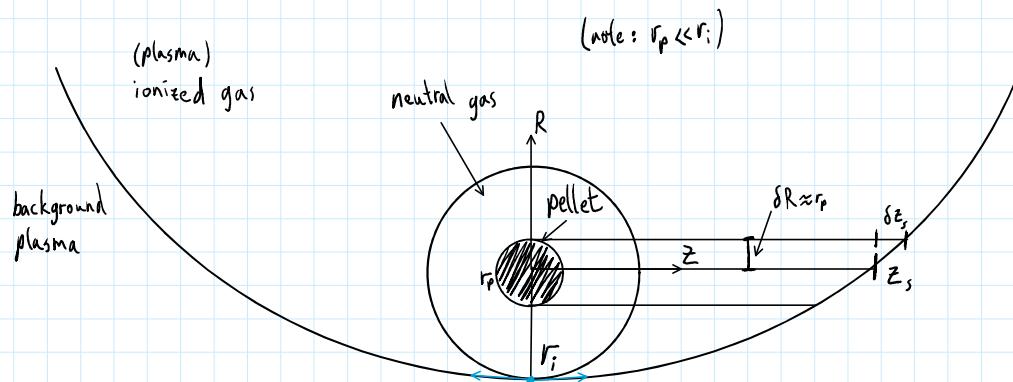


To be able to use the perturbation model of the neutral ablation cloud, it is important to model the heat flux which reaches the neutral gas together with the average energy of the incoming electrons.

Specifically the previously defined asymmetry factors  $q_{rel}, E_{rel}$  have to be determined.

The first step is to determine the shape of the ionized part of the ablation cloud and the shielding length which the electrons have to pass.

### Shielding geometry



Assume the neutral gas cloud has a spherical boundary and follows the pellet with the pellet velocity  $v_p$ .

At the ionization radius  $r_i$  there is a sharp boundary to an ionized gas of ionization degree  $\alpha$ .

The expansion velocity is at the speed of sound  $c_{so}$  but is stopped perpendicular to the magnetic field line direction  $z$ .

In Oskar's drift paper it is shown that the charged particles have a constant acceleration  $v_0$  along the major radius direction  $R$ .

To determine the plasma cloud boundary, calculate the trajectory of particles starting at  $(R = -r_i, z = 0)$

$$R = -r_i + v_p t + \frac{1}{2} \alpha c_{so}^2 t^2 \quad (1) \quad (\text{initial velocity } v_0 = v_p)$$

$\uparrow \quad \uparrow$   
follows pellet      drift

$$z = c_{so} t \quad (2)$$

$$(1) \Leftrightarrow t^2 + t \frac{v_p}{v_0} - \frac{2}{v_0} (R + r_i) = 0$$

$$\Rightarrow t = -\frac{v_p}{v_0} \pm \sqrt{\left(\frac{v_p}{v_0}\right)^2 + \frac{2}{v_0} (R + r_i)} \quad (\text{only } + \text{ is important})$$

insert into (2)  $\Rightarrow$

$$z = c_{s0} \left( -\frac{v_p}{v_0} + \sqrt{\left(\frac{v_p}{v_0}\right)^2 + \frac{2}{v_0} (R + r_i)} \right) \quad (3)$$

$$\Rightarrow \text{central shielding length} \quad z_s = z(R=0) - r_i = c_{s0} \left( -\frac{v_p}{v_0} + \sqrt{\left(\frac{v_p}{v_0}\right)^2 + \frac{2}{v_0} r_i} \right) - r_i \quad (4)$$

In the limit of a negligible background pressure (ideal gas  $n k_B T = p$ )

and  $\left(\frac{v_p}{c_{s0}}\right)^2 \ll \frac{R+r_i}{R_m}$  by inserting  $v_0$

$$\Rightarrow z \approx \sqrt{\frac{\gamma_e (Z + \gamma_i)}{1 + (Z + \gamma_i)}} \sqrt{R_m (R + r_i)} \quad \text{which is similar to the expression used by Serichenkov}$$

