Computational Continuum Physics

(TIF330/FYM330), 2023

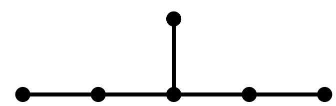
Problem set 1

Deadline: 23:59 on 2023-04-19

For submission requirements see guidance to written reports on the course page.

Problem 1. Explicit FDM with 5-point stencil for the heat equation. (10 points)

For one-dimensional heat equation obtain an expression that describes the explicit FDM with the highest possible accuracy for the stencil shown below. Determine the approximation order and analyze the stability of the obtained FDM.



Problem 2. FDTD modification without numerical dispersion along grid axis. (10 points)

Consider two-dimensional case of FDTD algorithm for computing the evolution of electromagnetic field in vacuum. Find a way to eliminate the numerical dispersion along one of the grid axes by extending the stencil that governs the advancement of field components perpendicular to the grid.

Hint: Consider augmenting the stencil by an additional pair of grid nodes from each side for each of two directions. Computing finite difference for each additional pair provides approximate values for the coordinate derivative other than that computed for the central pair. Similarly to the derivation of the Crank–Nicolson scheme, it is possible to take a weighted sum of computed derivatives and use the weights to control the properties of the constructed FDM.

Problem 3. 1D FDTD method for temporal evolution of an electromagnetic pulse. (15 points)

Develop a C++ implementation of 1D FDTD method for numerical solution of Maxwell's equations in vacuum:

$$\begin{cases} \dfrac{\partial E_y}{\partial t} = -\dfrac{\partial B_z}{\partial x} \\ \dfrac{\partial B_z}{\partial t} = -\dfrac{\partial E_y}{\partial x} \end{cases}$$

Simulate the propagation of a linearly polarized electromagnetic pulse along $x \in [-1, 1]$ towards positive x direction for a time interval $t \in [0, 2.5]$ starting from the following initial conditions:

$$E_y(x) = \begin{cases} \sin(20\pi x), & |x| < 0.1 \\ 0 & |x| \geq 0.1 \end{cases}$$

Consider the case of periodic boundary conditions and plot $E_y(x), B_z(x)$ at $t = 2.5$. Describe and show how one can implement the boundary conditions that model the surface of an ideal conductor. Propose and demonstrate a way to simulate a part of infinite space (e.g. to simulate the continuous emission of an antenna placed inside the simulation region).