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A macro traffic flow model accounting for road capacity and reliability analysis

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HIGHLIGHTS

- A macro traffic flow model with consideration of road capacity has been developed.
- The road capacity has effects on the risk coefficient and the system's reliability.
- The results have displayed the effects of the road capacity on traffic flow very well.

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ABSTRACT

Based on existing traffic flow models, in this paper we develop a macro traffic flow model taking into consideration road capacity to study the impact of the road capacity on traffic flow. The numerical results show that the road capacity destroys the stability of uniform flow and produces stop-and-go traffic under a moderate density and that the road capacity enhances the traffic risk coefficient and reduces the traffic system's reliability. In addition, the numerical results show that properly improving the road condition can enhance the road capacity, reduce the traffic risk coefficient and enhance the traffic system's reliability.

1. Introduction

To date, serious traffic problems (e.g., congestion, traffic accidents, traffic pollution, etc.) have attracted researchers to establish many traffic flow models to investigate complex traffic phenomena from different perspectives [1,2]. Roughly speaking, the existing traffic flow models can be divided into macro models [3–13] and micro models [14–25]. Although these traffic flow models can reproduce many complex traffic phenomena, they cannot be used to explore the effects of road capacity on traffic flow since this factor is not explicitly considered. In order to explore traffic phenomena resulting due to road capacity, Nagatani [26,27] proposed a traffic flow model to study the impact of road capacity on traffic flow, Weng et al. [28] proposed a traffic flow model to investigate the effects of weather conditions on the road capacity, and Shang et al. [29] used a cellular automaton model to explore the effects of right-turn vehicles on the road capacity. However, as the above models do not explicitly take into account capacity limitations they cannot reproduce many complex traffic phenomena in traffic systems. In fact, each traffic system has a capacity that will affect the driving behavior, so the road capacity should be explicitly taken into account in the traffic flow model. In this paper, we propose a new macro traffic flow model accounting for road capacity to study the impact of capacity limitations on traffic flow.

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2. Model

The first macro traffic flow model was developed independently by Lighthill and Whitham [3] and Richards [4] (it is called the LWR model), where the control equations can be formulated as follows:

$$\begin{cases} \rho_t + q_x = 0 \\ q = \rho v_e(\rho), \end{cases} \tag{1}$$

where ρ, q, $v_e(\rho)$ are respectively the density, flow and equilibrium speed that satisfy the following conditions:

(1) the equilibrium speed is a decreasing function of the density;
(2) $v_e(\rho_j) = 0$, $q(0) = q(\rho_j) = 0$, where ρ_j is the jam density;
(3) as the function of the density, $q(\rho)$ is a concave function.

Eq. (1) is simple and can reproduce the formation and propagation of shock, but it cannot be used to study non-equilibrium traffic flow because the speed cannot deviate from the equilibrium speed $v_e(\rho)$. To overcome the drawback, researchers later proposed various high-order models [5–13]. Roughly speaking, the traffic flow models [5–13] can be divided into DG (density-gradient) model and SG (speed-gradient) model, where the DG model can be formulated as follows:

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x = \dfrac{v - v_e}{T} - \dfrac{c^2(\rho)}{\rho}\rho_x, \end{cases} \tag{2}$$

where v is the speed, $c(\rho) > 0$ is the sonic speed, T is the reaction time. Although Eq. (2) can overcome the drawback of the LWR model, it produces backward motion under some specific conditions. In order to overcome the backward motion, researchers developed various SG models, where the simplest SG model can be formulated as follows [6]:

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x = \dfrac{v - v_e}{T} + c_0 v_x, \end{cases} \tag{3}$$

where c_0 is the propagation speed of small perturbations.

Jiang et al. [5] proved that Eq. (3) can overcome the shortcomings of Eqs. (1) and (2) and describe some complex traffic phenomena (e.g., local cluster, stop-and-go, etc.), but it cannot be used to study the influence of road capacity on traffic flow since it does not consider this factor. In fact, each road has a capacity that has the following properties:

(1) When the vehicle arrival rate is greater than the capacity, the real flow is equal to the road capacity and a queue occurs; at this time, the queue may have impeding effects on traffic flow.
(2) When the vehicle arrival rate is less than or equal to the capacity, the real flow is equal to the vehicle arrival rate and no queue occurs; at this time, the SG model [6] can describe the traffic flow.
(3) The road capacity can drop due to some specific reasons (e.g., closing lane, toll station, bus station, etc.).

