Notes for "GRAVITATION" - MTW

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abc

1. GEOMETRODYNAMICS

1.1. The parable of the apple

Space tells matter how to move and matter tells space how to curve.

1.2. Spacetime with and without coordinates

1.2.1. Hint for possible different characterization (P.6)

"But with all the daring in the world, how is one to drive a nail into spacetime to mark a point? Happily, nature provides its own way to localize a point in spacetime, as Einstein was the first to emphasize. Characterize the point by what happens there!"

1.2.2. Hint from idealized limit (P.10)

"A more detailed diagram would show a maze of world line and of light rays and the intersections between them. From such a picture, one can in imagination step to the *idealized* limit: an infinitely dense collection of light rays and of world lines of infinitesimal test particles."

1.2.3. Hint of breakdown of manifold description (P.12)

"Not so quantum general relativity or 'quantum geometrodynamics'. It predicts violent fluctuations in the geometry at distances on the order of the Planck length,... As nearly as one can estimate these fluctuations give space at small distances a 'multiply connected' or 'foam-like' character."

1.3. Weightlessness

1.3.1. Box 1.2 (P.16)

a. Lorand von Eötvös The forces mentioned are shown in Fig. 1, from which we directly observe that the

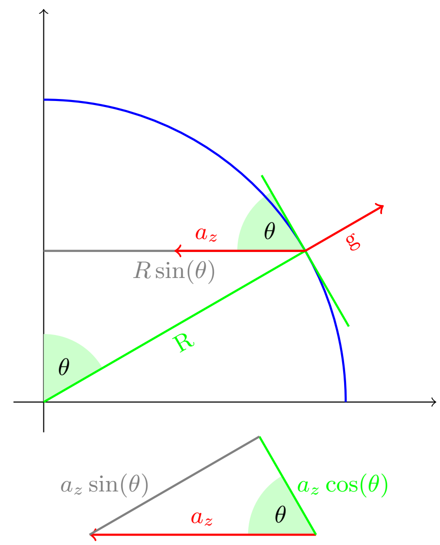

Figure 1: Forces mentioned in Box 1.2, *Lorand von Eötvös*, where R is the radius of the earth and θ is the angle measured from the north pole

centripetal acceleration is in total

$$a_z = \omega^2 R\sin(\theta),$$

which is *independent* of g in this way of viewing forces (ω is the rotation speed of the earth). From this we get the northward directed part of a_z to be

$$a_z\cos(\theta) = \omega^2 R\sin(\theta)\cos(\theta).$$

b. Beall We know the highest observed energies of the myon to be

$$E_\mu = 1\cdot 10^{13}\,\text{eV}.$$

While the upper threshold is mentioned to be

$$E_{\text{thresh}} = 1\cdot 10^3 mc^2$$

. If now the myon were to be "too light" we would have

$$E_\mu > E_{\text{thresh}}.$$

On the other hand we know the highest observed energies of photons to be

$$E_\gamma = 1\cdot 10^{13}\,\text{eV}.$$

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The transferred energy (to a photon) mentioned results in

$$E_\gamma \geq 2mc^2.$$

These two observations combined put an upper limit (Not "too heavy") on m.

1.3.2. "This assumption of exact physical equivalence makes it impossible for us to speak of the absolute acceleration of the system of reference, ..." ($P.17$)

This because any part of the acceleration could be due to a gravitational field.

1.4. Local Lorentz Geometry, with and without Coordinates

1.4.1. Hint : What about breakdown at small scales? ($P.21$)

"The geometry of spacetime is locally Lorentzian everywhere."

1.5. Time

Time is defined so that motion looks simple.

