EBS 289K: Topics in Agricultural Robotics and Automation

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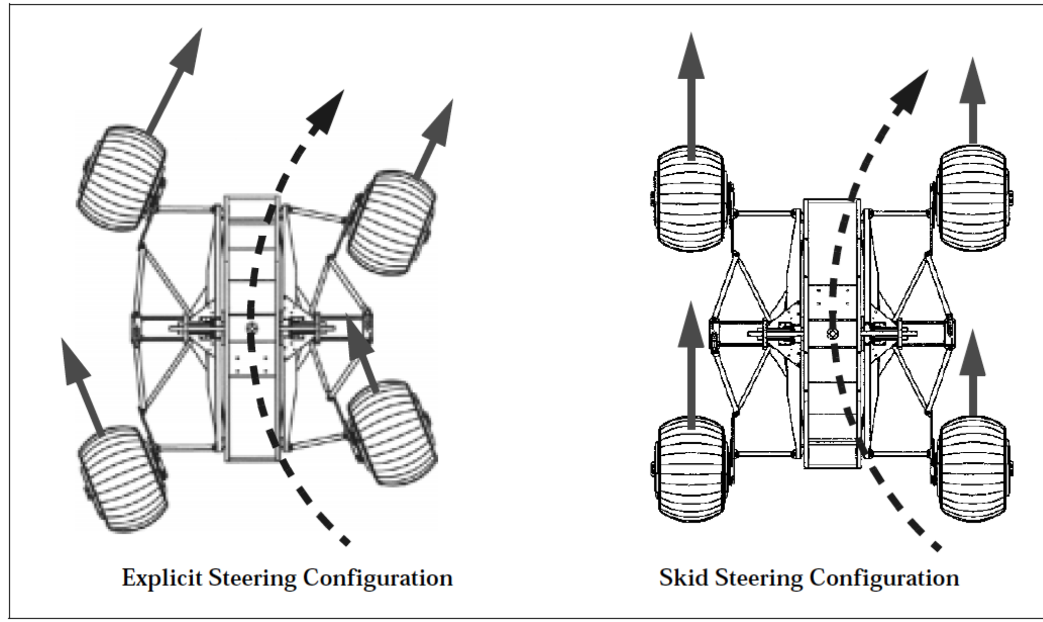
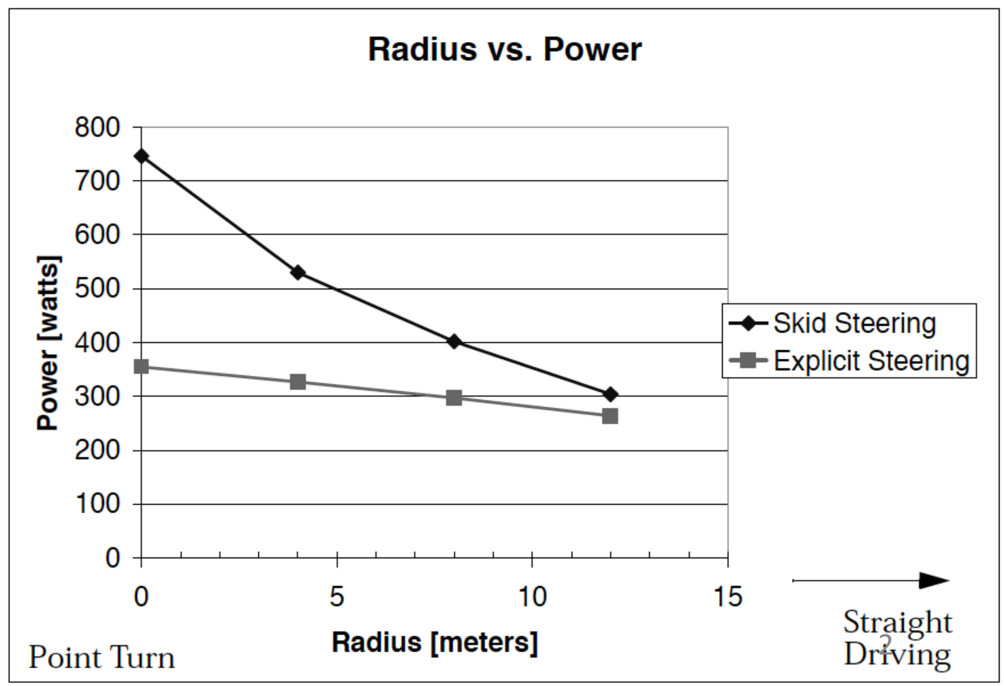
Energy expenditure