The above properties indicate that we should explicitly consider road capacity and that the SG model [6] can be extended to study the effects of the road capacity on traffic flow. Based on the above discussion, when the flow is greater than the road capacity, the flow can be defined as follows:

$$q = \begin{cases} C, & \text{if } \rho v > C \\ \rho v, & \text{if } \rho v \le C, \end{cases} \tag{4}$$

where q is the flow and C is the road capacity.

Based on Eq. (4), we can obtain that a queue will occur when $\rho v > C$ and that the length of the queue is equal to $\rho v - C$. At this time, the queue has impeding impacts on traffic flow, but the range over which the queue affects traffic flow is affected by many factors (e.g., the density). For simplicity, we here assume that the range is constant and independent of the density. Based on Ref. [30], the queue can be regarded as the input of an on-ramp. For convenience, we define the queue's length as $\max\{\rho v - C, 0\}$. Thus, we can develop a macro traffic flow model with road capacity, i.e.,

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x = \dfrac{v - v_e}{T} + c_0 v_x + G_1, \end{cases} \tag{5}$$

where G_1 is the friction effect resulting from the queue. Based on the above discussion, we can find that a queue will occur when the flow is greater than the road capacity and that the queue can be qualitatively explained as the inflow of an on-ramp. Based on Ref. [30], we find that the road capacity will have a friction effect G_1 on traffic flow that can be defined as follows:

$$G_1 = -\varepsilon \frac{\max\{0, \rho v - C\}}{L} \rho v, \tag{6}$$

where L is the length of the range over which the queue affects traffic flow and ε is the friction coefficient [30]. Note: the units of the right hand of Eq. (5) are consistent with those of the left hand of Eq. (5) [30]. In fact, the parameter ε has quantitative but no qualitative effects on the numerical results, so we do not calibrate it here; however, we will in the future use empirical data to calibrate it and study the effects of the road capacity on traffic flow.

Roughly speaking, the model accounting for road capacity is the same as the traffic flow model [30]. In fact, the two models have the following differences: the friction effect G_1 is related to the input of the on-ramp in the model [30]; the friction effect G_1 is related to the road capacity in Eq. (5). In comparison with existing traffic flow models, Eq. (5) can be used to study the effects of road capacity on traffic flow because it explicitly considers this factor.

3. Numerical tests

It is difficult to obtain the analytical solution of Eq. (5), so we will use a numerical scheme to discretize it in this paper. In fact, there are various numerical schemes to discretize Eq. (5), but the choice has no qualitative impact on the numerical results. However, Eq. (5) is deduced from the SG model [6], therefore, we use the upwind scheme to discretize Eq. (5) and compare the following numerical results with those of the SG model [6]. Thus, Eq. (5) can be discretized as follows:

$$\rho_i^{j+1} = \rho_i^j + \frac{\Delta t}{\Delta x}\rho_i^j\left(v_i^j - v_{i+1}^j\right) + \frac{\Delta t}{\Delta x}v_i^j\left(\rho_{i-1}^j - \rho_i^j\right), \tag{7}$$

where if $v_i^j < c_0$,

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}\left(c_0 - v_i^j\right)\left(v_{i+1}^j - v_i^j\right) + \frac{\Delta t}{T}\left(v_e\left(\rho_i^j\right) - v_i^j\right) + \Delta t G_{1,i}^j, \tag{8a}$$

otherwise

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}\left(c_0 - v_i^j\right)\left(v_i^j - v_{i-1}^j\right) + \frac{\Delta t}{T}\left(v_e\left(\rho_i^j\right) - v_i^j\right) + \Delta t G_{1,i}^j, \tag{8b}$$

where i, j are the road section and time interval, respectively; Δx, Δt are the lengths of the road section and time interval, respectively.