To determine the heatflux asymmetry calculate how much the shielding length varies over a small distance  $\delta R$  along the major radius

$$\delta z_s \approx \frac{dz}{dR} \Big|_{R=0} \delta R \quad (\text{assuming linearity over } \delta R)$$

$$\text{with } \frac{dz}{dR} = c_{s0} \frac{1}{v_0} \frac{1}{2} \left[ \left(\frac{v_p}{v_0}\right)^2 + \frac{2}{v_0} (R + r_i) \right]^{-\frac{1}{2}}$$

$$\Rightarrow \delta z_s \approx \frac{c_{s0}}{v_0} \left[ \left(\frac{v_p}{v_0}\right)^2 + \frac{2}{v_0} r_i \right]^{-\frac{1}{2}} \delta R$$

$$\Leftrightarrow \delta z_s \approx \frac{c_{s0} \delta R}{\sqrt{v_p^2 + 2 v_0 r_i}} \quad (5)$$

### Resulting heat flux asymmetry

Due to the complex dynamics of how the incoming electrons traverse the plasma cloud, a full description of the shielding mechanisms is outside the scope of this project.

As a first estimation the incoming heatflux is modelled by assuming that electrons with a mean free path  $\lambda_{mfp}$  shorter than the distance traveled through the plasma cloud  $d$  are fully shielded and the other electrons reach the neutral gas cloud unaffected.

The distance travelled is not exactly the same as the shielding length  $z$  because the electrons gyrate around the magnetic field lines.

because the electrons gyrate around the magnetic field lines.

This helical motion means the electrons vectorial velocity is neither parallel nor perpendicular to  $\vec{B}$ .

The angle between  $\vec{B}$  and  $\vec{v}_e$  is called the pitch angle  $\theta_p$

$$\vec{v}_e = \vec{v}_{\parallel} + \vec{v}_{\perp} \quad \Rightarrow \quad \xi := \cos \theta_p = \frac{v_{\parallel}}{v_e} \quad (6)$$

This pitch angle stays constant during the gyration.

Therefore the total distance travelled becomes clear when imagining to unroll the gyration

$$d \text{ (unrolled gyration)} \quad \Rightarrow \quad z = d \cos \theta_p \Leftrightarrow d = \frac{z}{\xi} \quad (7)$$

Therefore electrons reach the neutral cloud if

$$\frac{z}{\xi} \leq \lambda_{mfp} \quad (8)$$

The mean free path for the slowing down part of electron-electron collisions is  
(from Per's book)

$$\Rightarrow \lambda_{mfp} = \frac{v_e}{v_{ci}} = \frac{4\pi e_0^2 m_e^2 v_e^4}{n_e e^4 \ln \Lambda} \quad (9) \quad \lambda_{mfp} = \left( \frac{v_e}{v_{th}} \right)^4 \lambda_T \quad (\lambda_T \hat{=} \text{mean free path at thermal speed } v_{th})$$

$$v_{th} = \sqrt{2 \frac{k_B T}{m_e}}$$

as mentioned electrons contribute to the heat flux into the neutral cloud if

$\frac{z}{\xi} \leq \lambda_{mfp}$  or expressed in terms of a critical velocity

$$\Rightarrow v_e \geq v_c(\xi, z)$$

$$\Rightarrow v_c(\xi, z) = \left[ \frac{z}{\xi \lambda_T} \right]^{1/4} v_{th} \quad (10)$$

