Brown Representability

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Introduction

In this document we write an exposition presenting the topic of Brown representability and how it has appeared in different contexts – beginning first with some necessary preliminaries followed by a more topological perspective and culminating in a modern and categorical statement holds in triangulated categories.

Contents

1 Outline of Presentation

The presentation will demonstrate some proper subset of the information in this document. The current sketch is the following:

- Introduction and preliminaries
  - Background we are assuming (categories, functors, natural transformations, and limits)
  - What is representability and why is it useful/important?
  - (Co)homological functors
- Brown Representability in Simplicial/CW Complexes
  - The homotopy category of pointed connected topological spaces
  - Categorical notions (e.g. weak pushouts)
  - Statement of theorem
  - Application
- Brown Representability in Triangulated Categories
  - Why are triangulated categories what we want?
  - Definiton and categorical background (e.g. compact generation)
  - Statement of theorem
  - Applications

2 Some Background

In this section we begin with some preliminary definitions and theorems that serve as the machinery for Brown representability. The results that follow are fairly standard and the interested reader can find more information in [Nee01] and [Rie16]. For the sake of time, we will omit a discussion of these facts in our lecture but include them in this document in case someone is interested in them.

2.1 General Category Theory

2.1.1 Definition
A **category** $\mathcal{C}$ is a collection of **objects** along with, for every pair (X, Y) of objects in $\mathcal{C}$, a collection $\mathrm{Hom}_{\mathcal{C}}(X, Y)$ of **morphisms** from X to Y such that

- There is a distinguished map, the **identity map** $\mathrm{id}_X \in \mathrm{Hom}_{\mathcal{C}}(X, X)$ for each $X \in \mathcal{C}$ such that, for every $f \in \mathrm{Hom}_{\mathcal{C}}(X, Y)$ and every $g \in \mathrm{Hom}_{\mathcal{C}}(Y, X)$,

$$f = \mathrm{id}_X \circ f \qquad g = g \circ \mathrm{id}_X .$$

- For each X, Y, and Z in $\mathcal{C}$ there is a map $\circ : \mathrm{Hom}_{\mathcal{C}}(Y, Z) \times \mathrm{Hom}_{\mathcal{C}}(X, Y) \to \mathrm{Hom}_{\mathcal{C}}(X, Z)$ (called **composition**) that is associative. That is, for each $f \in \mathrm{Hom}_{\mathcal{C}}(X, Y)$, $g \in \mathrm{Hom}_{\mathcal{C}}(Y, Z)$, and $h \in \mathrm{Hom}_{\mathcal{C}}(Z, W)$,

$$(h \circ g) \circ f = h \circ (g \circ f).$$

2.1.2 Definition
The **opposite category** $\mathcal{C}^{op}$ of a category $\mathcal{C}$ is the category whose objects are precisely those of $\mathcal{C}$ and whose morphisms are as follows: for every $A, B \in \mathcal{C}$ and $f \in \mathrm{Hom}_{\mathcal{C}}(A, B)$, there is a map $f^* \in \mathrm{Hom}_{\mathcal{C}^{op}}(B, A)$ such that for all composable f and g in $\mathcal{C}$,

$$(f \circ g)^* = g^* \circ f^*.$$

2.2 Size Considerations

"Size" is often an issue when you are discussing category theory in that one can quickly get into trouble with Russell's paradox. To ameliorate this to some degree, it is often wise to restrict oneself to the categories which are suitably small (although often ideas can be extended with extra care to a more general setting).

Here we define some of the types of smallness that we are interested in. The general idea to keep in mind is that if X is small, then the collections associated to X are sets.

2.2.1 Definition
A category $\mathcal{C}$ is called **locally small** if for each $A, B \in \mathcal{C}$, $\mathrm{Hom}_{\mathcal{C}}(A, B)$ is a set.

2.2.2 Definition
A category $\mathcal{C}$ is called **small** if it is locally small and, in addition, has a set's worth of objects.

2.2.3 Definition
A **small (co)limit** (see [Rie16] for definition of (co)limits) is a limit of a diagram that is indexed by a small indexing category.

2.2.4 Remark: Since (co)products are (co)limits, this also gives us a definition for **small (co)products**. We can also just say they are (co)products indexed by a set.