Before the simulation, we must define the equilibrium speed of Eq. (5). For simplicity, we define the equilibrium speed as follows [31]:

$$v_e(\rho) = v_f\left(\left(1 + \exp\left(\frac{\rho/\rho_j - 0.25}{0.06}\right)\right)^{-1} - 3.72 \times 10^{-6}\right), \tag{9}$$

where v_f is the free flow speed and ρ_j is the jam density.

Next, we must define the road capacity. Fig. 1 is the empirical data for two of Beijing's roads, showing that different roads have different capacities [32]. In addition, each road's capacity has some stochastic properties. For simplicity, we define the road capacity as follows:

$$C = C_{\text{average}} + \xi, \tag{10}$$

where C_{average} is the road's average capacity and ξ is a stochastic variable. ξ is often related to many factors (e.g., density) in a real traffic system, but it has no qualitative influences on the numerical tests, so we assume that it obeys the normal distribution in this paper. As for Eq. (10), we here give the following notes:

(1) There may be some prominent differences between the definition of the road capacity and the real road capacity. However, this is reasonable since it is accordant with the empirical data shown in Fig. 1.
(2) Each road has its own capacity, with stochastic properties (the empirical data have shown this property), so we can apply probability theory to define the road capacity as Eq. (10), which shows that Eq. (10) is reasonable.
(3) In this paper, the variables C, ξ are dependent on the variables x, t, since the road capacity may vary with the variables x, t.

Using Fig. 1, we can conclude the following:

(1) On the Weizikeng segment, $C_{\text{average}} = 2103$, ξ is a stochastic digit belonging to $(-99, 97)$.
(2) On the Beitaipingzhuang segment, $C_{\text{average}} = 1433$, ξ is a stochastic digit belonging to $(-30, 27)$.
(3) The road capacity of the Weizikeng segment is greater than that of the Beitaipingzhuang segment since its road condition is better [32].

In this paper, we assume that the road is a ring whose length is 30 km and that the road's midpoint has capacity limitations. For simplicity, other parameters are defined as follows [6,30]:

$$\begin{aligned} &T = 10\ \text{s}, \quad c_0 = 11\ \text{m/s}, \quad v_f = 30\ \text{m/s}, \quad \rho_j = 0.2\ \text{veh/m}, \\ &\mu = 0.1, \quad \Delta x = L = 100\ \text{m}, \quad \Delta t = 1\ \text{s}. \end{aligned} \tag{11}$$

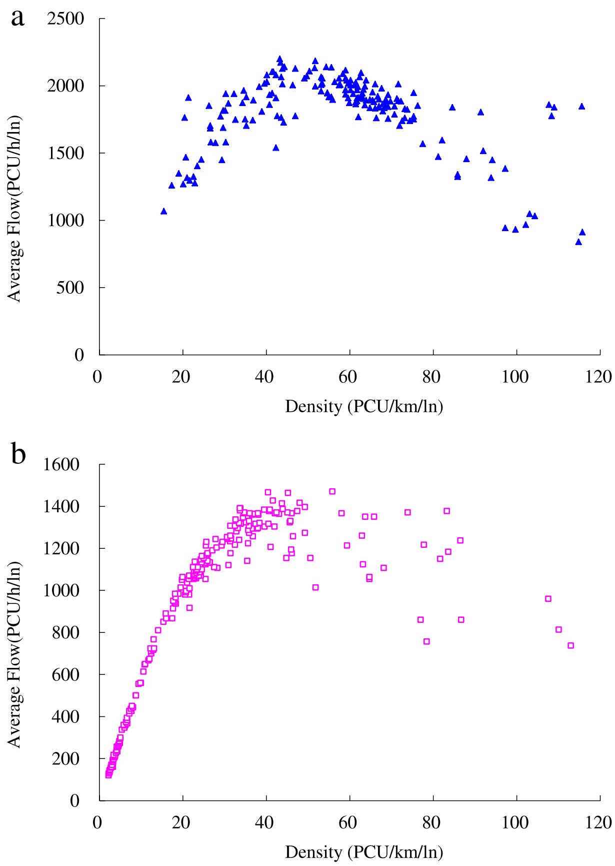

Fig. 1. The empirical data of two of Beijing's roads. (a) Weizikeng segment of the Badaling Highway. (b) Beitaipingzhuang segment of the third ring.