Then the total heat flux along one specific field line reaching the neutral cloud is  
(from one side)

$$q_p = \iiint q(\vec{v}_e) f(\vec{v}_e) d^3 v_e \quad \text{over allowed velocities} \quad (\text{normalized to 1})$$

$$\text{with the Maxwellian distribution } f(\vec{v}) = \left[ \frac{m_e}{2\pi k_B T} \right]^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) = \left( \frac{1}{\sqrt{2} v_{th}} \right)^3 \exp\left(-\left(\frac{v}{v_{th}}\right)^2\right)$$

$$\text{and the heat flux } q(\vec{v}) = \xi v_e \frac{m v_e^2}{2} n_e \quad (\text{remember } v_{\parallel} = \xi v_e)$$

integrate in "spherical coordinates"  $d^3 v = v_e^2 \sin \theta_0 dv_e d\theta_0 d\varphi$

$\varphi \hat{=} \text{gyration angle}$   
 $\theta_0 \hat{=} \text{pitch angle}$

integrals in "spherical coordinates"  $d^3 v_e = v_e^2 \sin\theta_p d\theta_p d\varphi$

$\varphi \hat{=} \text{gyration angle}$   
 $\theta_p \hat{=} \text{pitch angle}$

substitute  $\xi = \cos\theta_p \Leftrightarrow \frac{d\xi}{d\theta_p} = -\sin\theta_p \Leftrightarrow d\theta_p = -\frac{1}{\sin\theta_p} d\xi \Rightarrow d^3 v_e = -v_e^2 dv_e d\xi d\varphi$

the interval in  $\theta_p$  would be  $\theta_p \in [0, \frac{\pi}{2}] \Leftrightarrow \xi \in [1, 0] \rightarrow$  switching the bounds gets rid of the minus sign

$$\Rightarrow q_p = \int_0^{2\pi} d\varphi \int_0^1 d\xi \int_{v_c}^{\infty} dv_e v_e^2 \left( \xi v_e \frac{m v_e^2}{2} n_e \right) \left( \frac{1}{\sqrt{v_e} v_{th}} \right)^3 \exp\left(-\left(\frac{v_e}{v_{th}}\right)^2\right) \quad (11)$$

no  $\varphi$ -dependence  $\Rightarrow \int_0^{2\pi} d\varphi = 2\pi$

$$q_p = 2\pi \frac{m_e}{2} n_e \pi^{-\frac{3}{2}} v_{th}^2 \int_0^1 d\xi \xi \cdot \int_{v_c}^{\infty} dv_e \left( \frac{v_e}{v_{th}} \right)^5 \exp\left(-\left(\frac{v_e}{v_{th}}\right)^2\right)$$

substitute  $u = \frac{v_e}{v_{th}} \Rightarrow \frac{du}{dv_e} = \frac{1}{v_{th}} \Leftrightarrow dv_e = v_{th} du \quad u(v_e=\infty)=\infty, u(v_e=v_c)=\left[\frac{\xi}{\xi \lambda_T}\right]^{\frac{1}{4}} = (\xi \times)^{-\frac{1}{4}}$  with  $x := \frac{\lambda_T}{\xi}$

$$q_p = \pi m_e n_e \left( \frac{v_{th}}{\sqrt{\pi}} \right)^3 \int_0^1 d\xi \xi \int_{u(v_c)}^{\infty} du u^5 \exp(-u^2) \quad \text{use Wolfram Alpha}$$

$$= \pi m_e n_e \left( \frac{v_{th}}{\sqrt{\pi}} \right)^3 \int_0^1 d\xi \xi \left[ -\frac{1}{2} e^{-u^2} (u^4 + 2u^2 + 2) \right]_{u(v_c)}^{\infty}$$

$$= \pi m_e n_e \left( \frac{v_{th}}{\sqrt{\pi}} \right)^3 \int_0^1 d\xi \xi \left[ \frac{1}{2} e^{-(\xi x)^{\frac{1}{2}}} \left( (\xi x)^{-1} + 2(\xi x)^{\frac{1}{2}} + 2 \right) \right] \quad \text{use Wolfram Alpha}$$