2.3 Functors and Representability

2.3.1 Definition
A (covariant) **functor** between two categories $\mathcal{C}$ and $\mathcal{D}$ is a map $F : \mathcal{C} \to \mathcal{D}$ that maps objects and morphisms in $\mathcal{C}$ to those in $\mathcal{D}$, respectively, such that

- For each $X \in \mathcal{C}$, we have $F(\mathrm{id}_X) = \mathrm{id}_{F(X)}$
- For each $X, Y \in \mathcal{C}$ and $g \in \mathrm{Hom}_{\mathcal{C}}(X, Y)$, $F(g)$ is a morphism in $\mathrm{Hom}_{\mathcal{C}}(F(X), F(Y))$.
- For all X, Y, Z in $\mathcal{C}$ and $f \in \mathrm{Hom}_{\mathcal{C}}(X, Y)$ and $g \in \mathrm{Hom}_{\mathcal{C}}(Y, Z)$,

$$F(g \circ f) = F(g) \circ F(f)$$

2.3.2 Remark: Leveraging duality, one can define a **contravariant functor** $F : \mathcal{C} \to \mathcal{D}$ as a covariant functor $F : \mathcal{C}^{op} \to \mathcal{D}$.

2.3.3 Definition
An **equivalence of categories** $F : \mathcal{C} \to \mathcal{D}$ is a functor such that

- F is both **full** (injective on Homs) and **faithful** (surjective on Homs); and
- F is **essentially surjective** – for each $D \in \mathcal{D}$, there is a $D' \in \mathcal{D}$ such that $D \cong D'$ and, for some $C \in \mathcal{C}$, we have $F(C) = D'$.

An **auto-equivalence** for $\mathcal{C}$ is an equivalence of categories from $\mathcal{C}$ to itself.

2.4 Natural Transformations

A natural transformation is a “map between maps” that demonstrates some kind of nice compatibility between two functors. Often, when one says a property is “natural” what one actually means is that there is a natural transformation involved.

2.4.1 Definition
Let $F, G : \mathcal{C} \to \mathcal{D}$ be two functors. Then $\eta : F \to G$, defined to be a collection of **component maps** $\{\eta_\alpha : F(\alpha) \to G(\alpha)\}_{\alpha \in \mathcal{C}}$, is called a **natural transformation** if, for every $A, B \in \mathcal{C}$ and $f \in \mathrm{Hom}_{\mathcal{C}}(A, B)$, the following square (called a **naturality square**) commutes:

$$\begin{tikzcd} F(A) \arrow[r, "\eta_A"] \arrow[d, "F(f)"] & G(A) \arrow[d, "G(f)"] \\ F(B) \arrow[r, "\eta_B"] & G(B) \end{tikzcd}$$

If η has an inverse, then it is called a **natural isomorphism.**

2.5 Triangulated Categories

The primary context that we are interested is a class of categories called *triangulated categories.* Examples of these include $\mathbf{K}(\mathbf{mod}\text{-}R)$, the homotopy category of chain complexes of R-modules (modulo chain homotopy) and $\mathbf{D}(\mathbf{mod}\text{-}R)$, the derived category in which we (Verdier) localize the previous category at quasi-isomorphisms.

2.5.1 Definition
Let $\mathcal{C}$ be a category with auto-equivalence $\Sigma : \mathcal{C} \to \mathcal{C}$. A **triangle** in $\mathcal{C}$ is a diagram in $\mathcal{C}$:

$$X \to Y \to Z \to \Sigma X.$$

2.5.2 Definition
A **morphism of triangles** is a commutative diagram

$$\begin{tikzcd} X \arrow[r, "u"] \arrow[d, "f"] & Y \arrow[r, "v"] \arrow[d, "g"] & Z \arrow[r, "w"] \arrow[d, "h"] & \Sigma X \arrow[d, "\Sigma f"] \\ X' \arrow[r] & Y' \arrow[r] & Z' \arrow[r] & \Sigma X' \end{tikzcd}$$

2.5.3 Lemma
The collection of all triangles in $\mathcal{C}$ with triangle morphisms form a k-linear additive category denoted $\Delta(\mathcal{C})$

2.5.4 Definition
The **rotation** of a triangle $X \xrightarrow{u} Y \to Z \to \Sigma X$ is $Y \to Z \to \Sigma X \xrightarrow{-\Sigma u} \Sigma Y$. We use the notation $\mathscr{R}(\Delta)$ for the rotation of Δ and $\mathscr{R}^{-1}$ for the inverse rotation.