First, we study the impact of the road capacity on uniform flow in the Beitaipingzhuang segment and the Weizikeng segment (see Figs. 2 and 3). From the two figures, we can make the following conclusions:

(1) The road capacity has no prominent influence on uniform flow when the initial density is very low or very high (see Fig. 2(a), (e), Fig. 3(a), (d)). The lower critical density is 0.03 veh/m on the two segments, while the upper critical density on the Beitaipingzhuang segment is 0.07 veh/m and the upper critical density on the Weizikeng segment is 0.06 veh/m. The reasons are as follows: the road capacity of the Weizikeng segment is greater than the flow when the initial density is between 0.06 veh/m and 0.07 veh/m, so the road capacity has no influence on uniform flow; the road capacity on the Beitaipingzhuang segment is less than the flow when the initial density is between 0.06 veh/m and 0.07 veh/m, so the road capacity has impeding effects on uniform flow and will destroy the stability of uniform flow (see Fig. 2(d)).
(2) When the initial density is moderate, the road capacity has great effects on uniform flow and prominent stop-and-go traffic will occur (see Fig. 2(b)–(d), Fig. 3(b) and (c)). However, the stop-and-go traffic on the Beitaipingzhuang segment is slightly more serious than that on the Weizikeng segment under the same density because the road condition of the Weizikeng segment is better than that of the Beitaipingzhuang segment, which further validates that improving the road condition can improve the stability of traffic flow.
(3) The above traffic phenomena are qualitatively consistent with reality, showing that the proposed model can describe the impact of the road capacity on uniform flow from the qualitative perspective.

Next, we explore the effects of the road capacity on the reliability of traffic flow under uniform flow. Before exploring the impact, we should first define the reliability of traffic flow. Tang et al. [33] defined a traffic risk coefficient as follows:

$$\mu(\rho) = \sigma(\rho v)^{\theta-1}, \tag{12}$$

where μ is the traffic risk coefficient, and σ, θ are two coefficients. Based on the definition given by Tang et al. [33], the traffic risk coefficient can be regarded as the rear-collision probability.

In fact, when a rear-collision accident occurs in a single-lane system (the rear-collision probability is equal to 1), the system will be completely paralyzed, meaning that the traffic system's reliability is worst at this time (the reliability coefficient can be defined as zero). Based on the relationship between the rear-collision probability and the traffic system's

