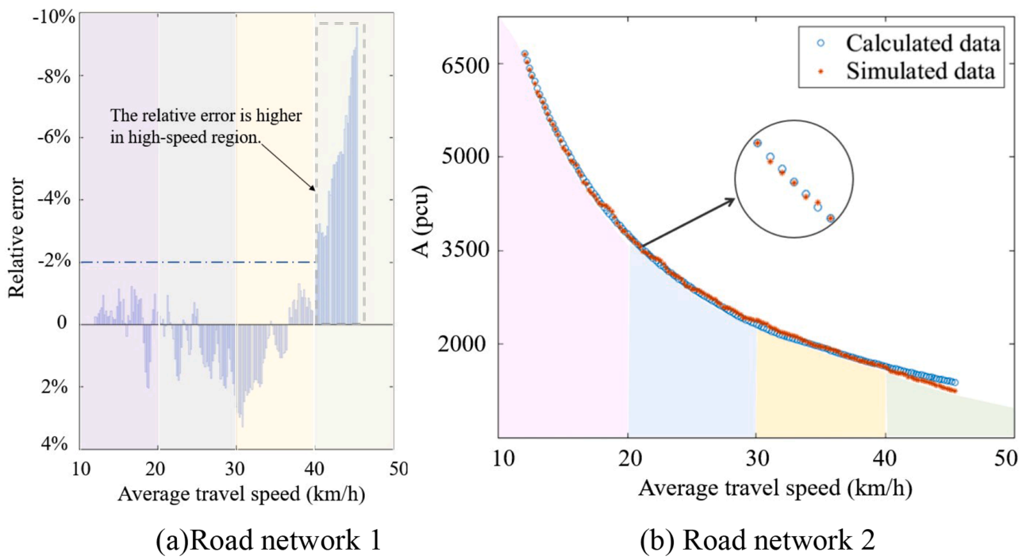


Fig. 10. Verification results of road network. (a) Road network 1. (b) Road network 2.

further explored the influence of different variables on carrying capacity, and then gave the guiding significance of carrying capacity on the improvement of road network service level. Finally, by comparing the characteristics and applicable environments of different types of capacity models, the advantages of the model proposed in this paper are summarized, and the practical application value for traffic control is elaborated.

4.1. Verification of carrying capacity model

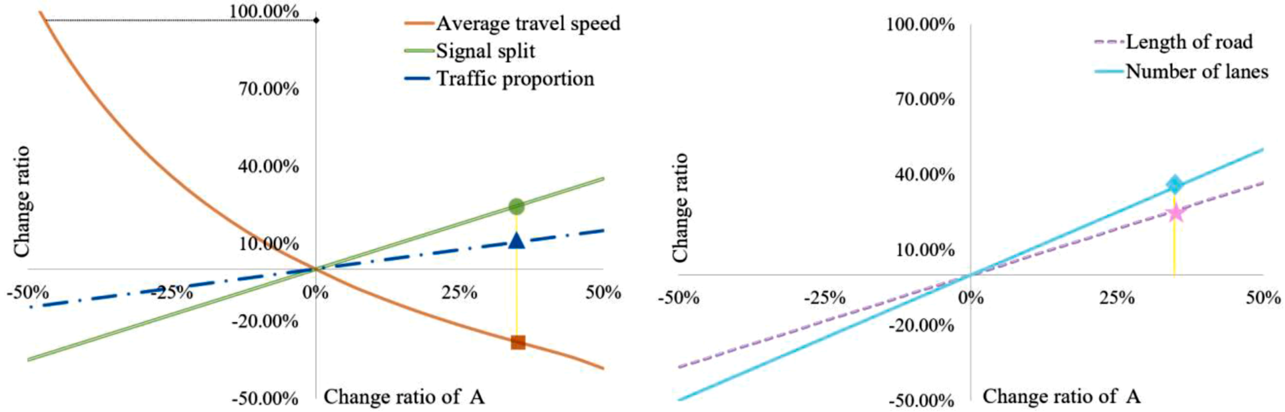
In this paper, VISSIM software based on VB programming is employed to build the simulation scenarios for verifying the proposed models. It includes two parts: Validation of equivalent lane and Validation of carrying capacity model.