2.5.5 Definition
Let $\mathcal{C}$ be a k-linear additive category with a **suspension** or **shift** functor (auto-equivalence, actually)

$$\Sigma : \mathcal{C} \to \mathcal{C}.$$

Then $(\mathcal{C}, \Sigma, D)$ is **pre-triangulated** category where D is a full, nonempty subcategory $D \subseteq \Delta(\mathcal{C})$ with shift functor Σ of D and we have the following axioms:

- (**TR0**) $0 \to X \xrightarrow{\mathrm{id}} X \to 0$ is in D for each $X \in \mathcal{C}$ and furthermore D is closed under both shifts and triangle isomorphisms.
- (**TR1**) [*Mapping Cone Axiom*] For any $f : X \to Y$ in $\mathcal{C}$, there is a triangle

$$X \xrightarrow{f} Y \to Z \to \Sigma X$$

- (**TR2**) [*Rotation Axiom*] If $F \in D$, then $\mathscr{R}(F), \mathscr{R}^{-1}(F) \in D$
- (**TR3**) [*Morphism Axiom*] Given two triangles

$$\begin{tikzcd} X \arrow[r] \arrow[d, "f"] & Y \arrow[r] \arrow[d, "g"] & Z \arrow[r] \arrow[d, dashed, "h"] & \Sigma X \arrow[d, "\Sigma f"] \\ X' \arrow[r] & Y' \arrow[r] & Z' \arrow[r] & \Sigma X' \end{tikzcd}$$

with maps f and g, there exists an $h : Z \to Z'$ such that the above diagram commutes.

2.5.6 Remark: The name "mapping cone axiom" in (**TR2**) above comes from the fact that in $\mathbf{K}(\mathbf{mod}\text{-}R)$, or in fact in any additive category with suspension, Z can be constructed as $\mathrm{cone}(f)$, the mapping cone of f.

2.5.7 Definition
A triple $(\mathcal{C}, \Sigma, D)$ is a **triangulated category** if it is pre-triangulated and, in addition, satisfies the axiom (**TR4**) (Verdier/octahedral axiom):

Suppose we are given three triangles: $X \xrightarrow{u} Y \to Z' \to \Sigma X$, $Y \xrightarrow{v} Z \to X' \to \Sigma Y$ and $X \xrightarrow{v\circ u} Z \to Y' \to \Sigma X$. Then there is a triangle $Z' \to Y' \to X' \to \Sigma Z'$ such that

$$\begin{tikzcd}
X \arrow[rr, bend left, "v\circ u"] \arrow[dr, "u"] & & Z \arrow[rr, bend left] \arrow[dr] & & X' \arrow[rr, bend left, dashed] \arrow[dr] & & \Sigma Z' \\
& Y \arrow[ur, "v"] \arrow[dr] & & Y' \arrow[ur, dashed] \arrow[dr] & & \Sigma Y \arrow[ur] & \\
& & Z' \arrow[ur, dashed] \arrow[rr, bend right] & & \Sigma X \arrow[ur] & &
\end{tikzcd}$$

2.5.8 Remark: There are two other representations of the octahedral axiom that are sometimes more helpful:

$$\begin{tikzcd}
X \arrow[r, "u"] \arrow[d, "\sim"] & Y \arrow[r] \arrow[d, "v"] & Z' \arrow[r] \arrow[d, dashed] & \Sigma X \arrow[d, "\sim"] \\
X \arrow[r, "v\circ u"] & Z \arrow[r] \arrow[d] & Y' \arrow[r] \arrow[d, dashed] & \Sigma X \arrow[d, "\Sigma u"] \\
& X' \arrow[r, "\sim"] \arrow[d] & X' \arrow[r] \arrow[d, dashed] & \Sigma Y \\
& \Sigma Y \arrow[r] & \Sigma Z' &
\end{tikzcd}$$

$$\begin{tikzcd}
& & X \arrow[dl, "u"] \arrow[dd, "v\circ u"] & \\
& Y \arrow[dr, "v"] \arrow[ddl] & & \\
& & Z \arrow[d] \arrow[dr] & \\
Z' \arrow[rr, dashed] & & Y' \arrow[r, dashed] & X'
\end{tikzcd}$$

We often want that our triangulated category $\mathcal{T}$ is slightly more controlled. For this, we introduce another condition:

2.5.9 Definition
If $\mathcal{T}$ is a triangulated category that is closed under small coproducts, then it is said to satisfy (**TR5**). If $\mathcal{T}^{op}$ satisfies (**TR5**) (or equivalently $\mathcal{T}$ is closed under small products), then we say $\mathcal{T}$ satisfies (**TR5***).

