UNIVERSITY OF BOLOGNA
*Department of Statistical Sciences*

*In Fulfillment of the Requirements for the Second cycle degree of Statistical, Financial and Actuarial Sciences*

# The Predictive Accuracy of Asymmetric GARCH Models During the COVID-19 Crisis: A Study of Stock Market Volatility Across Sectors

**Supervisor:** Prof. Giovanni Angelini
Department of Statistical Sciences

**Candidate:** Nicolò Amadori

Defense: December 12, 2024

Academic Year 2022/2023

Cari nonni, dedico questa tappa finale del mio percorso universitario, un traguardo che ho sempre sognato di raggiungere, sostenuto dal vostro amore e dalla vostra incrollabile fiducia in me.

Desidero scusarmi per il tempo che non vi ho dedicato, per i momenti in cui mi sono lasciato sopraffare dalle pressioni quotidiane, dimenticando l'importanza di fermarsi e apprezzare la vostra presenza. Vi prego di perdonarmi, perché ogni attimo insieme è un tesoro che voglio custodire nel cuore.

Al nonno Tony e alla nonna Mina, non riesco a trovare parole che possano esprimere la mia gratitudine per il ruolo fondamentale che avete nella mia vita. Siete il mio rifugio e la mia forza, e ogni volta che affronto le incertezze, mi ricordo di quanto siete stati un esempio di amore e resilienza. La vostra saggezza continua a guidarmi, e il calore della vostra presenza mi avvolge come un abbraccio, anche nei momenti più difficili.

Ai miei nonni paterni, che non sono più con noi, ma che continuano a vegliare su di me con affetto dall'alto. A voi, che mi avete insegnato a riconoscere il valore delle piccole cose, portandole nel profondo del mio essere. Quanto darei per un momento con voi, per sentirvi vicini, per un ultimo abbraccio e una ultima conversazione sincera, per poter guardare nei vostri occhi e vedere la gioia e l'orgoglio per il mio cammino.

La vostra eredità vive in me, e ogni passo che compio è un tributo al vostro amore, una celebrazione di ciò che siete stati e continuate a essere nella mia vita. Siete sempre nel mio cuore, e porterò con me i vostri insegnamenti mentre avanziamo insieme, in questo viaggio senza fine chiamato vita.

# Contents


**Thesis Details** **vi**

**I** **1**

**1 INTRODUCTION** **1**
1.1 RESEARCH INQUIRY . . . . . . . . . . . . . . . . . . . . . . . . 1

**II** **3**

**2 IMPACT OF THE COVID-19 PANDEMIC ON THE U.S. STOCK MARKET** **3**
2.1 Historical View . . . . . . . . . . . . . . . . . . . . . . . . . . 3
2.2 Impacts on the Stock Market and Volatility . . . . . . . . . . . . 3
2.3 Impacts on Different Economic Sectors . . . . . . . . . . . . . . . 6
2.4 Pharmaceutical Industry . . . . . . . . . . . . . . . . . . . . . . 6
2.5 So, What effects did the COVID-19 pandemic have on the U.S. stock market? . . . . . . . . . . . . . . . . . . . . . . . . . . 7

**III** **8**

**3 METHODOLOGY** **8**
3.1 Statistical Software in Thesis Development . . . . . . . . . . . . 8
3.2 Econometric tools used in the analysis . . . . . . . . . . . . . . . 8
3.2.1 GARCH($p$, $q$) . . . . . . . . . . . . . . . . . . . . . . 9
3.2.2 NGARCH($p$, $q$) . . . . . . . . . . . . . . . . . . . . . 9
3.2.3 TGARCH($p$, $q$) . . . . . . . . . . . . . . . . . . . . . 10
3.2.4 GJR GARCH($p$, $q$) . . . . . . . . . . . . . . . . . . . 11
3.2.5 EGARCH($p$, $q$) . . . . . . . . . . . . . . . . . . . . . 12
3.2.6 FIGARCH($p$, $q$) . . . . . . . . . . . . . . . . . . . . 12
3.3 Causal factors behind authors' adoption of these econometric models 13
3.4 Cluster Methodology . . . . . . . . . . . . . . . . . . . . . . . . 14
3.4.1 Cluster Analysis of Financial Time Series Based on Forecast Errors and Volatility Models . . . . . . . . . . . . . . . . . 14

**4 DESCRIPTIVE ANALYSIS** **16**
4.1 Descriptive Analysis of Sector ETFs Representing the S&P500 market 16
4.2 Absolute Autocorrelation Analysis . . . . . . . . . . . . . . . . . 19
4.3 Autocorrelation Analysis . . . . . . . . . . . . . . . . . . . . . . 22
4.4 Time series plots of the daily S&P 500 industry indices . . . . . . 24
4.4.1 General observations on the sectors during the 2016-2023 period . . . . . . . . . . . . . . . . . . . . . . . . . . 24
4.4.2 Sectors with the most negative impact during the pandemic 24
4.4.3 Sectors with more contained impacts or benefits . . . . . . 25
4.4.4 Energy sector: a prolonged downturn . . . . . . . . . . . . 26
4.4.5 Volatility clustering and persistence of volatility . . . . . . 26
4.4.6 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . 26
4.5 Distribution of S&P 500 Returns . . . . . . . . . . . . . . . . . . 27

**5 CONDITIONAL VOLATILITY** **29**
5.1 Sectoral ETF Performance During the 2020 COVID-19 Pandemic: A Comparative Analysis . . . . . . . . . . . . . . . . . . . . . . 32
5.1.1 Positive Performance and Strong Recovery . . . . . . . . . 32
5.1.2 Mixed or Weak Performance . . . . . . . . . . . . . . . . . 32
5.1.3 Negative Performance . . . . . . . . . . . . . . . . . . . . 33
5.1.4 Summary of Performance Analysis . . . . . . . . . . . . . 33
5.2 Identifying the Most Suitable Model for Financial Time Series Modeling . . . . . . . . . . . . . . . . . . . . 34
5.2.1 XLV Standardized Residuals . . . . . . . . . . . . . . . . . 34
5.2.2 XLY Standardized Residuals . . . . . . . . . . . . . . . . . 38
5.2.3 XLE Standardized Residuals . . . . . . . . . . . . . . . . . 42
5.3 Conditional Volatility Comparison (TGARCH Model) and Beta coefficients . . . . . . . . . . . . . . . . . . . . . . . . . . . . 48

**6 IDENTIFYING THE BEST FIT DISTRIBUTION FOR MODELING RETURNS** **50**
6.1 Is the Normal Distribution the Best Fit for Modeling Returns? . . 50
6.2 Comparison of Residual Distributions with and without GED . . 52
6.3 How Does the GED Respond to the Jarque-Bera Test? . . . . . . 54
6.4 Q-Q PLOT ETF . . . . . . . . . . . . . . . . . . . . . . . . . . 55

**IV** **58**

**7 SECTORS AND INDUSTRIES ANALYSIS** **58**
7.1 The role of MAPE in Forecast Accuracy . . . . . . . . . . . . . . 62
7.2 Correlations between COVID-19 and TGARCH model forecast errors 67
7.3 Single and Complete linkage Analysis . . . . . . . . . . . . . . . 68
7.4 Two-dimensional feature space maps . . . . . . . . . . . . . . . . 71

**8 CONCLUSION** **73**

**V** **76**

**9 REFERENCES** **76**
9.1 Webography . . . 76
9.1.1 Sector References . . . 76
9.2 Bibliography . . . 77
9.3 Appendix . . . 80
9.4 Disclaimer . . . 80

# Thesis Details

**Thesis Title:**
The Predictive Accuracy of Asymmetric GARCH Models During the COVID-19 Crisis: A Study of Stock Market Volatility Across Sectors

**Master Student:** Nicolò Amadori

**Supervisors:**
Prof. Giovanni Angelini, Department of Statistical Sciences, Unibo

The main body of this thesis consists of the following papers:

1. Jorge Caiado, Francisco Lúcio, **"Stock market forecasting accuracy of asymmetric GARCH models during the COVID-19 pandemic"** *The North American Journal of Economics and Finance*

In addition to the main papers, the following publications have also been made:

1. Personal Research on the Use of GARCH Models Based on Lars Stentoft's Paper , **" Pricing American options when the underlying asset follows GARCH processes"** *Journal of Empirical Finance 12 (2005)*
2. My bachelor's thesis reviewed by Professor Cristina Bernini, **"Big Pharma and the American Healthcare system"**

This thesis has been submitted for assessment in partial fulfilment of the Master degree. The thesis is based on the submitted or published scientific papers which are listed above. Parts of the papers are used directly or indirectly in the extended summary of the thesis.

# Abstract

This thesis, titled **"The Predictive Accuracy of Asymmetric GARCH Models During the COVID-19 Crisis: A Study of Stock Market Volatility Across Sectors"**, explores a new clustering methodology to analyze financial time series, applying it to investigate the effect of the COVID-19 pandemic on the U.S. stock market. I measure the forecast accuracy of asymmetric GARCH models applied to different S&P500 industries, utilizing forecast errors over various time horizons and threshold levels to build a distance matrix for stock indices. Hierarchical clustering techniques are then used to categorize the industries into distinct groups. My analysis identifies clusters of industries that share similar volatility patterns in response to COVID-19. Industries significantly impacted by the pandemic, which also exhibited less precise stock market predictions—such as Hotels, Airlines, Apparel, Accessories & Luxury Goods, and Automobiles—are distinct in Euclidean space from those that experienced a lesser impact and demonstrated more accurate forecasts, including Pharmaceuticals, Internet & Direct Marketing Retail, Data Processing, and Movies & Entertainment.

# Part I

# Chapter 1 INTRODUCTION

## 1.1 RESEARCH INQUIRY

The COVID-19 pandemic significantly impacted global financial markets, leading to high levels of uncertainty and volatility. In the United States, the first case was identified in January 2020, and by March, the World Health Organization had declared it a pandemic due to its rapid spread worldwide. The pandemic's uncertainty, including concerns over public health, governmental interventions, and changes in consumer and business behavior, caused notable fluctuations in U.S. stock market returns and an increase in market volatility. The U.S. stock market suffered a severe crash in March 2020, largely driven by government-imposed lockdowns and the temporary closure of many businesses as a response to the health crisis.

Various methodologies have been employed to analyze the impact of the COVID-19 pandemic on financial markets. Researchers have explored how news about the pandemic influenced market behavior, examining its effect on market volatility and stock prices. Some studies have focused on the pandemic's impact on different sectors, using advanced econometric models to capture the changes in market dynamics. These studies often utilize volatility models to predict future market behavior, testing different forecasting approaches to determine their effectiveness during periods of economic disruption. Others have assessed the predictive power of different economic indicators to forecast market volatility amid the ongoing uncertainty.

Despite the wide range of studies exploring the effects of the pandemic on financial markets, few have investigated using forecast errors from volatility models for clustering analysis. This thesis addresses this gap by introducing a novel clustering approach based on the forecast errors of volatility models. This approach aims to identify similarities and differences among industries listed on the stock market, providing insights into how different sectors responded to the economic impact of COVID-19.

In this research, we examine the effects of the COVID-19 pandemic on major S&P 500 industries, including Communication Services, Consumer Discretionary, Healthcare, Industrials, Energy, Financials, and Information Technology. We begin by analyzing the empirical characteristics and trends of various sub-industries within these categories. Next, we assess the forecast accuracy of Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH) models in predicting the behavior of these sub-industry indices. Our analysis involves calculating the mean absolute percentage errors (MAPE) for multi-step-ahead forecasts across several cut-off points during the pandemic period. Finally, we employ a model-based clustering method that utilizes the Euclidean distances between the forecast errors of TGARCH models to uncover common dynamic features among industries during the pandemic.

This study uses a clustering method to analyze time series with varying volatility over time, highlighting the dynamic behavior of financial markets. By focusing on out-of-sample forecast performance rather than traditional measures of model fit, this research aims to provide a more reliable tool for understanding market behavior. This approach allows us to identify industry clusters that share similar responses to the pandemic's economic shock, offering valuable insights into sector-specific volatility patterns.

Cluster analysis using stochastic volatility models is critical for several financial applications, including portfolio diversification, optimal asset allocation, and the study of market co-movements. Identifying similarities and differences among industrial sectors based on their response to the pandemic can provide valuable information for investors and policymakers, helping them make informed decisions in times of economic uncertainty. This thesis contributes to the literature by providing a comprehensive analysis of how different industries within the S&P 500 have been affected by COVID-19, using a novel clustering approach based on forecast errors.

# Part II

# Chapter 2 IMPACT OF THE COVID-19 PANDEMIC ON THE U.S. STOCK MARKET

## 2.1 Historical View

The COVID-19 pandemic, which emerged at the end of 2019 and was declared a global health emergency in the early months of 2020, had a devastating impact on many world economies, and the U.S. stock market was no exception. The first case of COVID-19 in the United States was recorded on January 21, 2020, marking the beginning of an unprecedented health and economic crisis. From the early stages of the pandemic, concerns about public health, economic uncertainty, and drastic containment measures adopted to limit the spread of the virus significantly influenced market behavior, with noticeable effects on stock price volatility. Beginning in mid-March 2020, stock markets experienced an unprecedented increase in volatility, which remained high even after a relative stabilization in April, still remaining above pre-pandemic levels. This phenomenon can be attributed to a combination of factors, including government-imposed restrictions such as lockdowns, travel bans, closure of non-essential businesses, and mandatory mask-wearing, as well as general uncertainty about the duration and severity of the pandemic itself.

## 2.2 Impacts on the Stock Market and Volatility

The increase in stock market volatility during the COVID-19 pandemic can be explained by various factors, including the rising number of infections and deaths, both globally and within the United States. Uncertainty surrounding the availability and effectiveness of vaccines, as well as the emergence of new virus variants, generated heightened anxiety among investors, leading to more intense and frequent market reactions. Additionally, the constant flow of news about the pandemic, often characterized by alarming or negative tones, further fueled uncertainty, negatively influencing stock returns and amplifying market fluctuations.

Table 2.1: Sectors' composition of the S&P500 index

| Sector | Industry / Sub-industry | TKR-ETF | Companies |
|---|---|---|---|
| **Communication Services** | Entertainment/Movies & Entertainment | XLC | Meta Platforms Inc (Class A) (META), Alphabet Inc (Class A) (GOOGL), Charter Communications Inc (Class A) (CHTR), Fox Corporation (class A) (FOXA), Fox Corporation (class B) (FOX), Live Nation Entertainment (LYV), Netflix (NFLX), The Walt Disney Company (DIS), ViacomCBS (VIAC) |
| **Consumer Discretionary** | Internet & Direct Marketing Retail/ Internet & Direct Marketing Retail | XLY | Amazon (AMZN), Booking Holdings (BKNG), eBay (EBAY), Etsy (ETSY), Expedia Group (EXPE) |
| | Automobiles/Automobile Manufacturers | XLY | Ford (F), Tesla (TSLA), GM (GM) |
| | Textiles, Apparel & Luxury Goods/Apparel, Accessories & Luxury Goods | XLY | Nike (NKE), PVH (PVH), Ralph Lauren Corporation (RL), Tapestry (TPR), Under Armour (class A) (UAA), Under Armour (class C) (UA), VF Corporation (VFC) |
| | Hotels, Restaurants & Leisure/Hotels, Resorts & Cruise Lines | XLY | Carnival Corporation (CCL), Hilton Worldwide (HLT), Marriott International (MAR), Royal Caribbean Group (RCL), Norwegian Cruise Line Holdings (NCLH) |
| | Hotels, Restaurants & Leisure/Restaurants | XLY | Chipotle Mexican Grill (CMG), Darden (DRI), Domino's (DPZ), McDonald's (MCD) |

| Sector | Industry / Sub-industry | TKR-ETF | Companies |
|---|---|---|---|
| **Healthcare** | Pharmaceuticals | XLV | Eli Lilly and Co (LLY), UnitedHealth Group Inc (UNH), Johnson & Johnson (JNJ), AbbVie Inc (ABBV), Merck & Co Inc (MRK), Thermo Fisher Scientific Inc (TMO), Abbott Laboratories (ABT), Danaher Corp (DHR), Pfizer Inc (PFE) |
| **Industrials** | Airlines/Airlines | XTN | Alaska Air Group (ALK), American Airlines Group (AAL), Delta Air Lines (DAL), Southwest Airlines (LUV), United Airlines (UAL) |
| **Energy** | Oil, Gas & Consumable Fuels/Oil & Gas Exploration & Production | XLE | Exxon Mobil Corp (XOM), Chevron Corp (CVX), EOG Resources Inc (EOG), APA Corporation (APA), Baker Hughes (BKR), Devon Energy (DVN), Diamondback (DVN), EOG Resources (EOG), Halliburton (HAL), Marathon Oil (MRO), Occidental Petroleum (OXY) |
| **Financial** | Banks/Diversified Banks | XLF | Berkshire Hathaway Inc (Class B) (BRK-B), The Goldman Sachs Group Inc (GS), Bank of America (BAC), Citigroup (C), JPMorgan Chase (JPM), U.S. Bancorp (USB), Wells Fargo (WFC) |

| Sector | Industry / Sub-industry | TKR-ETF | Companies |
|---|---|---|---|
| **Information Technology** | IT Services/Data Processing & Outsourced Services | FINX | Amdocs (DOX), ADP (ADP), Jack Henry & Associates (JKHY), Mastercard (MA), FIS (FIS), Paychex (PAYX), PayPal (PYPL), Visa (V) |

## 2.3 Impacts on Different Economic Sectors

The pandemic had varying effects on different economic sectors, with some suffering significant losses and others managing to capitalize on the new market conditions. For example, the airline industry experienced a drastic decline in passenger numbers due to travel restrictions and quarantine measures, leading to a significant reduction in revenue and a drop in stock prices of major airlines. Similarly, the hotel and cruise line sectors saw a sharp contraction in demand, with many operators facing extended periods of inactivity.

Conversely, the online retail sector saw exponential growth, as lockdown measures drove consumers to shop online rather than in physical stores. This trend benefited not only e-commerce retailers but also sectors related to logistics and digital payments. The entertainment sector experienced mixed impacts: while the closure of cinemas led to losses for theater chains, streaming platforms saw a significant increase in subscribers, driven by the rising demand for digital content.

The energy sector, particularly oil and gas, experienced significant fluctuations due to a drastic drop in global energy demand caused by economic slowdowns and travel restrictions. This led to a significant imbalance between supply and demand, resulting in volatile oil prices.

## 2.4 Pharmaceutical Industry

The pharmaceutical industry played a crucial role during the pandemic, both in the production of vaccines and therapies and in the provision of essential health products. While many economic sectors saw declining revenues and increased volatility, pharmaceutical companies often reported positive performance, becoming a focal point for investors seeking stability. The race to develop effective vaccines and the production of drugs for the treatment of COVID-19 led to increased research and development activities and greater investor confidence in the sector.

This context made pharmaceutical stocks relatively attractive, counteracting the negative trends observed in other sectors.

## 2.5 So, What effects did the COVID-19 pandemic have on the U.S. stock market?

The impact of the COVID-19 pandemic on the U.S. stock market was profound and varied, with volatility reaching historically high levels and sectors reacting differently depending on demand and supply dynamics. Containment measures, health and economic uncertainty, and investor sentiment played a crucial role in determining market fluctuations. While some sectors suffered heavy losses, others were able to adapt and even thrive in this new context. The pandemic highlighted the need for more robust risk management strategies and greater preparedness to face future global crises, emphasizing the importance of a diversified and flexible approach to investment management.

# Part III

# Chapter 3 METHODOLOGY

## 3.1 Statistical Software in Thesis Development

The analyses and presentation of the results in this study were carried out using the software R and Python. These tools were essential for data processing, generating forecasting models, and creating charts and tables that support the conclusions of the study. R was primarily used for statistical analysis, while Python facilitated data management and graphical visualization. The integration of these programs ensured precision and methodological rigor in the analysis of the phenomena studied.

## 3.2 Econometric tools used in the analysis

The paper explores various specifications of volatility that potentially manage asymmetric responses to positive and negative changes in returns. Although the GARCH model is known for handling conditional volatility in financial time series, there are other specifications in volatility models that deserve consideration. These alternative models aim to capture how volatility responds differently to changes in returns compared to what a classic GARCH model would predict.

Asymmetry in volatility responses refers to the fact that financial markets often exhibit different behaviors when negative changes occur compared to positive changes in stock prices or returns. Some specifications of these alternative models seek to capture this difference, recognizing that volatility may react differently to negative events compared to positive events.

Considering these asymmetries in the volatility response to price movements, such models offer advantages in better reflecting the reality of financial markets, particularly the leverage effect. The latter suggests that sharp decreases in stock prices are often accompanied by a more pronounced increase in volatility compared to sharp price increases. In other words, there is a tendency for stock prices to be negatively correlated with volatility, and these alternative models seek to capture such asymmetric dynamics in volatility to more accurately reflect the complexity of financial markets.

In this regard, the tools we observed are illustrated on the following page.

### 3.2.1 GARCH($p$, $q$)

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is employed to model conditional variance in financial or economic time series. This model is designed to capture the variation in volatility over time, allowing for the consideration of conditional heteroskedasticity, which refers to the variation of variance over time.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \tag{3.1}$$

Where:

- $\sigma_t^2$: conditional variance on time $t$,
- $\omega$: term of intercept,
- $\alpha_i$: coefficients of lagged standardized errors $(\varepsilon_{t-i}/\sigma_{t-i})$,
- $\beta_j$: coefficients of lagged conditional variances $(\sigma_{t-j}^2)$.

### 3.2.2 NGARCH($p$, $q$)

The first asymmetric GARCH model the authors consider is the Nonlinear Asymmetric GARCH (NGARCH) model. The NGARCH(p,q) model is a generalization of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, which includes a nonlinear component in modeling the conditional variance of a financial time series. The general formula for the NGARCH(p,q) model is expressed as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \varepsilon_{t-k}^2 \sigma_{t-k}^2$$

Where:

- $\sigma_t^2$ is the conditional variance on time $t$.

- $\omega$ term of constant.
- $\alpha_i$, $\beta_j$, e $\gamma_k$ are Model parameters to be estimated.
- $\varepsilon_{t-i}^2$ conditional squared error at time $t-i$.
- $\sigma_{t-j}^2$ The conditional variance squared at time $t-j$.
- $r$ Represents the number of lags used for the nonlinear term.

The nonlinear function is represented by the term $\gamma_k \varepsilon_{t-k}^2 \sigma_{t-k}^2$; This introduces a nonlinear dependency between the shock term and the conditional variance.

This model allows for a flexible modeling of the relationship between past errors and the conditional variance of returns. Unlike traditional GARCH models, NGARCH allows for non-linear forms in modeling volatility.

### 3.2.3 TGARCH($p$, $q$)

The TGARCH model, or Threshold GARCH, represents an evolution of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model. It distinguishes itself by its ability to consider the non-linear variation of conditional volatility in relation to a specific threshold or critical point.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{q} \gamma_k \left( \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \times I_{\{\frac{\varepsilon_{t-k}}{\sigma_{t-k}} < 0\}} \right). \tag{3.2}$$

Where:

- $\sigma_t^2$: is the conditional variance on time $t$,
- $\omega$: term of intercept,
- $\alpha_i$: coefficients of lagged standardized errors ($\varepsilon_{t-i}/\sigma_{t-i}$),
- $\beta_j$: coefficients of lagged conditional variances ($\sigma_{t-j}^2$),
- $\gamma_k$: coefficients of lagged standardized errors multiplied by their negative part:
  - $\varepsilon_{t-k}/\sigma_{t-k}$ multiplied by the indicator of the negative part of $\varepsilon_{t-k}/\sigma_{t-k}$.

Unlike traditional GARCH, TGARCH adds a fundamental element: a threshold term. This term enables the model to model conditional volatility non-uniformly concerning changes in returns. Essentially, TGARCH is designed to capture and more accurately reflect the difference in volatility response to positive and negative price movements.

The inclusion of this additional component allows the model to adapt to market conditions where asymmetric variations in volatility are observed, enabling the volatility process to react differently based on the direction and magnitude of price movements.

