Matrice des cofacteurs

$$A = \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix}$$

Matrice des cofacteurs

Elément $\Delta_{11} = Signe_{11} \times Mineur_{11}$

- $Signe_{11} = (-1)^{1+1} = (-1)^2 = +1$
- $Mineur_{11} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 1 & 2 & 0 \\ 3 & -2 & -2 \\ 1 & -3 & 1 \end{vmatrix} = \begin{matrix} (1 \times (-2) \times 1) \\ +(2 \times (-2) \times 1) \\ +(0 \times 3 \times (-3)) \\ -(0 \times (-2) \times 1) \\ -((-2) \times (-3) \times 1) \\ -(1 \times 2 \times 3) \end{matrix} = -18$
- $\Delta_{11} = (+1) \times (-18) = -18$

Matrice des cofacteurs

Elément $\Delta_{12} = Signe_{12} \times Mineur_{12}$

- $Signe_{12} = (-1)^{1+2} = (-1)^3 = -1$
- $Mineur_{12} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 3 & 2 & 0 \\ 1 & -2 & -2 \\ 0 & -3 & 1 \end{vmatrix} = \begin{matrix} (3 \times (-2) \times 1) \\ +(2 \times (-2) \times 0) \\ +(0 \times 1 \times (-3)) \\ -(0 \times (-2) \times 0) \\ -((-2) \times (-3) \times 3) \\ -(1 \times 2 \times 1) \end{matrix} = -26$
- $\Delta_{12} = (-1) \times (-26) = 26$

Matrice des cofacteurs

Elément $\Delta_{13} = Signe_{13} \times Mineur_{13}$

- $Signe_{13} = (-1)^{1+3} = (-1)^4 = +1$
- $Mineur_{13} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & -2 \\ 0 & 1 & 1 \end{vmatrix} =$

$$\begin{matrix} (3 \times 3 \times 1) \\ +(1 \times (-2) \times 0) \\ +(0 \times 1 \times 1) \\ -(0 \times (3) \times 0) \\ -((-2) \times 1 \times 3) \\ -(1 \times 1 \times 1) \end{matrix} \quad = 14$$

- $\Delta_{13} = (+1) \times (14) = 14$

Matrice des cofacteurs

Elément $\Delta_{14} = Signe_{14} \times Mineur_{14}$

- $Signe_{14} = (-1)^{1+4} = (-1)^5 = -1$
- $Mineur_{14} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 3 & 1 & 2 \\ 1 & 3 & -2 \\ 0 & 1 & -3 \end{vmatrix} =$

$$\begin{matrix} (3 \times 3 \times (-3)) \\ +(1 \times (-2) \times 0) \\ +(2 \times 1 \times 1) \\ -(2 \times (3) \times 0) \\ -((-2) \times 1 \times 3) \\ -((-3) \times 1 \times 1) \end{matrix} \quad = -16$$

- $\Delta_{14} = (-1) \times (-16) = 16$

Matrice des cofacteurs

Elément $\Delta_{21} = Signe_{21} \times Mineur_{21}$

- $Signe_{21} = (-1)^{2+1} = (-1)^3 = -1$
- $Mineur_{21} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 6 & 1 & -2 \\ 3 & -2 & -2 \\ 1 & -3 & 1 \end{vmatrix} = \begin{matrix} (6 \times (-2) \times 1) \\ +(1 \times (-2) \times 1) \\ +((-2) \times 3 \times (-3)) \\ -((-2) \times (-2) \times 1) \\ -((-2) \times (-3) \times 6) \\ -(1 \times 1 \times 3) \end{matrix} = -39$
- $\Delta_{21} = (-1) \times (-39) = 39$