3 Brown Representability for Simpilicial Complexes

One of the first incarnations of Brown representability comes from topology (as these things often do). First we need to single out the appropriate context.

3.1 Category of Interest

3.1.1 Definition
Let $\mathbf{Top}_*^c$ denote the category of connected pointed topological spaces. That is, the objects are pairs (X, p) where X is a connected topological space and p is a distinguished point in X. The maps are continuous maps that send the distinguished point of one pair to the distinguished point in the other.

3.1.2 Definition
$\mathrm{Ho}(\mathbf{Top}_*^c)$ is the **homotopy category of connected topological spaces.** The objects are the same as in $\mathbf{Top}_*^c$, but maps are *weak homotopy equivalence classes* (relative to the basepoint) of maps of pointed spaces.

3.1.3 Remark: In essence, $\mathrm{Ho}(\mathbf{Top}_*^c)$ consists of weak homotopy equivalence classes (recall that two objects are weak homotopy equivalent if they have the same homotopy groups).

3.1.4 Remark: This is a form of *localization of categories* that is outside the scope of this paper, but closely mirrors the idea we see in commutative algebra.

3.1.5 Remark: Since any topological space is weakly homotopy equivalent to a CW-complex, we can easily see that $\mathrm{Ho}(\mathbf{Top}_*^c) \simeq \mathrm{Ho}(\mathbf{CW}_*)$, the homotopy category of pointed CW complexes. This gives us some nice leverage for computation and tangibility.

3.2 Generalized (Eilenberg-Steenrod) Cohomology Theories

The idea here is that we can construct a general definition that generally captures what we mean when we say we are working with cohomology.

Notice that these theories notably do *not* include nonabelian cohomology theories and other theories of interest.

3.2.1 Definition
$\mathbf{Ab}^{\mathbb{Z}}$ is the functor category $\mathbf{Func}(\mathbb{Z}, \mathbf{Ab})$ or, equivalently, the category of $\mathbb{Z}$-graded Abelian groups.

3.2.2 Remark: In case you haven't seen it before, the category $\mathbb{Z}$ we're talking about has as its objects the integers and a (unique) morphism $m \to n$ whenever $n \geq m$. This is the category we can construct from any set that admits a preorder (reflexivity and transitivity).

Functorality here says that for any $k \in \mathbb{N}$ we can consider the degree k maps between the i^{th} graded piece of G and the $(i+k)^{th}$ graded piece of H for any $G, H \in \mathbf{Ab}^{\mathbb{Z}}$. If $\varphi : G \to H$ is any degree zero map and $h \in H^k$ is any degree k element in H, then an example of such a map is $\varphi'(g) = h \cdot \varphi(g)$.

3.2.3 Definition ((Reduced) Generalized Cohomology Theory)
A **reduced generalized cohomology theory** on $\mathcal{C} = \mathbf{Top}_*^{CW}$ is a family of functors:

$$\widetilde{E}^\bullet : \mathrm{Ho}(\mathcal{C})^{op} \to \mathbf{Ab}^{\mathbb{Z}}$$

along with a collection of natural isomorphisms

$$\eta^i : \widetilde{E}^i \to \widetilde{E}^{i+1} \circ \Sigma$$

subject to the following conditions:

- (*Homotopy Invariance*)If $f, g : X \to Y$ are homotopic maps (relative to their base points), then
$$\widetilde{E}^\bullet(f) = \widetilde{E}^\bullet(g)$$
- (*Exactness*) For the short exact sequence $A \overset{\iota}{\hookrightarrow} X \to \mathrm{cone}(\iota)$ the following is exact:
$$\widetilde{E}^\bullet(\mathrm{cone}(\iota)) \to \widetilde{E}^\bullet(X) \xrightarrow{\iota^*} \widetilde{E}^\bullet(A)$$

Reduced cohomology theories more succinctly capture the notions we expect when we are doing cohomology, but in order to do things like relative homology, one needs a little more generality.

