# MT5763_2_220021614

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```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.2 --
## v ggplot2 3.3.6      v purrr   0.3.4
## v tibble  3.1.8      v dplyr   1.0.10
## v tidyr   1.2.1      v stringr 1.4.1
## v readr   2.1.2      v forcats 0.5.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(parallel)
```

## Problem 1

Description: Consider the following independent random variables:
X ~ N($\mu$ = 4, $\sigma^2$ = 10)
Y ~ U(a = 2, b = 8).
Compute $Pr(X > Y)$
Use bootstrapping to derive the sampling distribution for your estimate of $Pr(X > Y)$
Show how the sample variance of this sampling distribution changes as a function of the number of Monte Carlo simulations.

---

For the underlying problem, a sample containing 1e+5 (100*1000) random deviates from X ~ N($\mu$ = 4, $\sigma^2$ = 10) and Y ~ U(a = 2, b = 8) is used. To simulate "real-world conditions", the solution is obtained from only the below given vectors for X and Y.

```
set.seed(0911)

probability_calculator <- function(n) {

  X <- rnorm(n, mean = 4, sd = sqrt(10))
  Y <- runif(n, min = 2, max = 8)

  # Calculating Pr(X>Y)
  Pr_hat <- sum(X > Y) / n

  output <- list(X = X,
```

```r
                 Y = Y,
                 Pr_hat = Pr_hat)

  return(output)
}


results <- probability_calculator(100000)

print(results[3])
```

```
## $Pr_hat
## [1] 0.39006
```

Calculating $\bar{P}r(X > Y)$ from the initial sample without any further methods, we derive a value of `rtoString(results[3])`. To derive the distribution of $\bar{P}r(X > Y)$, we use a non-parametric bootstrap. One benefit of this technique is that it does not rely on the assumption of normally distributed data.

```r
# creating function for bootstrap, using 8 cores (for completeness only).
bootstrap_multicore <- function(n_straps, vec1 = X, vec2 = Y) {
prob_vector <- unlist(mclapply(1:n_straps, function(n = n, vec1 = X, vec2 = Y) {
  resX <- vec1[sample(1 : length(vec1), length(vec1), replace = TRUE)]
  resY <- vec2[sample(1 : length(vec2), length(vec2), replace = TRUE)]

  Prob <- sum(resX > resY) / length(resX)
  return(Prob)
}, mc.cores = 8, mc.set.seed = TRUE))
return(prob_vector)
}
```

```r
# set seed for reproducibility
RNGkind("L'Ecuyer-CMRG")
set.seed(0112)


X <- unlist(results[1])
Y <- unlist(results[2])

