Comparison of p_star with Parks expressions

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Our expression for p_*: $$p_{*,our} = f_p \left[\frac{m (Q q)^2}{\Lambda_* r_p} \right]^{\frac{1}{3}}$$

with $$f_p = \frac{\lambda_*}{\gamma} \left(\frac{\tilde{r}_p}{4} \frac{(\gamma - 1)^2}{\tilde{q}^2} \right)^{\frac{1}{3}} \approx 0.14$$

and from Parks eq. 1: $$q = \frac{n}{4} \left(\frac{8T}{\pi m_e} \right)^{\frac{1}{2}} 2T = \sqrt{\frac{2}{\pi m_e}}\, n T^{3/2}$$

Parks assumes cgs units and in a way assumes all variables to be just numbers. Thus when he writes r_p he actually means $\frac{r_p}{cm}$.

Parks eq. 30 (with units):

$$p_{*,30} = 10.62 \left(\frac{m}{amu}\right)^{\frac{1}{3}} \frac{(\gamma - 1)^{2/3}}{\gamma} \lambda_* \frac{\tilde{r}_p^{\frac{1}{3}}}{\tilde{q}_{bc}^{\frac{2}{3}}} \frac{\left(Q \left(\frac{n}{cm^{-3}}\right)\right)^{2/3}}{\left(\frac{r_p}{cm}\right)^{1/3}} \left(\frac{T}{eV}\right) \left(\frac{\Lambda_*}{cm^2}\right)^{-\frac{1}{3}} (dyn/cm^2)$$

Parks eq. 39 (with units, here he set $\gamma = \frac{7}{5}$, $Q = 0.65$, $m = 4\,amu$):

$$p_{*,39} = 3.95 \cdot 10^{-8} \left(\frac{r_p}{cm}\right)^{-\frac{1}{3}} \left(\frac{n}{cm^{-3}}\right)^{2/3} \left(\frac{T}{eV}\right)^{1.54} (dyn/cm^2)$$

Compare eq. 30

$$p_{*,30} = 10.62 \left(\frac{m}{amu}\right)^{\frac{1}{3}} \frac{(\gamma - 1)^{2/3}}{\gamma} \lambda_* \frac{\tilde{r}_p^{\frac{1}{3}}}{\tilde{q}^{\frac{2}{3}}} \frac{\left(Q \left(\frac{n}{cm^{-3}}\right)\right)^{2/3}}{\left(\frac{r_p}{cm}\right)^{1/3}} \left(\frac{T}{eV}\right) \left(\frac{\Lambda_*}{cm^2}\right)^{-\frac{1}{3}} (dyn/cm^2)$$

(collect units, dimensionless and physical quantities)

$$= \underbrace{\left[10.62 \left(\frac{1}{amu}\right)^{\frac{1}{3}} \frac{\left(\frac{1}{cm^{-3}}\right)^{\frac{2}{3}}}{\left(\frac{1}{cm}\right)^{\frac{1}{3}}} \left(\frac{1}{eV}\right) \left(\frac{1}{cm^2}\right)^{-\frac{1}{3}} \frac{dyn}{cm^2} \right]}_{= 10.62\ amu^{-\frac{1}{3}}\ cm \frac{dyn}{eV}} \underbrace{\left[\frac{\lambda_*}{\gamma} (\gamma - 1)^{2/3} \frac{\tilde{r}_p^{\frac{1}{3}}}{\tilde{q}^{\frac{2}{3}}} \right]}_{= 4^{\frac{1}{3}} f_p \text{ (matches our } f_p)} \underbrace{\left[m^{\frac{1}{3}} \frac{(Q n)^{2/3}}{r_p^{1/3}} T \Lambda_*^{-\frac{1}{3}} \right]}_{= \left[\frac{m (Q q)^2}{\Lambda_* r_p} \right]^{\frac{1}{3}} \left(\frac{\pi m_e}{2} \right)^{\frac{1}{3}}}$$

$$\Rightarrow p_{*,30} = 10.62 \cdot 4^{\frac{1}{3}} \left(\frac{\pi}{2}\right)^{\frac{1}{3}} \underbrace{\left(\frac{m_e}{\text{amu}}\right)^{\frac{1}{3}}}_{=(5.486\cdot 10^{-4})^{\frac{1}{3}}} \underbrace{\frac{\text{cm}\cdot\text{dyn}}{\text{eV}}}_{=6.242\cdot 10^{11}} \cdot p_{*,our} \qquad \text{since } q^{\frac{2}{3}} = \left(\frac{2}{\pi m_e}\right)^{\frac{1}{3}} n^{\frac{2}{3}} T$$

$$\Rightarrow p_{*,30} = 1.001 \cdot 10^{12}\, p_{*,our} \Rightarrow \text{Parks missed a factor } 10^{-12} \text{ in eq. 30}$$