4.1.1. Validation of equivalent lane

To test whether the equivalent lane can replace the actual lane, the same number of vehicles are input on the equivalent road and let the vehicles continue to move while the average speed remains unchanged. By comparing the number of vehicles in the road network before and after the equivalence, the effectiveness and accuracy of equivalent lane can be validated.

Specifically, we constructed a two-way 6-lane * 8-lane intersection. After equivalence, it is equal to let the vehicle pass through the intersection in the form of the overpass (Fig. 6). Under the premise of identical infrastructure conditions, the accuracy and effectiveness of equivalent lane are tested by inputting different traffic flows (Fig. 7).

As can be seen from Fig. 7 that the number of vehicles on the road decreases with the increase of average travel speed. Comparing the number of vehicles before and after the equivalence, the average relative error is 1. 61% and the maximum error does not exceed 6.



(a) Dynamic variables (b) Static variables

Fig. 11. Sensitivity analysis of different variables. (a) Dynamic variables. (b) Static variables.

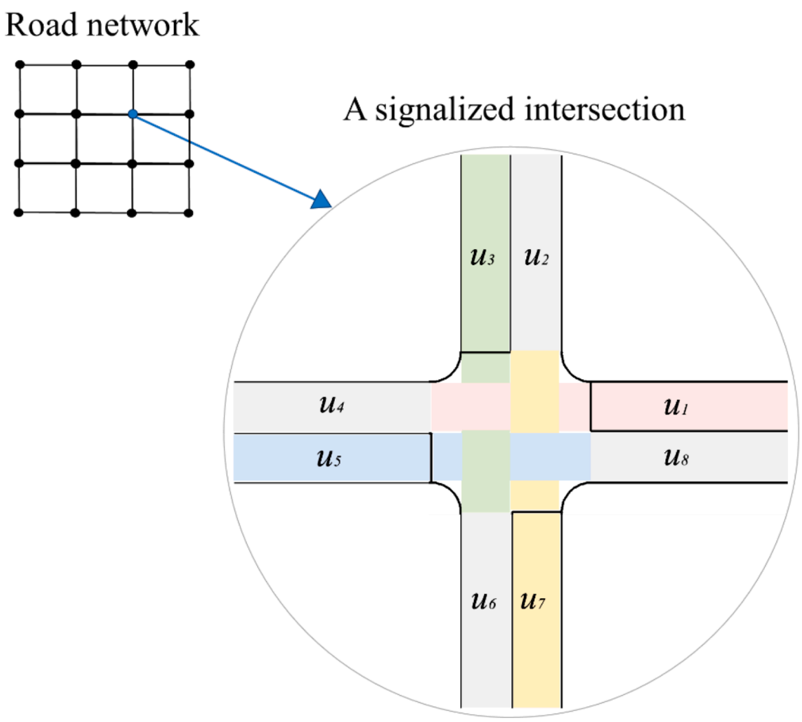


Fig. 12. The example of signalized intersection. .

Table 1
Speed change under different signal split combinations. .

Number	λ_1	λ_2	λ_3	λ_4	A(pcu)	Average travel speed (km/h)	Level of service
1	0. 35	0. 15	0. 35	0. 15	824	20. 00	E
2	0. 35	0. 15	0. 30	0. 20	824	20. 33	E
3	0. 35	0. 15	0. 25	0. 25	824	20. 66	E
4	0. 35	0. 15	0. 20	0. 30	824	20. 98	E
5	0. 35	0. 15	0. 15	0. 35	824	21. 30	E
6	0. 35	0. 15	0. 10	0. 40	824	21. 63	E
7	0. 35	0. 15	0. 05	0. 45	824	21. 95	E
8	0. 30	0. 20	0. 35	0. 15	824	20. 33	E
9	0. 30	0. 20	0. 30	0. 20	824	20. 66	E
10	0. 30	0. 30	0. 20	0. 20	824	21. 30	E
11	0. 25	0. 35	0. 15	0. 25	824	21. 95	E
12	0. 25	0. 25	0. 30	0. 20	824	20. 98	E
13	0. 25	0. 25	0. 25	0. 25	824	21. 30	E
14	0. 20	0. 30	0. 20	0. 30	824	21. 95	E
15	0. 20	0. 30	0. 15	0. 35	824	22. 27	D
16	0. 20	0. 40	0. 15	0. 25	824	22. 27	D
17	0. 15	0. 25	0. 20	0. 40	824	22. 27	D
18	0. 15	0. 35	0. 10	0. 40	824	22. 91	D
19	0. 15	0. 35	0. 05	0. 45	824	23. 23	D
20	0. 15	0. 25	0. 20	0. 40	824	22. 27	D
21	0. 15	0. 45	0. 15	0. 25	824	22. 59	D
22	0. 10	0. 50	0. 15	0. 25	824	22. 91	D
23	0. 10	0. 40	0. 20	0. 30	824	22. 59	D
24	0. 10	0. 40	0. 25	0. 25	824	22. 27	D
25	0. 10	0. 30	0. 25	0. 35	824	22. 27	D