- Skid-steering spends more energy





Ackermann Steering

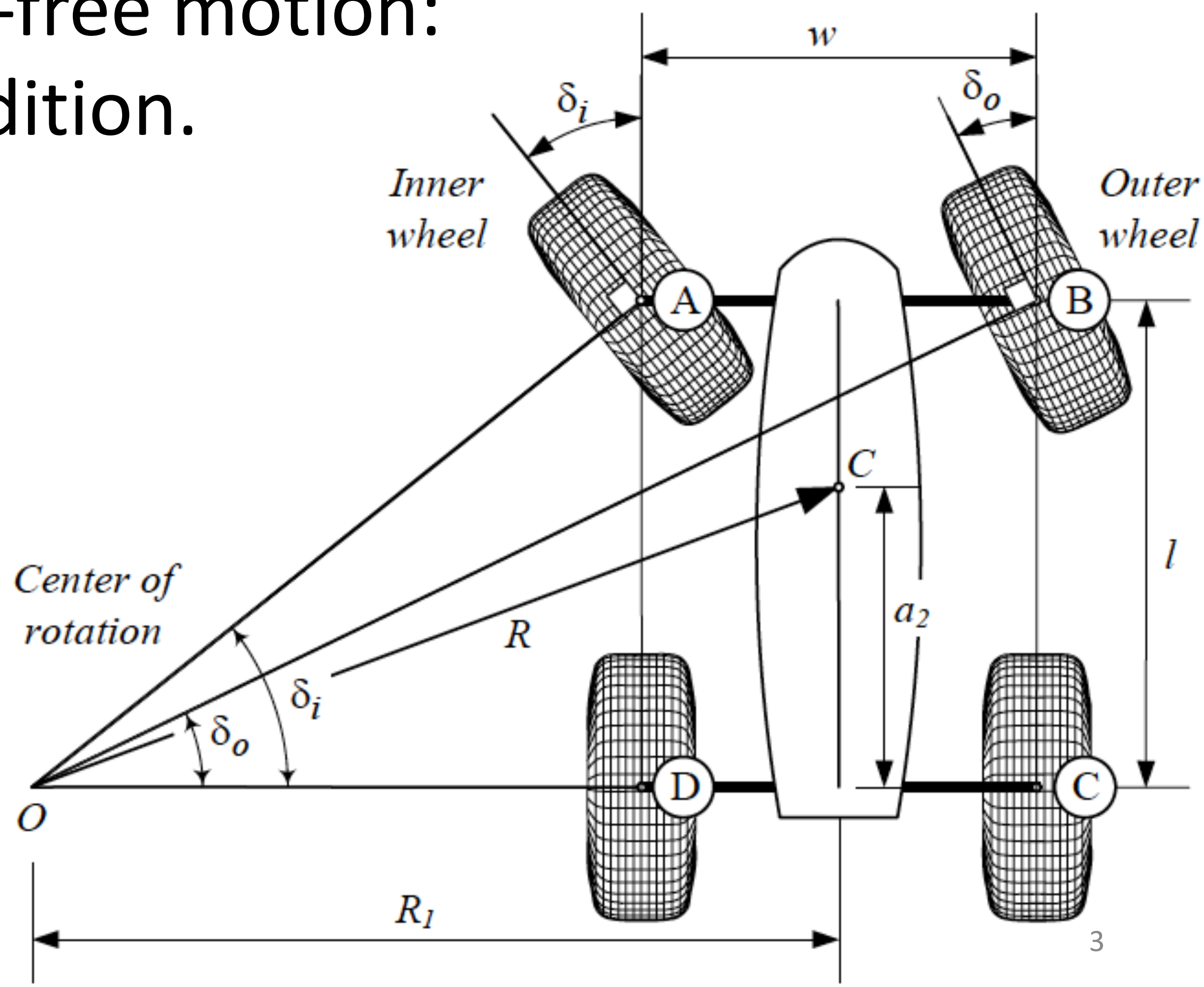
- Front-wheel steering.
- Slow speed, slip-free motion: Ackermann condition.

