Simple scaling law for the PRL paper

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$$F = \frac{4\pi r_p^2}{3} p_1(r_1) \qquad \dot{V} = \frac{F}{\frac{4}{3}\pi r_p^3 \rho_p}$$

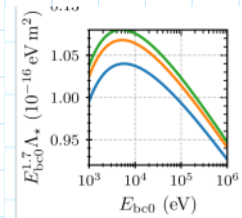
$$p_1(r_p) = p_* \, a\left(E_{rd} - b\, q_{rd}\right)$$

$$q_{bc0}^{\frac{2}{3}} = \left(\frac{2}{\pi m_e}\right)^{\frac{1}{3}} n_{bg}^{\frac{2}{3}} T_{bg} \qquad E_{bc0} = 2\, T_{bg}$$

$$p_* = f_p \left[\frac{m\,(Q\, q_{bc0})^2}{\Lambda_* r_p}\right]^{\frac{1}{3}}$$

since we plotted $E_{bc0}^{1.7}\Lambda_*$ and it is nearly constant

Let's define $\alpha = \dfrac{E_{bc0}^{1.7}\Lambda_*}{10^{-16}\,eV^{1.7}\,m^2} \approx 1.05 \longrightarrow$



$$\Leftrightarrow \Lambda_* = \alpha\, E_{bc0}^{-1.7}\, 10^{-16}\, eV^{1.7}\, m^2$$

$$q_{rd} = \frac{3}{2}\frac{\delta r}{T_{bg}}\partial_z T_{bg} + \frac{\delta r}{n_{bg}}\partial_z n_{bg}$$

$$E_{rd} = \frac{\delta r}{T_{bg}}\partial_z T_{bg}$$

let's label $\delta r = R \cdot r_p$

$R \approx 1$ to 2

Combine everything

$$p_* = f_p \left[\frac{m\,(Q\, q_{bc0})^2}{\Lambda_* r_p}\right]^{\frac{1}{3}}$$

with $\Lambda_* = \alpha\, E_{bc0}^{-1.7}\, 10^{-16}\, eV^{1.7}\, m^2$

$$q_{bc0}^{\frac{2}{3}} = \left(\frac{2}{\pi m_e}\right)^{\frac{1}{3}} n_{bg}^{\frac{2}{3}} T_{bg} \qquad E_{bc0} = 2\, T_{bg}$$

$u \hat{=}$ atomic mass unit

$m \hat{=}$ metre , M = mass of gas particles

$$p_* = f_p \left[\frac{M\, Q^2}{r_p\, \alpha\, (2T_{bg})^{-1.7}\, 10^{-16}\, eV^{1.7}\, m^2}\right]^{\frac{1}{3}} \left(\frac{2}{\pi m_e}\right)^{\frac{1}{3}} n_{bg}^{\frac{2}{3}} T_{bg}$$

$$= \frac{f_p Q^{2/3}}{\alpha^{1/3}} \underbrace{\left(2^{1.7}\cdot 10^{16}\cdot\frac{2}{\pi}\right)^{\frac{1}{3}}}_{\approx} \left(\frac{M}{m_e}\right)^{\frac{1}{3}} \left(\frac{T_{bg}}{eV}\right)^{\frac{1.7}{3}} \left(\frac{r_p}{m}\right)^{-\frac{1}{3}} m^{-1}\, n_{bg}^{\frac{2}{3}}\, T_{bg}$$

$$= \frac{f_p Q^{2/3}}{\alpha^{1/3}}\, 2.74501\cdot 10^5 \left(\frac{M}{u}\right)^{\frac{1}{3}} \left(\frac{u}{m_e}\right)^{1/3} \left(\frac{r_p}{m}\right)^{-\frac{1}{3}} \left(\frac{T_{bg}}{eV}\right)^{1.5\bar{6}} \left(\frac{n_{bg}}{m^{-3}}\right)^{2/3} \underbrace{m^{-3}\, eV}$$

$$= \frac{f_p\, Q^{2/3}}{\alpha^{1/3}} \;\dot{\approx}\; 2.74501\cdot 10^{5} \left(\frac{M}{u}\right)^{\frac{1}{3}} \underbrace{\left(\frac{u}{m_e}\right)^{1/3}}_{\approx 12.215747} \left(\frac{r_p}{m}\right)^{-\frac{1}{3}} \left(\frac{T_{bg}}{eV}\right)^{1.56} \left(\frac{n_{bg}}{m^{-3}}\right)^{2/3} \underbrace{m^{-3}\, eV}_{\approx 1.60218\cdot 10^{-19}\, Pa}$$

$$\boxed{P_* = 5.37249\cdot 10^{-13} \; \frac{f_p\, Q^{2/3}}{\alpha^{1/3}} \left(\frac{M}{u}\right)^{\frac{1}{3}} \left(\frac{r_p}{m}\right)^{-\frac{1}{3}} \left(\frac{T_{bg}}{eV}\right)^{1.5\bar{6}} \left(\frac{n_{bg}}{m^{-3}}\right)^{2/3} Pa}$$

! important note: this only holds when assuming Parks q_{bc0} and E_{bc0}, i.e. no plasmoid shielding!

$$p_1(r_p) = p_*\, a\left(E_{rel} - b\, q_{rel}\right) \;,\quad F = \frac{4\pi r_p^2}{3}\, p_1(r_p) \;,\quad \dot{V} = \frac{F}{\frac{4}{3}\pi r_p^3 \rho_p}$$

$$\dot{V} = \frac{p_1(r_p)}{r_p\, \rho_p} = \frac{1}{r_p \rho_p}\, P_*\, a\left(E_{rel} - b\, q_{rel}\right)$$

with

$$q_{rel} = \frac{3}{2}\frac{\delta r}{T_{bg}}\, \partial_z T_{bg} + \frac{\delta r}{n_{bg}}\, \partial_z n_{bg}$$

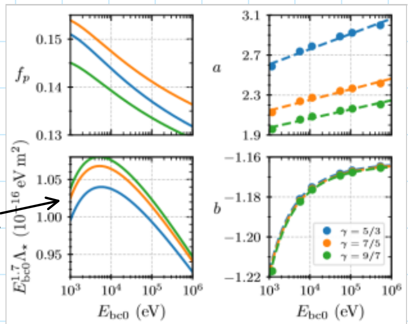
$$E_{rel} = \frac{\delta r}{T_{bg}}\, \partial_z T_{bg}$$

note: the $\delta r \approx r_p$ dependency cancels the $\frac{1}{r_p}$ in $\dot{V}$

defining $\delta r = R\, r_p$

$$\boxed{\begin{aligned} \dot{V} &= \frac{aR}{\rho_p}\, P_* \left[\frac{\partial_z T_{bg}}{T_{bg}}\left(1 - \frac{3}{2}b\right) - b\,\frac{\partial_z n_{bg}}{n_{bg}}\right] \\ \text{with } P_* &= 5.37249\cdot 10^{-13}\; \frac{f_p\, Q^{2/3}}{\alpha^{1/3}} \left(\frac{M}{u}\right)^{\frac{1}{3}} \left(\frac{r_p}{m}\right)^{-\frac{1}{3}} \left(\frac{T_{bg}}{eV}\right)^{1.5\bar{6}} \left(\frac{n_{bg}}{m^{-3}}\right)^{2/3} Pa \end{aligned}}$$

α



$Q \approx 0.65$ $\quad$ $1 \lesssim R < 2$

$M/u = 4$ for D_2