50%. The standard deviation of the error is 2. 84%.

Based on the above verification, the number of vehicles travelling in the road network remains basically unchanged before and after the equivalence. Therefore, the maximum number of vehicles travelling in the road network will also be the same. It further indicates that this idea is available and effective in calculating the carrying capacity.

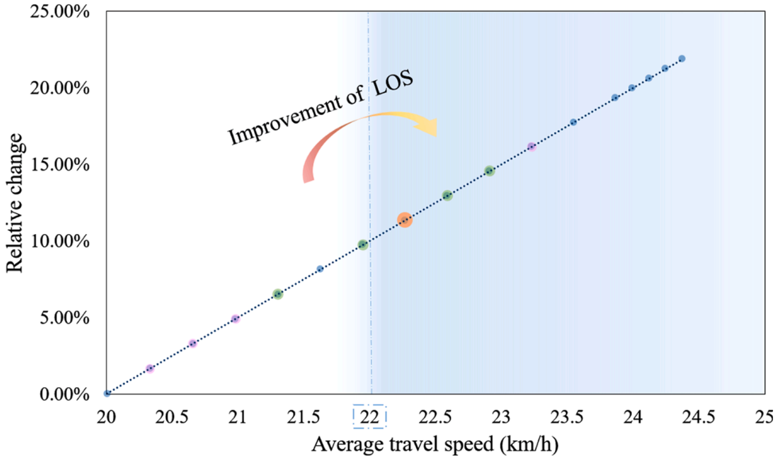


Fig. 13. Counts plot of change in average travel speed. .

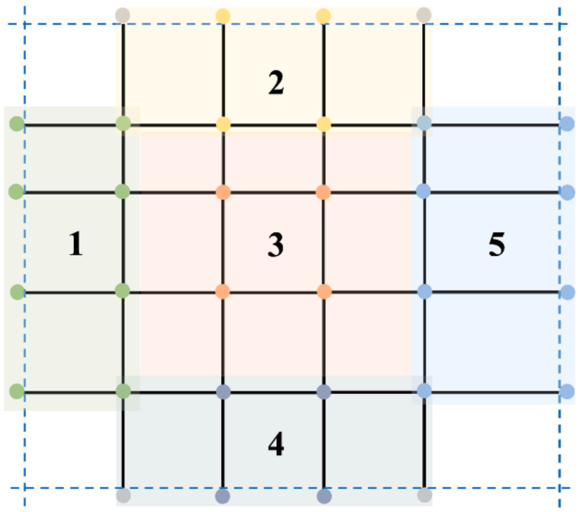


Fig. 14. Sub-region division diagram of road network 2. .