$$\cot\delta_0 - \cot\delta_i = \frac{w}{l}$$

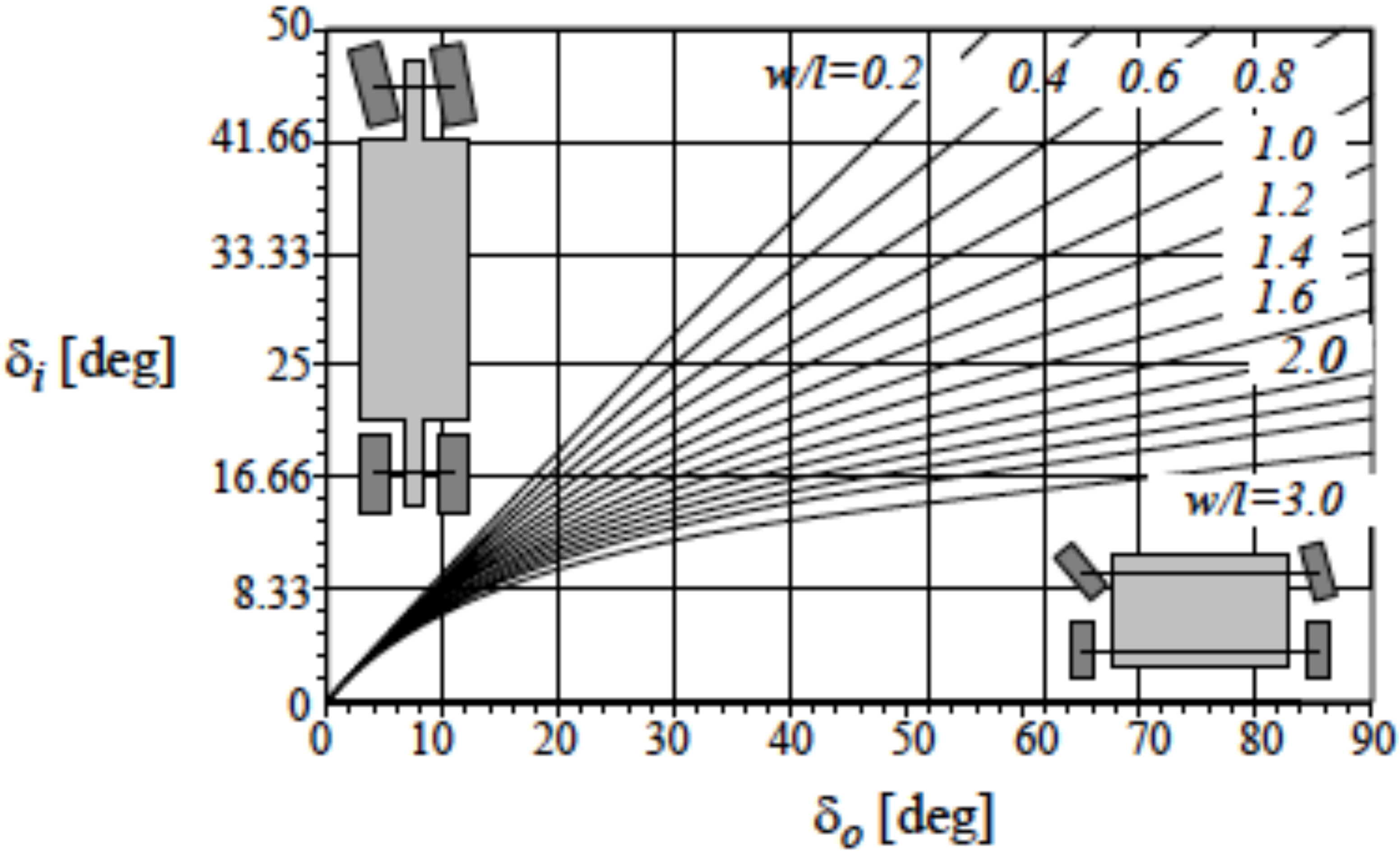
- Prove it!