Matrice des cofacteurs

Elément $\Delta_{22} = Signe_{22} \times Mineur_{22}$

- $Signe_{22} = (-1)^{2+2} = (-1)^4 = +1$
- $Mineur_{22} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 1 & -2 \\ 1 & -2 & -2 \\ 0 & -3 & 1 \end{vmatrix} = \begin{matrix} (0 \times (-2) \times 1) \\ +(1 \times (-2) \times 0) \\ +((-2) \times 1 \times (-3)) \\ -((-2) \times (-2) \times 0) \\ -((-2) \times (-3) \times 0) \\ -(1 \times 1 \times 1) \end{matrix} = 5$
- $\Delta_{22} = (+1) \times (5) = 5$

Matrice des cofacteurs

Elément $\Delta_{23} = Signe_{23} \times Mineur_{23}$

- $Signe_{23} = (-1)^{2+3} = (-1)^5 = -1$
- $Mineur_{23} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 6 & -2 \\ 1 & 3 & -2 \\ 0 & 1 & 1 \end{vmatrix} =$

$$\begin{matrix} (0 \times 3 \times 1) \\ +(6 \times (-2) \times 0) \\ +((-2) \times 1 \times 1) \\ -((-2) \times 3 \times 0) \\ -((-2) \times 1 \times 0) \\ -(1 \times 6 \times 1) \end{matrix} \quad = -8$$

- $\Delta_{23} = (-1) \times (-8) = 8$

Matrice des cofacteurs

Elément $\Delta_{24} = Signe_{24} \times Mineur_{24}$

- $Signe_{24} = (-1)^{2+4} = (-1)^{6} = +1$
- $Mineur_{24} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 6 & 1 \\ 1 & 3 & -2 \\ 0 & 1 & -3 \end{vmatrix} =$

$$\begin{matrix} (0 \times 3 \times (-3)) \\ +(6 \times (-2) \times 0) \\ +(1 \times 1 \times 1) \\ -(1 \times 3 \times 0) \\ -((-2) \times 1 \times 0) \\ -((-3) \times 6 \times 1) \end{matrix} = 19$$

- $\Delta_{24} = (+1) \times (19) = 19$

Matrice des cofacteurs

Elément $\Delta_{31} = Signe_{31} \times Mineur_{31}$

- $Signe_{31} = (-1)^{3+1} = (-1)^4 = +1$
- $Mineur_{31} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 6 & 1 & -2 \\ 1 & 2 & 0 \\ 1 & -3 & 1 \end{vmatrix} = \begin{matrix} (6 \times 2 \times 1) \\ +(1 \times 0 \times 1) \\ +((-2) \times 1 \times (-3)) \\ -((-2) \times 2 \times 1) \\ -(0 \times (-3) \times 6) \\ -(1 \times 1 \times 1) \end{matrix} = 21$
- $\Delta_{31} = (+1) \times (21) = 21$

Matrice des cofacteurs

Elément $\Delta_{32} = Signe_{32} \times Mineur_{32}$

- $Signe_{32} = (-1)^{3+2} = (-1)^5 = -1$
- $Mineur_{32} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 1 & -2 \\ 3 & 2 & 0 \\ 0 & -3 & 1 \end{vmatrix} = \begin{matrix} (0 \times 2 \times 1) \\ +(1 \times 0 \times 0) \\ +((-2) \times 3 \times (-3)) \\ -((-2) \times 2 \times 0) \\ -(0 \times (-3) \times 0) \\ -(1 \times 1 \times 3) \end{matrix} = 15$
- $\Delta_{32} = (-1) \times (15) = -15$

Matrice des cofacteurs

Elément $\Delta_{33} = Signe_{33} \times Mineur_{33}$

- $Signe_{33} = (-1)^{3+3} = (-1)^{6} = +1$
- $Mineur_{33} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 6 & -2 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} =$

$$\begin{matrix} (0 \times 1 \times 1) \\ +(6 \times 0 \times 0) \\ +((-2) \times 3 \times 1) \\ -((-2) \times 1 \times 0) \\ -(0 \times 1 \times 0) \\ -(1 \times 6 \times 3) \end{matrix} \quad = -24$$