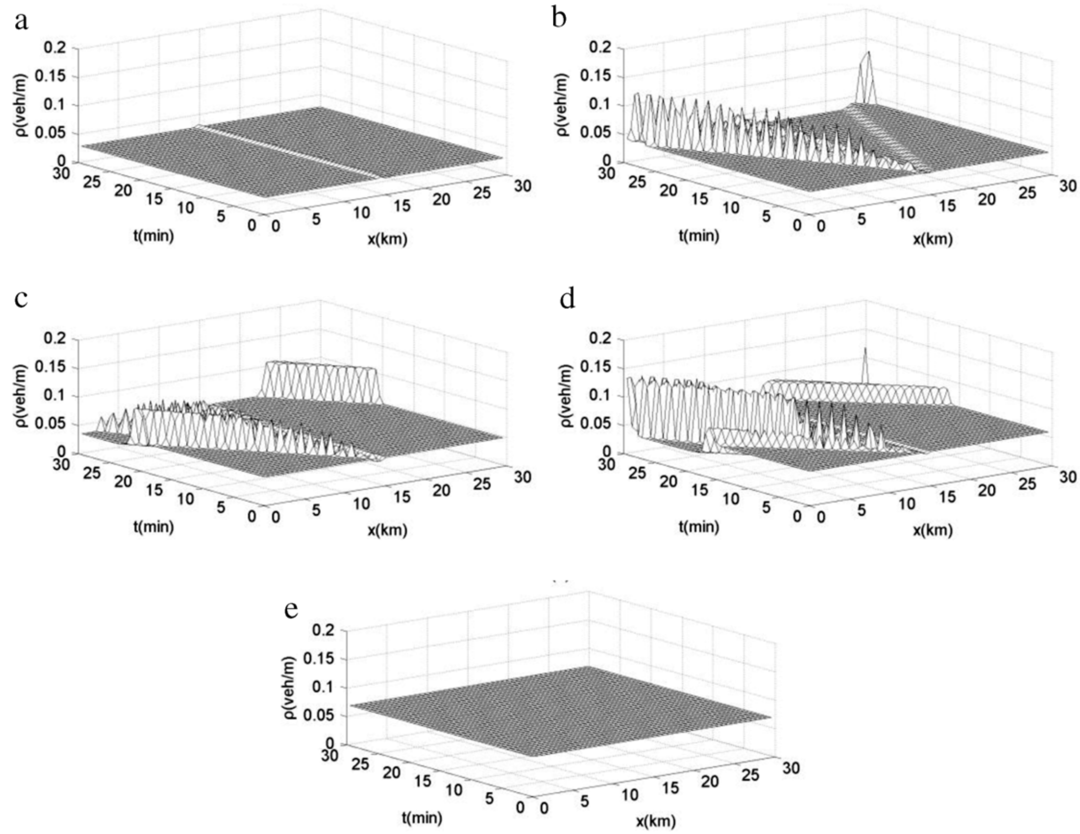

Fig. 2. The evolution of uniform flow on the Beitaipingzhuang segment under different initial densities, where the initial densities in (a)–(e) are respectively 0.03, 0.04, 0.05, 0.06 and 0.07.

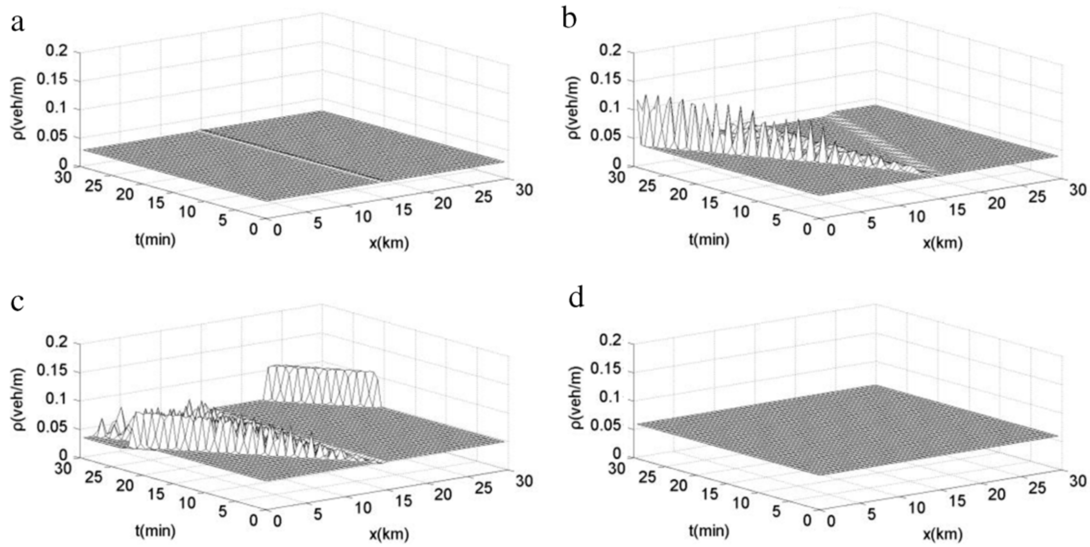

Fig. 3. The evolution of uniform flow on the Weizikeng segment under different initial densities, where the initial densities in (a)–(d) are respectively 0.03, 0.04, 0.05 and 0.06.

reliability, we can here define the traffic system's reliability coefficient as follows:

$$R(\rho) = 1 - \mu(\rho) = 1 - \sigma(\rho v)^{\theta - 1}, \tag{13}$$

where $R(\rho)$ is the traffic system's reliability coefficient.