$$\tan\delta_i = \frac{l}{R_1 - \frac{w}{2}}$$

$$\tan\delta_o = \frac{l}{R_1 + \frac{w}{2}}$$

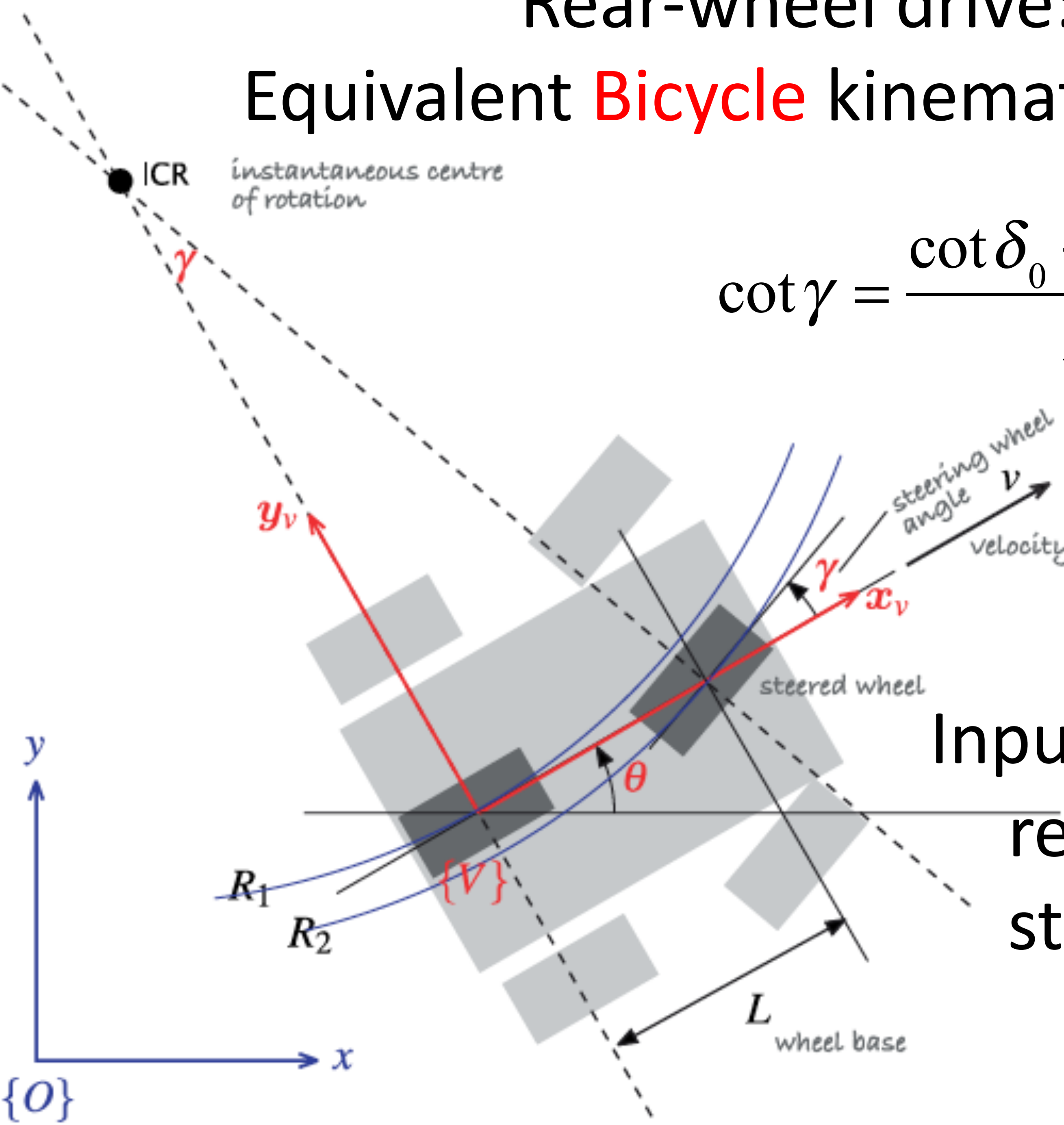


Effect of w/l on the Ackerman condition for front-wheel-steering vehicles



Rear-wheel drive: Equivalent Bicycle kinematic model

$$\cot\gamma = \frac{\cot\delta_0 + \cot\delta_i}{2}$$



Inputs:

rear wheel v,

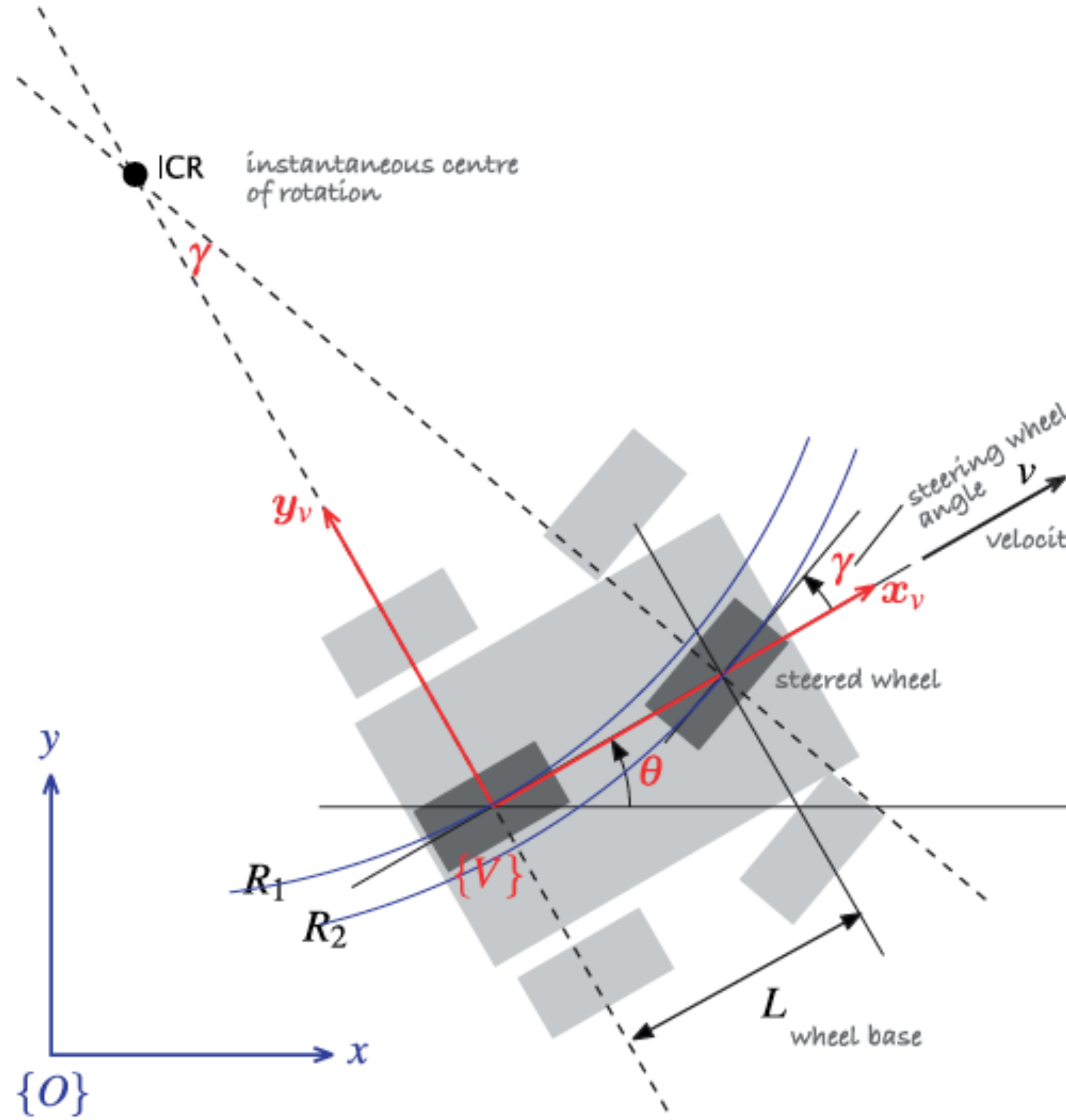
steering angle γ.