Let $\mathbf{Top}_{\hookrightarrow}^{CW}$ be the category of pairs (X, A) of CW complexes with inclusions $A \hookrightarrow X$ with subspace-preserving morphisms.

3.2.4 Definition ((Unreduced, Relative) Generalized Cohomology Theory) An unreduced, relative generalized cohomology theory for $\mathbf{Top}_{\hookrightarrow}^{CW}$ is a collection of functors

$$E^\bullet : \mathrm{Ho}(\mathbf{Top}_{\hookrightarrow}^{CW}) \to \mathbf{Ab}^{\mathbb{Z}}$$

along with natural isomorphisms

$$\eta_{(X,A)} : E^\bullet(A, \varnothing) \to E^{\bullet+1}(X, A)$$

such that it satisfies an analogous condition for homotopy invariance that we saw in 3.2.3 along with the following:

- (*Exactness*) For (X, A), the following is exact:

$$\cdots \to E^n(X, A) \to E^n(X, \varnothing) \to E^n(A, \varnothing) \xrightarrow{\eta_{(X,A)}} E^{n+1}(X, A) \to \cdots$$

  or, more succinctly,

$$\cdots \to E^n(X, A) \to E^n(X) \to E^n(A) \to E^{n+1}(X, A)$$

  which reflects the long exact sequence in relative cohomology that you may be familiar with (see your favorite book on algebraic topology, e.g. [Hat02, p.199]).

- (*Excision*) For (X, A) and (A, U) in $\mathbf{Top}_{\hookrightarrow}^{CW}$ such that $\overline{U} \subseteq \mathrm{Int}(A)$, the inclusion $\iota : (X \setminus U, A \setminus U) \hookrightarrow (X, A)$ induces an isomorphism

$$\iota^* : E^\bullet(X, A) \xrightarrow{\simeq} E^\bullet(X \setminus U, A \setminus U)$$

3.3 Stable Homotopy Categories

Here we will throw down a few definitions and ideas that illustrate the idea that *Eilenberg-Steenrod cohomology theories are natural constructions* in that they are precisely the cohomology of the $(\infty, 1)$ category of spectra.

3.4 Ancillary Definitions

3.4.1 Definition
A **weak pushout** of a diagram $X \xleftarrow{f} Y \xrightarrow{g} Z$ is an object W and maps $f' : X \to W$ and $g' : Z \to W$ such that for any object O and maps $\alpha : X \to O$ and $\beta : Z \to O$, such that $\alpha \circ f = \beta \circ g$, there exists a map $\gamma : W \to O$ such that the following diagram commutes:

$$\begin{tikzcd}
Y \arrow[r, "f"] \arrow[d, "g"] & X \arrow[d, "f'"] \arrow[ddr, bend left, "\alpha"] & \\
Z \arrow[r, "g'"] \arrow[drr, bend right, "\beta"] & W \arrow[dr, dashed, "\gamma"] & \\
& & O
\end{tikzcd}$$

A **weak pullback** is defined dually.

3.4.2 Remark: The "weakness" referred to in this definition is due to the fact that the map γ exists, but is not required to be unique (unlike the regular pullback and pushout).

3.5 The Theorem

The following result was proven by Brown and presented in [Bro65] and further generalized in [Ada71].

3.5.1 Theorem (Brown-Adams)
A contravariant functor $F : \mathrm{Ho}(\mathbf{Top}_*^c) \to \mathbf{Set}_*$ is representable precisely if

- F takes coproducts to products; and
- F takes weak pushouts to weak pullbacks.

