Ejercicios M.M.1

1. $\lambda = \frac{20}{60} = 0,33\overline{3}$ clientes / minuto

$\mu = \frac{1}{3} = 0,33\overline{3}$ clientes / minuto

$Ls = \frac{\lambda}{\mu - \lambda} = \frac{0,33\overline{3}}{0,33\overline{3} - 0,33\overline{3}} = \frac{0,33\overline{3}}{0} \rightarrow \nexists$ no existe

$Wq = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{0,33\overline{3}}{0,33\overline{3}(0,33\overline{3} - 0,33\overline{3})} = \frac{0,33\overline{3}}{0,33\overline{3} \cdot 0} = \frac{0,33\overline{3}}{0}$

$\nexists$ no existe

$P_0 = 1 - \rho \rightarrow \rho = \frac{\lambda}{\mu} = \frac{0,33\overline{3}}{0,33\overline{3}} = 1$

$P_0 = 1 - 1 = 0 \rightarrow 0\%$

2. $\lambda = \frac{15}{60} = 0,25$ autos / minuto

$\mu = \frac{1}{5} = 0,20$ autos / minuto

$P_{+2} = 1 - P_0 - P_1$

$\rho = \frac{\lambda}{\mu} = \frac{0,25}{0,20} = 1,25$

$P_0 = 1 - \rho = 1 - 1,25 = -0,25$

$P_1 = \left[1 - \frac{\lambda}{\mu}\right] \cdot \left[\frac{\lambda}{\mu}\right]^n = [1 - 1,25]\ [1,25]^1 = -0,31$

$P_{+2} = 1 - (-0,25) - (-0,31) = 1,56 \rightarrow 156\%$

$\rho = \frac{\lambda}{\mu} = \frac{0,25}{0,20} = 1,25$ como $\lambda > \mu$ hay congestionamiento

↳ Factor de uso

3. $\lambda = \frac{30}{60} = 0,5$ clientes / minuto

$\mu = \frac{1}{2} = 0,5$ clientes / minuto

$Ws = \frac{1}{\mu - \lambda} = \frac{1}{0,5 - 0,5} = \frac{1}{0} \rightarrow \nexists$ no existe

$P_{10} = \left[1 - \frac{\lambda}{\mu}\right] \cdot \left[\frac{\lambda}{\mu}\right]^n = \left[1 - \frac{0,5}{0,5}\right] \cdot \left[\frac{0,5}{0,5}\right]^{10} = 0 \rightarrow 0\%$

4- $\lambda = \frac{10}{60} = 0,16$ clientes / minuto

$\mu = \frac{1}{4} = 0,25$ clientes / minuto

$P = \frac{\lambda}{\mu} = \frac{0,16}{0,25} = 0,64 \rightarrow 64\%$

$P_0 = 1 - P = 1 - 0,64 = 0,36 \rightarrow 36\%$

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{0,16^2}{0,25(0,25 - 0,16)} = 1,137$ clientes

6- $\lambda = \frac{25}{60} = 0,42$ llamadas / minuto

$\mu = \frac{1}{2} = 0,5$ llamadas / minuto

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{0,42^2}{0,5(0,5 - 0,42)} = 4,16$ llamadas en espera

$P_{+4} = 1 - P_0 - P_1 - P_2 - P_3$

$P = \frac{\lambda}{\mu} = \frac{0,42}{0,5} = 0,84$

$P_0 = 1 - P = 1 - 0,84 = 0,16$

$P_1 = [1 - P] \cdot [P]^1 = [1 - 0,84] \cdot [0,84]^1 = 0,1344$

$P_2 = [1 - P] \cdot [P]^2 = [1 - 0,84] \cdot [0,84]^2 = 0,112896$

$P_3 = [1 - P] \cdot [P]^3 = [1 - 0,84] \cdot [0,84]^3 = 0,096$

$P_{+4} = 1 - 0,16 - 0,13, 0,11, 0,096 = 0,504 \rightarrow 50,4\%$

7- $\lambda = \frac{40}{60} = 0,66$ clientes / minuto

$\mu = \frac{1}{1} = 1$ clientes / minuto

Tenemos que hacer P_4 pq hay 4 en el sistema, 1 en servicio

$P = \frac{\lambda}{\mu} = \frac{0,66}{1} = 0,66$

$P_4 = [1 - P] \cdot [P]^4 = [1 - 0,66] \cdot [0,66]^4 = 0,066$

6,6%

8- $\lambda = \frac{100}{60} = 1,66$ clientes / minuto

$\mu = \frac{150}{60} = 2,5$ clientes / minuto

$\rho = \frac{\lambda}{\mu} = \frac{1,66}{2,5} = 0,664$

$P_0 = 1 - 0,664 = 0,336 \rightarrow 33,6\%$

$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{1,66^2}{2,5(2,5-1,66)} = 1,31$ clientes