Bicycle model kinematic equations

$$\dot{x} = v\cos\theta$$

$$\dot{y} = v\sin\theta$$

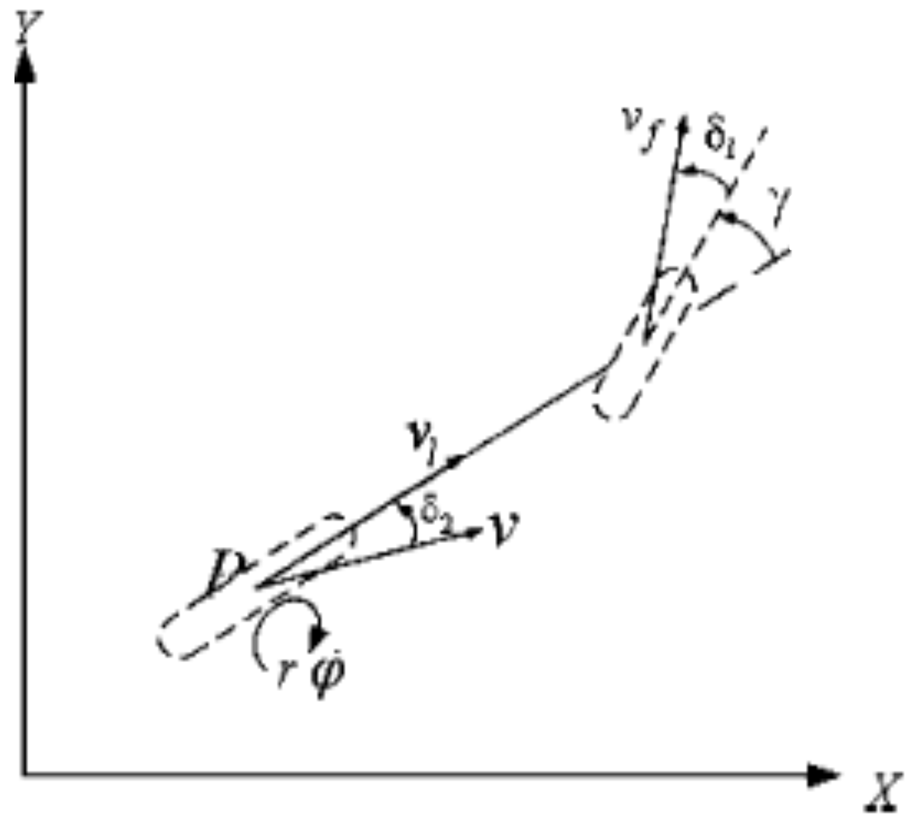
$$\dot{\theta} = v\frac{\tan\gamma}{L}$$



- Hint:
  Vehicle rotates about ICR.

Skidding and slipping

- Front wheel:
  - ❑ Skidding with slip angle δ_1.
- Rear (driven) wheel:
  - ❑ Longitudinal slip, s.
  - ❑ Skidding with slip angle δ_2.



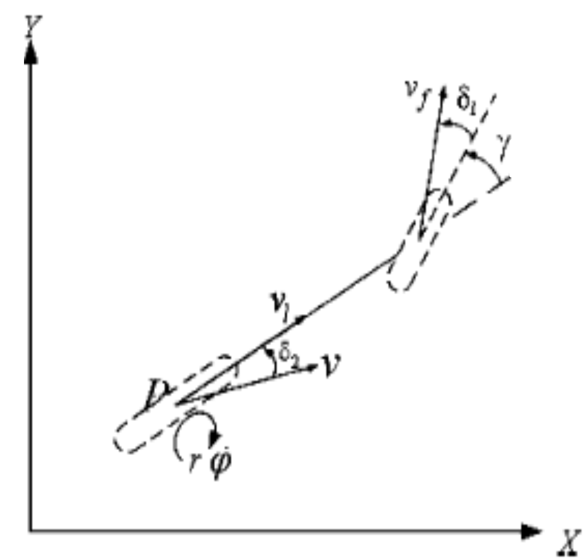
Skidding and slipping

- Kinematic model:
  - Same approach as with differential drive.

$$v_l = \begin{cases} (1-s)\,v & \text{if driving} \\ v \,/\, (1+s) & \text{if braking} \end{cases}$$

