General Exam Paper

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Abstract

We begin by going through a considerable amount of domain knowledge concerning representations of GL_n, representations of $\mathfrak{S}_n$, and strict polynomial functors all in service of understanding the Schur-Weyl functor that relates several of these categories. From there, we investigate recent work on the part of Krause and his students Aquilino and Reischuk on this functor and the fact that it is monoidal under reasonably natural monoidal structures on the categories in question. Finally we ask some questions about whether the monoidal structure on strict polynomial functors extends meaningfully to pathologies that arise in positive characteristic.

Contents

1 Introduction

The story of this project (more-or-less) begins with Schur's doctoral thesis [Sch01] in which he defined polynomial representations of GL_n—a theory which he developed more completely in his later paper *Über die rationalen Darstellungen der allgemeinen linearen Gruppe*[1] [Sch73]. In these papers, Schur develops the idea of a **polynomial representation of** GL_n, meaning a (finite dimensional) representation where the coefficient functions of the representing map

$$\rho : \mathrm{GL}_n \to \mathrm{GL}(V)$$

is polynomial in each coordinate. That is, if $V = \oplus_{i=1}^n k v_i$, then for every $1 \leq i, j \leq n$, we have the map $r_{v_i v_j} : \mathrm{GL}_n \to k$ such that

$$\rho(g) \cdot v_i = \sum_{i=1}^{n} r_{v_i v_j} v_j$$

2 Representations of GL_n and of $\mathfrak{S}_n$

3 Strict Polynomial Functors

4 The Schur-Weyl Functor

5 Questions and Extensions

References

[Sch01] Issai Schur. *Über eine Klasse von Matrizen, die sich einer gegebenen Matrix zuordnen lassen*. s.n., 1901. URL: http://hdl.handle.net/2027/hvd.32044091874271.

[Sch73] Issai Schur. "Über die rationalen Darstellungen der allgemeinen linearen Gruppe". In: *Gesammelte Abhandlungen*. Vol. 3. Springer-Verlag, 1973, pp. 68–85.

[1] *On the rational representations of the general linear group*