#!!! as I use 10,000 bootstraps, computation can take some time!!!
prob_vec <- bootstrap_multicore(n_straps = 10000)
```

The bootstrap algorithm above re-samples the vectors X and Y *with replacement*, calculates the resulting $\bar{P}r_i(X > Y)$, and repeats this procedure $n$ times. The algorithm generates a vector ($prob\_vector$) with $n$ probabilities.
The distribution of $Pr(X > Y)$ can now be evaluated

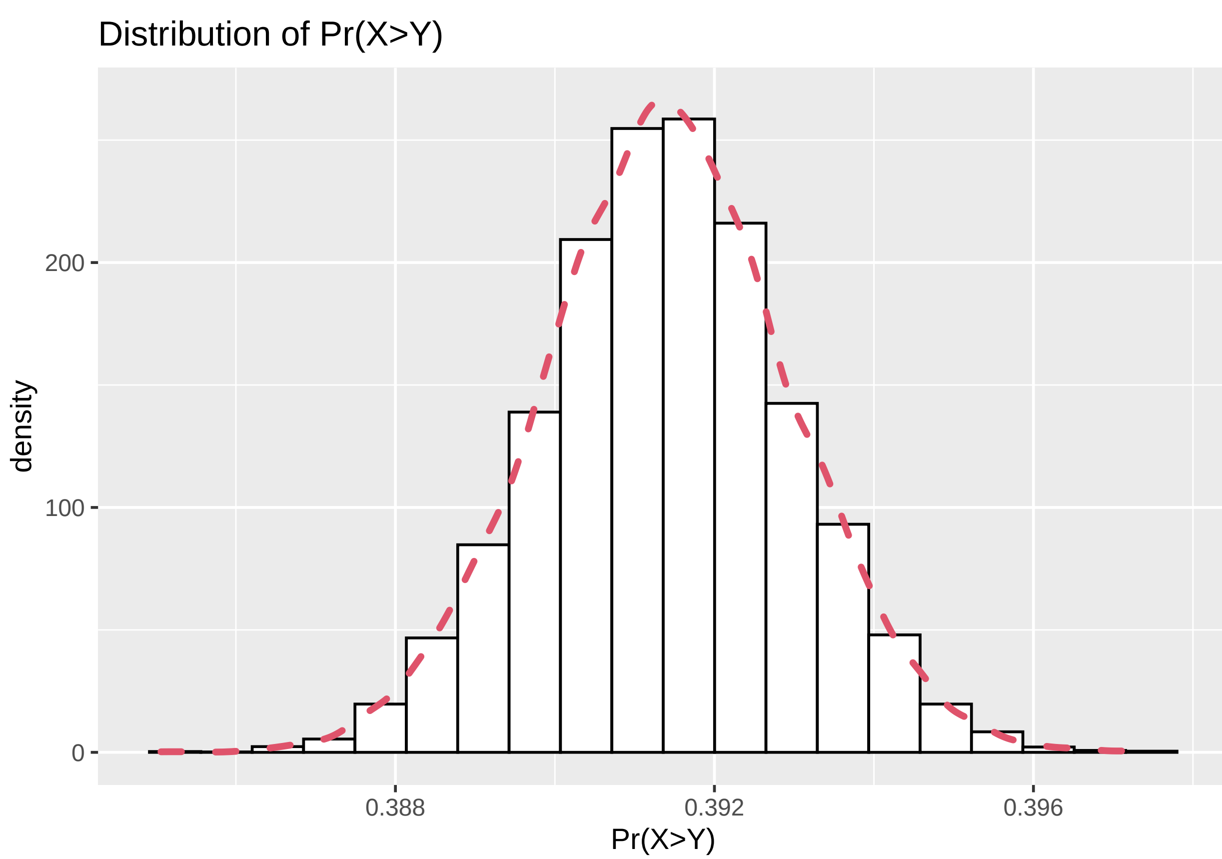
1. Using point estimates:

```r
quantile(prob_vec, c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%   97.5%
## 0.38833 0.39139 0.39446
```

2. Using visualization (histogram):

```
df_probabilities <- as.data.frame(prob_vec)