$$v_y = v_l \tan(\delta_2)$$

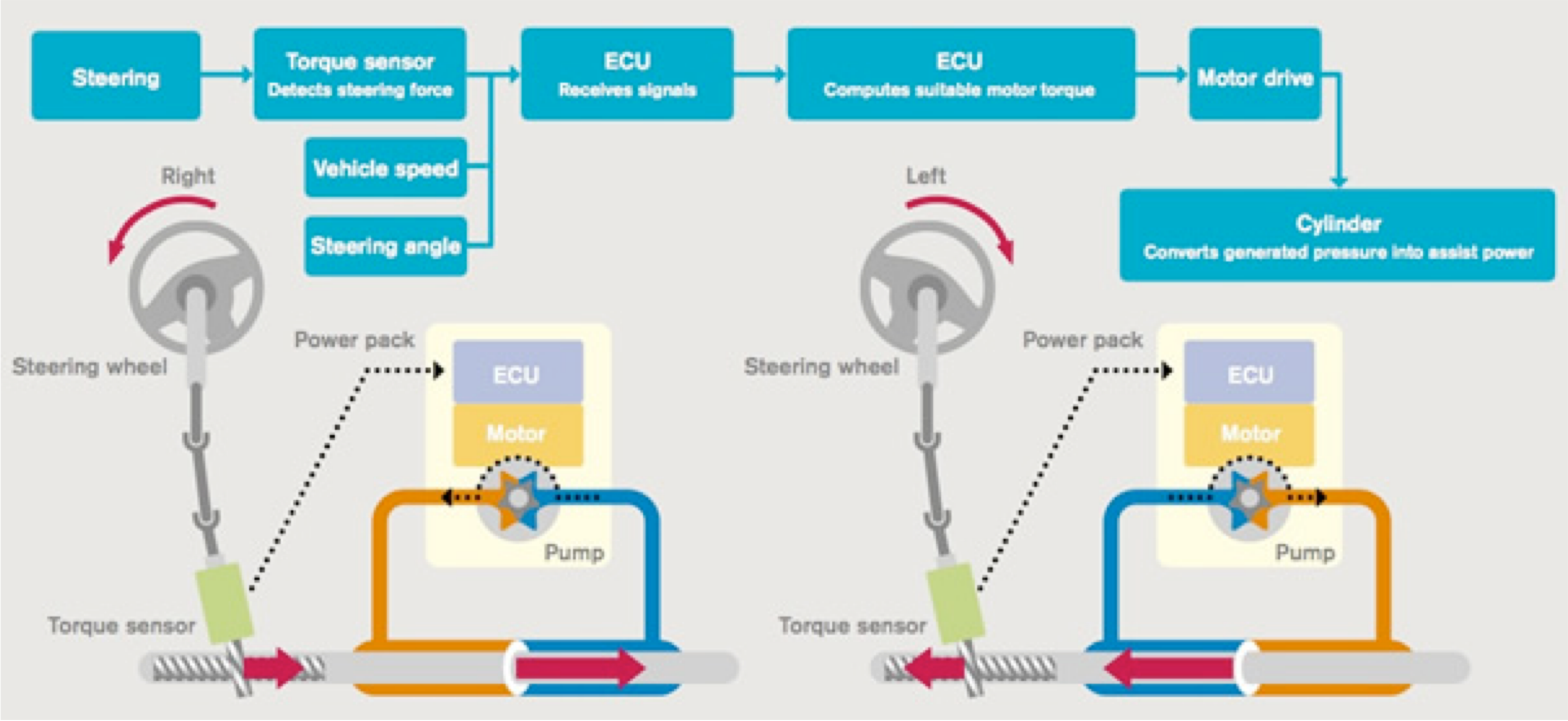
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v_l \cos\theta - v_y \sin\theta \\ v_l \sin\theta + v_y \cos\theta \\ \dfrac{v_l}{L}\tan(\gamma + \delta_1) - \dfrac{v_y}{L} \end{bmatrix}$$



Wang, D., & Low, C. B. (2008). Modeling and analysis of skidding and slipping in wheeled mobile robots: Control design perspective. *IEEE Transactions on Robotics*, *24*(3), 676-687.

Steering dynamics

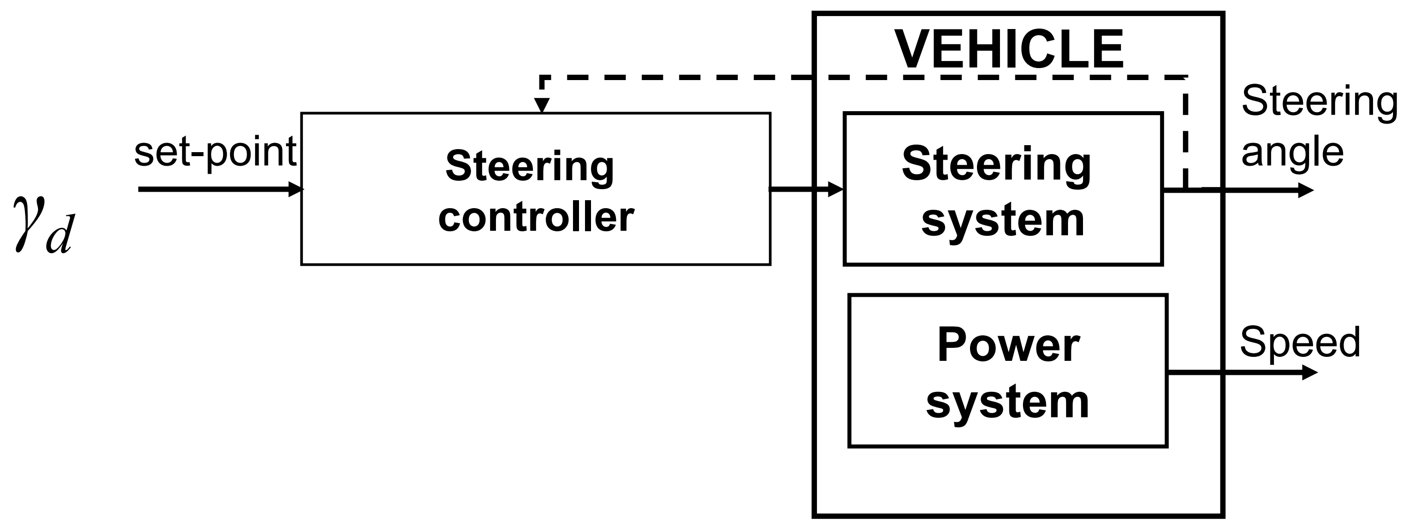
- If we want to change steering angle from γ_1 to γ_2 can it happen instantaneously?
- Tractors and off-road vehicles typically feature electro-hydraulic steering systems.
- Smaller vehicles involve gearing or chains.
- Linkages and tires have mass.