$$= \pi m_e n_e \left( \frac{v_{th}}{\sqrt{\pi}} \right)^3 \frac{1}{2} \cdot \frac{1}{2x^2} \left[ \exp(-(\xi x)^{\frac{1}{2}}) (2(\xi x)^{3/2} + 2(\xi x)^2 + (\xi x) - \sqrt{\xi x}) - Ei\left(-(\xi x)^{\frac{1}{2}}\right) \right]_0^1$$

with  $Ei(z) := - \int_{-z}^{\infty} \frac{e^{-t}}{t} dt \quad \text{with } Ei(-\infty) = 0$

$$q_p = \pi m_e n_e \left( \frac{v_{th}}{\sqrt{\pi}} \right)^3 \frac{1}{4} \frac{1}{x^2} \left[ \exp(-x^{\frac{1}{2}}) (2x^{3/2} + 2x^2 + x - \sqrt{x}) - Ei(-x^{\frac{1}{2}}) \right] \quad (12^*)$$

$$\text{with } v_{th} = \sqrt{2 \frac{k_B T}{m_e}} \quad x = \frac{\lambda_T}{z} \Rightarrow \frac{dx}{dz} = -\frac{\lambda_T}{z^2} = -\frac{x}{z}$$

$$q_p = \sqrt{\frac{(k_B T)^3}{2\pi m_e}} n_e \frac{1}{x^2} \left[ e^{-\frac{1}{x^2}} (2x^{3/2} + 2x^2 + x - \sqrt{x}) - Ei\left(-\frac{1}{\sqrt{x}}\right) \right] \quad (12)$$

in Parks paper  $q_{bg} = 2 \sqrt{\frac{(k_B T)^3}{2\pi m_e}} n_e$

$$\frac{dq_p}{dz} = \frac{\partial q_p}{\partial x} \frac{dx}{dz} = -\frac{x}{z} \sqrt{\frac{(k_B T)^3}{2\pi m_e}} n_e \frac{2}{x^3} \left( Ei\left(-\frac{1}{\sqrt{x}}\right) + \exp\left(-\frac{1}{\sqrt{x}}\right) \sqrt{x} \right) \quad (13)$$

The particle flux reaching the neutral cloud along one field line is

$$\Phi_p = \iiint \phi(\vec{v}_e) f(\vec{v}_e) d^3 v_e \quad \text{with } \phi(\vec{v}_e) = \xi v_e n_e \quad (\text{remember } v_{||} = \xi v_e)$$

$$\Phi_p = \int_0^{2\pi} d\varphi \int_0^1 d\xi \int_{v_c}^{\infty} dv_e v_e^2 \xi v_e n_e \left( \frac{1}{\sqrt{v_e} v_{th}} \right)^3 \exp\left(-\left(\frac{v_e}{v_{th}}\right)^2\right)$$

$$\phi_p = \int_0^{2\pi} \int_0^1 \int_{v_e}^{\infty} \frac{1}{v_e} v_e^2 \int_{v_e}^{\infty} v_e n_e \left( \frac{1}{\sqrt{\pi} v_{th}} \right)^3 \exp\left(-\left(\frac{v_e}{v_{th}}\right)^2\right)$$

$$= 2\pi n_e \pi^{-\frac{3}{2}} \int_0^1 \int_{v_e}^{\infty} \int_{v_e}^{\infty} \left( \frac{v_e}{v_{th}} \right)^3 \exp\left(-\left(\frac{v_e}{v_{th}}\right)^2\right)$$

substitute  $u = \frac{v_e}{v_{th}} \Rightarrow \frac{du}{dv_e} = \frac{1}{v_{th}}$   $\Rightarrow dv_e = v_{th} du$

$$= \frac{2}{\sqrt{\pi}} n_e v_{th} \int_0^1 \int_{u(v_e)}^{\infty} du u^3 \exp(-u^2) \quad \text{use Wolfram Alpha}$$