Compare eq. 39

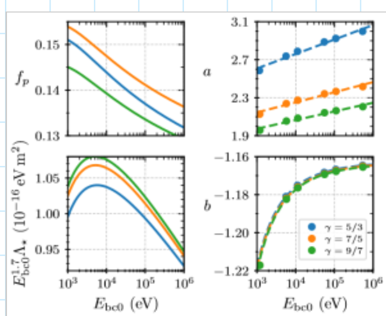
$$p_{*,39} = 3.95 \cdot 10^{-8} \left(\frac{r_p}{\text{cm}}\right)^{-\frac{1}{3}} \left(\frac{n}{\text{cm}^{-3}}\right)^{2/3} \left(\frac{T}{\text{eV}}\right)^{1.54} (\text{dyn}/\text{cm}^2)$$

$$p_{*,our} = f_p \left[\frac{m(Q\,q\;\;)^2}{\Lambda_*\, r_p}\right]^{\frac{1}{3}} \qquad \text{with} \quad q^{\frac{2}{3}} = \left(\frac{2}{\pi m_e}\right)^{\frac{1}{3}} n^{\frac{2}{3}} T$$

$$= f_p\, Q^{\frac{2}{3}} \left(\frac{m_{D_2}}{m_e}\right)^{\frac{1}{3}} \left(\frac{2}{\pi}\right)^{\frac{1}{3}} n^{\frac{2}{3}} T\, \Lambda_*^{-\frac{1}{3}}\, r_p^{-\frac{1}{3}}$$

the plot of $E_{bc0}^{1.7} \Lambda_*$ over E_{bc0} in the PRL paper suggests we can assume $\Lambda_* = \alpha (2T)^{-1.7}$ with $\alpha \approx (0.95 \text{ to } 1.05) \cdot 10^{-16}\, \text{eV}^{1.7} \text{m}^2$

(see plot)



$$\Rightarrow p_{*,our} = f_p\, Q^{\frac{2}{3}} \left(\frac{m_{D_2}}{m_e}\right)^{\frac{1}{3}} \left(\frac{2}{\pi}\right)^{\frac{1}{3}} \alpha^{-\frac{1}{3}} \left((2T)^{-1.7}\right)^{-\frac{1}{3}} \cdot T\, r_p^{-\frac{1}{3}}\, n^{\frac{2}{3}}$$

$$= \underbrace{f_p}_{\approx 0.15} \underbrace{Q^{\frac{2}{3}}}_{=0.15^{2/3}} \underbrace{\left(\frac{m_{D_2}}{m_e}\right)^{\frac{1}{3}}}_{\approx(7.292\cdot 10^3)^{\frac{1}{3}}} \left(\frac{2}{\pi}\right)^{\frac{1}{3}} \underbrace{\alpha^{-\frac{1}{3}}}_{\approx(10^{-16}\,\text{eV}^{1.7}\text{m}^2)^{-\frac{1}{3}}} 2^{1.7/3}\, r_p^{-\frac{1}{3}}\, n^{\frac{2}{3}}\, T^{1.57}$$

$$\Rightarrow p_{*,our} \approx 5.56 \cdot 10^5 \underset{\uparrow\,\text{metre}}{m^{-\frac{2}{3}}} r_p^{-\frac{1}{3}}\, n^{\frac{2}{3}} \left(\frac{T^{1.57}}{\text{eV}^{0.57}}\right)$$

$$p_{*,39} = 3.95 \cdot 10^{-8} \left(r_p\right)^{-\frac{1}{3}} \left(n\right)^{2/3} \left(T\right)^{1.54} \ldots$$

$$p_{*,39} = 3.95 \cdot 10^{-8} \left(\frac{r_p}{cm}\right)^{-\frac{1}{3}} \left(\frac{n}{cm^{-3}}\right)^{2/3} \left(\frac{T}{eV}\right)^{1.54} (dyn/cm^2)$$

$$= 3.95 \cdot 10^{-8} \; \frac{cm^{\frac{1}{3}} \cancel{cm^2}}{\cancel{cm^2}} \; \underbrace{\frac{dyn}{eV}}_{\approx 6.242 \cdot 10^{13} \, m^{-1}} \; r_p^{-\frac{1}{3}} \, n^{2/3} \left(\frac{T}{eV^{0.54}}\right)^{1.54}$$

$$p_{*,39} \approx 5.312 \cdot 10^{5} \; m^{-\frac{2}{3}} \; r_p^{-\frac{1}{3}} \, n^{2/3} \left(\frac{T}{eV^{0.54}}\right)^{1.54}$$

$\Rightarrow$ Parks' eq. 39 matches our expression reasonably well

Parks used $E_{bc0}^{1.62} \cdot \Lambda_* \approx$ const.

in my opinion $E_{bc0}^{1.7} \cdot \Lambda_* \approx$ const fits better but it is very similar