Steering dynamics

- Electro-hydraulics and wheel-soil interaction are very complex to model, and parameters are unknown.

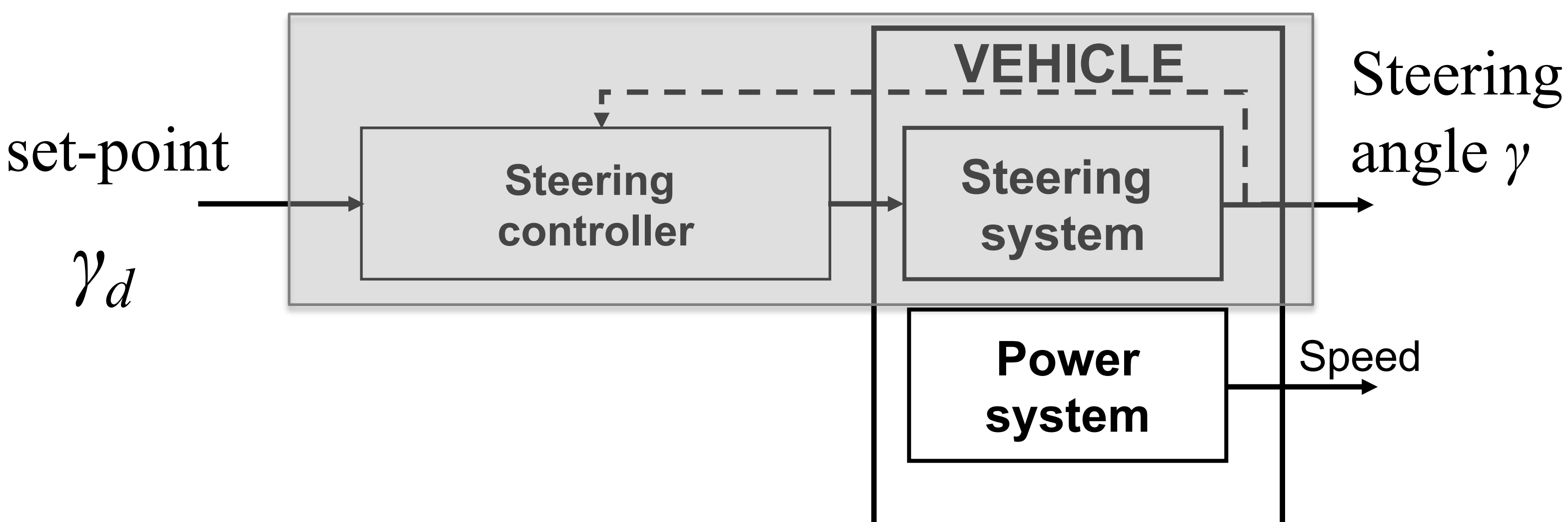
- Computer-controlled vehicles feature a low-level steering controller.



Steering dynamics

- The steering system closed loop transfer function can be approximated as a first order system.

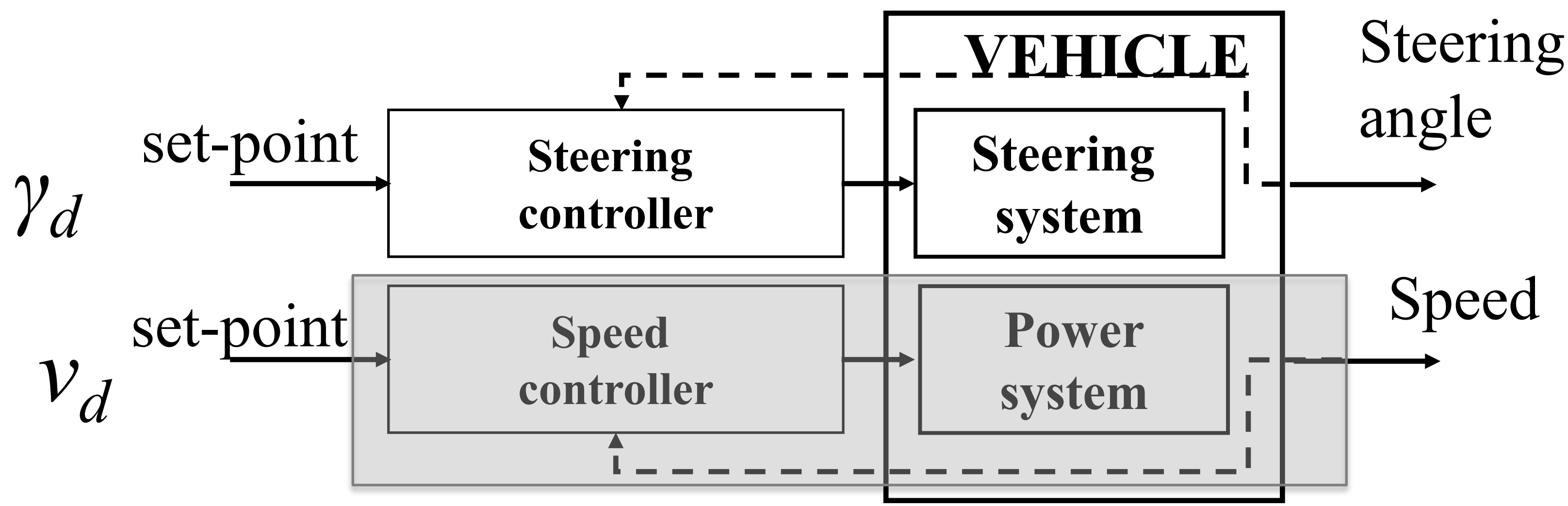
$$\dot{\gamma} = -\frac{1}{\tau_{\gamma}}\gamma + \frac{1}{\tau_{\gamma}}\gamma_d$$