$$= \frac{2}{\sqrt{\pi}} n_e v_{th} \int_0^1 \int_{u(v_e)}^{\infty} du \left[ -\frac{1}{2} \exp(-u^2) (u^2 + 1) \right]_{u(v_e)}^{\infty} \quad \text{with } u(v_e) = (\xi x)^{-\frac{1}{4}} \quad \text{with } x = \frac{\lambda_T}{z_s}$$

$$= \frac{1}{\sqrt{\pi}} n_e v_{th} \int_0^1 \int_{u(v_e)}^{\infty} du \exp(-(\xi x)^{\frac{1}{2}}) ((\xi x)^{\frac{1}{2}} + 1) \quad \text{use Wolfram Alpha}$$

$$= \frac{1}{\sqrt{\pi}} n_e v_{th} \frac{1}{4x^2} \left[ \exp(-(\xi x)^{\frac{1}{2}}) (2(\xi x)^{\frac{3}{2}} + 2(\xi x)^2 + \sqrt{\xi x} - (\xi x)) + Ei(-(\xi x)^{\frac{1}{2}}) \right]_0^1$$

$$\boxed{\phi_p = \frac{1}{\sqrt{\pi}} n_e v_{th} \frac{1}{4x^2} \left[ e^{-\frac{1}{\sqrt{x}}} (2x^{\frac{3}{2}} + 2x^2 + \sqrt{x} - x) + Ei\left(-\frac{1}{\sqrt{x}}\right) \right]} \quad (14)$$

Similar to Parks paper the effective energy is  $E_p = \frac{q_p}{\phi_p}$

$$(12^*): q_p = \frac{1}{\sqrt{\pi}} m_e n_e v_{th}^3 \frac{1}{4} \frac{1}{x^2} \left[ \exp(-x^{\frac{1}{2}}) (2x^{\frac{3}{2}} + 2x^2 + x - \sqrt{x}) - Ei(-x^{\frac{1}{2}}) \right]$$

$$\Rightarrow E_p = \frac{q_p}{\phi_p} = m_e v_{th}^2 \frac{e^{-\frac{1}{\sqrt{x}}} (2x^{\frac{3}{2}} + 2x^2 + x - \sqrt{x}) - Ei\left(-\frac{1}{\sqrt{x}}\right)}{e^{-\frac{1}{\sqrt{x}}} (2x^{\frac{3}{2}} + 2x^2 + x - \sqrt{x}) + Ei\left(-\frac{1}{\sqrt{x}}\right)} \quad (15)$$

in Parks paper  $E_{bg} = 2k_B T = m_e v_{th}$

$$\frac{dE_p}{dz} = \frac{dx}{dz} \frac{dE_p}{dx} = -\frac{1}{z^2} \frac{dE_p}{dx} \quad \leftarrow \text{very large expression in Wolfram Alpha}$$

Relation to the first order perturbation

Perturbation:  $q = q_0(r) + q_1(r, \theta)$

$$E = E_0(r) + E_1(r, \theta)$$

here we have  $q_p(z), E_p(z)$  where  $z$  is the shielding length at the specific field line

but  $z = z(R)$  and  $R = R(\theta) = -\delta R \cos \theta$  where  $\delta R$  is the radius we consider to be  $r_p$  (eq. (3))

Taylor around  $R=0$

$$q_p \approx q_p(R=0) + \left. \frac{dq_p}{dR} \right|_{R=0} \cdot R + O(R^2) \quad \text{higher orders would be different Legendre polynomial modes? (it would be } (\cos \theta)^n \text{ modes)}$$

$$\Rightarrow q_0(r \rightarrow \infty) = q_p(R=0) \quad q_1(r \rightarrow \infty, \theta) = \frac{dq_p}{dR} \Big|_{R=0} f \delta R \cos \theta + O(R^2)$$

$$\frac{dq_p}{dR} \Big|_{R=0} = \frac{dq_p}{dz} \Big|_{z=z_s} \frac{dz}{dR} \Big|_{R=0}$$