4.1.2. Validation of carrying capacity model

In this part, the model is verified by comparing the carrying capacity obtained by VISSIM simulation and the carrying capacity obtained by proposed method. Two example road networks as shown in Fig. 8 were constructed. The details are as follows: (1) Road network 1: There were 12 nodes including 8 external nodes providing traffic demands and 4 internal nodes controlled by traffic signals, and 24 links. The cycle of all nodes is 160 s. The green splits for through and left-turn in the east-west direction were all 0. 23 and 0. 17. The corresponding green splits in the north-south direction were all 0. 30 and 0. 20. (2) Road network 2: There were 32 nodes including 16 external nodes providing traffic demands and 16 internal nodes controlled by traffic signals, and 80 links. All directions were two-way 6 lanes. The cycle of all external nodes is 180 s. The green split of the external nodes in the through direction was 0. 28, and the left-turn direction was 0. 22. The cycle of all internal nodes is 140 s. The green split of the internal nodes in the through direction was 0. 29, and the left-turn direction was 0. 21. Among them, the determination of the green splits in different directions is based on the structural design of road network, traffic input and actual traffic survey experience. The road capacity of the two road networks was 1800pcu/h, and all intersections consisted of conventional four phases. The traffic composition in the example road networks is all of the car type. The right-turn traffic proportion was also given directly.

Before performing simulation verification, it is necessary to analyze the change process of the road network traffic state first, and then confirm how to find the simulation value of the carrying capacity. The number of vehicles in the road network cannot always be constant, but it will not change suddenly. The fluctuation of number of vehicles is accompanied by the fluctuation of average travel speed. This is the changing process of the road network traffic state (Fig. 9). In the process of inputting traffic on the simulated road network, different combinations of traffic at the input endpoints will cause differences in the number of vehicles. Then, this can lead to a situation where the same average travel speed corresponds to multiple numbers of vehicles. However, each average travel speed only corresponds to one maximum number of vehicles, that is, carrying capacity. In particular, the road network has a critical average travel