Longitudinal velocity

- Does it change instantly?
- Same concept can be applied to a low-level speed controller.
- In the presence of slip, the steady state is (1-s)v_{d}.

$$\dot{v}_l = -\frac{1}{\tau_v} v_l + \frac{1}{\tau_v}(1-s)v_d$$



Discretization

- Euler discretization

$$\gamma_{k+1} - \gamma_k = dt\left(-\frac{1}{\tau_\gamma}\gamma_k + \frac{1}{\tau_\gamma}\gamma_d\right) \Rightarrow$$

$$\gamma_{k+1} = \gamma_k\left(1 - \frac{dt}{\tau_\gamma}\right) + \frac{dt}{\tau_\gamma}\gamma_d$$

- Similarly for speed.

Kinematic + simple dynamic model for driving: Updated equations

- Inputs: γ_d and v_d
- Remember:

$v_y = v_l\, tan(\delta_2)$ (skidding).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v_l} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} v_l cos\theta - v_y sin\theta \\ v_l sin\theta + v_y cos\theta \\ (v_l \tan(\gamma + \delta_1) - v_y)/L \\ (-v_l + (1-s)v_d)/\tau_v \\ (-\gamma + \gamma_d)/\tau_\gamma \end{bmatrix}$$

These equations must be integrated via Euler.

Integration vs. controller sampling

- The kinematic model integration step $\mathrm{d}t$ must be small (e.g., 1-10 ms) to achieve high accuracy when integrating the motion model with the Euler method.
- However, the controller's sampling interval DT is typically much longer, in the order of 100 ms (10 Hz) or slower.
- Therefore, we must implement our models with <u>two distinct sampling intervals</u>.
  - Two nested loops.

Kinematic + simple dynamic model: Updated equations

$$q = \begin{bmatrix} x \\ y \\ \theta \\ v \\ \gamma \end{bmatrix}$$

- Attention:
  - You need to pass input **u=[**$\gamma_d \; v_d$ **]** and state (**q**) <u>constraints</u> to your model.
    - $\boldsymbol{U}_{min} \leq \boldsymbol{u} \leq \boldsymbol{U}_{max}$ (2 x 1 vectors)
    - $\boldsymbol{q}_{min} \leq \boldsymbol{q} \leq \boldsymbol{q}_{max}$ (5 x 1 vectors)
  - The state constraints must be enforced <u>inside the Euler loop</u> because the state changes during integration
    - For example, the steering angle γ is now a component of the state vector **q**, and it changes inside the loop.
  - The input constraints need only be enforced once, before the beginning of the loop, since **u** is constant from step *k* to *k*+1 (during DT).

Assignment 2 Part 1

- Implement the bicycle model in Matlab, with skid-slip and with two distinct sampling intervals (Euler integration dt & controller DT).
- Input: control vector **u=[**$\gamma_d \; v_d$ **]**, state vector **q**, and all parameters (e.g., dt, DT, *L*, s, δ_1, δ_2), control and state constraints.
  – Note: input for turning could also be ω, or R.
- ***<u>Assume zero slip and skid</u>***
- Tractor wheel radius = 0.5m;
  Wheelbase = 2.5 m; $|\gamma_{max}| \leq 45^o$; $|v_{max}| \leq 5$ m/s
- Assume dt=DT=0.01 s.
- Apply input to turn at headland from one row to the next; row-center distance =2 m.
- Does it work? Why? Try row-center distance = 3 m. Show traces.

Assignment 2 Part 2

- ***<u>Non zero slip and skid</u>***
- Use same parameters as before.
- Add small slip angles δ1 and δ2 (e.g., 4^o)
- Apply input to turn at headland from one row to the next; row-center distance =3 m.
- What happens as slip angles δ1 and δ2 (skidding) increase? (Assume zero slip, s, for rear driven wheel).
- What if skidding is zero (δ1= δ2=0) and only slipping affects the rear wheel?

Assignment 2 Part 3

- Augment bicycle model with 1st order closed-loop steering and speed dynamics.
- Plot vehicle path for γ_d step function (0 to γ_{max}) and:
- 1. Speed lag:
  τ_v = 0 s; τ_γ from 0 to 2 s (same plot).
  – (How will you implement τ_v or τ_γ = 0 ?)
- 2. Speed lag:
  τ_v = 1 s; τ_γ from 0 to 2 s (same plot).
- Comment on these plots.