### 3.2.4 GJR GARCH($p$, $q$)

Another asymmetric model considered is GJR, where the variance process is defined by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{q} \gamma_k \varepsilon_{t-k}^2 I_{\{\varepsilon_{t-k}<0\}} \tag{3.3}$$

Where:

- $\sigma_t^2$: is the conditional variance on time $t$,
- $\omega$: term of intercept,
- $\alpha_i$: coefficients of lagged squared errors ($\varepsilon_{t-i}^2$),
- $\beta_j$: coefficients of lagged conditional variances ($\sigma_{t-j}^2$),
- $\gamma_k$: coefficients of lagged squared errors multiplied by the indicator of the negative part of the errors ($\varepsilon_{t-k}^2 I_{\{\varepsilon_{t-k}<0\}}$),
- $p$: order of squared errors,
- $q$: order of conditional variance.

Similar to the Threshold GARCH (TGARCH), it adds an additional term to the GARCH specification to capture asymmetries in conditional volatility, known as the leverage term. The parameter gamma determines the leverage effect: if $\gamma < 0$, indicates a stronger volatility response to negative shocks compared to positive ones.

The leverage effect, reflected by the parameter gamma in the models, indicates an asymmetric sensitivity of volatility to directional price movements. If gamma is less than zero, it highlights a pronounced volatility response to negative shocks compared to positive ones. This means that during significant price declines, volatility increases more rapidly. Conversely, a positive value of gamma suggests a more pronounced response of volatility to positive shocks compared to negative ones. Essentially, the leverage effect represents the tendency of volatility to react differently to opposite directional changes in prices.

The NGARCH and GJR models can be reduced (concept of 'nesting') to the standard GARCH.

### 3.2.5 EGARCH($p$, $q$)

An additional nonlinear econometric model considered is the Exponential GARCH (EGARCH), in which the volatility process is defined by:

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2) + \sum_{j=1}^{q} \alpha_j g(\varepsilon_{t-j})$$

Where

$$\alpha_t = \sigma_t \varepsilon_t$$

$$g(\varepsilon_t) = \theta \varepsilon_t + \gamma \left[ |\varepsilon_t| - E(|\varepsilon_t|) \right]$$

Where:

- $\sigma_t^2$: is the conditional variance on time $t$,
- $\alpha_0$: term of intercept,
- $\alpha_i$: coefficients of lagged of the standardized errors ($\varepsilon_{t-i}/\sigma_{t-i}$),
- $\beta_j$: coefficients of lagged conditional variances ($\log(\sigma_{t-j}^2)$),
- $\gamma$: coefficients of lagged standardized errors multiplied by their absolute value.

The EGARCH model allows capturing asymmetry in conditional volatility.

In fact, in the error equation $g(\varepsilon_t)$, $\theta$ and $\gamma$ are two parameters that allow estimating the asymmetric and symmetric effects of the shock. However, this model requires knowledge of the errors $\varepsilon_t$, which often are not Gaussian. The asymmetric impact is observed when, after estimating the GARCH(p, q) model, a non-zero parameter $\theta$ is obtained.

If the coefficient $\theta$ is between -1 and 0, it suggests that significant price decreases generate a more pronounced increase in volatility compared to price increases of equal magnitude, indicating the presence of the leverage effect.

The EGARCH model cannot be transformed or reduced to a standard GARCH model (concept of "non-nesting").

### 3.2.6 FIGARCH($p$, $q$)

The last alternative model considered is the Fractionally Integrated GARCH (FIGARCH), specified as:

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left( \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - \frac{\gamma_i}{\sqrt{2\pi}} \right) + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{k=1}^{q} \gamma_k \log(\sigma_{t-k}^2) \times I_{\{\sigma_{t-k}^2 < 0\}}. \tag{3.4}$$

Where:

- $\sigma_t^2$: is the conditional variance on time $t$,
- $\alpha_i$: coefficients of lagged of the standardized errors $(\varepsilon_{t-i}/\sigma_{t-i})$,
- $\beta_j$: coefficients of lagged conditional log-variances $(\log(\sigma_{t-j}^2))$,
- $\gamma_k$: coefficients of lagged conditional log-variances multiplied by the indicator
$$\log(\sigma_{t-k}^2) \times I_{\{\sigma_{t-k}^2<0\}}.$$

The FIGARCH model extends traditional GARCH models by considering the possibility of long-term memory in financial data. This model introduces a fractional integration parameter, which can vary between 0 and 1. When it is between 0 and 0.5, it indicates long-term memory with slow decay, meaning a persistent long-term influence of shocks in the time series. If it is exactly 0.5, there is long-term memory but with faster decay compared to values below 0.5. When it is between 0.5 and 1, it still indicates long-term dependence but with faster decay, showing less persistent memory compared to values closer to 0.

In essence, this parameter in the FIGARCH model reflects how long the impacts of past events influence the time series, allowing capturing long-term dependencies in financial data.

## 3.3 Causal factors behind authors' adoption of these econometric models

The use of econometric tools such as NGARCH, GJR, TGARCH, EGARCH, and FIGARCH entails various reasons:

- Complex GARCH Processes: These models provide sophisticated tools for modeling the complexity of GARCH processes. As these processes are known for their ability to capture volatility in financial markets, the use of advanced models may allow for better adaptability to real data and increased precision in volatility modeling.
- Management of Volatility Asymmetries: Some of these tools, such as EGARCH and TGARCH, handle volatility asymmetries. This is crucial in financial options, where volatility changes in an upward or downward direction can impact option prices differently.
- Identification of Long-Term Phenomena: The FIGARCH model captures long-term phenomena in volatility, such as persistence or long-term memory

in financial time series. This can influence the analysis of American options, where the long-term dynamics of volatility can affect prices.

- Response to Extreme Events or Shocks: Models like GJR are useful for modeling sudden events in financial markets. This is relevant in the analysis of American options, where sudden changes or moments of high volatility can impact option prices.

## 3.4 Cluster Methodology

After establishing that the TGARCH model is the most suitable due to its superior ability to capture volatility asymmetries, thus providing a more accurate and detailed representation of the underlying dynamics in financial data, our research now shifts to the study of clustering financial time series based on forecast errors and volatility models.

### 3.4.1 Cluster Analysis of Financial Time Series Based on Forecast Errors and Volatility Models

In financial time series analysis, clustering based on time and frequency domain methods has been widely studied. One of the most recent contributions (Lício and Caiado, 2022) introduces a new distance metric based on autocorrelations of time-varying volatility models using the TGARCH (Threshold Generalized Autoregressive Conditional Heteroscedasticity) model. This method measures similarities between different industries' volatility during the COVID-19 pandemic by comparing forecast errors from multi-step-ahead predictions of stock market indices.

In this study, the authors extend Lício and Caiado's work by comparing and clustering financial time series according to their forecast error structures. They use TGARCH models to forecast stock market indices across various industries and evaluate the accuracy of these forecasts using the Mean Absolute Percentage Error (MAPE), which is defined as:

$$MAPE = \frac{1}{n-m} \sum_{t=m+1}^{n} \left| \frac{Y_t - F_t}{Y_t} \right| \times 100$$

where $Y_t$ is the actual value, and $F_t$ is the forecasted value. This formula computes the out-of-sample forecast accuracy based on a training sample and gradually incorporates future time periods into the training process.

For clustering, forecast errors are used to calculate dissimilarities between time series, represented by the ARMA-TGARCH process:

$$r_t = \mu + \sum_{i=1}^{q} \theta_i r_{t-i} + \epsilon_t$$
$$\epsilon_t = \sigma_t z_t$$
$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_1 I_{\epsilon_{t-1}<0})\epsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

Here, $\epsilon_t$ represents the innovations, and $\sigma_t$ is the conditional variance, capturing how bad news ($\epsilon_t < 0$) affects volatility more severely than good news ($\epsilon_t > 0$).

To compute the distance between forecast errors from different industries, the following formula is employed:

$$d_{MAPE}(x, y) = \sqrt{(MAPE_x - MAPE_y)' \mathbf{I}_\Theta (MAPE_x - MAPE_y)}$$

where $\mathbf{I}_\Theta$ is the identity matrix if errors are treated equally, or a weighted matrix if considering the variance of the errors. This distance can also be computed using the Mahalanobis distance or a variance-weighted distance, making it robust to different forecasting methods and time series behaviors.

The clustering approach considers different linkage methods:

- **Single linkage (nearest neighbor)** minimizes the distance between clusters.
- **Complete linkage (furthest neighbor)** maximizes the distance between clusters.

The study concludes by examining the dynamic relationships between industries' volatility and clustering these industries based on forecast errors. The dendrogram visualizes how clusters form and join, ultimately showing that industries hit hardest by the pandemic tend to form distinct clusters with similar volatility characteristics. The study emphasizes the utility of combining model-based and feature-based clustering methods for deeper insights into financial time series data.

# Chapter 4 DESCRIPTIVE ANALYSIS

## 4.1 Descriptive Analysis of Sector ETFs Representing the S&P500 market

During the observation period from January 1, 2016, to December 31, 2023, encompassing a total of 2,012 observations, important diagnostic findings emerged regarding the financial returns analyzed. First, the hypothesis of normality is rejected for all return series, consistent with the findings of the reference paper. This result is confirmed by the Jarque-Bera tests, a normality test that measures the deviation in skewness and kurtosis from a normal distribution. The low p-values obtained from this test indicate a significant departure from normality, suggesting that the returns do not follow a normal distribution, as evidenced by the leptokurtosis observed in all data series.

**Leptokurtosis**, markedly observed in the returns of the financial, energy, and healthcare sectors, is a phenomenon indicating a distribution with "fatter tails" compared to the normal distribution, meaning the presence of more frequent extreme events (kurtosis $> 3$). This implies that returns have a higher probability of deviating significantly from the mean, both positively and negatively. The ETFs of the three sectors mentioned above, in particular, exhibit more pronounced kurtosis than those of the other sectors within the S&P 500. This phenomenon can indicate a greater concentration of returns around the mean, but with the possibility of more pronounced extreme events.

Another relevant aspect is the **negative skewness** observed across all seven sectors under investigation. Negative skewness indicates that the distribution of returns is asymmetric, with a longer left tail compared to the right. This suggests that extreme negative returns are more frequent or severe than extreme gains, leading to a higher risk of losses for investors. In summary, negative skewness represents a higher risk of significant negative events, even when the overall distribution of returns is predominantly positive.

Regarding autocorrelation, the results of the **Ljung-Box test** confirm the

Table 4.1: ETFs descriptive statistics

| Statistics | XLC | XLY | XLV |
|---|---|---|---|
| **Mean** | 0.00042 | 0.00056 | 0.00045 |
| **Standard Deviation** | 0.01525 | 0.01388 | 0.01079 |
| **Skewness** | -0.35510 | -0.59943 | -0.22122 |
| **Excess Kurtosis** | 5.67003 | 8.45512 | 9.98610 |
| **Max** | 0.08990 | 0.09379 | 0.07705 |
| **Min** | -0.11279 | -0.12668 | -0.09861 |
| **ACF** | [ 1, -0.12317...] | [ 1, -0.06542...] | [ 1, -0.11644...] |
| **ACF Abs** | [1, 0.26368...] | [1, 0.30462...] | [1, 0.33901...] |
| **JB Test** | (1893.91, 0.0) | (6110.62, 0.0) | (8372.30, 0.0) |
| **LB-Q(20)** | (21.16, 4.212e-06) | (8.62, 3.321e-03) | (27.30, 1.733e-07) |
| | (25.23, 3.310e-06) | (19.91, 4.730e-05) | (45.59, 1.259e-10) |
| | (27.20, 5.329e-06) | (20.86, 1.122e-04) | (47.09, 3.321e-10) |
| | (29.17, 7.187e-06) | (21.10, 3.011e-04) | (55.37, 2.709e-11) |
| | (29.25, 2.067e-05) | (21.77, 5.775e-04) | (60.41, 9.986e-12) |
| **LB-Q(20) Squared** | (248.35, 5.939e-56) | (149.73, 1.985e-34) | (464.34, 5.452e-103) |
| | (461.30, 6.755e-101) | (542.05, 4.730e-05) | (990.54, 8.047e-216) |
| | (538.44, 2.221e-116) | (702.98, 1.966e-118) | (1222.00, 1.234e-264) |
| | (626.51, 2.830e-134) | (833.09, 5.211e-179) | (1429.89, 2.270e-308) |
| | (692.11, 2.493e-147) | (934.78, 7.891e-200) | (1738.97, 0.000e+00) |

presence of serial correlation in both returns and absolute returns. The Ljung-Box test assesses whether there is temporal dependence between observed values in a time series, by evaluating the correlation between the lags of the series. In our analysis, the extremely low p-values, close to zero, provide strong statistical evidence of autocorrelation, suggesting that past returns significantly influence future returns. The squared Ljung-Box test (Ljung-Box on squared returns) revealed even stronger serial correlation, indicating the presence of ARCH effects and persistent volatility in absolute returns—a typical phenomenon in financial markets characterized by volatility clustering. Again, the notably low p-values compared to the original paper confirm greater statistical significance in our analysis.

Table 4.2: ETFs descriptive statistics

| XTN | XLE | XLF | FINX |
|---|---|---|---|
| 0.00051 | 0.00053 | 0.000527 | 0.00049 |
| 0.01616 | 0.01984 | 0.01479 | 0.01822 |
| -0.26423 | -0.40579 | -0.22929 | -0.30171 |
| 5.47189 | 13.07091 | 14.16277 | 5.25331 |
| 0.12043 | 0.16037 | 0.13156 | 0.11154 |
| -0.10891 | -0.20141 | -0.13709 | -0.12840 |
| .[ 1, -0.00822...] | [ 1, -0.05275...] | [ 1, -0.12281...] | [ 1, 0.00224...] |
| .[1, 0.25075... ] | [1, 0.27424...] | [1, 0.42339...] | [1, 0.29940...] |
| .(2532.26, 0.0) | (14370.90, 0.0) | (16824.89, 0.0) | (2139.04, 0.0) |
| **LB** (0.13, 7.121e-01) | (5.60, 1.791e-02) | (30.37,3.557e-08) | (0.00, 9.231e-01) |
| .(5.48, 6.454e-02) | (10.48, 5.286e-03) | (54.17, 1.725e-12) | (4.09, 1.292e-01) |
| .(7.26, 6.401e-02) | (10.63, 1.384e-02) | (54.69, 7.993e-12) | (4.89,1.799e-01) |
| .(10.46, 3.324e-02) | (10.66, 3.060e-02) | (64.94, 2.643e-13) | (8.46, 7.589e-02) |
| .(11.72, 3.872e-02) | (14.73, 1.155e-02) | (69.17, 1.524e-13) | (9.61, 8.687e-02) |
| .(121.17, 3.493e-28) | (78.66, 7.347e-19) | (545.21, 1.382e-120) | ( 160.38, 9.333e-37) |
| .(375.65, 2.682e-82) | (187.96, 1.528e-41) | ( 940.55, 5.765e-205) | (544.92,4.696e-119) |
| .(553.65, 1.123e-119) | (372.18, 2.344e-80) | (1187.42, 3.918e-257) | (685.51, 2.904e-148) |
| .(772.98, 5.446e-166) | (592.25, 7.343e-127) | (1485.67, 0.000e+00) | (920.00, 7.722e-198) |
| .(879.25, 8.231e-188) | (745.36, 7.607e-159) | (1485.67, 0.000e+00) | (1047.10, 3.803e-224) |

In conclusion, the results show that the financial return series analyzed exhibit a non-normal distribution, with negative skewness and pronounced leptokurtosis. The autocorrelation in both returns and absolute returns, confirmed by the Ljung-Box and squared Ljung-Box tests, strengthens the evidence of persistent volatility effects, while negative skewness highlights a greater vulnerability to extreme losses. This complex of diagnostic evidence underscores the risk and volatility characteristics present in the financial return series analyzed.

## 4.2 Absolute Autocorrelation Analysis

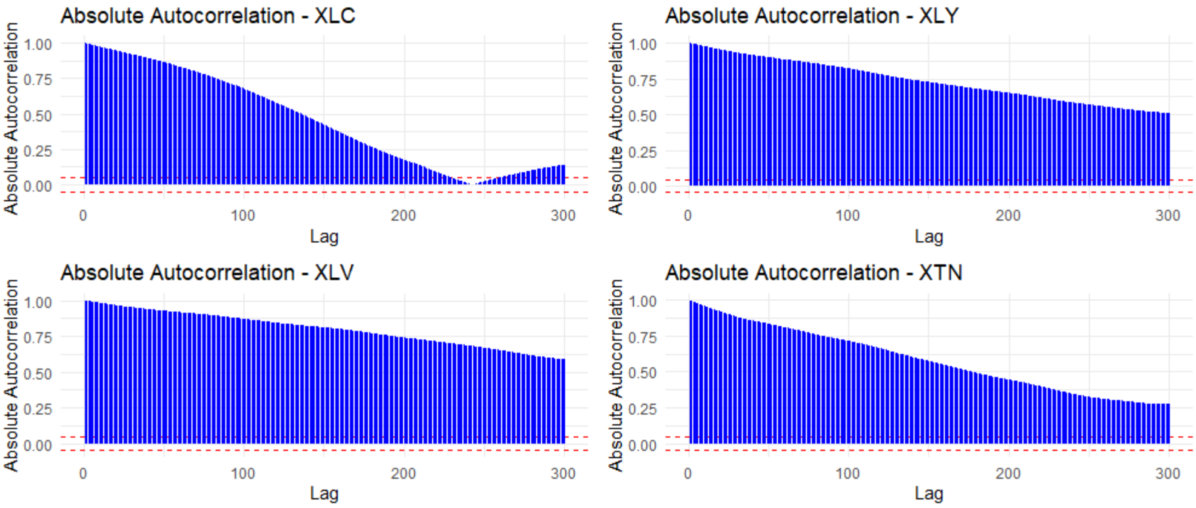


Figure 4.1: XLC-XLY-XLV-XTN Absolute Autocorrelation

The analysis of absolute autocorrelations in the returns of S&P500 sector ETFs, as shown in the provided charts, highlights significant temporal dynamics and a notable structural persistence in the time series of returns. The graphs display the absolute autocorrelation of returns for different sector categories, including Communication Services (XLC), Consumer Discretionary (XLY), Healthcare (XLV), Industrials (XTN), Energy (XLE), Financials (XLF), and Information Technology (FINX), for lags up to a maximum of 300 periods.

The absolute autocorrelations show, for all sectors examined, a strong correlation in the initial lags, with values above 0.75. This indicates that recent absolute returns tend to be highly correlated, suggesting short-term persistence. However, as the lags increase, the autocorrelation gradually decreases, although not uniformly. This phenomenon suggests that the memory of the time series dissipates but not quickly, highlighting a form of dependence that extends over time.

Despite the gradual reduction, it is interesting to note that autocorrelation remains positive and significant even for lags beyond 200 periods. This suggests a certain long-term persistence in the time series of absolute returns, consistent with the presence of phenomena such as autoregressive conditional heteroskedasticity (ARCH), where volatility tends to cluster in specific periods. Such persistence may indicate the presence of an intrinsic temporal structure in the return series.

Analyzing the various sectors, some relevant differences can be observed. For instance, the XLC ETF, representing the Communication Services sector, shows a more rapid decline in autocorrelations compared to other sectors, reaching very low values (close to zero) around 200 lags. This suggests that the temporal memory of returns in this sector is relatively short. In contrast, the XLY (Consumer Discretionary) and XLV (Healthcare) ETFs exhibit greater persistence, with autocorrelations remaining above 0.25 even beyond 300 lags, indicating a more pronounced temporal dependence in these sectors.

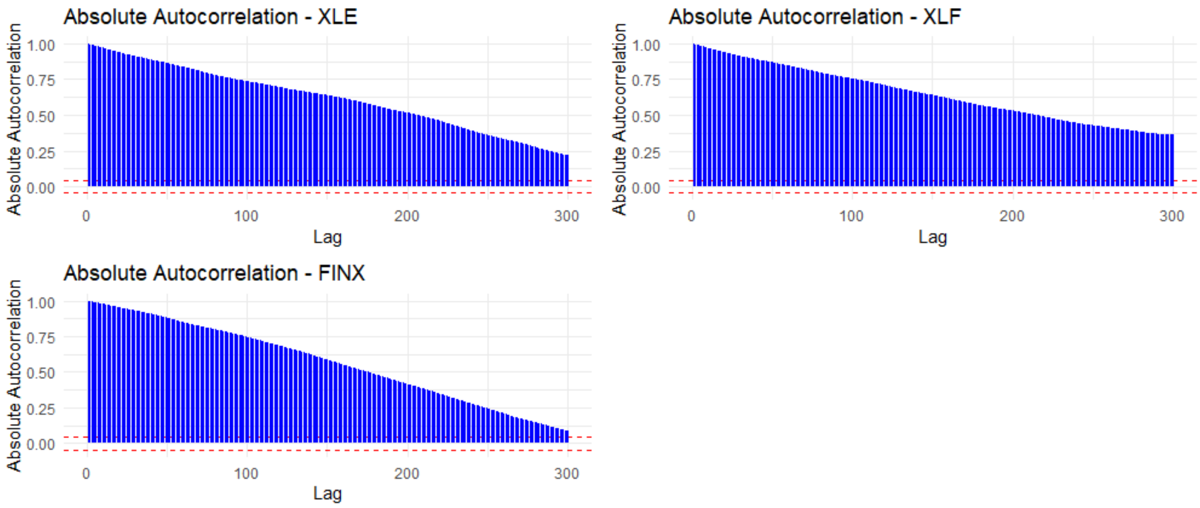


Figure 4.2: XLE-XLF-FINX Absolute Autocorrelation

The Energy (XLE) and Financials (XLF) ETFs show similar behavior, characterized by a more gradual reduction in autocorrelation compared to other sectors. This indicates that returns in these sectors are influenced by more persistent temporal dynamics. The FINX ETF, representing the Information Technology sector, also shows a slower decline in autocorrelations, indicating the presence of more pronounced structural dependencies over time. The Industrials (XTN) sector, finally, shows persistence comparable to the financial and energy sectors, suggesting that returns here are also influenced by past events over a longer period.

From an econometric standpoint, this analysis confirms the presence of significant serial correlation in absolute returns. The frequent crossing of the confidence bands in the charts indicates a clear sequential dependence in the returns, implying that current returns are partly influenced by past movements. This is a particularly relevant result in volatility modeling, as serial correlation is often an indicator of volatility clustering, a characteristic typically associated with ARCH and GARCH models.

Finally, the evidence of stronger serial correlation in absolute returns compared to the original untransformed returns is consistent with the idea that volatility manifests through periods of increased intensity, followed by calmer periods—a dynamic that might not be apparent in the raw return series.

In conclusion, this analysis of absolute autocorrelations for ETFs suggests that there is persistent temporal dependence in many sectors of the market. The more pronounced persistence in the financial, energy, and technology sectors indicates that these segments may be influenced by long-term macroeconomic factors or internal dynamics. These results highlight the importance of considering temporal structure in volatility models, particularly in portfolio management and risk management strategies.

## 4.3 Autocorrelation Analysis

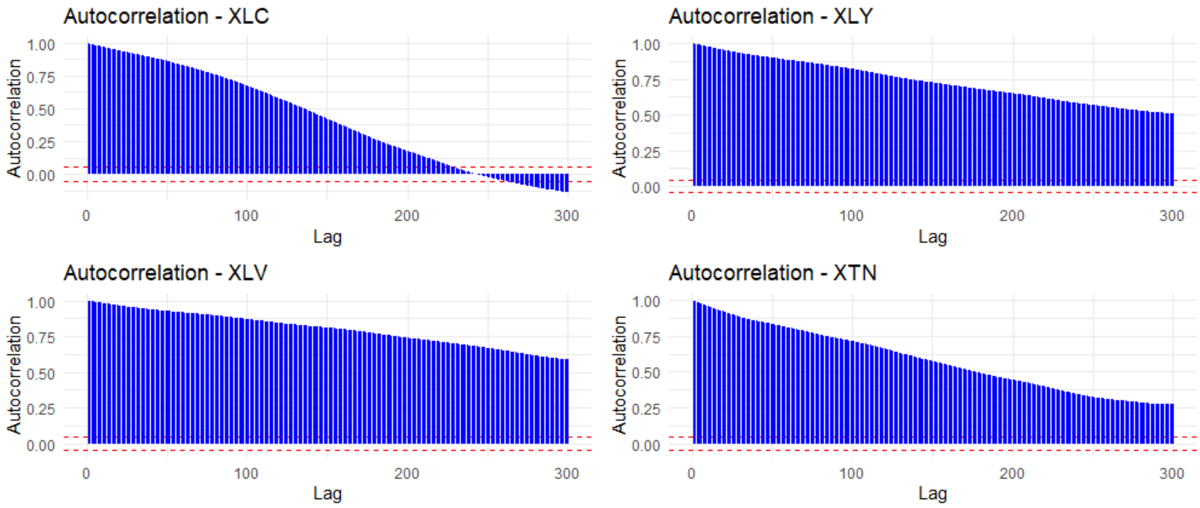


Figure 4.3: XLE-XLF-FINX Autocorrelation

The analysis of autocorrelations in the returns of S&P500 sector ETFs, as represented in the provided charts, highlights significant temporal dynamics within each sector. The charts show how the autocorrelation of returns gradually decreases as the lags increase, although not uniformly, suggesting that the time series are influenced by past events, with memory that tends to persist for a certain period.