$$\text{with } \frac{dz}{dR} = \frac{c_{s0}}{\dot{v}_o} \left[ \left( \frac{v_p}{\dot{v}_o} \right)^2 + \frac{2}{\dot{v}_o} (R + r_i) \right]^{-\frac{1}{2}}$$

from the beginning

similarly

$$E_0(r \rightarrow \infty) = E_p(R=0) \quad E_1(r \rightarrow \infty, \theta) = \frac{dE_p}{dR} \Big|_{R=0} f \delta R \cos \theta + O(R^2)$$

$$\text{with the definition } q_{rel} = \frac{Q_1(\infty)}{q_{bg}} \begin{matrix} \swarrow \cos \theta \text{ mode} \\ \searrow \text{spherical mode} \end{matrix}$$

$$q_{rel} = -\frac{1}{q_p(z=z_s)} \frac{dq_p}{dx} \Big|_{x=x_s} \frac{dx}{dz} \Big|_{z=z_s} \frac{dz}{dR} \Big|_{R=0} \delta R$$

$$x = \frac{\lambda_T}{z} \quad \frac{dx}{dz} = -\frac{\lambda_T}{z^2} = -\frac{x}{z}$$

$$E_{rel} = -\frac{1}{E_p(z=z_s)} \frac{dE_p}{dx} \Big|_{x=x_s} \frac{dx}{dz} \Big|_{z=z_s} \frac{dz}{dR} \Big|_{R=0} \delta R$$

## Summary

$$\text{shielding length } Z(R) = c_{s0} \left( -\frac{v_p}{\dot{v}_o} + \sqrt{\left( \frac{v_p}{\dot{v}_o} \right)^2 + \frac{2}{\dot{v}_o} (R + r_i)} \right) \quad z_s = Z(R=0)$$

with  $c_{s0} \hat{=} \text{sound speed}$   $v_p \hat{=} \text{pellet speed}$   $r_i \hat{=} \text{ionization radius}$

Drift acceleration  $\dot{v}_o$  given in Oskars paper

$$\text{defining the mean free path at thermal velocity } \lambda_T = \frac{4 \pi \epsilon_0^2 m_e^2 v_{th}^4}{n_e e^4 \ln \Lambda}$$

$$\text{with Coulomb logarithm } \ln \Lambda, \quad v_{th} = \sqrt{2 \frac{k_B T}{m_e}} \quad \text{where } T = T_{e,bg}$$

$$\text{and } x = \frac{\lambda_T}{z}, \quad x_s = x(z=z_s)$$

$$\text{Parks paper: } q_{bg} = 2 \sqrt{\frac{(k_B T)^3}{2 \pi m_e}} n_e, \quad E_{bg} = 2 k_B T$$

while solving the integrals leads to

$$q_0(\infty) = q_{bg} f_q(x_s) \quad \text{with } f_q(x) = 2 \frac{1}{x^2} \left[ e^{-\frac{1}{\sqrt{x}}} (2x^{3/2} + 2x^2 + x - \sqrt{x}) - E_i(-\frac{1}{\sqrt{x}}) \right]$$

$$E_0(\infty) = E_{bg} f_E(x_s) \quad \text{with } f_E(x) = \frac{e^{-\frac{1}{\sqrt{x}}} (2x^{3/2} + 2x^2 + x - \sqrt{x}) - E_i(-\frac{1}{\sqrt{x}})}{e^{-\frac{1}{\sqrt{x}}} (2x^{3/2} + 2x^2 - x + \sqrt{x}) + E_i(-\frac{1}{\sqrt{x}})}$$

$$q_{rel} = \frac{f'(x_s)}{f(x_s)} \frac{x_s}{z_s} \frac{dz}{dR} \Big|_{R=0} \delta R$$

where  $\delta R = r_p$  (assumption)

$$E_{rel} = \frac{f'_E(x_s)}{f_E(x_s)} \frac{x_s}{z_s} \frac{dz}{dR} \Big|_{R=0} \delta R$$