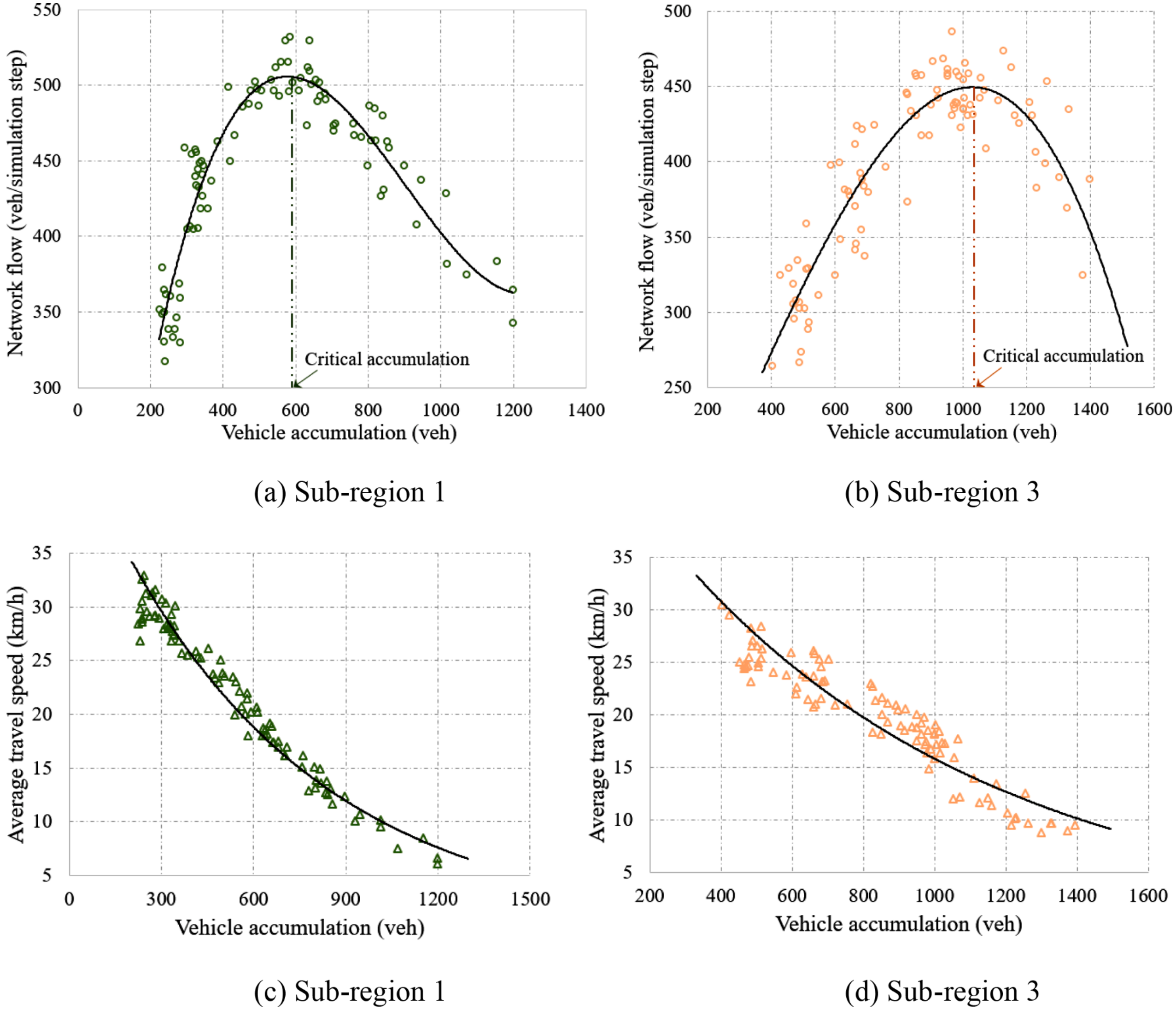


Fig. 15. The relationships between the vehicle accumulation and road network flow, average travel speed respectively. (a) Sub-region 1. (b) Sub-region 3. (c) Sub-region 1. (d) Sub-region 3.

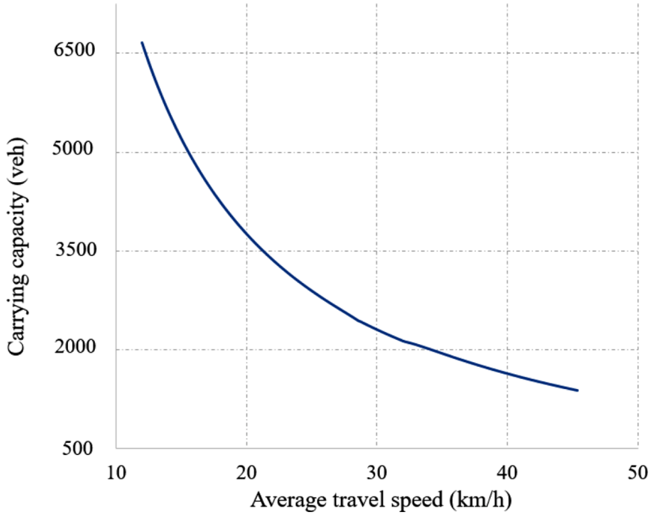


Fig. 16. The relationship between average travel speed and carrying capacity. .

speed ($\overline{V}_{\text{limit}}$) at high-speed condition. Under this condition, as long as the number of vehicles is not greater than n, vehicles can always travel freely at this speed. These are also consistent with the actual situation.

By means of simulation, the vehicles were input incrementally from external nodes. We recorded the number of vehicles corresponding to different average travel speeds of road network. Then the maximum number of vehicles travelling in the road network at different speeds, that is, carrying capacity can be got. Furthermore, compare the simulated value with the carrying capacity calculated by proposed model (Fig. 10). Among them, the simulation includes all types of flow input combinations, a total of 6016 sets: The number of experiments for road network 1 is 2389 and the number of sets corresponding to carrying capacity is 219; the number of experiments for road network 2 is 3627 and the number of sets corresponding to carrying capacity is 234.

It can be seen from the Fig. 10 that the simulated data and the calculated data have the same trend. As the average travel speed of road network increases, the carrying capacity decreases. Moreover, the relative error between the two is small. The average relative error is −0. 46% and the standard deviation of the error is 2. 50%. The maximum error is no more than 10%, indicating that the model has high accuracy and can meet the actual traffic demands. In addition, in the high-speed region, the relative error of model is large, which may be related to the existence of $\overline{V}_{\text{limit}}$. This is also the reason why it is more difficult to find the carrying capacity corresponding to the high-speed during the simulation.