In particular, the charts show that the first lags exhibit significant correlation, with autocorrelation values starting close to 1 and decreasing over time. This implies that the returns of previous observations have a strong impact on future returns in the short term. This type of behavior is consistent with temporal persistence phenomena, such as market trends or cyclicality, indicating that shocks or events affecting the market have an effect that lasts beyond the initial observation.

As the lags increase, autocorrelations tend to decrease, gradually falling within the confidence bands, suggesting that the influence of past events diminishes in the long term, though not entirely disappearing. This is especially evident in the sectors represented by the ETFs XLE (Energy), XLF (Financials), and FINX (Information Technology), where autocorrelation remains positive and significant for a larger number of lags compared to other sectors. Such behavior could reflect the nature of these sectors, often characterized by macroeconomic or technological factors that have more persistent effects over time.

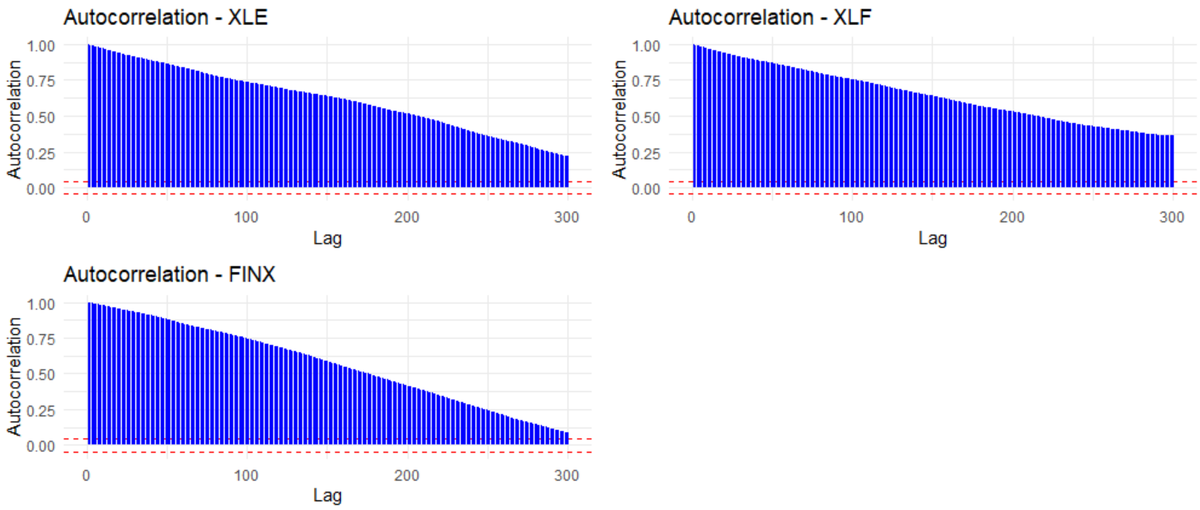


Figure 4.4: XLE-XLF-FINX Autocorrelation

On the other hand, sectors such as Communication Services (XLC) and Consumer Discretionary (XLY) show a faster decline in autocorrelations, suggesting a lower persistence of shocks or a shorter temporal memory. This could be due to greater sensitivity in these sectors to short-term events, such as changes in consumer preferences or fluctuations in the economic cycle.

The presence of autocorrelations in returns can be interpreted as a signal of the dependence of future observations on past values, which could have various implications. From an econometric perspective, strong autocorrelation in returns indicates the possibility of more effective forecasting models based on historical data. Moreover, the persistence of autocorrelations could suggest the opportunity to exploit investment strategies based on such dynamics, especially in contexts where investors seek to benefit from trend-following or mean-reversion phenomena.

Finally, the presence of autocorrelation in returns can also be linked to aggregate volatility over time (volatility clustering), a phenomenon characterizing many financial markets. The persistence of volatility can influence perceived risk by investors and impact risk management strategies. In conclusion, the analysis of autocorrelations in the returns of S&P500 sector ETFs shows how past events influence market behavior, with effects that can persist over time, varying by sector. These observations suggest that the nature of the dynamics within each sector can have important implications for return forecasting and portfolio management.

## 4.4 Time series plots of the daily S&P 500 industry indices

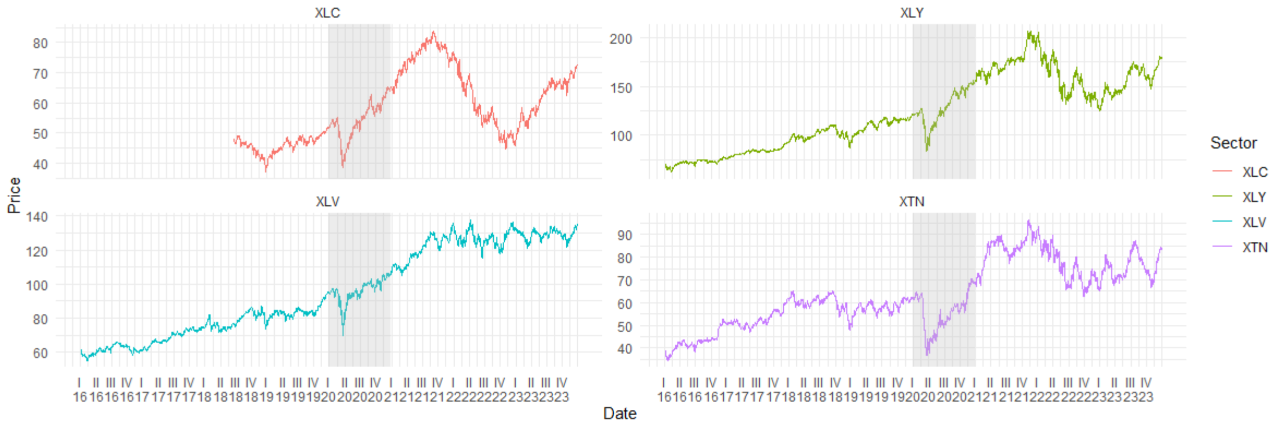


Figure 4.5: XLC-XLY-XLV-XTN Daily Returns

Analyzing the attached charts, which represent the daily returns of seven key sectors of the S&P500 (XLC, XLY, XLV, XTN, XLE, XLF, and FINX) from the first quarter of 2016 to the fourth quarter of 2023, we can draw several interesting conclusions about market movements and volatility that characterized this period.

### 4.4.1 General observations on the sectors during the 2016-2023 period

General observations on the sectors during the 2016-2023 period Overall, daily returns show some stability until 2020, when the COVID-19 pandemic disrupted global markets. This period is clearly visible in the charts, with a marked increase in volatility and a significant drop in prices across almost all sectors. This phenomenon of "volatility clustering," where periods of low volatility are followed by phases of high volatility, is typical during times of economic crisis or in the presence of exogenous shocks such as the pandemic.

### 4.4.2 Sectors with the most negative impact during the pandemic

The most evident effect of the COVID-19 pandemic can be observed in 2020. The most affected sectors seem to be FINX (Information Technology) and XLY (Consumer Discretionary), where a substantial drop in value is noted. These sectors, particularly sensitive to economic changes and lockdown measures, suffered an immediate impact. In the case of the technology sector, a sharp decline is observed, followed by a relatively quick recovery, indicative of the resilience and subsequent boom in demand for digital services and products.

Specifically, the FINX line shows a notable peak before the COVID-induced crash but also a strong recovery in the following years, likely due to the acceleration of digital transformation and the increasing reliance on technology during and after the lockdown.

The XLY sector, which includes consumer discretionary goods, experienced a sharp decline due to pandemic-related restrictions, which reduced spending on non-essential goods. The high volatility and steep declines indicate how vulnerable these sectors were to sudden changes in consumer demand.

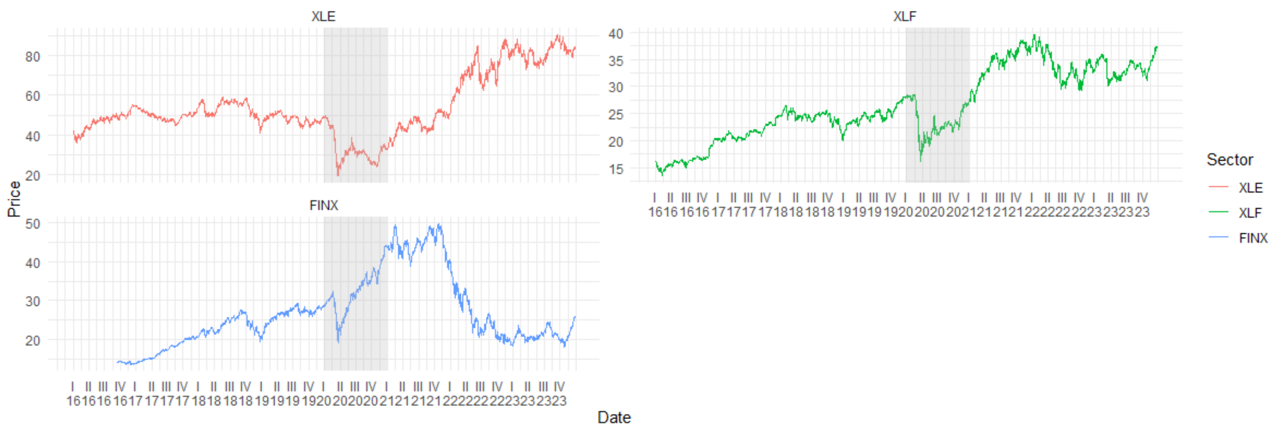


Figure 4.6: XLE-XLF-FINX Daily Returns

### 4.4.3 Sectors with more contained impacts or benefits

In contrast, XLV (Healthcare) and XLF (Financials) showed greater resilience. The healthcare sector benefited from the pandemic due to the increasing demand for medical services, vaccines, and pharmaceuticals, largely reflecting the stability and importance of this sector during a global health crisis. The XLV curve shows relative stability compared to other sectors, with a quick recovery after the initial drop. This suggests that investors saw healthcare as a safe haven, recognizing its central role during the crisis.

XLF (Financials) exhibits an interesting behavior. Although the financial sector felt the initial impact of the COVID crisis, with a visible drop in 2020, the recovery was fairly rapid. However, the financial sector's recovery was more gradual than other sectors, likely due to uncertainty about global economic conditions and the impact of monetary policies adopted by central banks to counter the pandemic's effects.

### 4.4.4 Energy sector: a prolonged downturn

The XLE (Energy) sector suffered one of the most drastic losses during 2020, highlighting its sensitivity to global economic shocks and energy demand. The collapse of oil prices, caused by both reduced global demand and supply issues, hit the sector hard. The XLE curve shows a significant decline in the months following the outbreak of the pandemic, reflecting the combined effect of decreased energy demand and the drop in commodity prices. The recovery has been slower and more gradual compared to other sectors, indicating the complexity of structural challenges the energy sector faced.

### 4.4.5 Volatility clustering and persistence of volatility

The presence of periods characterized by high volatility followed by periods of lower stability is visible in almost all sectors, particularly from 2020 onwards. This feature, known as "volatility clustering," is typical in financial markets during times of crisis. Events such as the pandemic triggered volatility clustering, with a prolonged impact on markets, signaling a period of uncertainty and heightened investor reactivity to global economic and health developments. Even after the initial crisis phase, markets continued to show volatility, especially in sectors more sensitive to the evolution of the pandemic and economic policies.

### 4.4.6 Conclusions

The analysis of the charts clearly shows that COVID-19 had a significant impact on the daily returns of the main S&P500 sectors. Sectors like XLY and FINX were hit hard during the initial phase of the pandemic, with a rapid recovery afterward, while the energy sector XLE faced a longer and more severe impact. The healthcare sector XLV showed remarkable resilience, reflecting the importance of healthcare during the pandemic.

The "volatility clustering" behavior observed in the charts suggests that, even during periods of relative economic recovery, volatility remains a dominant feature in the markets, especially during times of global crises.

## 4.5 Distribution of S&P 500 Returns

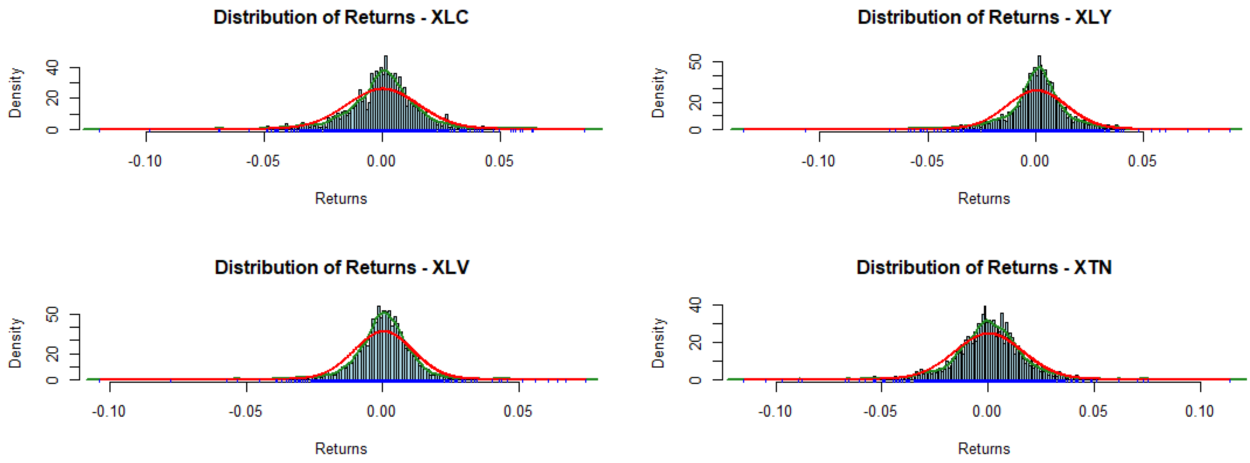


Figure 4.7: XLC-XLY-XLV-XTN distribution of returns

The analysis presented in Table 4.1, along with the preceding graphs, clearly shows that the returns of the studied stocks do not follow a normal distribution. This finding is supported by the high kurtosis observed in all seven series representing sectors within the S&P500, with particular emphasis on ETFs. Elevated kurtosis indicates the presence of fat tails in the return distribution, implying a higher likelihood of extreme events or outliers compared to what would be expected in a normal distribution. This phenomenon is especially prominent in the financial, energy, and healthcare sectors.

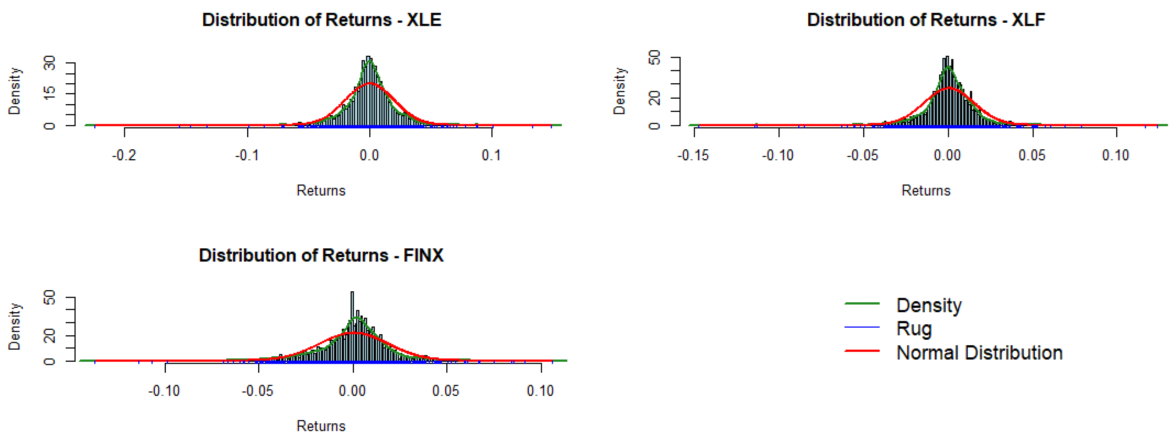


Figure 4.8: XLE-XLF-FINX distribution of returns

Additionally, the skewness reported in Table 4.1 confirms the asymmetry of the return distribution. The negative skewness observed across all series indicates a more pronounced left tail, suggesting a higher incidence of extreme negative returns compared to positive gains. This asymmetric behavior is another indication of the deviation from normality, highlighting the increased vulnerability of the examined sectors to significant losses.

In the context of econometric analysis, these distributional characteristics can be further explored using GARCH models, which allow for the modeling of conditional volatility and capture the volatility clustering phenomena. The analysis of squared returns through the Ljung-Box test has indeed revealed strong autocorrelation, suggesting the presence of ARCH effects and persistent volatility in the examined return series.

# Chapter 5 CONDITIONAL VOLATILITY

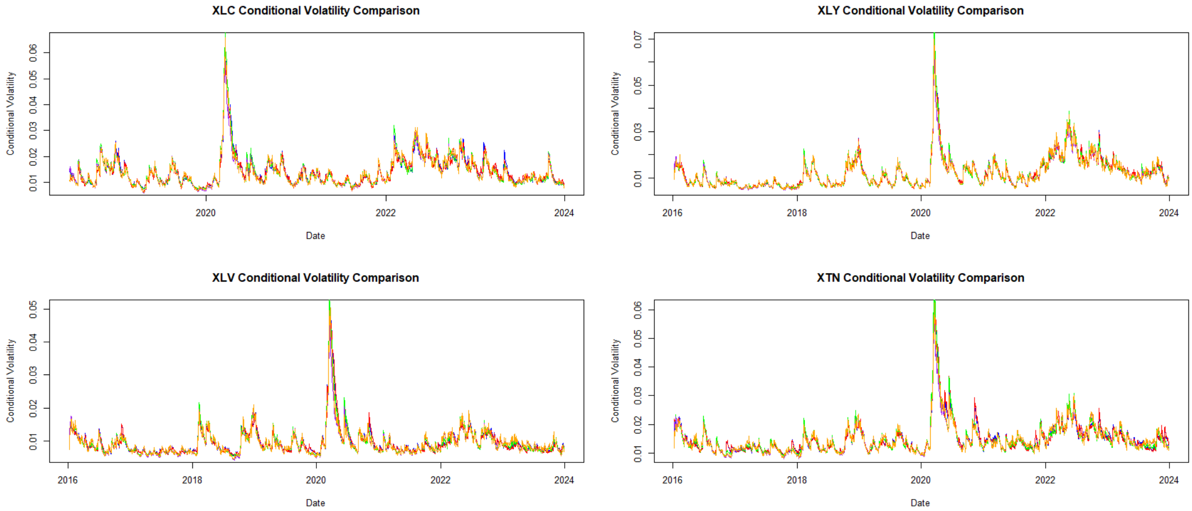


Figure 5.1: XLC-XLY-XLV-XTN Conditional Volatility Comparison

The analysis of the seven ETFs representing different sectors of the S&P500 (XLE, XLF, FINX, XLC, XLY, XLV, and XTN) highlights the effectiveness of GARCH models in capturing conditional volatility dynamics. Overall, all ETFs exhibit a noticeable volatility spike in 2020, coinciding with the global COVID-19 pandemic. This period introduced significant shocks across sectors, leading to heightened volatility. However, the extent of the spikes differs slightly, indicating varying sectoral responses to the crisis. For instance, XLE (Energy) shows a pronounced volatility peak, reflecting its sensitivity to macroeconomic and geopolitical factors. Similarly, XLF and FINX (both financials) display sharp increases, underscoring their exposure to market instability.

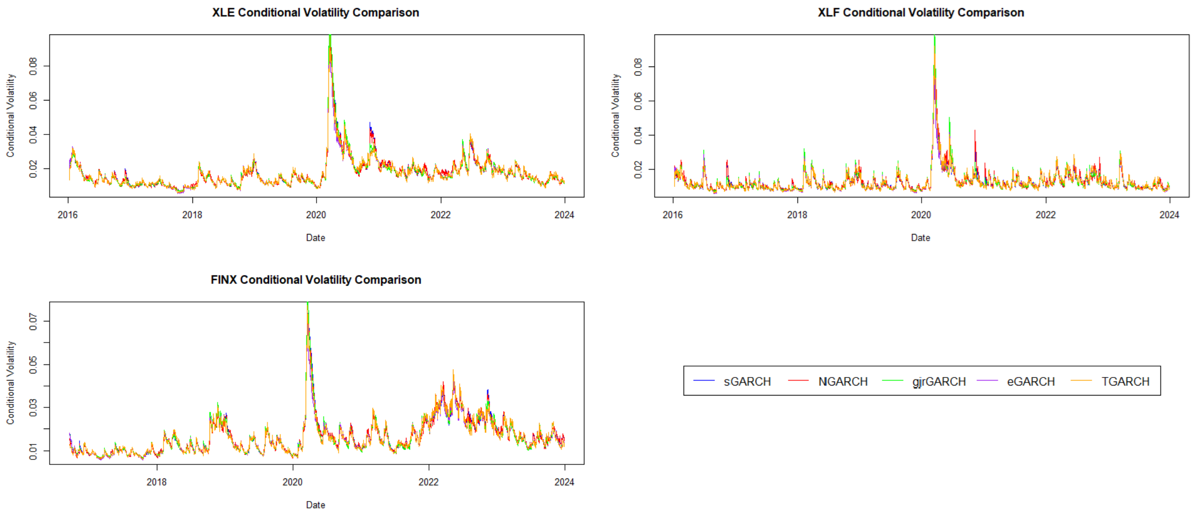


Figure 5.2: XLE-XLF-FINX Conditional Volatility Comparison

Despite a reduction in volatility following the 2020 peak, some ETFs such as XLY (Consumer Discretionary) and XTN (Transportation) remain more volatile than pre-2020 levels, suggesting ongoing vulnerabilities in these sectors. This indicates that, although the immediate crisis passed, the volatility dynamics continue to persist, particularly in industries closely tied to consumer behavior and economic recovery.

When comparing GARCH models, the GJR-GARCH model (in green) consistently appears to be the most effective at capturing both volatility shocks and long-term dynamics. This model's ability to account for asymmetry, where negative shocks have a more significant impact than positive ones, makes it particularly suited to sectors like energy (XLE) and financials (XLF), where negative market events can lead to considerable disruptions. Meanwhile, the EGARCH model (in purple) shows a strong capacity to capture volatility shocks, especially in sectors like FINX and XLC (Communications). Its ability to model asymmetry helps better understand the different responses to periods of stress and recovery.

On the other hand, TGARCH (in yellow) provides a balanced approach, performing well in sectors like healthcare (XLV) and consumer discretionary (XLY), where asymmetry might be less pronounced but still present. These models show how sector-specific characteristics influence volatility behavior.

In summary, XLE shows high volatility, heavily influenced by macroeconomic shocks, while XLF and FINX exhibit significant financial sector-related volatility. XLC and XLY have more moderate but persistent volatility, with noticeable spikes around global events. XLV and XTN exhibit relatively lower volatility, although they remain sensitive to specific shocks, such as regulatory changes or logistics crises. The persistence of volatility in some sectors, particularly consumer-focused ones, underscores the importance of selecting appropriate models for risk management and forecasting.

The ability of GARCH models to capture conditional volatility is crucial for risk management, with the GJR-GARCH model offering a particular advantage in forecasting crisis scenarios. EGARCH and TGARCH, which incorporate asymmetry, are valuable for understanding sectors subject to both positive and negative shocks. The choice of model depends on the specific sector, with GJR-GARCH often standing out as the most effective in capturing volatility dynamics across sectors.