9- $\lambda = \frac{18}{60} = 0,3$ clientes / minuto

$\mu = \frac{1}{6} = 0,166$ vehículos / minuto

$W_s = \frac{1}{\mu - \lambda} = \frac{1}{-0,134} = -7,46$ minutos

10- $\lambda = \frac{25}{60} = 0,416$ visitantes / minuto

$\mu = \frac{1}{3} = 0,\overline{33}$

$P_{45} = 1 - P_0 - P_1 - P_2 - P_3 - P_4$

$\rho = \frac{\lambda}{\mu} = \frac{0,416}{0,\overline{33}} = 1,26$

$P_1 = [1-\rho] \cdot [\rho]^1 = [1-1,26] \cdot [1,26]^1 = -0,3276$

$P_2 = [1-\rho] \cdot [\rho]^2 = [1-1,26] \cdot [1,26]^2 = -0,4127$

$P_3 = [1-\rho] \cdot [\rho]^3 = [1-1,26] \cdot [1,26]^3 = -0,52$

$P_4 = [1-\rho] \cdot [\rho]^4 = [1-1,26] \cdot [1,26]^4 = -0,6553$

$P_{45} = 1 - (-0,3276) - (-0,4127) - (-0,52) - (-0,6553)$

$P_{45} = 2,9156 \rightarrow 291\%$

11- $\lambda = \frac{20}{60} = 0,\overline{33}$ unidades / minuto

$\mu = \frac{30}{60} = 0,5$ unidades / minuto

$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{0,\overline{33}}{0,5(0,5-0,\overline{33})} = 3,88$ minutos

12. $\lambda = \frac{2}{1} = 2$ $\quad P = 0,5 \rightarrow 50\%$

$P = \frac{\lambda}{\mu} \rightarrow \mu = \frac{\lambda}{P} = \frac{2}{0,5} = 4$

$Lq = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{2^2}{4(4-2)} = \frac{1}{2}$

Ejercicio Parcial:

$P_1 = (1-P) \cdot P = 0,22$

$P - P^2 = 0,22$

$-P^2 + P - 0,22 = 0$

$P = 0,673 \quad \vee \quad P = 0,326$

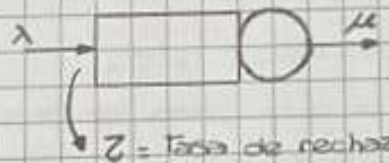
- Para $P = 0,673$

$P(4) = (1-0,673) \cdot (0,673)^4 = 0,067$

- Para $P = 0,326$

$P(4) = (1-0,326) \cdot (0,326)^4 = 0,0076$

MM1N



Z = Tasa de rechazo

$Z = \lambda \cdot P_B$

$P_B = \frac{P^{n}(1-P)}{1-P^{n+1}}$ (n: cliente en espera; n+1: clientes en el sistema)

Rendimiento de entrada	Rendimiento de salida	Rendimiento del sistema
$\gamma_i = \lambda \cdot Z$	$\gamma_o = \mu - (\mu \cdot P_0)$	$\gamma_i = \gamma_o = \gamma$
$\gamma_i = \lambda - (\lambda \cdot P_B)$	$\gamma_o = \mu(1 - P_0)$	
$\gamma_i = \lambda(1 - P_B)$		

Tasa de llegada efectiva → Los que realmente entran al sistema sin que este se bloquee

$$\bar{\lambda} = \lambda \cdot (1 - P_B)$$

$$\bar{\lambda} = \lambda \left[1 - \left(\frac{P^n \cdot (1-P)}{1-P^{n+1}}\right)\right] \quad (P_B)$$

Utilización efectiva del sistema → $\bar{P} = \frac{\bar{\lambda}}{\mu}$

Saturación → $P = \frac{\lambda}{\mu}$

Probabilidad → $P_n = P^n \cdot P_0 \qquad P_0 = \frac{1-P}{1-P^{n+1}}$

Tiempo promedio de permanencia en el sistema

↳ Efectiva → $W_s = \frac{L_s}{\bar{\lambda}}$ (Lamba efectiva) → $W_s = \frac{L_s}{\lambda}$; $W_s = W_q + \frac{1}{\mu}$

Tiempo de espera en cola → $W_q = \frac{L_q}{\lambda}$; $W_q = W_s - \frac{1}{\mu}$