- $\Delta_{33} = (+1) \times (-24) = -24$

Matrice des cofacteurs

Elément $\Delta_{34} = Signe_{34} \times Mineur_{34}$

- $Signe_{34} = (-1)^{3+4} = (-1)^{7} = -1$
- $Mineur_{34} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 6 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & -3 \end{vmatrix} =$

$$\begin{matrix} (0 \times 1 \times (-3)) \\ +(6 \times 2 \times 0) \\ +(1 \times 3 \times 1) \\ -(1 \times 1 \times 0) \\ -(2 \times 1 \times 0) \\ -((-3) \times 6 \times 3) \end{matrix} = 57$$

- $\Delta_{34} = (-1) \times (57) = -57$

Matrice des cofacteurs

Elément $\Delta_{41} = Signe_{41} \times Mineur_{41}$

- $Signe_{41} = (-1)^{4+1} = (-1)^5 = -1$
- $Mineur_{41} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 6 & 1 & -2 \\ 1 & 2 & 0 \\ 3 & -2 & -2 \end{vmatrix} =$

$$\begin{matrix} (6 \times 2 \times (-2)) \\ +(1 \times 0 \times 3) \\ +((-2) \times 1 \times (-2)) \\ -((-2) \times 2 \times 3) \\ -(0 \times (-2) \times 6) \\ -((-2) \times 1 \times 1) \end{matrix} \quad = -6$$

- $\Delta_{41} = (-1) \times (-6) = 6$

Matrice des cofacteurs

Elément $\Delta_{42} = Signe_{42} \times Mineur_{42}$

- $Signe_{42} = (-1)^{4+2} = (-1)^{6} = +1$
- $Mineur_{42} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 1 & -2 \\ 3 & 2 & 0 \\ 1 & -2 & -2 \end{vmatrix} = \begin{matrix} (0 \times 2 \times (-2)) \\ +(1 \times 0 \times 1) \\ +((-2) \times 3 \times (-2)) \\ -((-2) \times 2 \times 1) \\ -(0 \times (-2) \times 0) \\ -((-2) \times 1 \times 3) \end{matrix} = 22$
- $\Delta_{42} = (+1) \times (22) = 22$

Matrice des cofacteurs

Elément $\Delta_{43} = Signe_{43} \times Mineur_{43}$

- $Signe_{43} = (-1)^{4+3} = (-1)^{7} = -1$
- $Mineur_{43} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 6 & -2 \\ 3 & 1 & 0 \\ 1 & 3 & -2 \end{vmatrix} =$

$$\begin{matrix} (0 \times 1 \times (-2)) \\ +(6 \times 0 \times 1) \\ +((-2) \times 3 \times 3) \\ -((-2) \times 1 \times 1) \\ -(0 \times 3 \times 0) \\ -((-2) \times 6 \times 3) \end{matrix} \quad = 20$$

- $\Delta_{43} = (-1) \times (20) = -20$

Matrice des cofacteurs

Elément $\Delta_{44} = Signe_{44} \times Mineur_{44}$

- $Signe_{44} = (-1)^{4+4} = (-1)^{8} = +1$
- $Mineur_{44} \rightarrow \begin{pmatrix} 0 & 6 & 1 & -2 \\ 3 & 1 & 2 & 0 \\ 1 & 3 & -2 & -2 \\ 0 & 1 & -3 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 0 & 6 & 1 \\ 3 & 1 & 2 \\ 1 & 3 & -2 \end{vmatrix} =$

$$\begin{matrix} (0 \times 1 \times (-2)) \\ +(6 \times 2 \times 1) \\ +(1 \times 3 \times 3) \\ -(1 \times 1 \times 1) \\ -(2 \times 3 \times 0) \\ -((-2) \times 6 \times 3) \end{matrix} = 56$$

- $\Delta_{44} = (+1) \times (56) = 56$

Matrice des cofacteurs

$$\Delta = \begin{pmatrix} -18 & 26 & 14 & 16 \\ 39 & 5 & 8 & 19 \\ 21 & -15 & -24 & -57 \\ 6 & 22 & -20 & 56 \end{pmatrix}$$