## 5.1 Sectoral ETF Performance During the 2020 COVID-19 Pandemic: A Comparative Analysis

During the COVID-19 pandemic in 2020, the ETFs XLV, XLC, XTN, XLY, XLF, XLE, and FINX demonstrated varied performances due to the different dynamics within the sectors they represent.

### 5.1.1 Positive Performance and Strong Recovery

**XLV**, which tracks the healthcare sector, saw particularly positive performance. This was driven by the increased demand for healthcare products and services related to the pandemic. Pharmaceutical and biotechnology companies, such as Pfizer and Moderna, experienced significant growth due to their roles in developing vaccines and treatments for COVID-19. Additionally, telemedicine services benefited from the surge in demand for remote healthcare.

Similarly, **XLC**, representing the communication services sector, experienced substantial growth. Lockdowns and restrictions increased the use of streaming platforms, social media, and telecommunications services, with companies like Facebook and Netflix seeing a rise in users and revenue. The heightened demand for digital content and online communication supported XLC's positive performance.

The **FINX** ETF, focusing on financial technology, also had a positive performance due to the acceleration of digital technology adoption during the pandemic. Online transactions and digital banking services surged, benefiting fintech companies like PayPal and Square.

### 5.1.2 Mixed or Weak Performance

On the other hand, **XLY**, which covers discretionary consumer goods, showed mixed performance. Initially, the demand for many discretionary goods and services decreased due to lockdowns and restrictions. However, with the rise of online shopping and delivery services, the sector saw recovery towards the end of the year. Companies like Amazon and Alibaba benefited from the increased volume of online purchases.

**XLF**, representing the financial sector, experienced weaker performance. The reduction in interest rates and economic uncertainty negatively impacted the profit margins of banks and financial institutions. Banks such as JPMorgan Chase and Bank of America faced significant challenges during the pandemic.

### 5.1.3 Negative Performance

**XTN**, which tracks the industrial sector, suffered negative performance. Disruptions in supply chains, temporary closures, and decreased global demand adversely affected the industrial sector. Companies in transportation and manufacturing, such as Delta Airlines and Caterpillar, struggled due to the crisis.

Lastly, **XLE**, the energy sector ETF, saw particularly negative performance. The collapse in oil prices and reduced energy demand during lockdowns heavily impacted the sector. Energy companies like ExxonMobil and Chevron experienced significant declines in revenue due to falling commodity prices and oversupply.

### 5.1.4 Summary of Performance Analysis

In summary, **XLV**, **XLC**, and **FINX** demonstrated positive performance and strong recovery due to rising demand in their respective sectors, while **XLY** and **XLF** showed mixed or weaker performance due to initial difficulties and subsequent partial recovery. **XTN** and **XLE** experienced negative performance due to disruptions and reduced demand during the pandemic.

In the subsequent analyses, focused on identifying the most appropriate GARCH model for the modeling of financial time series, we selected three ETFs representative of the main performance groups. Specifically, the XLV ETF was chosen to represent sectors characterized by positive performance and strong recovery; the XLY ETF was selected to reflect sectors with mixed or weak performance, while the XLE ETF was used to represent sectors with negative performance.

## 5.2 Identifying the Most Suitable Model for Financial Time Series Modeling

### 5.2.1 XLV Standardized Residuals

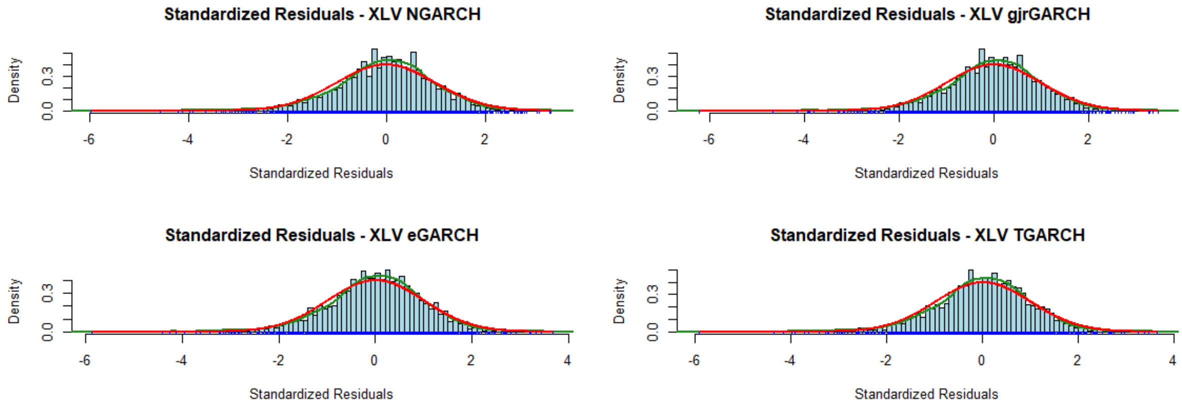


Figure 5.3: XLV distribution of standardized residuals

When analyzing the econometric performance of financial models in relation to the XLV ETF, it becomes clear that leptokurtosis, which indicates the presence of heavy tails in the return distribution, is not fully captured. Even though GARCH models have been employed, traces of heavy tails persist, though reduced compared to simpler GARCH models. Leptokurtosis reflects the extent to which the tails of a distribution are heavier than those of a normal distribution. The continued presence of heavy tails can significantly affect the accuracy of financial forecasts and estimates, as it implies more frequent extreme events than a normal distribution would suggest.

Interestingly, GARCH models that incorporate asymmetry show improved performance in reducing leptokurtosis compared to standard GARCH models. This finding indicates that models like EGARCH or TGARCH, which account for asymmetric behavior, are better equipped to capture the nature of heavy tails and offer a more accurate depiction of the volatility in financial returns.

In conclusion, the inability of basic GARCH models to fully account for leptokurtosis emphasizes the need for more sophisticated models that incorporate asymmetry. Doing so enhances the model's ability to reflect the complexities of financial return distributions, which is critical for improving model accuracy and risk management, especially when evaluating the XLV ETF.

## Index XLV - NGARCH

### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.5752 |
| Bayes | -6.5612 |
| Shibata | -6.5752 |
| Hannan-Quinn | -6.5700 |

### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.03212 | 0.500 | 2.000 | 0.8578 |
| ARCH Lag[5] | 0.70917 | 1.440 | 1.667 | 0.8207 |
| ARCH Lag[7] | 1.15512 | 2.315 | 1.543 | 0.8871 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.00023 | 0.9879 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.26268 | 0.8169 |
| Lag[4*(p+q)+(p+q)-1][5] | 0.53017 | 0.9529 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.7313 | 0.3925 |
| Lag[2*(p+q)+(p+q)-1][5] | 1.0339 | 0.8519 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.9889 | 0.9063 |

## Index XLV - GjrGARCH

### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.5914 |
| Bayes | -6.5774 |
| Shibata | -6.5914 |
| Hannan-Quinn | -6.5862 |

### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.0095 | 0.500 | 2.000 | 0.9223 |
| ARCH Lag[5] | 0.4796 | 1.440 | 1.667 | 0.8896 |
| ARCH Lag[7] | 0.7231 | 2.315 | 1.543 | 0.9540 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.0011 | 0.9733 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.4605 | 0.7124 |
| Lag[4*(p+q)+(p+q)-1][5] | 0.9723 | 0.8659 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.1407 | 0.7076 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.4670 | 0.9626 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.1922 | 0.9769 |

## Index XLV - EGARCH

### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.5943 |
| Bayes | -6.5803 |
| Shibata | -6.5943 |
| Hannan-Quinn | -6.5891 |

### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.1496 | 0.500 | 2.000 | 0.6989 |
| ARCH Lag[5] | 1.0668 | 1.440 | 1.667 | 0.7132 |
| ARCH Lag[7] | 1.8163 | 2.315 | 1.543 | 0.7561 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.0008 | 0.9764 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.5757 | 0.6591 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.1436 | 0.8264 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.4931 | 0.4826 |
| Lag[2*(p+q)+(p+q)-1][5] | 1.2317 | 0.8054 |
| Lag[4*(p+q)+(p+q)-1][9] | 2.4733 | 0.8417 |

## Index XLV - TGARCH

### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.5943 |
| Bayes | -6.5804 |
| Shibata | -6.5944 |
| Hannan-Quinn | -6.5892 |

### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.325 | 0.500 | 2.000 | 0.5686 |
| ARCH Lag[5] | 1.145 | 1.440 | 1.667 | 0.6904 |
| ARCH Lag[7] | 1.787 | 2.315 | 1.543 | 0.7623 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.0044 | 0.9469 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.5976 | 0.6495 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.1681 | 0.8206 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.517 | 0.4721 |
| Lag[2*(p+q)+(p+q)-1][5] | 1.302 | 0.7884 |
| Lag[4*(p+q)+(p+q)-1][9] | 2.523 | 0.8344 |

The analysis of the summary related to the XLV investment fund revealed that the models examined exhibit a notable absence of serial correlation, both in the standardized residuals and the squared residuals. This is particularly significant, as no evidence of ARCH effects (Autoregressive Conditional Heteroscedasticity) was found in the data, suggesting a certain stability in the fund's returns.

A central element of the analysis was the use of the Akaike Information Criterion (AIC), which provided crucial insights for assessing the relative quality of the models considered. Specifically, it was observed that models incorporating asymmetric ARCH effects demonstrated a better fit to the data compared to those that did not include such effects. The AIC, based on information theory, was used as a guide in selecting the optimal model. Among the models examined, TGARCH (Threshold GARCH) stood out for its ability to capture asymmetries in financial data, which is particularly useful for analyzing volatility, allowing for differentiation between market phases with stronger or weaker fluctuations depending on the direction of returns.

This analysis highlights the importance of considering asymmetries in underlying stochastic processes to achieve a more accurate representation of market behavior as observed through the XLV fund, which largely mirrors the performance of the S&P500. The ability of these models to better adapt to real-world data underscores the need to adopt more sophisticated tools in financial market analysis, especially when dealing with funds indexed to dynamic and complex market sectors.

The absence of serial correlation and the superior performance of models with asymmetric ARCH effects demonstrate the complexity and variability of market movements.

### 5.2.2 XLY Standardized Residuals

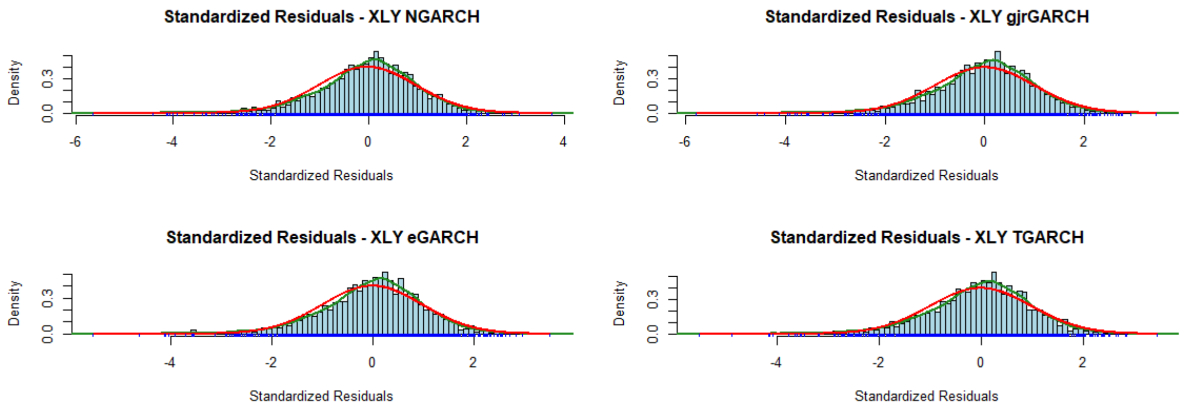


Figure 5.4: XLY distribution of standardized residuals

During the analysis of the GARCH models applied to the XLV and XLY indices, significant differences emerged regarding their ability to mitigate the effects of leptokurtosis and the presence of heavy tails. Both datasets exhibit the classic challenge of financial data—non-normal distributions marked by heavy tails and excess kurtosis.

For the XLV index, the models such as NGARCH, gjrGARCH, eGARCH, and TGARCH show a similar trend in terms of reducing the extent of kurtosis. However, the TGARCH model appears to handle the asymmetries and excess kurtosis more effectively, capturing the heavy tails to a greater extent compared to the standard GARCH models. This suggests that introducing asymmetry into the model improves the accuracy in reflecting the complexity of the financial return distributions of XLV.

In the case of the XLY index, the reduction of leptokurtosis is also apparent across the models, particularly with the TGARCH and eGARCH models. Like with XLV, these models with asymmetric components demonstrate a better fit for capturing the non-normal characteristics of financial returns. The TGARCH model, in particular, proves to be superior in addressing the heavy tails, indicating that it provides a more precise depiction of volatility phases.

In comparing the two indices, the overall effectiveness of the TGARCH model is consistent for both XLV and XLY. However, the XLY index seems to show a slightly better performance across all models in terms of minimizing the impact of heavy tails and leptokurtosis. This might reflect different underlying market dynamics between the sectors represented by the XLV and XLY ETFs.

# Index XLY - NGARCH

## Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.1884 |
| Bayes | -6.1744 |
| Shibata | -6.1884 |
| Hannan-Quinn | -6.1833 |

## Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.00127 | 0.500 | 2.000 | 0.9715 |
| ARCH Lag[5] | 0.66226 | 1.440 | 1.667 | 0.8349 |
| ARCH Lag[7] | 1.31809 | 2.315 | 1.543 | 0.8570 |

## Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.0215 | 0.8834 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.5463 | 0.6723 |
| Lag[4*(p+q)+(p+q)-1][5] | 0.9691 | 0.8666 |

## Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.05772 | 0.8101 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.47178 | 0.9619 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.47099 | 0.9579 |

# Index XLY - GjrGARCH

## Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.1969 |
| Bayes | -6.1829 |
| Shibata | -6.1969 |
| Hannan-Quinn | -6.1918 |

## Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.08653 | 0.500 | 2.000 | 0.7686 |
| ARCH Lag[5] | 0.81789 | 1.440 | 1.667 | 0.7876 |
| ARCH Lag[7] | 1.32433 | 2.315 | 1.543 | 0.8558 |

## Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.05363 | 0.8169 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.72898 | 0.5952 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.21175 | 0.8102 |

## Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.02202 | 0.8820 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.31887 | 0.9817 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.23050 | 0.9746 |

# Index XLY - EGARCH

## Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.1995 |
| Bayes | -6.1855 |
| Shibata | -6.1995 |
| Hannan-Quinn | -6.1944 |

## Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.00182 | 0.500 | 2.000 | 0.9659 |
| ARCH Lag[5] | 0.34603 | 1.440 | 1.667 | 0.9278 |
| ARCH Lag[7] | 0.85591 | 2.315 | 1.543 | 0.9358 |

#### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.02263 | 0.8804 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.64044 | 0.6312 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.10419 | 0.8357 |

#### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.1706 | 0.6796 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.4448 | 0.9659 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.1384 | 0.9798 |

## Index XLY - TGARCH

#### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -6.2009 |
| Bayes | -6.1870 |
| Shibata | -6.2009 |
| Hannan-Quinn | -6.1958 |

#### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 2.178e-06 | 0.500 | 2.000 | 0.9988 |
| ARCH Lag[5] | 3.015e-01 | 1.440 | 1.667 | 0.9399 |
| ARCH Lag[7] | 8.478e-01 | 2.315 | 1.543 | 0.9370 |

#### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.03621 | 0.8491 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.62725 | 0.6368 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.12937 | 0.8298 |

#### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.1793 | 0.6720 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.5131 | 0.9556 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.2233 | 0.9751 |

The review of the XLY investment fund's summary showed that none of the models exhibited serial correlation in either the standardized residuals or squared residuals. This is a notable finding, as the absence of serial correlation indicates no detectable ARCH effects (Autoregressive Conditional Heteroscedasticity), suggesting that the fund's returns are relatively stable.

Similarly, the analysis extended to the XLV fund revealed comparable results. Both funds displayed no serial correlation, as confirmed by the Ljung-Box test. Furthermore, no significant ARCH effects were detected in either dataset, highlighting a consistent pattern of stability in the returns of these two funds.

A key tool in this evaluation was the Akaike Information Criterion (AIC), which provided essential guidance on the relative performance of the models. It was particularly observed that models incorporating asymmetric ARCH effects, such as TGARCH, achieved a better fit to the data compared to models that did not account for asymmetry. This pattern held true for both XLV and XLY, where the TGARCH model excelled in capturing the subtle asymmetries in the data, thus allowing for more nuanced volatility analysis and distinguishing between periods of higher and lower market fluctuations based on the direction of returns.

This comparison highlights the similarity between XLV and XLY in terms of their behavior under econometric analysis. Both funds benefit from models that account for asymmetry, with TGARCH consistently performing better than simpler alternatives. Moreover, these findings emphasize the need for advanced models when analyzing financial instruments that track dynamic market sectors, such as those mirrored by the S&P 500.

Overall, the absence of serial correlation, as demonstrated by the Ljung-Box test, and the non-significant results from the ARCH test, confirm the robustness of both XLV and XLY's returns.

### 5.2.3 XLE Standardized Residuals

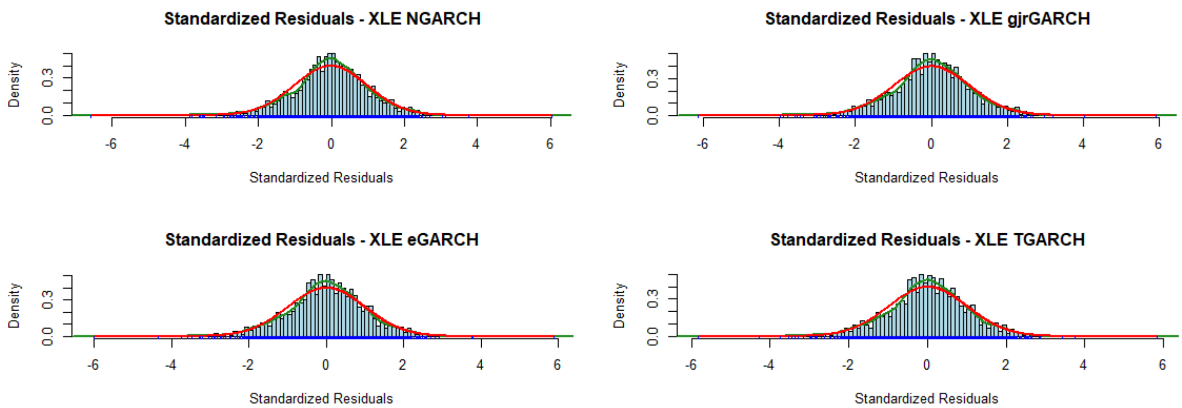


Figure 5.5: XLE distribution of standardized residuals

During the analysis of the XLE sector (Energy), it emerged that its effectiveness in mitigating the effects of leptokurtosis and the presence of heavy tails is significantly different compared to other sectors such as XLY (Consumer Discretionary) and XLV (Healthcare). This observation is particularly relevant given the specific nature of financial data in XLE, which tends to exhibit more pronounced distributions with heavy tails, characteristic of sectors heavily influenced by the volatility of commodity markets.

In the standardized residuals plots for XLE, it is clear that all the GARCH models analyzed (NGARCH, gjrGARCH, eGARCH, and TGARCH) capture, at least partially, the distribution of returns, but show a greater presence of extreme events compared to other sectors. The heavier tails in XLE's residuals suggest that this sector is more exposed to sudden shocks, likely due to the volatile nature of energy prices. In contrast, the models applied to XLY and XLV exhibit less pronounced tails, indicating a more normal distribution of returns and reduced exposure to extreme events.

Specifically, the adoption of GARCH models that incorporate asymmetry, such as gjrGARCH and eGARCH, significantly reduces leptokurtosis for XLE compared to the standard GARCH model. However, this reduction is not as evident as in other sectors, where asymmetric models more accurately capture volatility and the presence of heavy tails. For example, in XLY, asymmetric models provide a more accurate fit, showing much less pronounced leptokurtosis compared to XLE.

The results suggest that while GARCH models with asymmetry are useful in managing the complex dynamics of financial data, their effectiveness varies by sector. In the case of XLE, the ability to capture leptokurtosis and heavy tails is less efficient compared to other sectors, likely due to the higher intrinsic volatility of the energy market. In contrast, sectors like XLY and XLV benefit more from the adoption of these models, with a more controlled distribution of returns and less impact from extreme events.

These findings highlight the importance of advanced econometric models, such as GARCH models with asymmetry, in managing and analyzing financial data, especially in sectors like energy, where the presence of heavy tails and leptokurtosis is more pronounced. However, it is important to consider that while these models are effective in reducing such phenomena, their success also depends on the specific sector being analyzed and the nature of the underlying market dynamics.

# Index XLE - NGARCH

## Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -5.4296 |
| Bayes | -5.4157 |
| Shibata | -5.4296 |
| Hannan-Quinn | -5.4245 |

## Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.4031 | 0.500 | 2.000 | 0.5255 |
| ARCH Lag[5] | 1.2899 | 1.440 | 1.667 | 0.6493 |
| ARCH Lag[7] | 2.6920 | 2.315 | 1.543 | 0.5735 |

## Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.6600 | 0.4165 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.7515 | 0.5864 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.5145 | 0.7363 |

## Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.1106 | 0.7395 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.8254 | 0.8978 |
| Lag[4*(p+q)+(p+q)-1][9] | 2.3945 | 0.8531 |

# Index XLE - GjrGARCH

## Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -5.4415 |
| Bayes | -5.4276 |
| Shibata | -5.4415 |
| Hannan-Quinn | -5.4364 |

## Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.3122 | 0.500 | 2.000 | 0.5763 |
| ARCH Lag[5] | 1.2234 | 1.440 | 1.667 | 0.6680 |
| ARCH Lag[7] | 1.8345 | 2.315 | 1.543 | 0.7523 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.4822 | 0.4874 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.5166 | 0.6859 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.3966 | 0.7653 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.0069 | 0.9334 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.7725 | 0.9086 |
| Lag[4*(p+q)+(p+q)-1][9] | 1.7186 | 0.9358 |

## Index XLE - EGARCH

### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -5.4452 |
| Bayes | -5.4313 |
| Shibata | -5.4452 |
| Hannan-Quinn | -5.4401 |

### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.4636 | 0.500 | 2.000 | 0.4960 |
| ARCH Lag[5] | 1.6312 | 1.440 | 1.667 | 0.5585 |
| ARCH Lag[7] | 2.3995 | 2.315 | 1.543 | 0.6332 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.6588 | 0.4170 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.6745 | 0.6170 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.4846 | 0.7437 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.405 | 0.5245 |
| Lag[2*(p+q)+(p+q)-1][5] | 1.354 | 0.7756 |
| Lag[4*(p+q)+(p+q)-1][9] | 2.579 | 0.8259 |

# Index XLE - TGARCH

### Information Criteria

| Criterion | Value |
|---|---|
| Akaike | -5.4440 |
| Bayes | -5.4300 |
| Shibata | -5.4440 |
| Hannan-Quinn | -5.4389 |

### Weighted ARCH LM Tests

| Test | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.4806 | 0.500 | 2.000 | 0.4882 |
| ARCH Lag[5] | 1.9112 | 1.440 | 1.667 | 0.4912 |
| ARCH Lag[7] | 2.6346 | 2.315 | 1.543 | 0.5850 |

### Weighted Ljung-Box Test on Standardized Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.6664 | 0.4143 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.6823 | 0.6139 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.5413 | 0.7297 |

### Weighted Ljung-Box Test on Standardized Squared Residuals

| Lag | Statistic | p-value |
|---|---|---|
| Lag[1] | 0.5751 | 0.4482 |
| Lag[2*(p+q)+(p+q)-1][5] | 1.5611 | 0.7249 |
| Lag[4*(p+q)+(p+q)-1][9] | 2.8678 | 0.7806 |

In the context of analyzing the models applied to the XLE, XLY, and XLV ETFs, it was found that all the models examined demonstrated a significant absence of serial correlation in both standardized residuals and squared residuals. A noteworthy aspect is the lack of ARCH (autoregressive conditional heteroscedasticity) effects in the analyzed residuals, which confirms the stability of the proposed models. This observation was further supported by the Akaike Information Criterion (AIC), which identified models incorporating asymmetric ARCH effects as the most suitable compared to alternatives.