4.2. Analysis of carrying capacity model

4.2.1. Sensitivity analysis

In order to further explore the influence of different variables on the carrying capacity model, sensitivity analysis is performed on dynamic variables (average travel speed, traffic proportion, and signal split) and static variables (length of road and number of lanes) (Fig. 11). In terms of static factors, the influence of number of lanes is greater than length of road, and the two are directly proportional to the carrying capacity. In terms of dynamic factors, average travel speed has the greatest impact on the model, followed by signal split and traffic proportion. The average travel speed is inversely proportional to the carrying capacity, which is also consistent with the model verification results. In addition, it can also be found that the carrying capacity of road network can be increased by changing the signal split when the average travel speed is unchanged. And the maximum increase in carrying capacity is 35. 11% of the original value. This is of great significance to the improvement of urban traffic supply and demand.

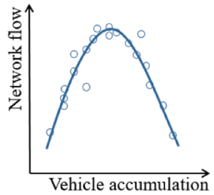
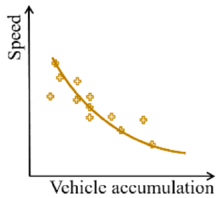
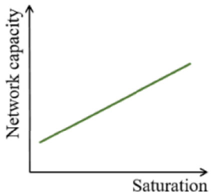
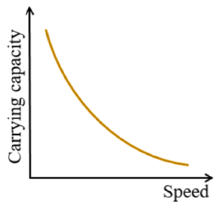
4.2.2. Effects of signal timing

In the actual road network, the static factors are unchanged, and the traffic proportion in the dynamic factor cannot be controlled artificially. In this case, only the signal split can be adjusted, which in turn changes the average travel speed and affects the carrying capacity of road network. It is known from the sensitivity analysis that when the average travel speed is constant, signal split can be changed to increase the carrying capacity. If the carrying capacity of road network is determined, how to improve the level of service by changing signal split?

In order to achieve it, this paper selects one intersection in the road network as an example. The tested intersection includes eight basic units (Fig. 12). Among them, the signal splits related to u_1, u_3, u_5, u_7 are λ_1, λ_2, λ_3 and λ_4 respectively. The signal splits of remaining basic units are fixed. The right-turn traffic proportion is also given. All directions are two-way eight lanes. The length of each link is 1. 5 km.

By changing signal splits of four phases, the combinations with different speed changes are obtained (Table 1). In Table 1, the first line is the original value of road network parameters. Among them, the average travel speed is 20 km/h, and carrying capacity is 824

Table 2
The differences among various capacity models. .

Category		Vehicle accumulation in MFD	Narrow road network capacity	Generalized road network capacity	Traffic carrying capacity proposed in this paper
Definition		The number of vehicles travelling in the road network involved in the MFD[1]	The maximum number of vehicles passing through critical sections of road network[5-6]	The maximum number of vehicles that road network can accommodate[9-10]	The maximum number of vehicles travelling simultaneously in the road network under a certain service level
Calculation period		In a certain period	In a certain period	In a certain period	None (independent of the time dimension)
Relation to other parameters	Traffic flow		Sum of traffic flow passing through critical sections	Unrelated	Unrelated
	Service level (saturation /speed)			No research yet	
Main function		Determine the critical accumulation corresponding to outflow-maximizing	Maximize the throughput of network	Maximize the vehicle accommodation of network	Maximize the operation of network
Features of the calculation model		—The model is simple and easy-to-operate; —The model is constrained by the homogeneity of the road network; —The model requires many experiments to determine uncalibrated variables; —The model only concerns flow-maximizing, and does not consider the problem that the actual travel demand cannot match the adjusted flow; —Speed and accumulation are not the only mapping relationship, the same speed can correspond to multiple accumulations.	— The model provides various solving algorithms from different aspects; — The model has strict constraints and no exact solutions.	—The model is simple and easy to understand; — The model has many parameters to be calibrate; —There is a deviation in the calculation result, which is related to the survey period.	—The model is practical and easy-to-operate; — The model is not restricted by the road network homogeneity; — The model has few parameters that need to be determined; — According to the requirements of service level, the maximum number of vehicles in the road network can be directly calculated, and the actual traffic demand can be accurately met; — There is a unique correspondence between speed and carrying capacity.

pcu. It can be seen from Table 1 and Fig. 13 that the signal split has a significant effect on increasing the average travel speed of road network. The relative change of average travel speed can be up to 21. 86%. At the same time, the level of service can rise from E to D under the premise of ensuring the same carrying capacity. In practical applications, traffic managers can choose the appropriate signal split combination corresponding to the D-degree level of service. Here, the grade of service level is based on HCM [39].