1.6. Curvature

1.6.1. Box 1.6 ($P.32$)

a. Figure C The formula

$$r = \frac{d^2}{8a},$$

where r is the radius of curvature, d is the "direct" distance between start and end point and a is the raise of the track of e.g. the ball, can be derived as follows:

Denote by α the angle inscribed by the shown cone under consideration, then

$$\sin(\alpha) = \frac{d}{r} \tag{1}$$

Further denote by x the distance between the center of curvature and the intersection point of the radial line at $\alpha/2$ with the "direct" line of connection between start and end point. Or in formulas

$$x = r - a.$$

Then we know

$$\begin{aligned}
\cos\left(\frac{\alpha}{2}\right) &= \frac{x}{r} \\
\Rightarrow a = r - x \\
&= r\left(1 - \cos\left(\frac{\alpha}{2}\right)\right) \\
&\overset{Eq.\ 1}{\approx} r\left(1 - \sqrt{1 - \left(\frac{d}{2r}\right)^2}\right) \\
&\approx r\left(1 - \left(1 - \frac{d^2}{8r^2}\right)\right) \\
&= \frac{d^2}{8r} \\
\Rightarrow r = \frac{d^2}{8a}
\end{aligned}$$

1.6.2. Eq. (1.13) ($P.37$)

The summation only happens over the spatial components i.e. j because we are here considering $v \ll c$, such that the separation in the time coordinate is negligible.

1.7. Effect of Matter on Geometry

1.7.1. Box 1.9 ($P.38$)

a. Galileo Galilei (1638) What is meant here is uniform acceleration leading to

$$s = \frac{1}{2}at^2 \Rightarrow s \propto t^2.$$

1.7.2. Hint : GR only being valid for larger (average) scales? ($P.42$)

"the field equation shows how the stress-energy of matter generates an *average* curvature in its neighborhood."

1.8. Exercises

1.8.1. Exercise 1.1 ($P.44$)

By unrolling the cylinder it is obvious that geodesics, which are parallel at one point, are parallel at any other point, i.e. they suffer no geodesic deviation:

$$\begin{aligned}
\frac{\mathrm{d}^2\xi}{\mathrm{d}s^2} &= 0 \overset{Eq.\ (1.6)}{=} -R\xi \\
\Rightarrow R &= 0
\end{aligned}$$

Given the formula

$$R = \frac{1}{\varrho_1 \varrho_2},$$

where ϱ_i are the principal radii of curvature at the point in question. We then get for a cylinder

$$\varrho_1 = \infty \qquad \varrho_2 = r \quad \Rightarrow R = 0,$$

where r is the radius of the cylinder in the 3-dimensional euclidean embedding space.

1.8.2. *Exercise* 1.2 (*P*.44)

Using Eq. (1.14) the values in question can be calculated.

1.8.3. *Exercise* 1.3 (*P*.44)

First fix some notation:

$$\begin{aligned} M &= \text{Mass of satellite} \\ \omega &= \text{Angular frequency of satellite} \\ r &= \text{Radius of orbit of satellite} \\ m &= \text{Mass of central object} \\ \varrho_{\text{Kepler}} &= \frac{m}{\frac{4\pi}{3} r^3} = \text{Kepler density} \end{aligned}$$

By setting the centripetal acceleration of the satellite equal to the gravitational acceleration (in Newtonian mechanics), we get:

$$\begin{aligned} M\omega^2 r &= G\frac{mM}{r^2} \\ \Rightarrow \omega^2 &= G\frac{m}{r^3} = \frac{4\pi G}{3}\varrho_{\text{Kepler}} \end{aligned}$$

2. FOUNDATIONS OF SPECIAL RELATIVITY

2.1. Quantum Mechanics

2.2. Symmetries

2.2.1. *"For this to be unitary and linear, t must be Hermitian and linear"* (*P*.51)

Linearity is trivial and hermiticity follow from the following observation:

$$\begin{aligned} \langle U\Psi | U\Phi\rangle &= \langle (1+i\varepsilon t)\Psi | (1+i\varepsilon t)\Phi\rangle \\ &= \langle\Psi|\Phi\rangle + \varepsilon i\left(\langle\Psi|t\Phi\rangle - \langle t\Psi|\Phi\rangle\right) + \mathcal{O}\big(\varepsilon^2\big) \\ &\overset{Eq.\,(2.2.2)}{\Leftrightarrow} \langle\Psi|t\Phi\rangle = \langle t\Psi|\Phi\rangle \\ &\overset{Eq.\,(2.1.5)}{\Leftrightarrow} t^\dagger = t \end{aligned}$$

2.2.2. *Eq.* (2.2.19) (*P*.54)

f^a_{bc} and f^a have to be real as θ^a are real.

3. THE ELECTROMAGNETIC FIELD

3.1. "In" and "Out" States

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