df_probabilities %>%
ggplot(aes(x = prob_vec)) +
  geom_histogram(aes (y = ..density..),
                 bins = 20,
                 colour = 1,
                 fill = "white")+
  geom_density(lwd = 1.2,
               linetype = 2,
               colour = 2)+
  xlab("Pr(X>Y)")+
  ggtitle("Distribution of Pr(X>Y)")
```



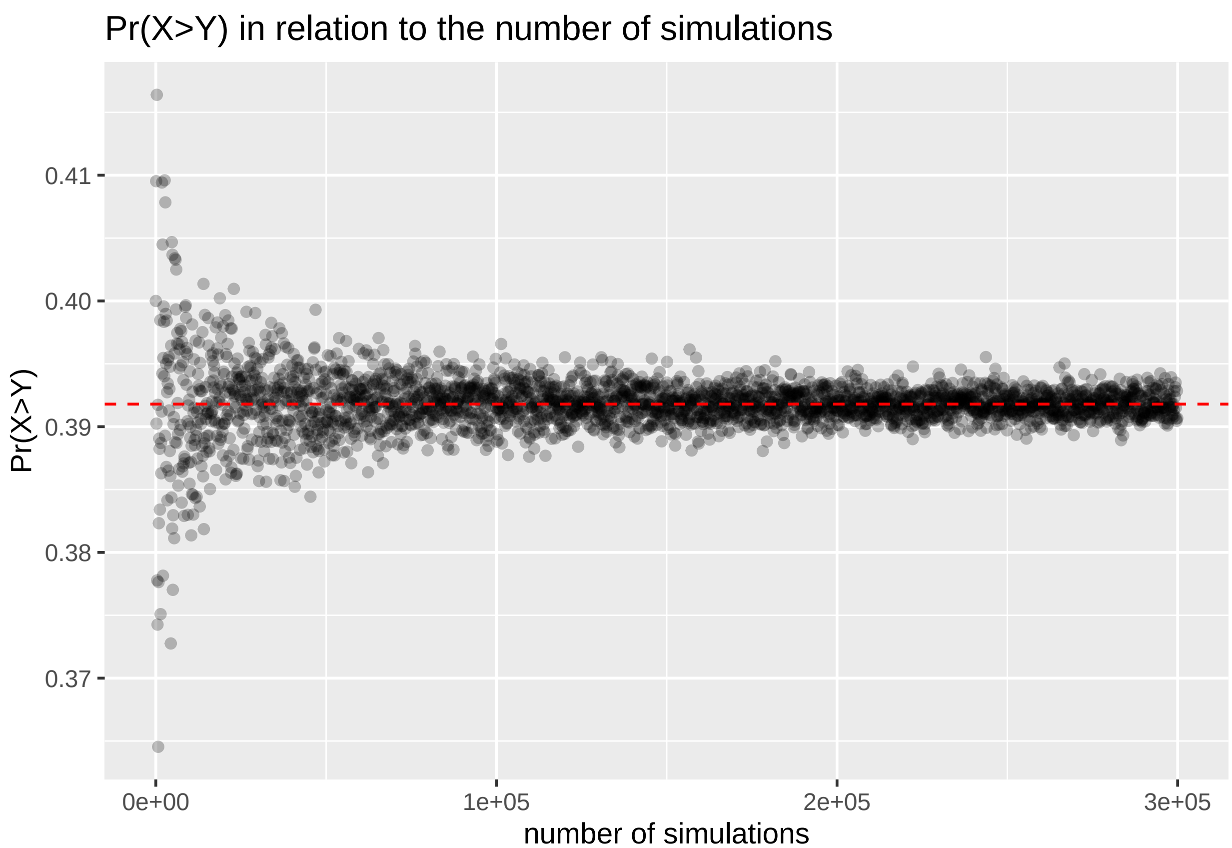

Lastly, it is of interest how the sample variance of the sampling distribution changes with dependence on the bootstraps

```
RNGkind("L'Ecuyer-CMRG")
set.seed(149)
```

```r
n_deviates <- seq(5, 300000, by = 100)

prob <- unlist(mclapply(n_deviates, function(i) {

    x <- rnorm(i, mean = 4, sd = sqrt(10))
    y <- runif(i, min = 2, max = 8)

    prob <- sum(x>y) / length(x)
    deviates <- i



    return(prob)

}, mc.cores = 8, mc.set.seed = TRUE)
)
```

```r
results_1.3 <- data.frame(prob, n_deviates)