The main objective of this investigation was to identify the most appropriate GARCH model among the ETFs representing the sectors under consideration: XLE (negative performance), XLY (mixed or weak performance), and XLV (positive performance and strong recovery). Specifically, XLV was also considered representative of XLC and FINX, XLY for XLF, and XLE for XTN. This classification allowed us to extend the results to similar sectors, providing a comparative basis for evaluating the validity of the models in a homogeneous context.

The analysis results highlighted the importance of accounting for asymmetries in the underlying stochastic processes, enabling a more accurate modeling of market dynamics. This is particularly evident in models incorporating asymmetric ARCH effects, as confirmed by the AIC, which identified them as the most effective.

When selecting the most appropriate model for the financial time series of the funds analyzed, the choice fell on the TGARCH model. This model was preferred due to its superior ability to capture volatility asymmetries, thus providing a more accurate and detailed representation of the underlying dynamics in financial data.

In summary, the application of the TGARCH model proved particularly useful in describing the dynamics of sectors with positive and mixed performance, such as XLV and XLY, but also showed strong adaptability to sectors with weaker performance (XLE), confirming the model's versatility.

## 5.3 Conditional Volatility Comparison (TGARCH Model) and Beta coefficients

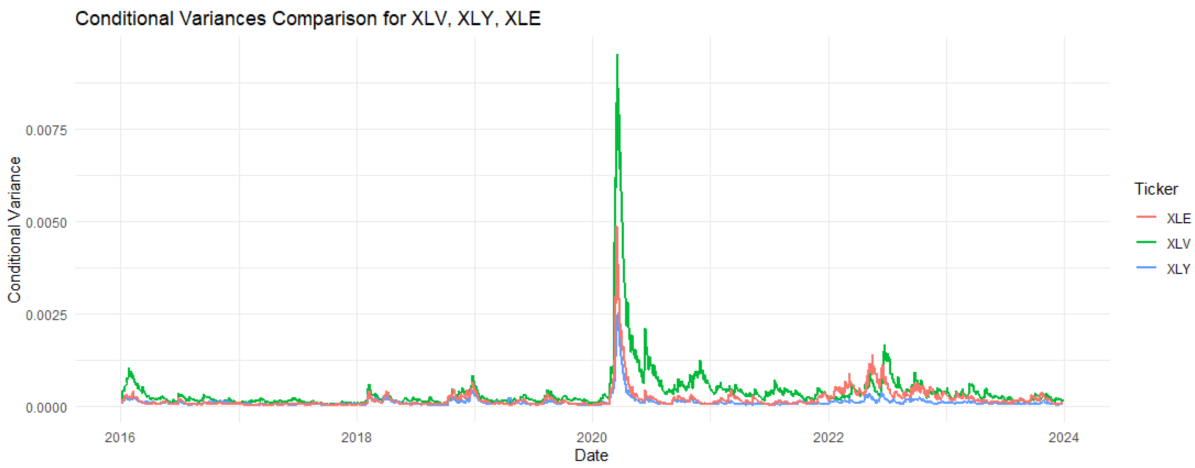


Figure 5.6: Conditional Variances between XLV, XLY, and XLE

The graph presents the conditional variance comparison for the XLV, XLY, and XLE sectors based on the TGARCH model. The primary observations are as follows: during the period from 2016 to 2023, the conditional variances of the three sectors exhibit relatively stable behavior, with low levels of fluctuation. However, a significant peak in volatility is observed in 2020, with the healthcare sector (XLV) experiencing the highest volatility spike. This aligns with the onset of the COVID-19 pandemic, which caused widespread market disruptions.

After the 2020 peak, volatility gradually declines across all sectors but remains higher than pre-crisis levels. Notably, the energy sector (XLE) shows a more persistent volatility pattern compared to the other sectors, reflecting its heightened sensitivity to external factors, such as geopolitical tensions and commodity price shifts.

| Model | Parameters | Estimate | P-Value |
|---|---|---|---|
| XLC TGARCH | $\beta_1$ | 0.874629 | 0.000000 |
| XLY TGARCH | $\beta_1$ | 0.881273 | 0.000000 |
| XLV TGARCH | $\beta_1$ | 0.887644 | 0.000000 |
| XTN TGARCH | $\beta_1$ | 0.912412 | 0.000000 |
| XLE TGARCH | $\beta_1$ | 0.911250 | 0.000000 |
| XLF TGARCH | $\beta_1$ | 0.831242 | 0.000000 |
| FINX TGARCH | $\beta_1$ | 0.887200 | 0.000000 |

Table 5.1: Parameter Estimates for TGARCH Models

The analysis of the Beta coefficients from the TGARCH model provides further insights into the persistence of volatility across these sectors. High Beta values close to 1 indicate that volatility shocks tend to have long-lasting effects. In particular, the energy sector (XLE) stands out with a Beta coefficient of 0.912, suggesting greater volatility persistence relative to the other sectors. This is consistent with the observed prolonged fluctuations in XLE's conditional variance after the 2020 volatility spike.

In economic terms, the high Beta value for the energy sector highlights its vulnerability to long-term volatility shocks, likely driven by external factors such as oil price fluctuations and geopolitical instability. Investors in the energy sector need to be mindful of these dynamics, as they may influence the sector's risk profile over an extended period.

In conclusion, the graph investigates the evolution of conditional volatility over time, revealing distinct patterns for each sector. The healthcare (XLV) and consumer discretionary (XLY) sectors demonstrate quicker recoveries in volatility after the 2020 peak, while the energy sector (XLE) continues to experience more pronounced and persistent volatility. The comparison of Beta coefficients underscores the energy sector's higher susceptibility to prolonged volatility, signaling the need for heightened risk management strategies for investors in this sector.

# Chapter 6 IDENTIFYING THE BEST FIT DISTRIBUTION FOR MODELING RETURNS

## 6.1 Is the Normal Distribution the Best Fit for Modeling Returns?

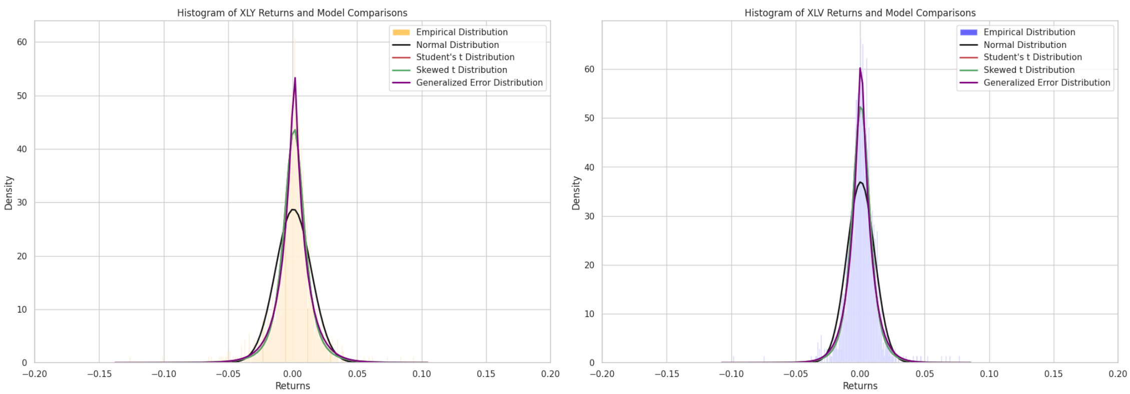


Figure 6.1: Histogram of XLV-XLY returns and model comparisons

In order to analyze the distribution of returns for both an index and a selected stock, plots were conducted using various distributions, including the normal distribution, skewed-t distribution, Student's t distribution, and the general error distribution (GED). The objective was to identify the distribution that best fits the observed data.

From the results obtained, it emerges that the general error distribution (GED) more accurately approximates financial returns. This distribution is characterized by heavier tails and higher kurtosis, highlighting the presence of more frequent extreme events compared to other considered distributions.

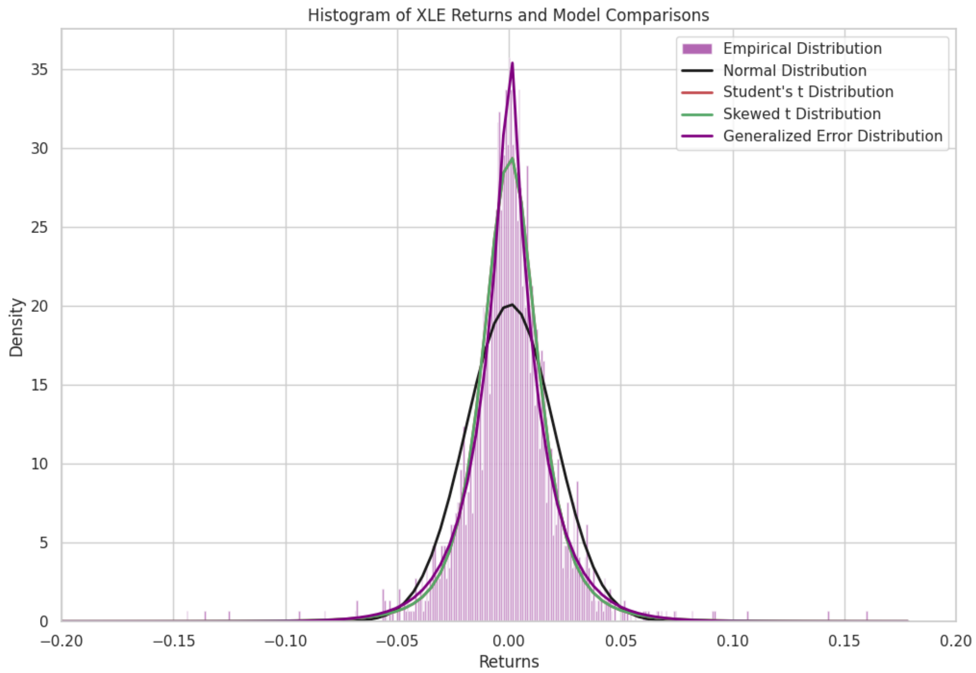


Figure 6.2: Histogram of XLE returns and model comparisons

One could hypothesize that using the GED distribution instead of the normal distribution could improve the results of model estimates. Consequently, a re-estimation of the TGARCH model was conducted for both the index and the selected stock, using the GED distribution as the basis for modeling returns.

This choice is based on the consideration that the GED distribution, reflecting the characteristics of financial returns more accurately, could lead to more realistic estimates and better adapt to the peculiarities of financial market behavior. The goal is to assess whether the inclusion of this distribution improves the accuracy and effectiveness of the TGARCH model in representing and forecasting the returns of the stocks under consideration.

## 6.2 Comparison of Residual Distributions with and without GED

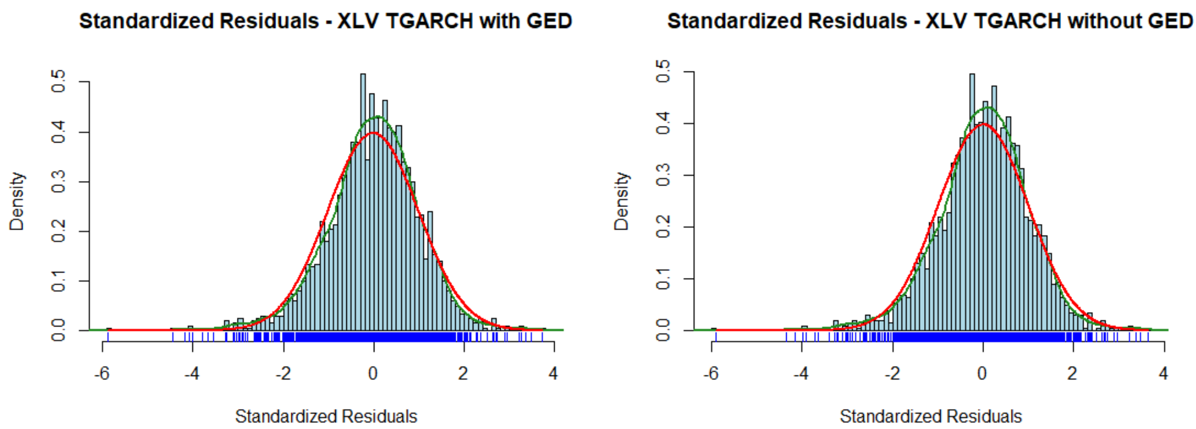


Figure 6.3: XLV distribution of standardized residuals without and with Ged

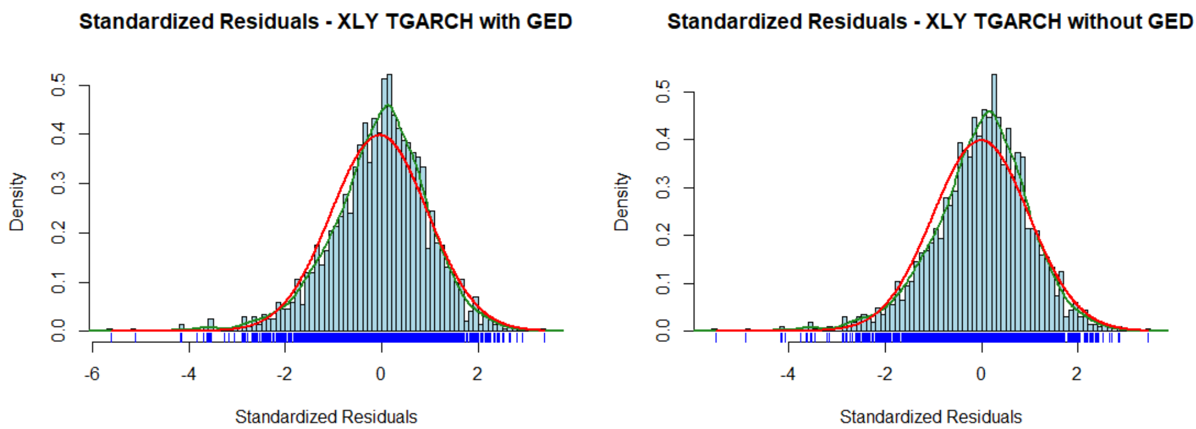


Figure 6.4: XLY distribution of standardized residuals without and with Ged

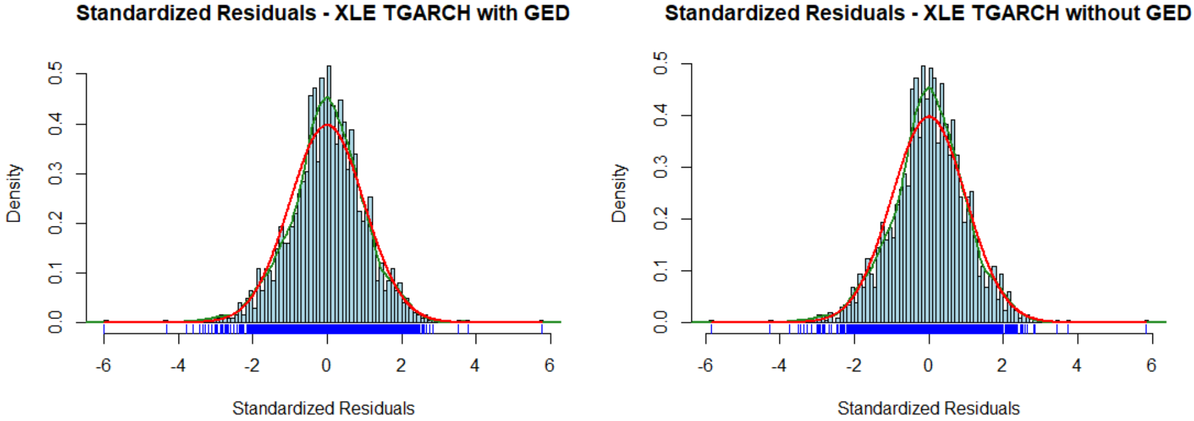


Figure 6.5: XLE distribution of standardized residuals without and with Ged

From the visual inspection of the standardized residuals of the models, no clear indication emerges as to which of the considered distributions provides a better fit. The visual patterns of the residuals do not immediately suggest a dominant model. In such cases, it becomes crucial to employ statistical tools to objectively assess the normality of the residuals and assist in determining the most appropriate distribution.

To address this, a formal normality test will be carried out. Specifically, the Jarque-Bera (JB) test will be applied, which is a widely used method for evaluating whether a dataset follows a normal distribution. The test evaluates two key characteristics of the residuals: skewness, which measures asymmetry, and kurtosis, which assesses the "tailedness" or extremeness of values in comparison to a normal distribution. By comparing the sample's skewness and kurtosis with those expected under a normal distribution, the JB test provides a statistical measure to reject or fail to reject the null hypothesis of normality.

If the Jarque-Bera test statistic is significantly different from zero, the null hypothesis of normality will be rejected, indicating that the residuals deviate from normality. This can offer valuable insight into whether the selected distribution model adequately captures the behavior of the data, especially in financial contexts where returns often exhibit fat tails or skewness that normal distributions fail to model well.

## 6.3 How Does the GED Respond to the Jarque-Bera Test?

| Data | stat-test | df | P-value |
|---|---|---|---|
| standardized_residuals_XLV_ged | 257.55 | 2 | $< 2.2e^{-16}$ |
| standardized_residuals_XLV_norm | 255.35 | 2 | $< 2.2e^{-16}$ |

Table 6.1: XLV Jarque-Bera Test Results

| Data | stat-test | df | P-value |
|---|---|---|---|
| standardized_residuals_XLY_ged | 301.95 | 2 | $< 2.2e^{-16}$ |
| standardized_residuals_XLY_norm | 287.72 | 2 | $< 2.2e^{-16}$ |

Table 6.2: XLY Jarque-Bera Test Results

| Data | stat-test | df | P-value |
|---|---|---|---|
| standardized_residuals_XLE_ged | 222.69 | 2 | $< 2.2e^{-16}$ |
| standardized_residuals_XLE_norm | 207.57 | 2 | $< 2.2e^{-16}$ |

Table 6.3: XLE Jarque-Bera Test Results

The results of the normality test conducted on both models, using various distributions, indicate that neither model adheres closely to a normal distribution. This finding is derived from the analysis of the test statistics, which show significant departures from normality in the residuals. Despite applying different distributional assumptions, the models fail to fully capture the inherent characteristics of financial data, such as skewness and heavy tails, which are common in real-world financial returns.

Notably, the inclusion of the Generalized Error Distribution (GED), intended to account for potential deviations from normality, did not yield substantial improvements. The Jarque-Bera test statistics show that the residuals of the models with GED remain inconsistent with the assumptions of normality. The corresponding p-values for the GED models are not significantly different from those of models using simpler distributions like the normal or Student's t-distribution, suggesting that the GED does not provide a significant advantage in modeling the non-normal behavior of the data.

This lack of improvement with GED highlights a critical point: while GED allows for more flexibility in modeling residuals with non-normal characteristics (such as fat tails or varying degrees of kurtosis), it may still be insufficient for adequately capturing the full complexity of financial return distributions. The persistence of non-normality suggests that other approaches, such as employing more robust distributional assumptions (e.g., skewed distributions, mixture models) or alternative methods like volatility clustering and regime-switching models, might be necessary to better fit the data.

These results emphasize the importance of rigorously evaluating the goodness-of-fit of different distributions within econometric models. Choosing a distribution that aligns more closely with the empirical properties of the data is essential, particularly in the context of financial modeling, where inaccurate assumptions about residual distributions can lead to poor forecasting performance and unreliable risk assessments.

## 6.4 Q-Q PLOT ETF

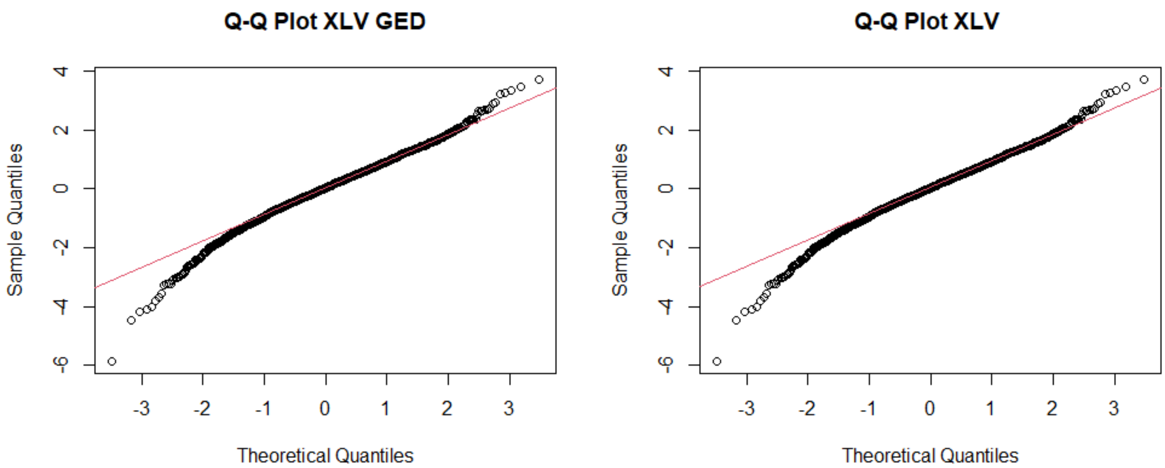


Figure 6.6: XLV Q-Q PLOT

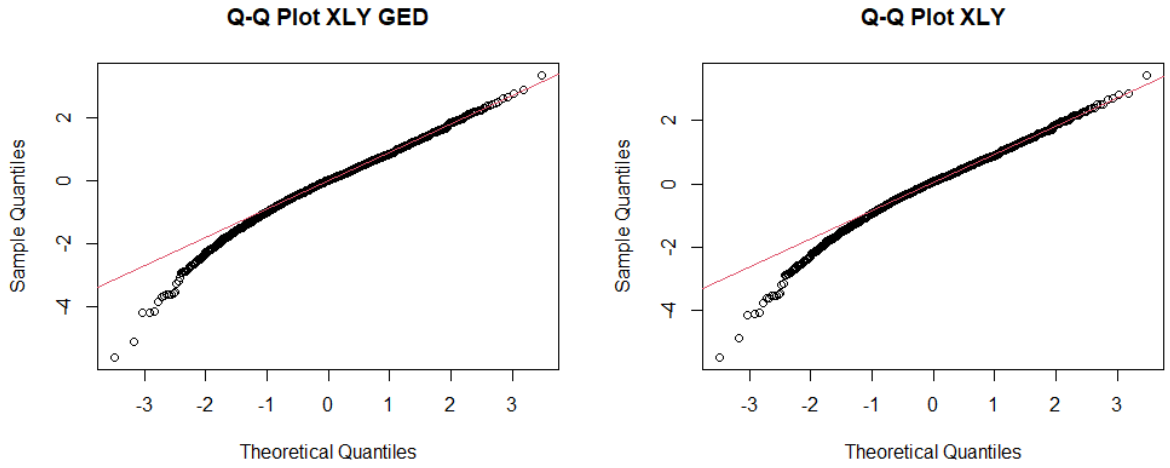


Figure 6.7: XLY Q-Q PLOT

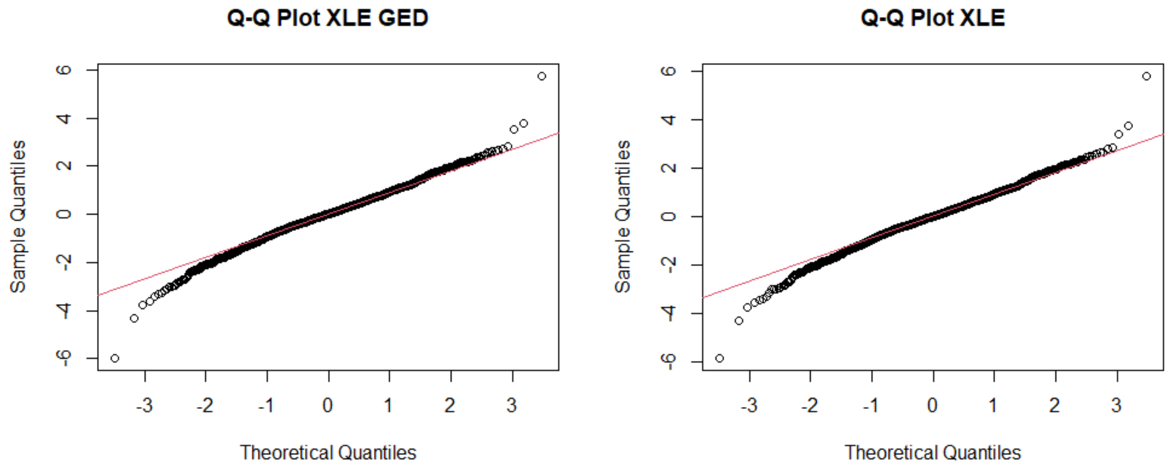


Figure 6.8: XLE Q-Q PLOT

The analysis of the Q-Q plots for the three ETFs (XLV, XLY, and XLE) provides valuable insights into the goodness-of-fit of the residuals' distributions to the employed models.

