MT5763_2_220021614

Nico Herrig

2022-10-15

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.2 --
## v ggplot2 3.3.6      v purrr   0.3.4
## v tibble  3.1.8      v dplyr   1.0.10
## v tidyr   1.2.1      v stringr 1.4.1
## v readr   2.1.2      v forcats 0.5.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(parallel)
```

Problem 1

Description: The problem presents two variables X and Y, where X ~ N($\mu = 4$, $\sigma^2 = 10$) and Y ~ U(a = 2, b = 8).
Compute $Pr(X > Y)$ and use bootstrapping to to derive the sampling distribution for your estimate of $Pr(X > Y)$. Show how the sample variance of this sampling distribution changes as a function of the number of Monte Carlo simulations.

For the underlying problem, a sample containing 1e+6 (100*10000) random deviates from X ~ N($\mu = 4$, σ^2 = 10) and Y ~ U(a = 2, b = 8) is used. To simulate "real-world conditions", the solution is obtained from only the below given vectors for X and Y.

```
# set seed for reproducibility
RNGkind("L'Ecuyer-CMRG")
set.seed(0911)

# Using parallel computing techniques, we generate 100 sets of 100 random deviates
X <- unlist(mclapply(1:100, function(i) {
  rnorm(100, mean = 4, sd = sqrt(10))
}, mc.cores = 8, mc.set.seed = TRUE))


Y <- unlist(mclapply(1:100, function(i) {
  runif(100, min = 2, max = 8)
}, mc.cores = 8, mc.set.seed = TRUE))
```

```r
Pr_hat <- sum(X > Y) / (100*100)

print(Pr_hat)
```

```
## [1] 0.3906
```

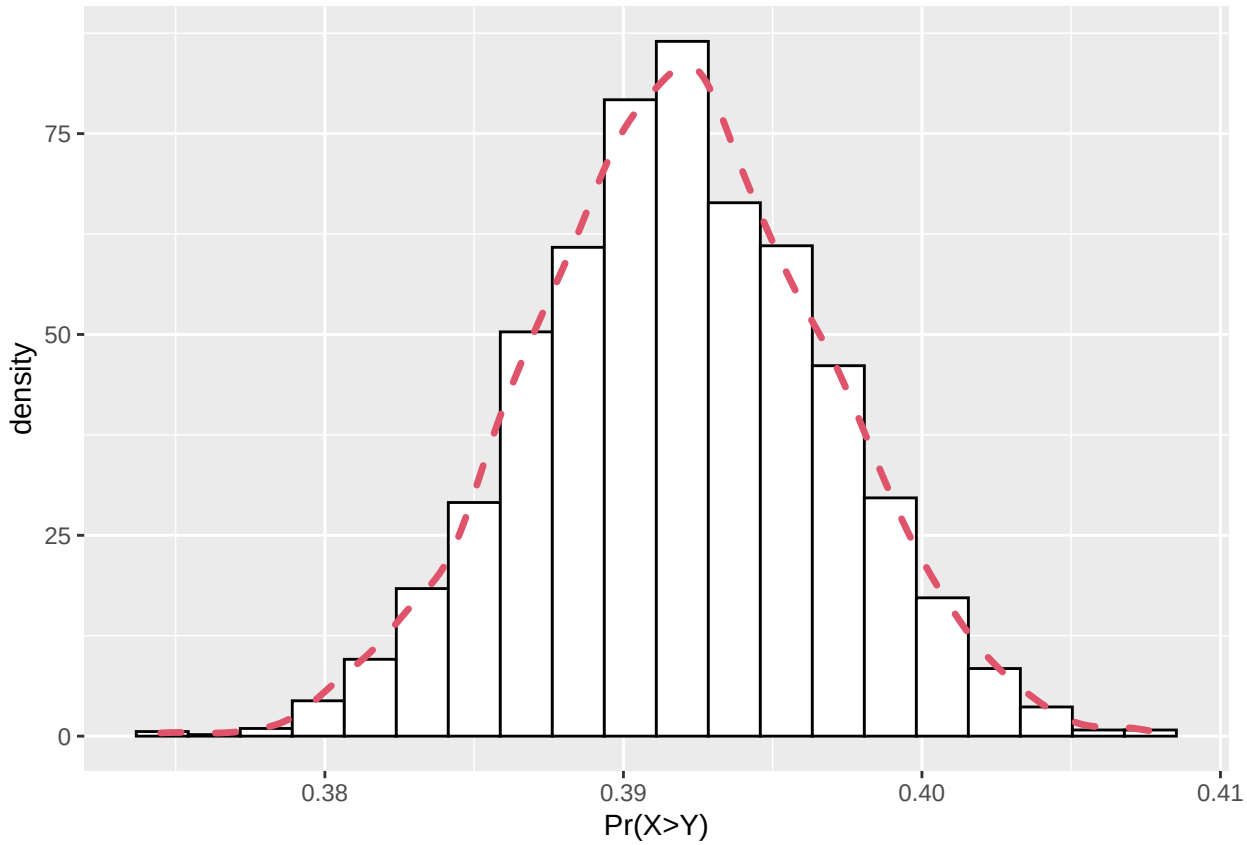
Calculating $\bar{Pr}(X > Y)$ from the initial sample without any further methods, we derive a value of 0.3906. To derive the distribution of 0.3906

A non-parametric bootstrapping is now used to simulate the distribution of $\bar{Pr}(X > Y)$ from the given sample.

```r
bootstrap <- function(n_bootstraps, vec1 = X, vec2 = Y) {

  vector_prob <- rep(NA, times = n_bootstraps)

  for (i in 1 : n_bootstraps) {
    X_resampled <- vec1[sample(1 : length(vec1), length(vec1), replace = TRUE)]
    Y_resampled <- vec2[sample(1 : length(vec2), length(vec2), replace = TRUE)]

    vector_prob[i] <- sum(X_resampled > Y_resampled) / length(X)
  }
  return(vector_prob)

}
```

The bootstrap algorithm re-samples the vectors X and Y *with replacement*, calculates the resulting $\bar{Pr}_i(X > Y)$, and repeats this procedure n times. The function puts out a vector with n generated probabilities. Using the bootstrap function, we now can evaluate the distribution of $Pr(X > Y)$

```r
set.seed(123)
df_probabilities <- data.frame(p_hat =bootstrap(n_bootstraps = 3000))

df_probabilities %>%
ggplot(aes(x = p_hat)) +
  geom_histogram(aes (y = ..density..),
                 bins = 20,
                 colour = 1,
                 fill = "white")+
  geom_density(lwd = 1.2,
               linetype = 2,
               colour = 2)+
  xlab("Pr(X>Y)")
```


density
75
50
25
0
0.38
0.39
0.40
0.41
Pr(X>Y)