results_1.3 %>%
  ggplot(aes(x = n_deviates, y = prob)) +
  geom_point(alpha = 1/4)+
  geom_hline(yintercept = median(prob), colour = "red", lty='dashed', lwd=0.5)+
  ylab("Pr(X>Y)")+
  xlab("number of simulations")+
  ggtitle("Pr(X>Y) in relation to the number of simulations")
```

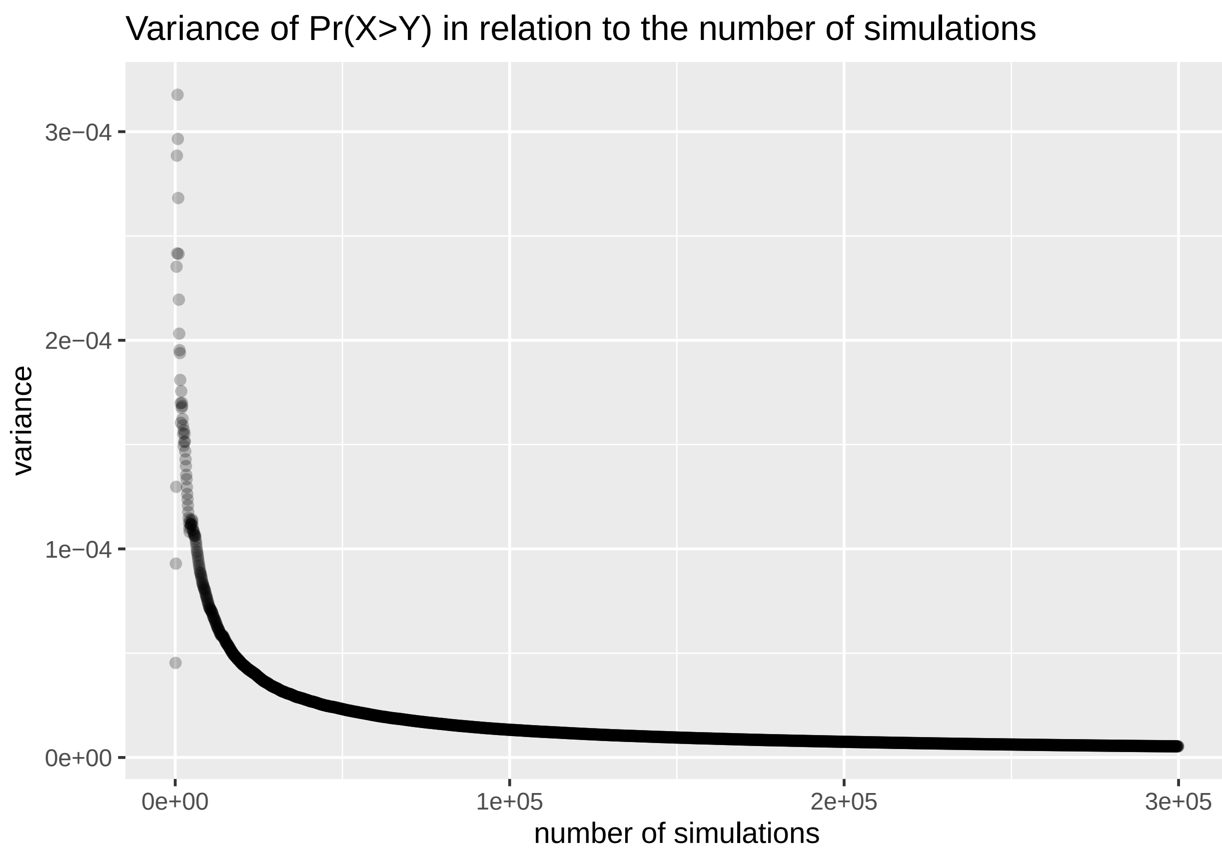



```
variance <- c()

for (i in 1 : (length(prob) - 1)) {
  variance[i] <- var(prob[1 : (i + 1)])
}

results_1.3.2 <- data.frame(variance, n_deviates[2:3000])

results_1.3.2 %>%
  ggplot(aes(x = n_deviates.2.3000., y = variance))+
  geom_point(alpha = 1/4)+
  ylab("variance")+
  xlab("number of simulations")+
  ggtitle("Variance of Pr(X>Y) in relation to the number of simulations")
```




### Problem 2

Description: Consider the following football tournament format: a team keeps playing until they accrue 7 wins or 3 losses (whichever comes first - no draws allowed). Assume a fixed win rate P (element of) [0, 1] across all rounds (they are paired at random).
Plot how the total number of matches played (i.e. wins + losses) varies as a function of $p$.
Comment on the observed win rate relative to the assumed win rate $p$ (i.e. if a team obtains 2 wins - 3 losses, the maximum likelihood point estimate for their win rate is 40%). Specifically, focus on the effect driven by the format of this tournament.

---

First, an algorithm simulating the above described tournament is needed. The code below shows a function for simulating the process.