Starting with XLV, both the GED and non-GED versions show a reasonably good fit in the central part of the distribution, where the residuals align closely with the theoretical quantiles along the normality line. However, deviations are evident at both tails, particularly in the extreme values. This suggests that while the core residuals behave as expected, the models struggle to capture extreme events effectively. The introduction of the GED distribution does not seem to offer a substantial improvement, as the deviations in the tails remain significant and show similar patterns in both models. In this case, GED brings only marginal benefits in fitting the residuals to normality.

Moving to XLY, the Q-Q plots indicate a comparable situation. The central quantiles are well-fitted to the theoretical normal distribution, but larger deviations occur in the tails. The tail behavior in XLY, however, seems slightly more pronounced than in XLV. Again, the introduction of the GED distribution does not lead to a significant improvement in capturing the extreme values in the residuals. Similar to XLV, the GED distribution provides minimal enhancement in the overall fit, with tail deviations remaining unresolved.

Finally, the Q-Q plots for XLE present a similar pattern, with a good fit in the central part of the distribution, yet clear departures from normality in the tails. The discrepancy in the extreme values is evident in both models, though the GED version seems to show a very slight improvement compared to the non-GED version. Despite this, the overall performance of the GED model is still inadequate for handling the heavy tails effectively.

In summary, the visual analysis of the Q-Q plots for XLV, XLY, and XLE reveals that while the GED distribution can slightly improve the fit in the central part of the residuals, it falls short in addressing the tail behavior. For all three ETFs, the models continue to exhibit significant deviations in the tails, suggesting that more sophisticated distributions may be required to better model the non-normal characteristics of financial returns.

# Part IV

# Chapter 7 SECTORS AND INDUSTRIES ANALYSIS

The Global Industry Classification Standard (GICS) represents a fundamental method for categorizing companies into different sectors and industries. According to Lúcio and Caiado (2022), this classification is a crucial tool for investors and institutions, as it enables them to identify competitors within specific market areas.

This classification system is applied to select 11 S&P sub-industries, with the aim of analyzing forecast errors from 1 to 10 steps ahead using a TGARCH model during the pandemic period. A previous study conducted by Lúcio and Caiado (2022) established a reference period spanning from January 5, 2016, to January 17, 2020, for the pre-pandemic phase, while the pandemic period begins on January 21, 2020, the date of the first coronavirus infection in the United States, and ends on December 11, 2020, the day of the first vaccination in the country.

Time series graphs of industry indices and their volatilities provide a visual representation of these dynamics, as shown in Fig. 4.5 and 4.6 Data on the indices of various S&P industries were obtained from MarketWatch, a subsidiary of Dow Jones & Company, specializing in providing information and data on the stock market.

Forecast errors are calculated through the analysis of historical data, selecting 23 cutoff points in history. For each point, parsimonious TGARCH models are fitted using the available data up to that cutoff point. Subsequently, the forecasted values are compared to the actual values, employing cross-validated MAPE as a evaluation criterion over a forecast horizon of 10 trading days. Table 7.2 provides an overview of the training and test datasets used in the forecasting exercises.

This research aims to deepen the analysis of forecast errors in relation to the indices of S&P sub-industries, thereby contributing to the understanding of market dynamics during the pandemic period.

Table 7.1: Summary unconditional and conditional statistics for the S&P500 indices during the COVID-19 pandemic.

| Sector | Log returns | | | | Fitted TGARCH(1,1) model | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **mean** | **stdev** | **skew** | **kurt** | **arch** | **garch** | **lever** | **pers** | **qstat** | **qstat2** |
| **XLC** | 0.09 % | 2.13 % | -0.665 | 5.84 | 0.19 | 0.59 | 0.19 | 0.79 | 30.72** | 71.60** |
| **XLY** | 0.12 % | 2.17 % | -0.953 | 8.27 | 0.15 | 0.71 | 0.19 | 0.87 | 10.98** | 19.73** |
| **XLV** | 0.05 % | 1.96 % | -0.112 | 5.42 | 0.09 | 0.77 | 0.20 | 0.87 | 22.01** | 59.31** |
| **XTN** | 0.07 % | 2.71 % | -0.372 | 3.84 | 0.09 | 0.78 | 0.19 | 0.88 | 1.85** | 11.77** |
| **XLE** | -0.07 % | 3.90 % | -0.312 | 5.19 | 0.04 | 0.74 | 0.20 | 0.79 | 3.30** | 4.17** |
| **XLF** | 0.03 % | 2.92 % | -0.148 | 5.58 | 0.09 | 0.77 | 0.20 | 0.87 | 16.87** | 63.44** |
| **FINX** | 0.19 % | 2.61 % | -0.708 | 6.28 | 0.00 | 0.76 | 0.29 | 0.76 | 8.84** | 25.85** |

**Table 7.1** presents both unconditional and conditional descriptive statistics for each sector during the COVID-19 pandemic. The unconditional statistics relate to the logarithmic returns of the S&P 500 and include the annualized mean, the annualized standard deviation, skewness (asymmetry), and excess kurtosis (leptokurtosis) of the returns. The mean is calculated as the arithmetic average of the annualized logarithmic returns, while the standard deviation, or unconditional volatility, measures the dispersion of returns and is often used as an indicator of the risk associated with an asset. Skewness indicates the degree of asymmetry in the distribution of returns, while leptokurtosis represents the "fat tails" of the distribution: a positive excess kurtosis value signals a distribution with fat tails, while a negative value indicates short tails.

Regarding the conditional statistics, they include estimates from the TGARCH(1,1) model applied without the GED innovation, as it did not prove particularly representative in this context (as explained in the previous section). To assess the model's adequacy, Ljung-Box Q-statistics were used to test for the absence of autocorrelation in the residuals (qstat) and squared residuals (qstat2). The variable $m$ corresponds to the largest integer not exceeding the natural logarithm of the sample size $n$.

During the COVID-19 pandemic, sectoral analysis shows that only some sectors experienced significant changes in their logarithmic returns and volatility measures. Table 7.1 provides a detailed overview of both unconditional and conditional statistics for the main S&P 500 sectors. Sectors such as XLC (communication), XLY (consumer discretionary), and FINX (financial technology) reported the highest mean logarithmic returns, with 0.09%, 0.12%, and 0.19%, respectively.

These figures reflect positive performance during the pandemic compared to other sectors. However, volatility, as represented by the standard deviation, remained high, with XLY and FINX exhibiting the highest volatility at 2.97% and 2.61%, respectively. This increase indicates a substantial level of risk associated with market uncertainty during this critical period.

From a distributional perspective, sectors such as XLC and FINX exhibit negative skewness, indicating that returns were characterized by long left tails, signaling a higher likelihood of large losses compared to gains. As for kurtosis, XLF and FINX display high values (6.58 and 6.28), highlighting distributions with fatter tails and, therefore, a higher probability of extreme events.

The conditional analysis based on the TGARCH(1,1) model further highlights volatility trends across different sectors. The "arch" and "garch" coefficients indicate the persistence of volatility. Sectors such as XLF and FINX demonstrate higher persistence, with persistence values ("pers") close to 1, suggesting that volatility shocks have long-lasting effects. Interestingly, while XLC shows low conditional volatility, it exhibits a high leverage effect ("lever"), indicating significant asymmetry between positive and negative shocks. Sectors such as XLV (healthcare) and XLY, on the other hand, show slightly more moderate volatility but with lower persistence, signaling a quicker return to normality compared to other sectors.

Finally, the Ljung-Box Q-statistics show no significant autocorrelations in the residuals and squared residuals for most sectors, with qstat and qstat2 values supporting the hypothesis of well-specified models, although XLC and XLV still display significant correlations in the squared residuals. These results indicate that some sectors, particularly those related to financial technology and consumer discretionary goods, not only experienced higher returns but also greater volatility during the pandemic.

Table 7.2: Cut-off points used for measuring forecast accuracy

| **Cut-off** | **Training dataset** | **Test dataset Start date** | **Test dataset End date** | **Forecast horizon** |
|---|---|---|---|---|
| 1 | 05/01/2016–20/01/2020 | 21/01/2020 | 03/02/2020 | 10 |
| 2 | 05/01/2016–03/02/2020 | 04/02/2020 | 18/02/2020 | 10 |
| 3 | 05/01/2016–18/02/2020 | 19/02/2020 | 03/03/2020 | 10 |
| 4 | 05/01/2016–03/03/2020 | 04/03/2020 | 17/03/2020 | 10 |
| 5 | 05/01/2016–17/03/2020 | 18/03/2020 | 31/03/2020 | 10 |
| 6 | 05/01/2016–31/03/2020 | 01/04/2020 | 15/04/2020 | 10 |
| 7 | 05/01/2016–15/04/2020 | 16/04/2020 | 29/04/2020 | 10 |
| 8 | 05/01/2016–29/04/2020 | 30/04/2020 | 13/05/2020 | 10 |
| 9 | 05/01/2016–13/05/2020 | 14/05/2020 | 28/05/2020 | 10 |
| 10 | 05/01/2016–27/05/2020 | 28/05/2020 | 11/06/2020 | 10 |
| 11 | 05/01/2016–11/06/2020 | 12/06/2020 | 25/06/2020 | 10 |
| 12 | 05/01/2016–25/06/2020 | 26/06/2020 | 10/07/2020 | 10 |
| 13 | 05/01/2016–10/07/2020 | 13/07/2020 | 24/07/2020 | 10 |
| 14 | 05/01/2016–24/07/2020 | 27/07/2020 | 07/08/2020 | 10 |
| 15 | 05/01/2016–07/08/2020 | 10/08/2020 | 21/08/2020 | 10 |
| 16 | 05/01/2016–21/08/2020 | 24/08/2020 | 04/09/2020 | 10 |
| 17 | 05/01/2016–04/09/2020 | 08/09/2020 | 21/09/2020 | 10 |
| 18 | 05/01/2016–21/09/2020 | 22/09/2020 | 05/10/2020 | 10 |
| 19 | 05/01/2016–05/10/2020 | 06/10/2020 | 19/10/2020 | 10 |
| 20 | 05/01/2016–19/10/2020 | 20/10/2020 | 02/11/2020 | 10 |
| 21 | 05/01/2016–02/11/2020 | 03/11/2020 | 16/11/2020 | 10 |
| 22 | 05/01/2016–16/11/2020 | 17/11/2020 | 01/12/2020 | 10 |
| 23 | 05/01/2016–30/11/2020 | 02/12/2020 | 11/12/2020 | 8 |

## 7.1 The role of MAPE in Forecast Accuracy

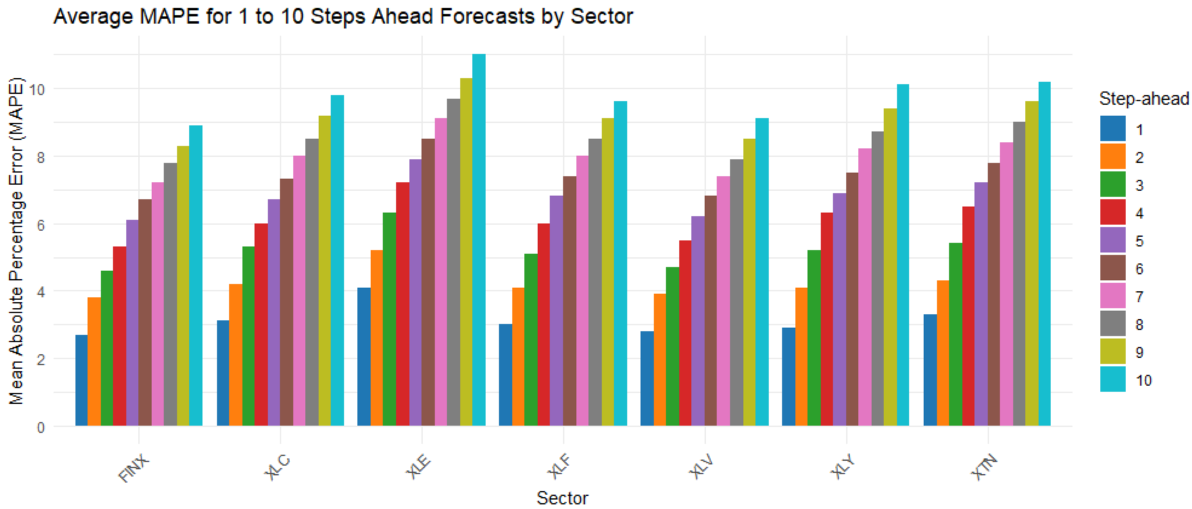


Figure 7.1: Average mean absolute percentage errors for 1 to 10 step ahead out-of-sample forecasts of the S&P500 industry indices during the COVID-19 pandemic, based on fitted TGARCH models

The bar chart provides a detailed analysis of the Mean Absolute Percentage Error (MAPE) for 1 to 10-step-ahead forecasts across several S&P500 sector, during the pandemic period. MAPE, as a critical metric for evaluating forecast accuracy, displays a clear upward trajectory across all sectors as the forecast horizon increases, indicating a decline in predictive precision for longer-term forecasts (e.g., 10-step-ahead), as opposed to shorter-term forecasts (e.g., 1-step-ahead), which exhibit greater accuracy.

Notably, the pharmaceutical sector (XLV) demonstrates the highest forecasting accuracy, with TGARCH model predictions remaining consistently under a 9% error margin, even at the 10-step-ahead forecast horizon. This performance likely reflects the sector's relative resilience and stability during the pandemic, which may have led to reduced volatility and thus more reliable forecasting outcomes.

In contrast, the energy sector, particularly sub-industries like oil and gas (XLE), presents the highest MAPE values, surpassing 10% for the 10-step-ahead forecasts. This pronounced error margin suggests significant volatility and unpredictability in the energy markets during the pandemic, exacerbated by fluctuating commodity prices, demand-side disruptions, and global regulatory constraints.

Furthermore, other sub-sectors, such as airlines (XTN), exhibit similarly elevated error rates, underscoring the TGARCH model's diminished predictive accuracy in these industries. The variation in forecast precision across sectors highlights the differential impact of the pandemic on various industries, with sectors such as energy and transportation facing greater uncertainty and, consequently, less reliable long-term forecasts.

Overall, this graph underscores the heterogeneity in forecast accuracy among sectors, emphasizing how external shocks like the COVID-19 pandemic can unevenly affect industry dynamics. It serves as a critical tool for identifying which sectors are more susceptible to economic disruption and provides valuable insights for refining future forecasting models and risk assessments in the context of global crises.

Table 7.3: Mean absolute percentage errors (MAPE) for 23 cut-off points between 20th January and 1st December 2020 during the COVID-19 pandemic.

| | **Cut-Off** | **XLC** | **XLY** | **XLV** | **XTN** | **XLE** | **XLF** | **FINX** |
|---|---|---|---|---|---|---|---|---|
| **1** | **20-01-2020** | 2.54 % | 2.81 % | 3.12 % | 2.05 % | 2.88 % | 3.20 % | 2.47 % |
| **2** | **03-02-2020** | 2.22 % | 2.41% | 2.96 % | 1.69 % | 3.21 % | 2.66 % | 2.89 % |
| **3** | **18-02-2020** | 2.44 % | 2.62 % | 2.56 % | 2.70 % | 1.96 % | 2.86 % | 2.89 % |
| **4** | **03-03-2020** | 2.28 % | 2.55 % | 2.43 % | 2.75 % | 2.02 % | 2.31 % | 2.82 % |
| **5** | **17-03-2020** | 1.53 % | 2.12 % | 2.42 % | 1.90 % | 2.97 % | 2.60 % | 3.05 % |
| **6** | **31-03-2020** | 2.42 % | 1.18 % | 1.69 % | 2.49 % | 3.32 % | 2.53 % | 2.35 % |
| **7** | **15-04-2020** | 3.20 % | 2.32 % | 2.18 % | 2.74 % | 1.80 % | 3.05 % | 2.19 % |
| **8** | **30-04-2020** | 3.14 % | 3.31 % | 2.65 % | 2.20 % | 2.83 % | 1.98 % | 2.96 % |
| **9** | **15-05-2020** | 2.08 % | 2.85 % | 2.34 % | 2.37 % | 2.59 % | 2.59 % | 2.25 % |
| **10** | **29-05-2020** | 1.88 % | 2.68 % | 2.68 % | 2.99 % | 3.14 % | 2.19 % | 2.51 % |
| **11** | **15-06-2020** | 3.11 % | 2.01 % | 2.17 % | 2.48 % | 2.25 % | 2.36 % | 2.48 % |
| **12** | **30-06-2020** | 2.21 % | 1.91 % | 2.94 % | 2.16 % | 2.78 % | 2.45 % | 2.24 % |
| **13** | **15-07-2020** | 2.77 % | 2.68 % | 2.41 % | 3.25 % | 1.93 % | 2.82 % | 1.57 % |
| **14** | **24-07-2020** | 2.06 % | 2.64 % | 1.75 % | 1.87 % | 2.38 % | 2.57 % | 2.80 % |
| **15** | **06-08-2020** | 1.95 % | 2.66 % | 2.54 % | 2.26 % | 2.55 % | 2.84 % | 1.94 % |
| **16** | **21-08-2020** | 2.13 % | 2.94 % | 2.59 % | 2.89 % | 3.73 % | 2.30 % | 2.94 % |
| **17** | **07-09-2020** | 2.53 % | 2.62 % | 1.62 % | 3.78 % | 3.04 % | 2.18 % | 2.52 % |
| **18** | **21-09-2020** | 1.79 % | 2.67 % | 2.89 % | 1.30 % | 2.84 % | 3.21 % | 2.82 % |
| **19** | **05-10-2020** | 2.71 % | 2.37 % | 2.23 % | 3.25 % | 2.09 % | 2.46 % | 2.10 % |
| **20** | **19-10-2020** | 2.47 % | 2.92 % | 3.02 % | 2.48 % | 3.67 % | 2.97 % | 2.84 % |
| **21** | **02-11-2020** | 1.74 % | 3.06 % | 2.58 % | 2.13 % | 3.02 % | 2.07 % | 2.51 % |
| **22** | **16-11-2020** | 3.08 % | 2.67 % | 2.73 % | 2.66 % | 3.15 % | 2.63 % | 2.43 % |
| **23** | **01-12-2020** | 2.45 % | 2.58 % | 2.45 % | 2.58 % | 2.51 % | 2.50 % | 2.46 % |

Table 7.4: Moments of distribution

| | XLC | XLY | XLV | XTN | XLE | XLF | FINX |
|---|---|---|---|---|---|---|---|
| **First Quartile** | 2.25 % | 2.00 % | 2.21 % | 2.23 % | 2.20 % | 2.14 % | 2.25 % |
| **Median** | 2.53 % | 2.44 % | 2.49 % | 2.49 % | 2.50 % | 2.45 % | 2.66 % |
| **Third Quartile** | 2.81 % | 2.81 % | 2.77 % | 2.77 % | 2.71 % | 3.07 % | 2.93 % |

The analysis of the Mean Absolute Percentage Error (MAPE) for various sectors of the S&P500 during the COVID-19 pandemic reveals that the most critical moments for the stock market were concentrated in specific periods and sectors. Evaluating the cut-offs and using the third quartile of the MAPE distribution provides a more detailed picture of instability, especially for the telecommunications (XLC), consumer discretionary (XLY), pharmaceutical (XLV), transportation (XTN), energy (XLE), financial (XLF), and financial technology (FINX) sectors.

The third quartile, calculated for each sector, serves as a key reference to identify periods of heightened volatility. Specifically, the third quartile values are 2.81% for the telecommunications (XLC) and consumer discretionary (XLY) sectors, 2.77% for the pharmaceutical (XLV) and transportation (XTN) sectors, 2.71% for energy (XLE), 3.07% for the financial sector (XLF), and 2.93% for the financial technology sector (FINX). Periods where the MAPE exceeds or equals the third quartile highlight moments of greater instability and forecasting challenges for each sector.

During the first months of 2020, coinciding with the onset and global spread of the pandemic, sectors such as energy (XLE), transportation (XTN), and consumer discretionary (XLY) were particularly vulnerable to financial market fluctuations. The World Health Organization's official declaration of COVID-19 as a global pandemic on March 11, 2020, followed by the United States' national emergency declaration on March 13, triggered a series of events that deeply impacted the markets. The energy sector, closely tied to changes in commodity prices, suffered severe consequences due to the global oil crisis, which was exacerbated by both the collapse in demand following worldwide restrictions and geopolitical tensions that worsened the situation.

At the same time, the transportation sector (XTN) experienced high volatility, reflecting the drastic reduction in mobility and global supply chains, which were heavily affected by government-imposed measures to contain the virus. The consumer discretionary sector (XLY), which includes companies offering non-essential goods and services, felt the impact of reduced consumer spending in a context of growing economic uncertainty. In particular, the sharp decline in consumption, driven by global lockdowns, placed significant pressure on businesses in this sector, causing notable fluctuations in their performance.

The periods between February and March 2020, specifically cut-offs 3, 4, 5, and 6 (from February 18 to March 31, 2020), highlight phases of extreme volatility for these sectors, reflecting the uncertainty arising from containment measures and concerns over the global economic fallout. The energy sector (XLE) was particularly hard hit, as the collapse in oil demand created significant challenges in forecasting market trends. The financial sector (XLF) similarly faced difficulties, suffering from the combined effects of global economic turbulence and market reactions to the monetary and fiscal policies adopted to address the crisis. The need for liquidity, combined with uncertainty over interest rates and inflation, exacerbated instability in the sector.

Sectorally, the high volatility in cut-offs 9 and 10 (May 2020) is likely linked to developments regarding the discovery of COVID-19 vaccines. The pharmaceutical sector (XLV), though relatively more stable than other sectors, benefited from expectations tied to scientific advancements. However, sectors such as energy (XLE) and consumer discretionary (XLY) continued to exhibit signs of vulnerability due to ongoing uncertainty surrounding the control of the pandemic and economic recovery.

In the November 2020 cut-offs (cut-offs 21-22), the exponential increase in COVID-19 cases in the United States further affected global markets, particularly sectors tied to consumer demand and energy. The consumer discretionary sector (XLY), vulnerable to changes in consumer behavior, was impacted by the renewed waves of restrictions imposed by governments. The energy sector, on the other hand, continued to experience volatility due to unpredictable oil demand dynamics and uncertainty over supply policies.

In conclusion, the pandemic's impact on stock markets manifested differently across sectors. The elevated MAPE values in the energy, transportation, and consumer discretionary sectors reflect the extreme difficulty in making accurate forecasts in such a globally uncertain context. However, sectors like pharmaceuticals, more closely tied to scientific advancements in controlling the pandemic, showed greater stability, demonstrating how the crisis had a varied and multifaceted impact on the global economy.

## 7.2 Correlations between COVID-19 and TGARCH model forecast errors

| ETF | Correlation with Cases | Correlation with Growth Factor |
|---|---|---|
| XLC | -0.12830 | 0.09156 |
| XLY | -0.01765 | 0.10523 |
| XLV | -0.04016 | 0.05539 |
| XTN | -0.16335 | 0.01623 |
| XLE | -0.02699 | 0.08793 |
| XLF | -0.01106 | 0.12414 |
| FINX | -0.03337 | 0.10044 |

Table 7.5: Correlation between Forecast Errors and COVID-19 Data between 21st January and 11th December 2020

According to Table 7.5, a moderate negative correlation is observed between the average number of new COVID-19 cases over the past 10 days and the forecast errors. However, when examining the growth factor of the average new cases during this period, the correlation with the forecast errors shifts to positive and is highly significant across most sectors and industries, as specified by the paper, including Movie & Entertainment (XLC), Data Processing & Outsourcing Services (FINX), Pharmaceuticals (XLV), Apparel, Accessories & Luxury Goods, Automobile Manufacturers, Hotels and Resorts & Cruise Lines, Oil & Gas Exploration & Production, Diversified Banks, and Restaurants.