Nro promedio de clientes en la cola para un sistema ocupado

↳ $L_b = \frac{L_q}{1-P_0}$

Tiempo promedio de clientes en la cola para un sistema ocupado

↳ $W_b = \frac{W_q}{1-P_0}$

$L_s = \frac{n}{2}$ cuando $P = 1$ → promedio de clientes en el sistema

$$L_s = \frac{P}{(1-P)} - \frac{(n+1)\cdot P^{n+1}}{1-P^{n+1}} \quad \text{cuando } P \neq 1$$

$$L_q = \frac{n\cdot(n-1)}{2(n+1)} \quad \text{cuando } P = 1$$

$$L_q = L_s - \frac{(1-P^n)\cdot P}{1-P^{n+1}} \quad \text{cuando } P \neq 1$$

14. $\lambda = 18$ vehículos / hora

$\mu = 6$ vehículos / hora

$n = 4$ vehículos

$\rho = \frac{18}{6} = 3 \rightarrow \neq 1$

a. $P_0 = \frac{1-\rho}{1-\rho^{n+1}} \rightarrow \frac{1-3}{1-3^{4+1}} = 1/121 = 0,0085 = 0,85\%$

b. $Ls = \frac{\rho}{(1-\rho)} - \frac{(n+1)\cdot\rho^{n+1}}{1-\rho^{n+1}} \rightarrow \frac{3}{(1-3)} - \frac{(4+1)\cdot 3^{4+1}}{1-3^{4+1}}$

$Ls = 3,52$ vehículos

c. $Ws = \frac{Ls}{\bar{\lambda}} = \frac{3,52}{5,95} = 0,59$ horas

↳ $\bar{\lambda} = \lambda\left[1-\left(\frac{\rho^n\cdot(1-\rho)}{1-\rho^{n+1}}\right)\right]$

$\bar{\lambda} = 18\cdot\left[1-\left(\frac{3^4(1-3)}{1-3^{4+1}}\right)\right] = 5,95$

d. $Wq = Ws - \frac{1}{\mu}$

$Wq = 0,59 - \frac{1}{6} = 0,423$ horas

e. $Lq = Ls - \frac{(1-\rho^n)\cdot\rho}{1-\rho^{n+1}} \rightarrow 3,52 - \frac{(1-3^4)\cdot 3}{1-3^{4+1}}$

$Lq = 2,52$ vehículos

15. $\lambda = 20$ clientes / hora

$\mu = 5$ clientes / hora

1 cliente ___ 12 min

x ___ 60 min

$n = 9$

a. Realiza 5 consultas por hora

b. $\bar{\lambda} = \lambda\cdot\left[1-\left(\frac{\rho^n\cdot(1-\rho)}{1-\rho^{n+1}}\right)\right]$ $\rho = \frac{\lambda}{\mu} \rightarrow \frac{20}{5} = 4$

$\bar{\lambda} = 20\left[1-\left(\frac{4^9(1-4)}{1-4^{9+1}}\right)\right] = 4,99$

c. $Ls = \frac{4}{(1-4)} - \frac{(9+1)\cdot 4^{9+1}}{1-4^{9+1}} = 8,66$ clientes

$Ws = \frac{Ls}{\lambda} \rightarrow \frac{8,66}{20} = 0,43$ horas

16. $\lambda = 5$ clientes / hora

$\mu = 6$ clientes / hora

$n = 3$

1 cliente ___ 10 minutos
x ___ 60 minutos

a. Atende 6 clientes por hora

b. $\rho = \frac{\lambda}{\mu} \rightarrow \frac{5}{6} = 0,83$

$\bar{x} = 5\left[1 - \left(\frac{0,83^3 (1-0,83)}{1-0,83^{3+1}}\right)\right] = 4,074$ clientes / hora

$\bar{\rho} = \frac{\bar{\lambda}}{\mu} \rightarrow \frac{4,074}{6} = 0,679$

c. $Ls = \frac{0,83}{(1-0,83)} - \frac{(3+1)\cdot 0,83^{3+1}}{1-0,83^{3+1}} = 1,27$

$Ws = \frac{Ls}{\lambda} = \frac{1,27}{5} = 0,254$ horas

$Lq = 1,27 - \frac{(1-0,83^3)\cdot 0,83}{1-0,83^{3+1}} = 0,595$ clientes

d. $Ws = 0,254$

e. $Po = \frac{1-\rho}{1-\rho^{n+1}} \rightarrow \frac{1-0,83}{1-0,83^{3+1}} = 0,407$

17. $\lambda = 30$ clientes / hora

$\mu = 3$ clientes / hora

1 cliente ___ 20 minutos
x ___ 60 minutos

$n = 20$

a. $\rho = \frac{\lambda}{\mu} = \frac{30}{3} = 10$

$Ls = \frac{10}{(1-10)} - \frac{(20+1)\cdot 10^{20+1}}{1-10^{20+1}} = 19,88$ clientes