```
# function for simulating the tournament
tournament_sim <- function(p_win) {
  p_loss <- 1 - p_win #loss rate
  outcome <- c("win", "loss")
  results_storage <- c() #empty vector for storing match results

# as you never play more than 9 games
# (6 wins + 3 losses (=9) or 2 losses and 7 wins (=9))
for (i in 1 : 9) {
```

```
  # simulating matches with sampling from c("win", "loss") with given
  # probabilities and replacement.
  # Adding the result to storage vector.
  results_storage <- c(sample(outcome, size = 1,
                          replace = TRUE,
                          prob = c(p_win, p_loss)), results_storage)

  # Conditions for winning or loosing the tournament
    if (length(results_storage[results_storage == "win"]) == 7) {
      break
    }

    if (length(results_storage[results_storage == "loss"]) == 3) {
      break
   }
  }

  #binding output together as a list
  output <- list(played_matches = length(results_storage),
                 match_results = results_storage,
                 overview = table(results_storage))

  return(output)
}
```

To solve the problem, a fixed win rate $p$ has to be defined. The following solution assumes a fixed win rate per round of 75%, so $p = 0.75$.

```
# declaring p
p <- 0.75

set.seed(0911)
#simulations one tournament
results <- tournament_sim(p_win = p)
```

Playing one tournament, the observed results are as following:/ / 1. Result of the tournament:

```
results$match_results
```

```
## [1] "win"  "win"  "win"  "win"  "win"  "loss" "win"  "win"
```

2. Overall results:

```
results$overview
```

```
## results_storage
## loss  win
##    1    7
```

3. Number of matched played:

```
results$played_matches
```

```
## [1] 8
```

It now should be taken in consideration how the *number of total matches* changes if we alter the winning rate $p$. This problem can be separated into several steps./ First,

```
# Alternating the sim function so that output is solely the
# number of played games
n_games <- function(p_win) {
  p_loss <- 1 - p_win
  outcome <- c("win", "loss")
  results_storage <- c()

  for (i in 1 : 9) {

    results_storage <- c(sample(outcome, size = 1,
                              replace = TRUE,
                              prob = c(p_win, p_loss)), results_storage)

      if (length(results_storage[results_storage == "win"]) == 7) {
        break
      }

      if (length(results_storage[results_storage == "loss"]) == 3) {
        break
    }
  }

  played_matches  <-  length(results_storage)

  return(played_matches)
}
```

Win rates from 5% to 95% are taken into consideration, in steps by 5%.

```
# generating storage matrix
win_rates <- seq(from = 0.05, to = 0.95, by = 0.05)

storage <- matrix(nrow = length(win_rates), ncol = 1000)
```

```
for (i in win_rates){
  res <- n_games(i)
  print(res)
}
```

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## [1] 3
## [1] 3
## [1] 3
## [1] 3
```

```
## [1] 4
## [1] 4
## [1] 7
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## [1] 5
## [1] 6
## [1] 9
## [1] 6
## [1] 7
## [1] 7
## [1] 8
## [1] 9
## [1] 7
## [1] 8
```

```
mclapply(1 : 1000, function(i) {
  p_win = 0.5
  p_loss <- 1 - p_win
  outcome <- c("win", "loss")
  results_storage <- c()

  for (i in 1 : 9) {

    results_storage <- c(sample(outcome, size = 1,
                              replace = TRUE,
                              prob = c(p_win, p_loss)), results_storage)

      if (length(results_storage[results_storage == "win"]) == 7) {
        break
      }

      if (length(results_storage[results_storage == "loss"]) == 3) {
        break
    }
  }

  played_matches  <-  length(results_storage)

  return(played_matches)
}, mc.cores = 8)
```

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