Taking these processes as examples, on the one hand, the carrying capacity can be increased by adjusting signal timing and infrastructure. On the other hand, while maintaining the carrying capacity unchanged, the service level of road network can be improved by adjusting the timing, thereby increasing the traffic efficiency of city. In short, the model proposed can provide the basis for the traffic managers to calculate the urban road supply and determinate the improvement range of road network service level, and provide directions for the optimization of urban traffic resources.

4.3. Comparison with other capacity models

This part mainly summarizes the differences in definition and characteristics of the four capacity types (Vehicle accumulation in MFD, Narrow road network capacity, Generalized road network capacity, and Traffic carrying capacity), and then draws the advantages and disadvantages of various models and their applicable environments. This can provide a basis for traffic managers or planners to choose a suitable road network capacity calculation model.

In order to display the characteristics of different types of capacity more clearly and quantitatively, specific experiments were carried out on the four capacity models with the road network 2 in Section 4. 1. 2 as an example.

Regarding vehicle accumulation in MFD, its function is to determine the maximum road network flow based on MFD. Before determining the MFD of road network, the homogeneity of road network needs to be considered. For the non-homogeneous road network, the sub-regions need to be divided first. Therefore, based on the existing research [40], the road network 2 was divided into 5 sub-regions, as shown in Fig. 14. After determining the road network sub-regions, hundreds of experiments were carried out to determine the MFD curves of different sub-areas. Taking sub-regions 1 and 3 as examples, the MFD functions are: $Q = 1 \times 10^{-6} \times Y^3 - 0.0028 \times Y^2 + 2.2271 \times Y - 36.193$ and $Q = (-3) \times 10^{-7} \times Y^3 + 0.0002 \times Y^2 + 0.4441 \times Y + 84.402$ respectively (Q is the network flow, and Y is the vehicle accumulation). The fitted relationship diagram between vehicle accumulation and road network flow, and the relationship diagram between vehicle accumulation and average travel speed are shown in Fig. 15. From Fig. 15(a)-(b), it can be seen that when using MFD for coordinated control of traffic flow, it focuses on seeking the critical accumulation to maximize network flow, and does not consider whether the adjusted number of vehicles is in line with the actual travel demand. In other words, when travel demand is restricted and cannot be adjusted to the critical accumulation, MFD is ineffective in this case. From Fig. 15(c)-(d), speed and accumulation are not the only mapping relationship, and the same speed can correspond to multiple accumulation values. Therefore, it is not feasible to use the relationship between the two to optimize the flow of network sub-regions.

Regarding the narrow road network capacity, this paper takes the literature [27] as an example to conduct a preliminary qualitative analysis. As can be seen from the model in [27], the calculation formulas are complex and numerous, with a total of 58; the constraints of model are strict, and the model is a non-convex function with no exact solution.

Regarding the generalized road network capacity, take the model of space-time consumption method in Section 2. 2. 2 as an example to calculate the capacity under different traffic conditions in one hour. When the average travel time of traffic individual is 30 km/h and 22 km/h, and the average travel time in both cases is 800 s, the corresponding capacities are 2945 veh and 4821 veh, respectively. This further confirms that the maximum number of vehicles in the road network has nothing to do with the average travel time of traffic individuals. The change in the number of vehicles is not due to the difference in calculation period, but the difference of traffic condition in the calculation period.