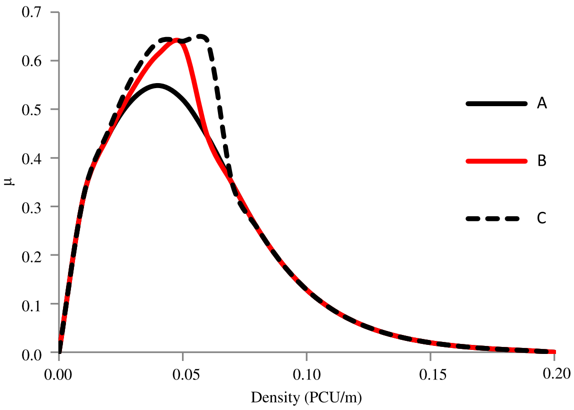

Fig. 4. The traffic risk coefficient under three different situations: (A) the traffic risk coefficient without road capacity, (B) the traffic risk coefficient on the Weizikeng segment and (C) the traffic risk coefficient on the Beitaipingzhuang segment.

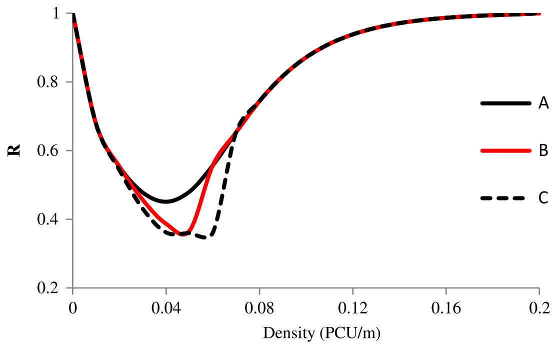

Fig. 5. The reliability coefficient under three different situations: (A) the reliability coefficient without road capacity, (B) the reliability coefficient on the Weizikeng segment and (C) the reliability coefficient on the Beitaipingzhuang segment.

Using Eqs. (12) and (13), we can obtain the traffic risk coefficient and the traffic system's reliability coefficient (see Figs. 4 and 5), where $\sigma = 0.01$, $\theta = 1.5$ [33] and other related parameters are the same as those in Figs. 2 and 3. From the two figures, we can obtain the following results:

(1) When the density is very low or very high, the road capacity has no effect on the traffic risk coefficient and the traffic system's reliability coefficient since the flow is less than the road capacity; when the density is moderate, the road capacity has an influence on the traffic risk coefficient and the traffic system's reliability coefficient because the road capacity is less than the flow: at this time, the traffic risk coefficient is increased and the traffic system's reliability coefficient is reduced (see Figs. 4 and 5).
(2) When the density is moderate, under the same density, the traffic risk coefficient of the Beitaipingzhuang segment is larger than that of the Weizikeng segment, while the traffic system's reliability coefficient of the Beitaipingzhuang segment is less than that of the Weizikeng segment (see Figs. 4 and 5), which shows that the Weizikeng segment is safer than the Beitaipingzhuang segment and that the reliability of the Weizikeng segment is higher than that of the Beitaipingzhuang segment.
(3) In addition, the above results show that improving the road condition can enhance both the traffic safety and the traffic system's reliability.

As for the double wave crests of curve C in Figs. 4 and 5, we give the following notes:

On the Beitaipingzhuang segment, other disturbance factors (e.g., bus stations) exist besides the interactions among vehicles; on the Weizikeng segment, no disturbance factors exist except the interaction among vehicles. This makes the stochastic variable ξ on the Beitaipingzhuang segment more complex than its equivalent on the Weizikeng segment. It is the stochastic variable ξ that produces the double wave crests of the traffic risk curve and reliability curve on the Beitaipingzhuang segment (see Figs. 4 and 5).

4. Conclusions

Although many traffic flow models have been developed to explore various complex traffic phenomena and obtained many important results, they cannot be used to directly study the effects of road capacity on traffic flow because they do not explicitly consider this factor. In this paper, we propose a macro traffic flow model to study the impact of road capacity on uniform flow (including the stability, the traffic risk coefficient and the traffic system's reliability). The numerical results show that the proposed model can describe some complex traffic phenomena due to the road capacity from a qualitative perspective.