3.6 What This Means

What should we take away from Brown representability? If we accept the hypothesis that generalized cohomology theories are reasonable approximations of what we are looking for in cohomology, this implies that each such incarnation of a cohomology theory $H^\bullet$ admits a spectrum object $E_\bullet$ in the stable homotopy category such that $H^\bullet \cong \mathrm{Ho}(\mathbf{Top}_*^{CW})(-, E_\bullet)$ such that the suspension isomorphism $\eta : H^n(-) \to H^{n+1}(\Sigma(-))$ is induced from the structure morphisms $\sigma_n : E_n \to \Omega E_{n+1}$ of the spectrum:

$$H^n(-) \cong \mathrm{Ho}(\mathbf{Top}_*^{CW})(-, E_n) \xrightarrow{(\sigma_n)} \mathrm{Ho}(\mathbf{Top}_*^{CW})(-, \Omega E_{n+1}) \cong \mathrm{Ho}(\mathbf{Top}_*^{CW})(\Sigma(-), E_{n+1}) \cong H^{n+1}(\sigma(-))$$

4 Brown Representability for Triangulated Categories

4.1 Additional Notions

Along with the standard categorical definitions we saw in section one, we need one further restriction on our categories as formulated in [Nee96]:

4.1.1 Definition (Compact Object)
Let $X \in \mathcal{C}$ be an object in a triangulated category. Then X is called **compact** if for every coproduct of objects in $\mathcal{C}$

$$\mathrm{Hom}_{\mathcal{C}}\left(X, \coprod_{\lambda\in\Lambda} t_\lambda\right) = \coprod_{\lambda\in\Lambda} \mathrm{Hom}_{\mathcal{C}}(X, t_\lambda)$$

4.1.2 Remark: The idea here is that when the codomain is a product, this equality always exists. The projections give you a product of maps into the factors and the universal property of products gives you a (unique!) map into the product if you have maps into each of the factors.

This does not hold in general for coproducts, however! Recall that in additive categories one has a canonical isomorphism between finite products and coproducts. Since triangulated categories are additive, this is akin to saying that X is well-behaved in that the maps from X still split even when the codomain is an arbitrary coproduct.

4.1.3 Definition (Compactly Generated Triangulated Category)
Let $\mathcal{C}$ be a triangulated category. Then $\mathcal{C}$ is called **compactly generated by a set** G of its elements if $\mathcal{C}$ is closed under small coproducts and G consists of compact objects of $\mathcal{C}$ such that for all $X \in \mathcal{C}$,

$$\mathrm{Hom}_{\mathcal{C}}(G, X) = 0 \quad \Rightarrow \quad X = 0.$$

4.1.4 Remark: Here the generators of $\mathcal{C}$ are elements that have "sufficient complexity" to detect all nonzero elements.

In the more general context where $\mathcal{C}$ is not necessarily additive, we can generalize this definition to a set of objects such that for every generator $E \in G$ and any $X, Y \in \mathcal{C}$, if for every $f, g \in \mathrm{Hom}(X, Y)$ and $h \in \mathrm{Hom}(E, X)$ we have

$$f \circ e = g \circ e$$

then

$$f = g$$

and here we would say that G has "sufficient complexity" to differentiate between any two non-identical maps.

Example 4.1
In R-**mod**, a set of generators is the collection of all free R-modules or, equivalently, the collection

$$\{R^S | S \in \mathbf{Set} \setminus \varnothing\}$$

where if $S \subset M \in R$-**mod** is any nonempty subset, then R^S is the product of $|S|$ copies of R, where a nonzero map to M is the map which sends the identity in factor to a different $s \in S$.

4.1.5 Definition
Let $F : \mathcal{C} \to \mathbf{Set}$ be a (covariant) functor. Then F is called **representable** if there exists an object $A \in \mathcal{C}$ such that F is naturally isomorphic to $\mathrm{Hom}_{\mathcal{C}}(-, A)$.

4.1.6 Definition
Let R be a ring and let $\mathcal{T}$ be a triangulated category. Then a functor $F : \mathcal{T} \to R$-**mod** is called **cohomological** if F sends each triangle $A \to B \to C \to \Sigma A$ to an exact sequence in R-**mod**.

4.1.7 Remark: This definition extends to functors from triangulated categories to any Abelian category, but due to the Freyd-Mitchell embedding it suffices to use R-**mod** for all small examples.

4.2 The Statement

Finally we come to the full-powered version of the Brown Representability due to Neeman in [Nee96]:

4.2.1 Theorem (Brown)
Let $\mathcal{T}$ be a compactly generated triangulated category. For a functor $H : \mathcal{T}^{op} \to \mathbf{Ab}$, the following are equivalent:

- The functor H is cohomological (def 4.1.6) and preserves small coproducts.
- H is representable (def 4.1.5).

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