Regarding the traffic carrying capacity model, the mapping relationship between carrying capacity and average travel speed in the road network 2 can be easily obtained by using formulas (30)-(32), as shown in Fig. 16. Because the relationship has a one-to-one correspondence, no matter what kind of traffic demand (service level) can be accurately met. Compared with MFD, the process of determining the critical cumulative value is omitted. In addition, because the definition of carrying capacity itself has the attribute of optimal value, it can be directly used for coordinated control of the road network perimeter. In the process of coordinated control, according to the demonstration in Section 3. 5, the regional homogeneity has no effect on the carrying capacity, so there is no need to divide the control sub-regions.

The characteristics of each model are summarized in Table 2. It can be seen from Table 2 that vehicle accumulation in MFD is different from the other three types and does not involve the meaning of maximum value. Its main function is to find the maximum output flow. In the latter three types of capacity, the performance of road network capacity is discussed from the perspectives of throughput, accommodation and operation. For the carrying capacity model proposed in this paper, it overcomes the complexity of solving the narrow capacity model, solves the problem of period deviation in generalized capacity calculation, and breaks the constraints on the homogeneity of the road network in the MFD model. Overall, its calculation process is simple and easy to operate, with few constraints and high practicability. By calculating the carrying capacity of road network, it has an important guiding role in the estimation of urban car ownership and the planning of road infrastructure.

5. Conclusions

Targeting at tackling the imbalance of traffic supply-demand and optimizing the design of transportation system, this paper developed a calculation model of traffic carrying capacity of road network. The proposed model utilized the novel ideas of lane equivalence and traffic flow abstraction to establish the relationship between average travel speed, signal timing parameters and traffic carrying capacity. With the help of VISSIM simulation software, through comparison and verification with a large amount of simulation data, the average relative error of carrying capacity model is −0. 46% and the standard deviation of the error is 2. 50%. These examples show that the model has high accuracy and can meet actual traffic demand. Among them, one of the contributions of the model is that it fully considers the existing problems of difficult parameter calibration and inexact solutions, and provides a practical and easy-to-operate calculation process. Moreover, it overcomes the limitations of existing methods on the homogeneity of road network, and can directly calculate the carrying capacity of non-homogeneous road network under a given service level.

In addition, through the analysis of carrying capacity model and the comparison with other capacity models, other conclusions can be drawn. (i) In terms of static factors, the number of lanes has a greater influence on carrying capacity than road length, and the two variables are directly proportional to carrying capacity. In terms of dynamic factors, average travel speed has the greatest impact on the carrying capacity model, followed by signal split and traffic proportion. (ii) Signal timing is of great significance to carrying capacity. By adjusting the signal split, on the one hand, the carrying capacity can be increased under the condition that the average travel speed is unchanged, and the maximum increase in carrying capacity is 35. 11% of the original value; on the other hand, the service level can also be improved without changing the carrying capacity. (iii) For narrow road network capacity, generalized road network capacity, and traffic carrying capacity of road network, they explored the performance of road network capacity from the perspective of throughput, accommodation and operation. Urban managers can dynamically select capacity models based on actual requirements and application environments.

This paper is just a preliminary attempt to calculate road network carrying capacity. It only discusses the guiding role of carrying capacity model in improving road network service level, but has not yet studied its application in the field of urban traffic system optimization. Therefore, in the future, the application of the carrying capacity model in the urban traffic system optimization should be further explored, such as the coordinated control of road network sub-region perimeter. Specifically, based on the model, the remaining carrying capacity of each sub-region at their average travel speeds is first calculated. Then, the priority order of sub-region coordination is determined according to the remaining carrying capacity: the sub-region with the smallest remaining carrying capacity is adjusted first, and the other sub-regions are adjusted in sequence. Furthermore, by coordinating and optimizing the signal timing of intersections at the perimeter of sub-regions, the traffic flow in the sub-region with less remaining capacity is adjusted to enter the sub-region with more remaining capacity, thereby achieving the effect of optimizing the overall road network operation mode. In addition, the effectiveness and availability of the proposed model should be examined through more case studies, such as different network topologies, trip modes, and road network scales, etc.

CRediT authorship contribution statement

Yuhong Gao: Conceptualization, Methodology, Writing – original draft. **Zhaowei Qu:** Supervision, Conceptualization, Funding acquisition. **Xianmin Song:** Writing – review & editing, Funding acquisition. **Zhenyu Yun:** Software, Validation, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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