$Lq = 19,88 - \frac{(1-10^{20})\cdot 10}{1-10^{20+1}} = 18,88$ clientes

b. $P_{20} = 10^{20}\cdot\left(\frac{1-10}{1-10^{20+1}}\right) = 0,9$

c. $Ws = \frac{19,88}{30} = 0,6629$ horas

d. $W_q = \frac{18,88}{30} = 0,629$ horas

18. λ = 10 personas / hora

μ = 10 personas / hora

n = 50

1 persona — 6 min
x — 60 min

a. $P_0 = \frac{1-1}{1-1^{50+1}} = \nexists$

b. $L_s = \frac{50}{2} = 25$

$L_q = \frac{50(50+1)}{2(50+1)} = 24,02$

c. $L_s = 25$

d. $W_s = \frac{25}{10} = 2,5$ horas

e. $W_q = \frac{24,02}{10} = 2,4$ horas

19. λ = 8 bicicletas / hora

μ = 12 bicicletas / hora

n = 20

1 bicicleta — 5 min
x — 60 min

a. $P = \frac{8}{12} = 0,667$

$L_s = \frac{0,667}{(1-0,667)} - \frac{(20+1).0,667^{20+1}}{1-0,667^{20+1}} = 1,87$

$L_q = 1,87 - \left(\frac{(1-0,667^{20}).0,667}{1-0,667^{20+1}}\right) = 1,20$ bicicletas

b. L_s = 1,87 bicicletas

c. $W_s = \frac{1,87}{8} = 0,233$ horas

d. $W_q = \frac{1,20}{8} = 0,15$ horas

e. $P_0 = \frac{1-0,667}{1-0,667^{20+1}} = 0,337$

20. $\lambda = 5$ pacientes / hora

$\mu = 6$ pacientes / hora

1 paciente — 10 min
x — 60 min

$n = 7$ $\quad \rho = \frac{5}{6} = 0,833$

a. $Ls = \frac{0,833}{1-0,833} - \frac{(7+1) \cdot 0,833^{7+1}}{1-0,833^{7+1}} = 2,573$ pacientes

$Lq = 2,573 - \frac{(1-0,833^{7}) \cdot 0,833}{1-0,833^{7+1}} = 1,791$ pacientes

b. $P_4 = 0,833^{7} \cdot \left(\frac{1-0,833}{1-0,833^{7+1}}\right) = 0,0605$

c. $Ls = 2,573$ pacientes

d. $Ws = \frac{2,573}{5} = 0,5146$ horas

e. $Wq = \frac{1,791}{5} = 0,3582$ horas

21. $\lambda = 12$ clientes / hora

1 cliente — 15 min
x — 60 min

$\mu = 4$ clientes / hora

$n = 15$ $\quad \rho = \frac{12}{4} = 3$

a. $P_0 = \frac{1-3}{1-3^{15+1}} = 4,64 \times 10^{-8}$

c. $Ls = \frac{3}{1+3} - \frac{(15+1) \cdot 3^{15+1}}{1-3^{15+1}} = 14,5$ clientes

b. $Ws = \frac{14,5}{12} = 1,208$ horas

d. $Lq = 14,5 - \frac{(1-3^{15}) \cdot 3}{1-3^{15+1}} = 13,5$ clientes

e. $Wq = \frac{13,5}{12} = 1,125$ horas

24- λ = 6 clientes / hora

μ = 1,33 clientes / hora

1 cliente — 45 min
x — 60 min

n = 10 $\rho = \frac{6}{1,33} = 4,51$

a- $P_{10} = 4,51^{10} \cdot \left(\frac{1-4,51}{1-4,51^{10+1}}\right) = 0,778 \rightarrow 77,8\%$

b- $Ls = \frac{4,51}{(1-4,51)} - \frac{(10+1) \cdot 4,51^{10+1}}{1-4,51^{10+1}} = 9,71$ clientes

c- $Ws = \frac{9,71}{6} = 1,61$ horas

d- $Lq = 9,71 - \left(\frac{(1-4,51^{10}) \cdot 4,51}{1-4,51^{10+1}}\right) = 8,71$ clientes

e- $Wq = \frac{8,71}{6} = 1,451$ horas

23- λ = 40 clientes / hora

μ = 2 clientes / hora

1 cliente — 30 min
x — 60 min

n = 25 $\rho = \frac{40}{2} = 20$

d- $Ls = \frac{20}{1-20} - \frac{(25+1) \cdot 20^{25+1}}{1-20^{25+1}}$

Ls = 24,947 clientes

b- $Lq = 24,947 - \frac{(1-20^{25}) \cdot 20}{1-20^{25+1}} = 23,947$ clientes

a- $Wq = \frac{Lq}{\lambda} = \frac{23,947}{40} = 0,5986$ horas

c- $Ws = \frac{Ls}{\lambda} = \frac{24,947}{40} = 0,6236$ horas

e- $P_{25} = 20^{25} \cdot \left(\frac{1-20}{1-20^{25+1}}\right) = 0,95 \rightarrow 95\%$