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CS330 – Reyzin

CS330 Homework Assignment #1

** I conceptually collaborated with Sean Smith on some of the questions **

Problem 1

1.) Here is a set of people and their preferences where every possible matching for these people contains an instability.

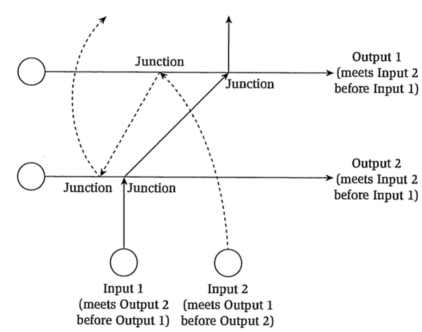
People: x1, x2, x3, x4
Preferences:
x1: x2, x3, x4
x2: x3, x1, x4
x3: x1, x2, x4
x4: x1, x2, x3

Case 1:
X1 ~ X3 & X2 ~ X4
    Instability: X1 and X2 want to be with each other

Case 3:
X1 ~ X4 & X2 ~ X3
    Instability: X1 and X3 want to be with each other

Case 3:
X1 ~ X2 & X3 ~ X4
    Instability: X2 and X3 want to be with each other

Problem 2



Correlation: While this problem seemed daunting at first, it later becomes clear that the junctions on each output line are just "lists" of preferences for the outputs according to each input. An input's list of preferences corresponds to the distance to its junctions.

Example Above:
Input 1 Preferences: Output 1, Output 2
Input 2 Preferences: Output 2, Output 1

Conclusion: From this, we can see that if the inputs are matched stable-y with their corresponding outputs, you can always find a match that works.

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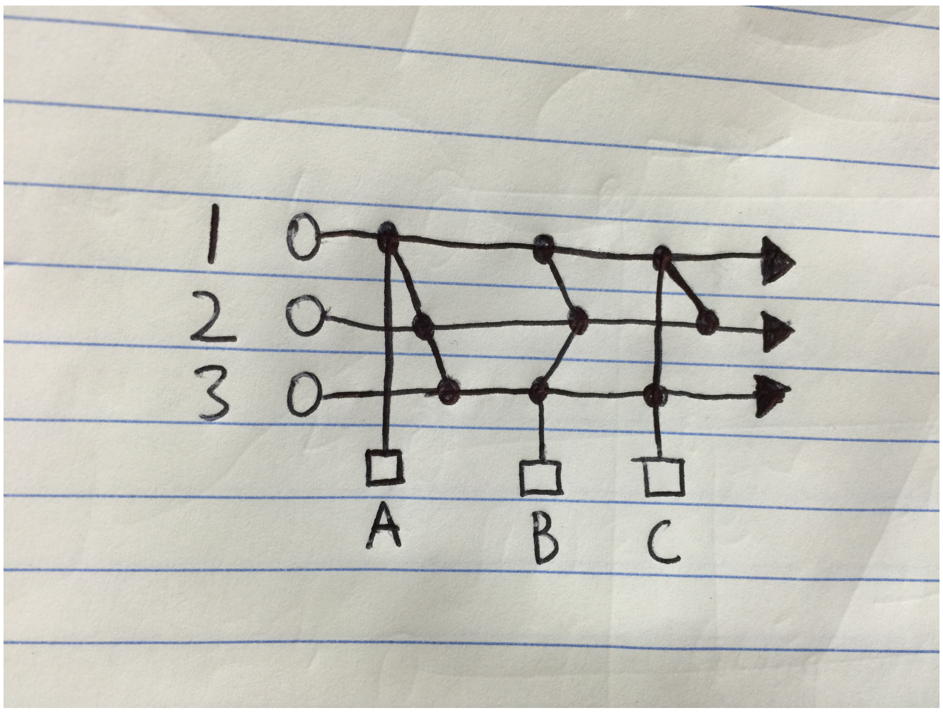
"The Algorithm": Approach the problem the same as the matching problem and map the preferences. From this, "find a stable match" and set the switches to the corresponding matches.

More In Depth:

If we approach this problem recursively, we can go from preference of each input down until we have no collisions. An example:



1: A3

- B3 *Collision*
  ****A3 is Out****

2: A2

- B2 ***Collision***
- B3
  - C3 ***Collision***
    ****B1 is Out****

  ****A2 is Out****

2: A1

- B1 ***Collision***
- B2
  - C2 ***Collision***
  - C3 **SUCCESS**

Conclusion of In-depth: Eventually, the algorithm will find a valid preference set for whatever set up is given

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Problem 3

$n^{\frac{1}{\lg(n)}}$ -> $\underline{\lg(n)}$ -> $\underline{log_3(5n)}$ -> $(\lg(n))^2$ -> $\underline{\left((2)^{\frac{1}{2}}\right)^{\lg(n)} \sim (n)^{\frac{1}{2}}}$ -> n -> $n \lg n$ -> $\lg(n!)$ -> $\underline{n^2 \sim \binom{n}{2} + n \lg n \sim 4^{\lg(n)}}$ -> n^3 -> $\underline{2^{100 \lg(n)} \sim n^{100}}$ -> $n^{\lg(\lg(n))}$ -> $2^{\lg^{1.001}(n)}$ -> $n * 2^n$ -> 2^{2^n} -> $2^{2^{n+1}}$

Important: $\sim$ indicates equivalent (I also underlined them though)

Problem 4

a.) $f(n) = o\big(g(n)\big) and\ f(n) \neq \theta(g(n))$

Ideas:
- f(n) increments slower than g(n)
- f(n) and g(n) do not increment at a rate roughly equal

Example: $f(n) = \log(n)$ and $g(n) = n^2$

b.) $f(n) = \theta\big(g(n)\big) and\ f(n) = o(g(n))$

Ideas:
- f(n) and g(n) do increment at a rate roughly equal
- f(n) increments slower than g(n)

Example: **None**

- Reasoning: The first equation basically says that $lim_{n \to \infty} \frac{f(n)}{g(n)}$ would equal 0, yet the second basically says that $lim_{n \to \infty} \frac{f(n)}{g(n)}$ would equal something like this: $0 < x < \infty$ … This is a clear contradiction.

c.) $f(n) = \theta\big(g(n)\big) and\ f(n) \neq O(g(n))$

Ideas:
- f(n) and g(n) do increment at a rate roughly equal
- f(n) does not increment at a rate less than or roughly equal to g(n)

Example: $f(n) = \log(n)$ and $g(n) = 2\log(n)$

d.) $f(n) = \Omega\big(g(n)\big) and\ f(n) \neq O(g(n))$

Ideas:
- f(n) does increment at a rate greater than or roughly equal to g(n)
- f(n) does not increment at a rate less than or roughly equal to g(n)

Example: $f(n) = n^2$ and $g(n) = log(n)$

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e.) $f(n) = \Omega(g(n)) and\ f(n) \neq o(g(n))$

Ideas:
- f(n) does increment at a rate greater than or roughly equal to g(n)
- f(n) does not increment slower than g(n)

Example: $f(n) = n^2$ and $g(n) = log\ (n)$

f.) $f(n) = \omega(g(n)) and\ g(n) \neq o(f(n))$

Ideas:
- f(n) does increment at a rate greater than g(n)
- g(n) does not increment slower than f(n)

Example: **None**

- Reasoning: The first equation says that $lim_{n \to \infty} \frac{f(n)}{g(n)}$ would equal ∞, yet the second basically says that $lim_{n \to \infty} \frac{f(n)}{g(n)}$ could equal anything OTHER than ∞… This is a clear contradiction.