This suggests that the growth factor in new cases may be a contributing factor influencing the accuracy of the forecasts. The positive correlations for this growth factor indicate that as the daily increase in new COVID cases rises, the MAPE values also increase. This observation is reasonable given the uncertainty stemming from reports of a significant rise in COVID cases, which can lead to heightened volatility in stock prices.

## 7.3 Single and Complete linkage Analysis

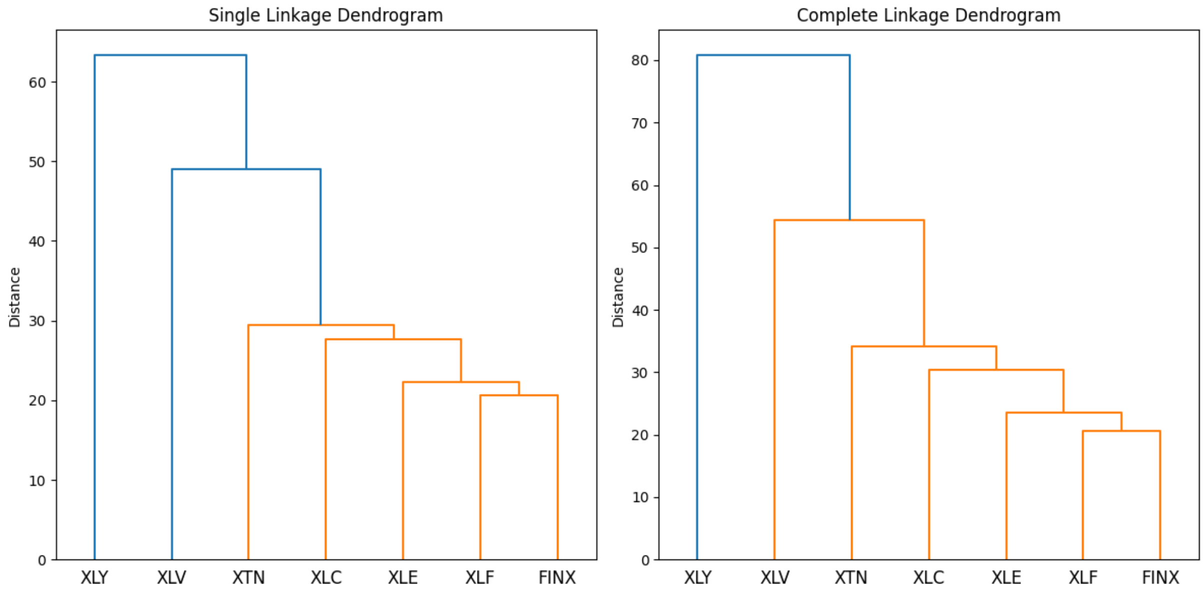


Figure 7.2: Single linkage and complete linkage dendrograms of S&P500 Sector, based on multi-step-ahead forecast errors during the COVID-19 pandemic.

The analysis of hierarchical single and complete linkage dendrograms provides complementary insights for both the sectors of S&P500 ETFs and the specific industries within these sectors. These graphs are based on calculating Euclidean distances between different sectors and industries, highlighting similarities and dissimilarities based on metrics such as volatility or forecast errors.

The **Single linkage method** measures the distance between two clusters by considering the closest two elements. This leads to less compact clusters, where groups tend to stretch out, allowing connections even with distant elements. On the other hand, the **Complete linkage method** considers the maximum distance between two points of the clusters, resulting in more homogeneous and compact groups.

These methods are used to identify relationships between different sectors and sub-sectors. In the case of the **S&P500 ETF sectors**, we see that the single linkage shows less defined clusters, where some sectors, such as Consumer Discretionary (XLY) and Healthcare (XLV), are quite distant from the rest. This indicates a lower correlation with other sectors. On the other hand, sectors like Energy (XLE), Financials (XLF), and FinTech (FINX) show higher correlation, as they form close clusters. The complete linkage, on the contrary, provides a more compact and homogeneous view, showing a clearer separation between groups of sectors like XLY and XLV compared to the others. This suggests that, overall, sectors like Energy and Financials behave similarly, while sectors like Consumer Discretionary and Healthcare are more isolated in their behavior.

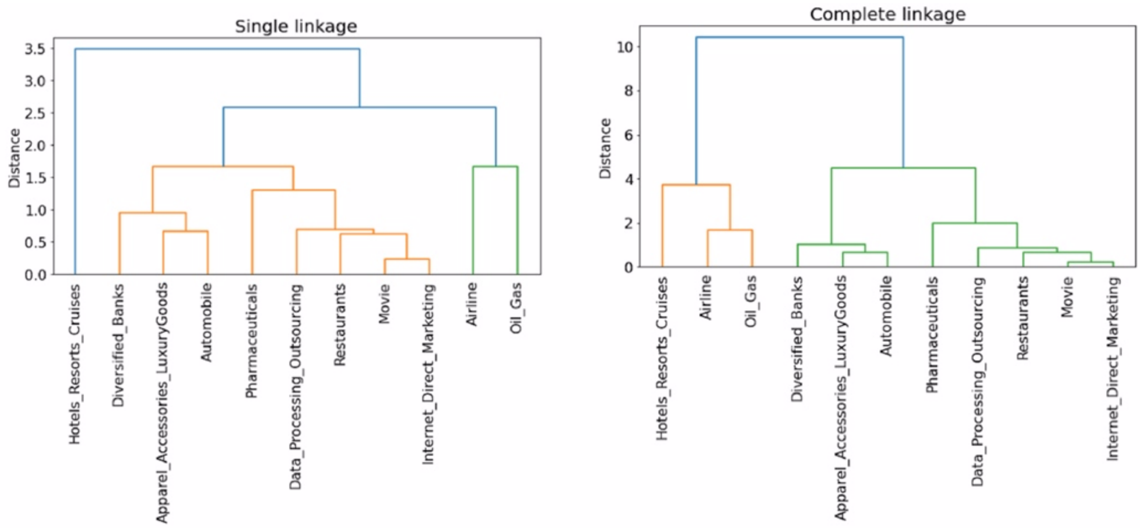


Figure 7.3: Single linkage and complete linkage dendrograms of S&P500 Industries, based on multi-step-ahead forecast errors during the COVID-19 pandemic.

Moving on to the **industries within the sectors**, the graph taken from the paper (Figure 7.3) provides a detailed analysis of how different industrial sectors reacted to TGARCH volatility forecasts, particularly in relation to forecast errors during the COVID-19 pandemic. The complete linkage identifies three main clusters: the first is composed of the Airline, Oil & Gas, and Hotels industries, the second cluster includes the Automobile, Apparel and Accessories Luxury Goods, and Diversified Banks industries, while the third cluster contains various industries such as Pharmaceuticals, Restaurants, and Data Outsourcing.

In the **Single linkage method**, it is evident that the Hotels, Resorts & Cruises industry is an outlier, confirming that this was the industry with the least accurate TGARCH forecasts, classified as an anomaly. Although the Airline and Oil & Gas industries are grouped together in both dendrograms, their forecast error, while high, is not as pronounced as in the Hotels industry.

The industries that experienced a lower impact during the pandemic, such as Restaurants, Pharmaceuticals, Data Processing & Outsourcing, Movie, and Internet Direct Marketing, show closer proximity to each other in Euclidean distance and lower forecast errors. Overall, the identified clusters confirm the differentiated impact of the pandemic on various industries: those related to travel and tourism (hotels and airlines) were heavily affected, while technology and online marketing industries, along with pharmaceuticals, showed greater resilience.

In the **S&P500 ETF graph**, we can observe that sectors such as Consumer Discretionary (XLY) and Healthcare (XLV) are more distant from other sectors, indicating lower correlation with them. In contrast, sectors like Energy (XLE), Financials (XLF), and FinTech (FINX) form tighter clusters, indicating greater similarity in historical price behavior. This structure confirms how some sectors are more connected to each other, while others show independent dynamics.

In conclusion, the **Complete linkage method** tends to form more homogeneous and well-defined groups, while **Single linkage** highlights connections even between more distant elements. From the ETF graph, it emerges that sectors like Financials and Energy tend to behave similarly, while others, such as Consumer Discretionary and Healthcare, follow more isolated dynamics. This structure is consistent with what was observed in the industrial sub-sectors graph, where technological and pharmaceutical industries show greater resilience compared to those linked to tourism, which were more affected by external events like the pandemic.

## 7.4 Two-dimensional feature space maps

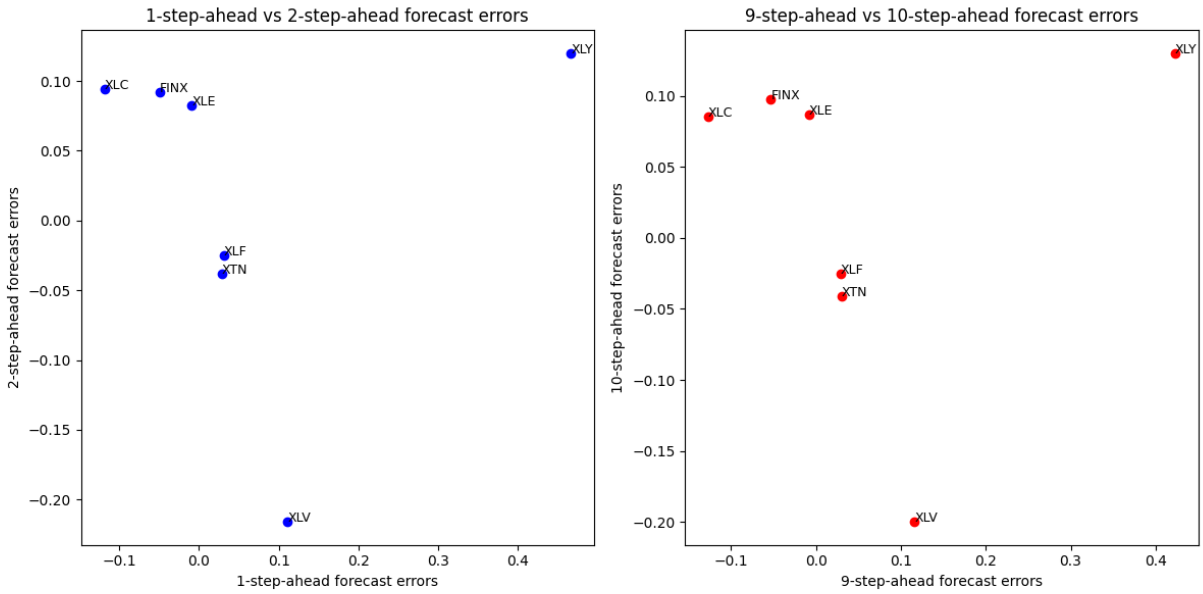


Figure 7.4: Two-dimensional feature maps of the S&P500 industry cluster solution: 1-step-ahead versus 2-step-ahead forecast errors

The step-ahead forecast error graphs provide a visual indication of how stable the predictions are over different time horizons. If the differences between the errors are minimal, we can conclude that the model is robust and the predictions are reliable. Conversely, large differences between one step and the next signal instability or difficulties in accurately predicting future trends.

Figures 7.4 and 7.5 present a representation of step-ahead forecast errors for two categories: sector ETFs of the S&P 500 and the industries within each sector. Each figure includes two graphs: one showing the difference between the 1-step-ahead and 2-step-ahead forecasts, and one comparing the 9-step-ahead and 10-step-ahead forecasts. The step-ahead forecast errors show how much predictions at a given horizon differ from the subsequent ones, revealing the reliability and precision of the forecasts.

In the graph in Figure 7.4, which relates to the ETFs, the points represent the different sectors of the S&P 500. In the 1-step-ahead vs 2-step-ahead graph, most sectors show limited errors, with only a few standing out significantly, such as XLY (Consumer Discretionary) and XLV (Health Care), suggesting greater volatility or difficulty in forecasting for these sectors. In the 9-step-ahead vs 10-step-ahead graph, we observe a similar pattern. Sectors like XLY continue to show larger errors, indicating consistent volatility even over longer time horizons.

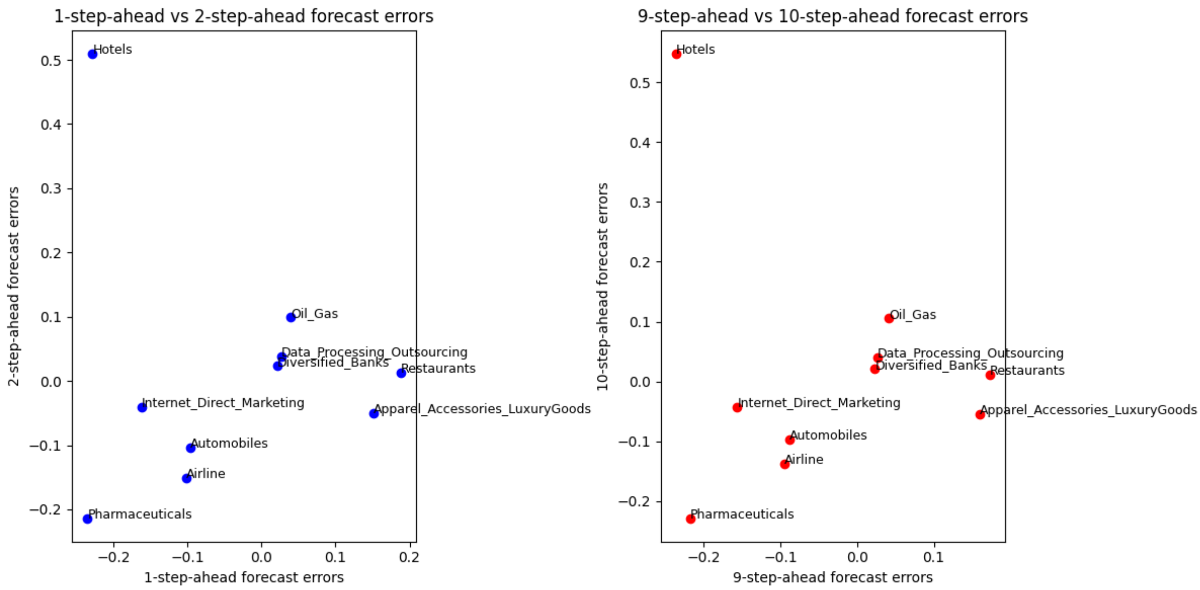


Figure 7.5: Two-dimensional feature maps of the S&P500 industry cluster solution: 9-step-ahead versus 10-step-ahead forecast errors

In the graph in Figure 7.5, which analyzes individual industries, each point represents an industry within a sector. In the 1-step-ahead vs 2-step-ahead graph, industries such as Hotels show significantly higher forecast errors compared to others, indicating strong uncertainty in predictions. On the other hand, industries like Pharmaceuticals or Airlines show very low errors, suggesting greater stability in short-term forecasts. In the 9-step-ahead vs 10-step-ahead graph, we see that some industries, such as Hotels, continue to show high errors, while more stable industries like Pharmaceuticals maintain consistent forecasts even over longer time horizons.

An important element is the consistency between the two graphs. We observe that sectors like Consumer Discretionary (XLY), which exhibit volatile behavior in the ETF graphs, are also reflected in the predictions for the industries within them, such as Hotels. This indicates that less reliable forecasts at the sector level also affect the forecasts for the underlying industries. Conversely, more stable sectors, such as Health Care (XLV for ETFs and Pharmaceuticals for industries), show consistency in terms of lower error variability, signaling more solid predictability.

The graphs reveal a significant consistency between the performance of sectors and the industries that comprise them. In sectors characterized by high volatility, such as Consumer Discretionary, we find industries with similarly high forecast errors, highlighting how macro-level uncertainty propagates to the micro level. Conversely, more stable sectors maintain consistent forecasts both at the sector and industry level. This suggests that sectors with lower forecast errors, such as Health Care, might be safer for long-term predictions, whereas more volatile sectors may require constant monitoring and more complex forecasting models.

# Chapter 8 CONCLUSION

The analysis of the step-ahead forecasting error charts provides a crucial tool for evaluating the stability of predictions during the COVID-19 pandemic, especially for sector-specific ETFs within the S&P500 and the industries related to them. By representing the differences between 1-step vs. 2-step forecasts and 9-step vs. 10-step forecasts, these charts clearly highlight the dynamics of model reliability, shedding light on sectors and industries that exhibited higher volatility and uncertainty throughout the pandemic.

Figures 7.4 and 7.5 offer a detailed visualization of multi-step forecasting errors, revealing the differentiated response of various sectors and industries during the critical phases of the pandemic. These charts, particularly useful for identifying forecasting discrepancies, show how the pandemic accentuated the difficulties in making accurate predictions over different time horizons.

In the case of sector ETFs (Figure 7.4), we see that sectors like Consumer Discretionary (XLY) and Healthcare (XLV) registered higher errors, particularly in the transition from short-term (1-step vs. 2-step forecasts) to medium-long-term forecasts (9-step vs. 10-step). The Consumer Discretionary sector (XLY), especially represented by volatile components like tourism and luxury, was severely impacted by economic lockdowns, travel restrictions, and global uncertainty. This made it extremely challenging for models to predict future trends with precision. The persistent forecasting error suggests that, during the pandemic, consumer discretionary goods experienced sharp fluctuations due to the uneven global economic recovery and unpredictable consumer behavior.

In contrast, the Healthcare sector (XLV), while also showing significant errors, exhibited a degree of resilience. During the pandemic, the demand for drugs, vaccines, and medical care increased considerably, providing the sector with more stable fundamentals. However, the high pressure on supply chains and the large-scale production of vaccines caused some variability in forecasts, especially during transitional phases between pandemic waves and the emergence of new virus variants.

Figure 7.5, which analyzes individual industries within sectors, confirms the patterns seen at the sector level. For example, within the Consumer Discretionary sector, the Hotel industry recorded some of the highest forecast errors, reflecting the ongoing uncertainty and volatility in global tourism. The sharp reduction in travel and the closure of borders made it nearly impossible for models to predict the hospitality sector consistently, which is highly dependent on global tourism. The evolution of restrictions and perceptions of health risks significantly influenced this industry's recovery, resulting in large forecasting margins of error.

On the other hand, sectors such as Pharmaceuticals, represented in the charts by ETFs like XLV, demonstrated greater stability and consistency in their forecasts both at the sector and industry levels. The forecasts for industries like Pharmaceuticals were more accurate even over longer time horizons (9-step vs. 10-step), suggesting that this sector, despite the pressures of increased production, managed uncertainties better during the pandemic. This may be due to the relatively predictable demand for drugs and vaccines, supported by significant governmental efforts and large-scale public investment. The pharmaceutical industry benefited from structured planning and long-term visibility, particularly linked to global vaccination campaigns.

A deeper analysis reveals that sectors such as Airlines and Hotels, among the hardest hit by the pandemic, emerged as clear outliers in the clustering process. The Threshold GARCH model used to analyze these sectors showed significant difficulty in producing accurate forecasts, especially during the critical periods of the pandemic (February-May 2020). The sharp contraction in demand and the rapid evolution of containment measures, such as international border closures and flight suspensions, made the behavior of these industries highly unpredictable, positioning them in distinct clusters compared to more resilient sectors.

Another significant cluster is formed by industries within the Oil and Gas sector, which, although initially affected by the global drop in energy demand, experienced a more gradual recovery compared to industries like tourism and travel. The Oil industry benefited from the progressive reopening of global economic activity, but its forecast remains influenced by external factors such as economic slowdowns or political decisions regarding energy production and consumption.

Overall, the analysis leads to clear conclusions. On the one hand, sectors such as Consumer Discretionary and tourism-related industries show significant unreliability in their forecasts, with high errors due to volatility and the unpredictable nature of pandemic-related restrictions. These sectors and their industries require constant monitoring and increasingly sophisticated predictive models to adapt to unexpected and dynamic events.

On the other hand, sectors like Healthcare and Pharmaceuticals have proven to be more stable and predictable, reflecting their ability to better adapt to the pandemic's dynamics. This suggests that investors, during periods of global uncertainty, might find refuge in sectors with stronger fundamentals and less exposure to external shocks.

In summary, the COVID-19 pandemic has amplified divergences between different economic sectors, highlighting the need for thorough and detailed analysis to understand market dynamics. ETFs, as instruments reflecting the performance of entire sectors, have proven useful in analyzing market volatility on a broad scale, while specific industries within these ETFs have demonstrated how macro-level forecasts can propagate to the micro level. The results confirm that stable sectors like Healthcare and Pharmaceuticals offer greater forecasting reliability, while more volatile sectors like Consumer Discretionary require more flexible and dynamic approaches to managing uncertainty.

# Part V

# Chapter 9 REFERENCES

## 9.1 Webography


1. **Morningstar:** www.morningstar.com
2. **Yahoo Finance:** finance.yahoo.com
3. **Company Financial Statements:** www.sec.gov/edgar/searchedgar/companysearch.html
4. **ETF XLV Overview:** https://www.morningstar.com/etfs/arcx/xlv/performance
5. **ETF Big Pharma:** https://www.etf.com/XLVoverview


### 9.1.1 Sector References


1. **Communication Services ETF:** https://finance.yahoo.com/quote/XLC/
2. **Consumer Discretionary ETF:** https://finance.yahoo.com/quote/XLY/
3. **Healthcare ETF:** https://finance.yahoo.com/quote/XLV/
4. **Industrials ETF:** https://finance.yahoo.com/quote/XTN/
5. **Energy ETF:** https://finance.yahoo.com/quote/XLE/
6. **Financial ETF:** https://finance.yahoo.com/quote/XLF/
7. **Information Technology ETF:** https://finance.yahoo.com/quote/FINX/

## 9.2 Bibliography


1. Achille, A., & Zipser, D. (2020). A perspective for the luxury-goods industry during—and after—coronavirus. McKinsey & Company.

2. Albulescu, C. T. (2021). COVID-19 and the United States financial markets' volatility. *Finance Research Letters*, 38, Article 101699.

3. Baek, S., Mohanty, S. K., & Glambosky, M. (2020). COVID-19 and stock market volatility: An industry level analysis. *Finance Research Letters*, 37, Article 101748.

4. Baker, S., Bloom, N., Davis, S., Kost, K., Sammon, M., & Viratyosin, T. (2020). The Unprecedented Stock Market Reaction to COVID-19. *The Review of Asset Pricing Studies*, 10, 742–758.

5. Bastos, J. A., & Caiado, J. (2014). Clustering financial time series with variance ratio statistics. *Quantitative Finance*, 14, 2121–2133.

6. Bastos, J. A., & Caiado, J. (2021). On the classification of financial data with domain agnostic features. *International Journal of Approximate Reasoning*, 138, 1–11.

7. Bureau of Transportation Statistics (2021). "U.S. Airlines 2020 Net Profit Down $35 Billion from 2019". https://www.bts.gov/newsroom/us-airlines-2020-net-profit-down-35-billion-2019 (Accessed: 6 January 2022).

8. Caiado, J. and Crato, N. (2007). "A GARCH-based method for clustering of financial time series: International stock markets evidence", in: Recent Advances in Stochastic Modeling and Data Analysis (Edited by C. H. Skiadas), World Scientific Publishing, 542-551.

9. Caiado, J., & Crato, N. (2010). Identifying common dynamic features in stock returns. Quantitative Finance, 10, 797–807.

10. Caiado, J., Maharaj, E. A., & D'Urso, P. (2015). Time Series Clustering. In C. Henning, M. Meila, F. Murtagh, & R. Rocci (Eds.), Handbook of Cluster Analysis (pp. 241–263). CRC Press, Taylor & Francis Group.

11. Caiado, J., Crato, N., & Poncela, P. (2020). A fragmented-periodogram approach for clustering big data time series. Advances in Data Analysis and Classification, 14, 117–146.

12. Carter, D., Mazumder, S., Simkins, B., & Sisneros, E. (2022). The stock price reaction of the COVID-19 pandemic on the airline, hotel, and tourism industries. Finance Research Letters, 44, Article 102047.

13. Chaudhary, R., Bakhshi, P., & Gupta, H. (2020). Volatility in international stock markets: An empirical study during COVID-19. Journal of Risk and Financial Management, 13(9), 208.