However, this paper has the following limitations:

(1) Apart from the road capacity, the other parameters and equilibrium speed are not defined on the basis of empirical data.
(2) The proposed model is deduced directly from the existing models [6,21] and not established on the basis of empirical data or phenomena.
(3) We study only the effects of road capacity on uniform flow.

In view of the above limitations, we will use empirical data to study the influences of traffic flow (including non-uniform flow) and develop a more exact traffic flow model accounting for road capacity in the future.

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References

[1] D. Chowdhury, L. Santen, A. Schadschneider, Phys. Rep. 329 (2000) 199.
[2] D. Helbing, Rev. Modern Phys. 73 (2001) 1067.
[3] M.J. Lighthill, G.B. Whitham, Proc. R. Soc. Lond. Ser. A 229 (1955) 317.
[4] P.I. Richards, Oper. Res. 4 (1956) 42.
[5] H.J. Payne, in: G.A. Bekey (Ed.), Mathematical Models of Public System, in: Simulation Councils Proceedings Series, vol. 1, 1971, p. 51.
[6] R. Jiang, Q.S. Wu, Z.J. Zhu, Transp. Res. B 36 (2002) 405.
[7] A.K. Gupta, V.K. Katiyar, J. Phys. A 38 (2005) 4069.
[8] A.K. Gupta, V.K. Katiyar, Physica A 368 (2006) 551.
[9] A.K. Gupta, V.K. Katiyar, Physica A 371 (2006) 674.
[10] A.K. Gupta, V.K. Katiyar, Transportmetrica 3 (2007) 73.
[11] A.K. Gupta, S. Sharma, Chin. Phys. B 19 (2010) 110503.
[12] A.K. Gupta, S. Sharma, Chin. Phys. B 21 (2012) 015201.
[13] A.K. Gupta, Int. J. of Mod. Phys. C 24 (2013) 1350018.
[14] W.X. Zhu, Commun. Theor. Phys. 50 (2008) 753.
[15] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, Y. Sugiyama, Phys. Rev. E 51 (1995) 1035.
[16] D. Helbing, B. Tilch, Phys. Rev. E 58 (1998) 133.
[17] T. Nagatani, Phys. Rev. E 60 (1999) 6395.
[18] R. Jiang, Q.S. Wu, Z.J. Zhu, Phys. Rev. E 64 (2001) 017101.
[19] X.M. Zhao, Z.Y. Gao, Eur. Phys. J. B 47 (2005) 145.
[20] L.B. Meuleners, D. Hendrie, A.H. Lee, Traffic Injury Prevention 12 (2011) 201.
[21] W.X. Zhu, R.L. Yu, Physica A 391 (2012) 954.
[22] W.X. Zhu, L.D. Zhang, Physica A 391 (2012) 4597.
[23] H.X. Ge, X.P. Meng, J. Ma, S.M. Lo, Phys. Lett. A 377 (2012) 9.
[24] Q.N. La, A.H. Lee, L.B. Meuleners, D.V. Duong, Accident Analysis and Prevention 50 (2013) 451.
[25] N. Sugiyama, T. Nagatani, Phys. Lett. A 376 (2012) 1803.
[26] T. Nagatani, Appl. Math. Model. 37 (2012) 1823.
[27] T. Nagatani, Physica A 391 (2012) 3266.
[28] J.C Weng, L.L. Liu, J. Rong, Discrete Dyn. Nature Soc. (2013) 791743.
[29] H.Y. Shang, H.J. Huang, W.X. Wu, Int. J. Mod. Phys. C 23 (2012) 1250010.
[30] T.Q. Tang, H.J. Huang, S.C. Wong, Z.Y. Gao, Y. Zhang, Commun. Theor. Phys. 51 (2009) 71.
[31] B.S. Kerner, P. Konhäuser, Phys. Rev. E 48 (1993) R2335.
[32] T.Q. Tang, Y.P. Wang, X.B. Yang, Y.H. Wu, Nonlinear Dynam. 70 (2012) 1397.
[33] T.Q. Tang, H.J. Huang, H.Y. Shang, Y. Xue, Chin. Phys. Lett. 26 (2009) 108901.