14. Deloitte. (2020a). Understanding COVID-19's impact on the automotive sector. Accessed 5 February 2022.

15. Deloitte. (2020b). Understanding the sector impact of COVID-19: Banking & Financial Markets. Accessed 1 April 2020.

16. Deloitte. (2020c). Understanding the sector impact of COVID-19: Oil & Gas. Accessed 5 February 2022.

17. Ding, W., Levine, R., Lin, C., & Xie, W. (2021). Corporate immunity to the COVID-19 pandemic. Journal of Financial Economics, 141, 802–830.

18. Engelhardt, N., Krause, M., Neukirchen, D., & Posch, P. N. (2021). Trust and stock market volatility during the COVID-19 crisis. Finance Research Letters, 38, Article 101873.

19. Ertugrul, H. M., Güngör, B. O., & Soytas, U. (2020). The Effect of Covid-19 Outbreak on Turkish Diesel Consumption Volatility Dynamics. University Library of Munich. MPRA Paper 110166.

20. Everitt, B., Landau, S., & Leese, M. (2001). Cluster Analysis (4th ed.). Edward Arnold: London.

21. Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. Journal of Finance, 48, 1779–1801.

22. Krishnan, V., Mann, R., Seitzman, N., & Wittkamp, N. (2020). Hospitality and COVID-19: How long until 'no vacancy' for US hotels? Mckinsey & Company.

23. Maharaj, E. A., D'Urso, P., & Caiado, J. (2019). Time Series Classification and Clustering, CRC Press. United States: Taylor & Francis Group.

24. MarketWatch (2021). Market Data Center [Database], New York. Accessed 7 February 2022.

25. Mazur, M., Dang, M., & Vega, M. (2021). COVID-19 and the March 2020 stock market crash: Evidence from S&P1500. Finance Research Letters, 38, Article 101690.

26. Mittal, S., & Sharma, D. (2021). The Impact of COVID-19 on Stock Returns of the Indian Healthcare and Pharmaceutical Sector. Australasian Accounting, Business and Finance Journal, 15, 5–21.

27. Onali, E. (2020). Covid-19 and stock market volatility. Available at SSRN: https://ssrn.com/abstract=3571453 or https://doi.org/10.2139/ssrn.3571453.

28. Otranto, E. (2008). Clustering heteroskedastic time series by model-based procedures. Computational Statistics & Data Analysis, 52, 4685–4698.

29. Otranto, E. (2010). Identifying financial time series with similar dynamic conditional correlation. Computational Statistics & Data Analysis, 54, 1–15.

30. Papadamou, S., Fassas, A., Kenourgios, D., & Dimitriou, D. (2020). Direct and indirect effects of COVID-19 pandemic on implied stock market volatility: Evidence from panel data analysis. University Library of Munich. MPRA Working Paper 100020.

31. Lúcio, F., & Caiado, J. (2022). COVID-19 and Stock Market Volatility: A Clustering Approach for S&P 500 Industry Indices. Finance Research Letters, 49, Article 103141.

32. Song, H. J., Yeon, J., & Lee, S. (2021). Impact of the COVID-19 pandemic: Evidence from the US restaurant industry. International Journal of Hospitality Management, 92, Article 102702.

33. Statista (2021a). Weekly flights change of global airlines due to COVID-19 as of January 2021. Accessed 7 February 2022.

34. Statista (2022a). Development of e-commerce shares in total retail sales in selected countries before and after the coronavirus (COVID-19) pandemic as of January 2021. Accessed 6 January 2022.

35. Statista (2022b). Netflix subscribers count worldwide 2011-2022. Accessed 6 January 2022.

36. Wang, J., Lu, X., He, F., & Ma, F. (2020). Which popular predictor is more useful to forecast international stock markets during the coronavirus pandemic: VIX vs EPU? International Review of Financial Analysis, 72, Article 101596.

37. Zakoian, J.-M. (1994). Threshold heteroskedastic models. Journal of Economic Dynamics and Control, 18, 931–955.

## 9.3 Appendix

1. **Analysis until Cap 7:**
   github.com/NicoAmadori/Master$_{T}hesis/blob/main/TESI.R$
2. **Analysis from Cap 7:**
   github.com/NicoAmadori/Master$_{T}hesis/blob/main/TESI_{2}.R$
3. **Descriptive statistics and Models Comparison:**
   github.com/NicoAmadori/Master$_{T}hesis/blob/main/Stat\&ModelComparison.ipynb$
4. **Last Graphics Chapter 7:**
   github.com/NicoAmadori/Master$_{T}hesis/blob/main/Stats\&LastGraphics.ipynb$

## 9.4 Disclaimer

Grammar error and text have been corrected with OpenAI

# Acknowledgments

È strano come il tempo scivoli tra le dita, due anni già volati, eppure ricordo come fosse ieri il momento in cui scrivevo i ringraziamenti della mia triennale. Riguardando indietro, scopro quanto io sia cambiato. Non sono più quel ragazzo di allora, anche se a volte stento a riconoscere le trasformazioni che il tempo e le esperienze hanno portato. È come se, passo dopo passo, pezzi di me si fossero rimodellati: la mia visione del mondo, le mie ambizioni, persino il modo in cui affronto il quotidiano. Ho perso persone lungo la strada, alcune vicinissime, altre di passaggio, e ho capito che forse è naturale così. Forse, come diceva qualcuno, i rapporti si stagliano sul nostro cammino per insegnarci qualcosa di essenziale, anche se a volte il loro viaggio accanto a noi è destinato a interrompersi.

A questo punto, voglio ringraziare chi mi è stato accanto con pazienza e ostinazione, chi ha saputo tenere saldo il proprio posto al mio fianco, anche nei giorni in cui, incerto e confuso, non sapevo neppure io quale fosse la direzione. So di non essere sempre una persona facile: a tratti cupo, tragicamente insicuro, a volte diviso tra mille pensieri, ma ho anche dei difetti. Prima di esprimere la mia gratitudine più sincera, sento il bisogno di condividere alcune scelte che hanno segnato il mio cammino, una su tutte l'Erasmus.

L'Erasmus è stata una decisione istintiva, un bisogno di fuggire dalla routine soffocante della mia città e dalla sensazione di mancanza di significato. Iniziando il percorso della magistrale, sentivo dentro di me una crescente inquietudine, una sete di qualcosa di nuovo e sconosciuto. Così, senza certezze, sono partito per la Polonia, sperando di trovare uno spazio dove respirare a pieni polmoni e riscoprire parti di me perdute nel tempo.

I primi mesi sono stati difficili. Il peso dello shock culturale ha riportato a galla insicurezze e paure represse, facendomi crollare. Ho vissuto momenti di solitudine e incomprensione, ho pianto come non ricordavo di poter fare. Tuttavia, paradossalmente, in quella fragilità ho trovato una nuova vitalità. Ho scoperto il conforto di chi, come me, stava attraversando le stesse sfide. Insieme, lontani da casa, abbiamo trovato una seconda famiglia e un senso di appartenenza che ha reso la Polonia una seconda casa.

Questa esperienza mi ha trasformato profondamente. Ho affrontato momenti

difficili, ma anche attimi di pura gioia e una leggerezza che non credevo possibile. Sono grato alla mia determinazione, alla forza che ho riscoperto dentro di me e a quella parte di me che ha scelto di non mollare, di restare anche quando era più semplice fuggire. Ora, guardando indietro, vedo le ombre dei timori superati e le cadute che mi hanno insegnato a rialzarmi, e mi rendo conto di essere cresciuto, diventando una persona più forte, consapevole e pronta a guardare con fiducia verso il futuro.

Ai miei genitori, il mio più sincero e profondo ringraziamento. Siete stati il mio sostegno silenzioso, il punto fermo che mi ha accompagnato in ogni passo di questo cammino. Senza di voi, senza il vostro sguardo discreto e incoraggiante, molte delle mie conquiste sarebbero rimaste solo sogni. Mi avete permesso di crescere seguendo il mio percorso, rispettando ogni mia scelta e lasciandomi la libertà di imparare, anche dagli errori, senza mai farmi sentire il peso del fallimento. In ogni momento di incertezza, quando la strada sembrava difficile e i miei dubbi prendevano il sopravvento, voi avete sempre avuto fiducia in me. La vostra presenza ha rappresentato una forza stabile, una sicurezza che mi ha permesso di andare avanti anche nei momenti più complessi, ricordandomi chi sono e di cosa sono capace. Se oggi sono qui, lo devo a voi, alla vostra forza e alla pazienza con cui mi avete accompagnato. Mi avete dato il coraggio di guardare oltre i miei limiti e la libertà di crescere senza paure. Mi avete insegnato a mantenere salde le mie radici senza mai spegnere il desiderio di esplorare, e così ho trovato la mia strada senza mai smarrirmi davvero. Vi sono grato per avermi dato la possibilità di essere la persona che sono oggi, costruendo un cammino di cui posso essere fiero. Spero che ogni mio traguardo possa riflettere, anche solo in parte, la fiducia che avete sempre riposto in me.

Qualcuno ha detto: "La vita senza una sorella è noiosa, ma la vita con una sorella... è un campo di battaglia." Non ci sono parole migliori per descrivere il nostro rapporto. Se la mia esistenza è sempre così agitata e incasinata, il merito è tutto tuo. Sì, proprio tuo, che te ne sei andata via di casa per costruirti una famiglia e mi hai lasciato da solo in questo covo di matti. Mi hai persino spinto a rifugiarmi in Polonia per sei mesi pur di scappare dalle mille attenzioni di mamma e babbo! Ma, nonostante tutto, devo ammettere che la tua assenza è stata solo temporanea. Anche lontana, sei sempre riuscita a farmi sentire parte di qualcosa. E le tue "lezioni di vita" non sono certo mancate: prendendomi cura dei tuoi figli, ho imparato la virtù della pazienza – una dote che, ne sono certo, tornerà utile non solo con mamma e babbo, ma anche con i miei futuri figli, la fidanzata e pure al lavoro. Certo, Biboz e la Ludo sono adorabili, ma diciamocelo: nessuno, nemmeno al mio peggior nemico, augurerei il "privilegio" di svegliarsi alle 8 del mattino con la domanda: "Zio, a cosa giochiamo?". In quei momenti capisci che la giornata sarà lunga... e che la tua sanità mentale è in pericolo.

Eppure, dietro tutto questo amore-odio, c'è un legame che va ben oltre le piccole rivalità. Ti devo dire grazie. Grazie perché, quando ero lontano, durante l'Erasmus, sei stata tu a mantenere viva la nostra famiglia per me. Le tue videochiamate, i messaggi di gruppo, le foto dei bimbi... mi hanno fatto sentire meno solo e mi hanno evitato almeno in parte il "terzo grado" di mamma. In ogni passo importante, anche da lontano, ci sei sempre stata. Forse non te lo dico mai, ma sono grato per questo legame che, tra alti e bassi, rimane una delle cose più forti e autentiche che ho. Grazie per essere sempre stata lì, per tenermi saldo anche quando ero lontano.

Se non sbaglio, c'è un famosissimo film di Aldo, Giovanni e Giacomo che si intitola: "Tre uomini e una laurea"... ecco, mi sento in dovere di dedicare qualche riga – ma solo qualche riga, perché davvero non se le meritano – ai miei Giovanni e Giacomo: **Matte e Zigno**.
Con voi, questo percorso ha preso un significato tutto nuovo. Nonostante l'ultimo anno di magistrale ci abbia sparpagliati in ogni angolo d'Europa, non abbiamo mai smesso di sentirci e di supportarci. È stato un viaggio a dir poco comico: tra i drammi di Matte con tutte le sue conquiste universitarie, i racconti da soap opera dell'Eli, a cui io e Zigno abbiamo assistito, con le sue telefonate minacciose a un tipino che non rispondeva, e le discussioni con la Bea su come "il meno va sempre prima del per", ogni momento è stato memorabile. Ci siamo spaccati sui libri, e puntualmente solo uno di noi si incasinava da solo agli orali (eh sì, quello ero io). Ho fatto bene a dare Tecniche all'estero dopo essere stato bocciato ben cinque volte a Matematica Finanziaria... per aver parlato troppo. Ma alla fine, ce l'abbiamo fatta: l'università è finita, niente più ansia da esami, né paura di rovinare la media. Ora l'unica vera paura che ci resta... è iniziare a lavorare!

Ai miei amici del king, o meglio, ai miei king, con una dedica speciale per **Cerri e Carlo**, gli stacanovisti per eccellenza. Loro, che incarnano la puntualità e che sanno bene cosa significa rispettare gli orari—anzi, quasi sempre mi insegnano l'arte del "quarto d'ora accademico". E no, per loro non si tratta di un quarto d'ora, ma di un'ora accademica intera, che sfruttano fino all'ultimo secondo. La puntualità, insomma, è un concetto relativo: diciamo che hanno un'interpretazione tutta loro.
Sempre loro, che a calcetto hanno riposto così tanta fiducia nelle mie doti atletiche da portarmi a strafare... risultato? Cinque bei punti di sutura e un naso in saldo! Una vittoria in pieno stile, una "tombola" che non dimenticherò tanto facilmente. E che dire del trasporto nella tratta kick off-ospedale? Loro che, con grande lungimiranza, hanno deciso di farmi andare con l'unica persona meno affidabile del gruppo—perché le probabilità di non arrivare in ospedale, diciamocelo, erano davvero alte. Amici affidabili, proprio.
Grazie poi per l'inesauribile ondata di proposte sane e virtuose, tipo l'allenamento domenicale. Domenica! Che concetto rivoluzionario, come se non esistessero divani e serie TV! Vi dico che per certe cose bisogna essere fisicamente preparati,

e diciamocelo: tutto questo non è davvero per me. La vita è già una maratona, ragazzi!
Infine, un pensiero all'esperienza in Danimarca, dove la cultura locale, o meglio, il Tinder danese mi ha illuminato. Diciamo che l'esplorazione antropologica delle bionde nordiche mi ha convinto, ma... beh, questo dettaglio magari lo teniamo per noi, vero?
Grazie, per il vostro stile unico, la vostra costanza nel mettermi in situazioni di pura sopravvivenza, e per gli inviti irrinunciabili che continuerò a... ehm... ignorare.

Un grazie speciale ai **Fagiolini**, che in tutti questi anni sono stati il mio porto sicuro... o forse sarebbe meglio dire la mia nave alla deriva! Con voi ogni giornata è diventata una commedia di assurdità e disastri epici, e il quotidiano stress è stato magicamente sostituito da un'altra forma di stress—quella delle risate senza sosta, delle battute che solo noi possiamo capire, e dei drammi che forse sono troppo assurdi per essere veri.
Il nostro rapporto è un capolavoro di complicità e alleanza, con quella punta di follia che solo chi ci conosce davvero può apprezzare. Siamo diventati maestri nel sostenerci... anche se il nostro concetto di "sostegno" a volte somiglia più a una serie di insulti benevoli e provocazioni. Ma alla fine, è questo che ci unisce: non c'è nulla che non ci diremmo in faccia, nulla che non ci faremmo a pezzi a vicenda... per poi ritrovarci sempre lì, a ridere e a ricominciare daccapo.
Abbiamo affrontato momenti indimenticabili, dalle avventure disastrose ai drammi senza senso, alle serate infinite e alle risate che, puntualmente, ci facevano dimenticare ogni altra preoccupazione. Questo legame va oltre la semplice amicizia: è una fratellanza di teste calde e di spiriti liberi che si capiscono senza bisogno di parole (anche perché le parole tra di noi sono spesso... be', diciamo "creative").
Vi ringrazio per essere sempre lì, anche quando siamo sparsi ovunque, e per essere la dose perfetta di leggerezza e follia che rende la vita un po' più divertente e, chissà, forse anche un po' più facile. Grazie, Fagiolini, per essere quello che siete: autentici, folli, e insostituibili. Non cambiate mai... o forse sì, ma giusto un po'!

Se sia il destino o meno che ci ha fatto incontrare, non lo so. Anzi, forse proprio non ci credo. Pensare che sia stato solo il normale corso degli eventi a portarci insieme... beh, fa ridere, no? Ragazzi così diversi, eppure così uguali, veri poli opposti che inspiegabilmente si attraggono. Un po' per caso ci siamo ritrovati, un po' per scelta siamo rimasti. Le persone entrano e escono dalle nostre vite come su un bus di linea, ma noi, nonostante un sacco di turbolenze e fermate improvvise, siamo rimasti a bordo.
Non è stato subito facile capirci: ogni tanto ci si è trovati a discutere, ad alzare i muri e sbattere porte. Ma poi, puntualmente, si tornava indietro – sarà per nostalgia o per puro masochismo – a fare pace, ammettere errori, e rientrare nell'arena. Siamo cresciuti insieme, un po' come un gregge di caproni che impara, a suon di testate, a volersi bene. Alcuni, poi, ci si sono affezionati fin troppo, finendo in vere e proprie gelosie morbose per il gruppo... eh sì, Ari e Betu, non facciamo nomi, ma tutti sanno di chi sto parlando.
Certo, puoi maturare quanto vuoi, ma i limiti sono lì a ricordarci chi siamo. Prendiamo Sintu, per esempio: lui sul pezzo non c'è mai stato e mai ci sarà. Gli puoi dire mille volte "Ci troviamo alle 20," e puntualmente te lo ritrovi che alle 19.50 ti chiama dicendo "Dove? A che ora, stasera?". E quando, per sicurezza, lo chiami dieci minuti prima dell'incontro, è sempre pronto con la scusa del secolo. Poi c'è Belle, il ritardatario cronico e campione nell'arte di farti innervosire: se decide che tu non hai ragione, è impossibile fargli cambiare idea. La nostra esperienza a Catania lo dimostra: se solo avesse prolungato qualche minuto in più la discussione, sarebbe finito giù dalle mura di Arci Castello... anche se devo ammettere che, a conti fatti, aveva ragione lui – la serata afro è stata... come dire... sicuramente unica...
E poi, a proposito di orgogliosi, chi se non la Betu! Io e te sembriamo cane e gatto, e litigare con te è diventato uno sport olimpico. Il problema, però, è che tu, Betu, hai un caratterino che non ti lascia più scampo una volta che parte. Ho imparato una lezione fondamentale: mai dirti "Scazzati, ne parliamo poi", ma sempre "Hai ragione, Teso, sono io quello in torto". Con questa tecnica, la convivenza diventa decisamente più facile! Lo stesso vale per Corso, con il quale, ridendo e battibeccando, siamo riusciti a trovare la nostra armonia. Tra una battuta e l'altra, mi hai insegnato che le vacanze possono essere una vera lotta di sopravvivenza. A proposito: "Arianna, hai finito di smarmellare quel cambio?" – una domanda ormai indelebile nella mia testa.
Solo tu, Ari, potevi sopportare cinque uomini – o meglio, cinque caproni. Ti abbiamo messa alla prova in mille modi, ma ne sei uscita più forte che mai. E pensare che da anni minacciamo di "far fuori" Happy, eppure l'unica che ha rischiato di far fuori tutti noi sei stata proprio tu con le tue leggendariamente letali torte di Natale. Solo Sintu è in grado di mangiarle senza riportare conseguenze.
E Dome, il nostro "buono" per eccellenza, che in realtà buono non è per niente. Spiegatemi quale "bravo amico" vi porterebbe sulle piste avanzate da sci senza sapere sciare, e poi ti lascia lì, a scendere di faccia. E quando finisce la giornata? Ride delle tue disgrazie senza offrirti una mano. E non parliamo di quella volta a Trento, quando ha sbagliato strada, portandoci in una marcia infinita che ha

rischiato di far fuori Fappi. Ma non si può dire che Fappi non sia uno che non sappia restituire il favore: tempo dopo infatti ci ha offerto uno dei suoi famosi pranzi, la pasta al polpo... con solo "un po'" di piccante.
Ah, Fappi, il nostro chef ufficiale e mascotte, esperto di ristoranti come nessun altro. È anche il più testardo: se si impunta su qualcosa, puoi fare tutte le controproposte che vuoi, tanto è inutile – alla fine si farà come dice lui. E nonostante gli dica un orario, Fappi fa sempre di testa sua e si presenta con ore di anticipo, per poi lamentarsi se non siamo pronti! Eppure, non possiamo non volerti bene. In fondo, vuoi sembrare distaccato, ma sei il più pacioccone di tutti. Sì, anche io sono un po' ritardatario – il secondo in classifica dopo Belle – ma devo dire che il ritardo è quasi un marchio di fabbrica, una questione di... personalità. Ho letto uno studio una volta che diceva che i ritardatari cronici sono i più intelligenti. Se è così, posso tranquillamente considerarmi Einstein.
Quando avete ordinato il pacchetto "amico normale," sfortunatamente non avete fatto il reso, quindi vi è toccato tenermi, con tutti i miei difetti (numerosi) e pregi (pochi). Scherzi a parte, grazie per avermi sempre sopportato e per non avermi mai giudicato, anche quando sapevate, forse meglio di me, che stavo prendendo una strada sbagliata. Mi siete stati accanto anche quando mi sentivo lontano e perso. E se, nell'ultimo anno, sono stato un po' distante, vi chiedo scusa: è stato un periodo incasinato, e so che non sempre ho ricambiato come avrei voluto. Grazie per non aver mai mollato la presa, per essermi rimasti accanto, nonostante tutto. So di essere un "caso umano," ma voi siete degli esperti in materia.

Vi amo, Passatello. Un abbraccio dal vostro Nicky: Ari, Sintu, Betu, Dome, Corso, Fappi, Belle.

Come disse una persona che mi è davvero cara: "Le cose migliori si lasciano per ultime." E stavolta, non parliamo di cibo, ma di te, Cate. Eravamo solo due ragazzi in un bar di Forlì, seduti l'uno di fronte all'altra, con quella consapevolezza sospesa che presto le nostre vite avrebbero preso direzioni opposte. Due poli lontani d'Europa ci aspettavano, tu a ovest, io a est. Ed eccoci lì, entrambi pronti a partire, eppure con quella fragile incertezza di chi non è davvero pronto a lasciarsi tutto alle spalle.
Diciamocelo: chi avrebbe mai scommesso su noi due? Se ci fosse stata una previsione sensata, nessuno ci avrebbe dato chance. Eppure, con un'ironia quasi beffarda, hai colto qualcosa che forse io stesso non riuscivo a vedere. Hai sempre detto che avevi un'idea su di noi, e allora mi viene da chiederti: discendi forse da una stirpe di veggenti? Perché, arrivati a questo punto, non mi stupisco più di niente. Con una pazienza che ancora mi lascia senza parole, sei riuscita a scavalcare ogni barriera, a scoprire quel mondo che tentavo così duramente di proteggere dietro una facciata a volte ruvida e distaccata. E in quel mondo tu sei entrata, riempiendolo con un pizzico di follia. Se mi domandassi quando è nato tutto questo, ti direi che è iniziato quel giorno, il nostro primo incontro. Ma non in un bar, né al cinema, né in un contesto che potesse sembrare normale. È tutto così straordinariamente assurdo, inaspettato, speciale. Eppure è reale. Abbiamo volato per ore, macinato migliaia di chilometri solo per avere qualche ora insieme e conoscerci meglio. Parigi, Varsavia, Lituania, Lettonia... sfidando coincidenze, rischiando voli, per quei minuti in più che sembravano sempre troppo pochi. Mi hai donato leggerezza, un'aria nuova che non sapevo di poter respirare. Con te ho messo da parte ansie e preoccupazioni, ho trovato pace persino nel caos. Ci siamo sorretti durante interminabili sessioni, abbiamo affrontato difficoltà che credevamo insormontabili, scoprendo una forza che non immaginavamo. Uno era il sostegno dell'altro, nei momenti in cui tutto sembrava traballare.
Sai, fino ad oggi sono stato tragicamente pessimistico, sempre pronto a vedere il bicchiere mezzo vuoto. Ma adesso voglio permettermi il lusso di essere ottimista, di guardare ogni giorno con gratitudine e, soprattutto, di non preoccuparmi di dove andremo. Non importa quanto durerà, se ci sarà un "per sempre" o no. Voglio solo vivere ogni attimo di questa felicità senza riserve, senza timori.

Perché a volte la felicità è un sogno a cui bisogna concedersi.

## WHEN THEY ASK ME WHAT I THINK ABOUT UNIVERSITARIAN PATH:

SCANNAMI