Blatt 4 Abgabe

Freitag, 21. Mai 2021 13:33

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9)

a) $$PDF(\Delta\Psi) = N \cdot \exp(-|\Delta\Psi| \cdot k) \qquad \Delta\Psi \in [-\pi, \pi]$$

Normierung:

$$\int_{-\pi}^{\pi} PDF(\Delta\Psi)\, d(\Delta\Psi) \stackrel{!}{=} 1$$

$$= \int_{-\pi}^{\pi} N \cdot \exp(-|\Delta\Psi| \cdot k)\, d(\Delta\Psi)$$

$$= \int_{-\pi}^{0} N \cdot \exp(+\Delta\Psi \cdot k)\, d(\Delta\Psi) + \int_{0}^{\pi} N \cdot \exp(-\Delta\Psi \cdot k)\, d(\Delta\Psi)$$

$$= N\left[\frac{1}{k}\exp(\Delta\Psi \cdot k)\right]_{-\pi}^{0} + N\left[-\frac{1}{k}\exp(-\Delta\Psi \cdot k)\right]_{0}^{\pi}$$

$$= \frac{N}{k}\left[1 - \exp(-\pi k)\right] - \frac{N}{k}\left[\exp(-\pi k) - 1\right]$$

$$= \frac{2N}{k}\left[1 - \exp(-\pi k)\right] \stackrel{!}{=} 1$$

$$\Leftrightarrow N = \frac{k}{2}\left[1 - \exp(-\pi k)\right]^{-1}$$

b) $$CDF(\Delta\Psi) = \int_{-\pi}^{\Delta\Psi} PDF(x)\, dx$$

$$= N \int_{-\pi}^{\Delta\Psi} \exp(-|x| \cdot k)\, dx$$

Fallunterscheidung:

$\Delta\Psi \leq 0$

$$CDF(\Delta\Psi) = N \int^{\Delta\Psi} \exp(x \cdot k)\, dx$$

$$CDF(\Delta\Psi) = N \int_{-\pi}^{\Delta\Psi} \exp(x \cdot k)\, dx$$

$$= N \left[\frac{1}{k} \exp(x \cdot k) \right]_{-\pi}^{\Delta\Psi}$$

$$= \frac{N}{k} \left[\exp(\Delta\Psi \cdot k) - \exp(-\pi \cdot k) \right]$$

$\Delta\Psi > 0$

$$CDF(\Delta\Psi) = N \int_{-\pi}^{0} \exp(x \cdot k)\, dx + N \int_{0}^{\Delta\Psi} \exp(-x k)\, dx$$

$$= N \left[\frac{1}{k} \exp(x \cdot k) \right]_{-\pi}^{0} + N \left[-\frac{1}{k} \exp(-x \cdot k) \right]_{0}^{\Delta\Psi}$$

$$= \frac{N}{k} \left[1 - \exp(-\pi k) \right] - \frac{N}{k} \left[\exp(-\Delta\Psi \cdot k) - 1 \right]$$

$$= \frac{N}{k} \left[2 - \exp(-\pi k) - \exp(-\Delta\Psi k) \right]$$

$$\Rightarrow CDF(\Delta\Psi) = \begin{cases} \frac{N}{k} \left(\exp(+\Delta\Psi \cdot k) - \exp(-\pi k) \right) & \text{für } \Delta\Psi \leq 0 \\ \frac{N}{k} \left(2 - \exp(-\pi k) - \exp(-\Delta\Psi k) \right) & \text{für } \Delta\Psi > 0 \end{cases}$$

$$\text{mit } N = \frac{k}{2} \left[1 - \exp(-\pi k) \right]^{-1}$$

c) CDF invertieren u sind gleichverteilte Zahlen zwischen 0 und 1

$$PPF(u) = CDF^{-1}(u)$$

$\Delta\Psi \leq 0$

$$CDF(\Delta\Psi) = \frac{N}{k} \left(\exp(\Delta\Psi k) - \exp(-\pi k) \right) = u$$

$$\Leftrightarrow \exp(\Delta\Psi k) = \frac{k}{N} u + \exp(-\pi k)$$

$$\Leftrightarrow \Delta\Psi = \frac{1}{k} \ln\left(\frac{k}{N} u + \exp(-\pi k) \right)$$

$$\Delta\Psi > 0$$

$$\mathrm{CDF}(\Delta\Psi) = \frac{N}{k}\left(2 - \exp(-\pi k) - \exp(-\Delta\Psi\, k)\right) = u$$

$$\Leftrightarrow\ -\exp(-\Delta\Psi k) = \frac{k}{N}u - 2 + \exp(-\pi k)$$

$$\Leftrightarrow\ -\Delta\Psi k = \ln\left(2 - \frac{k}{N}u - \exp(-\pi k)\right)$$

$$\Delta\Psi = -\frac{1}{k}\ln\left(2 - \frac{k}{N}u - \exp(-\pi k)\right)$$

welche Bereiche gelten für u ?

Grenze bei $\Delta\Psi = 0$

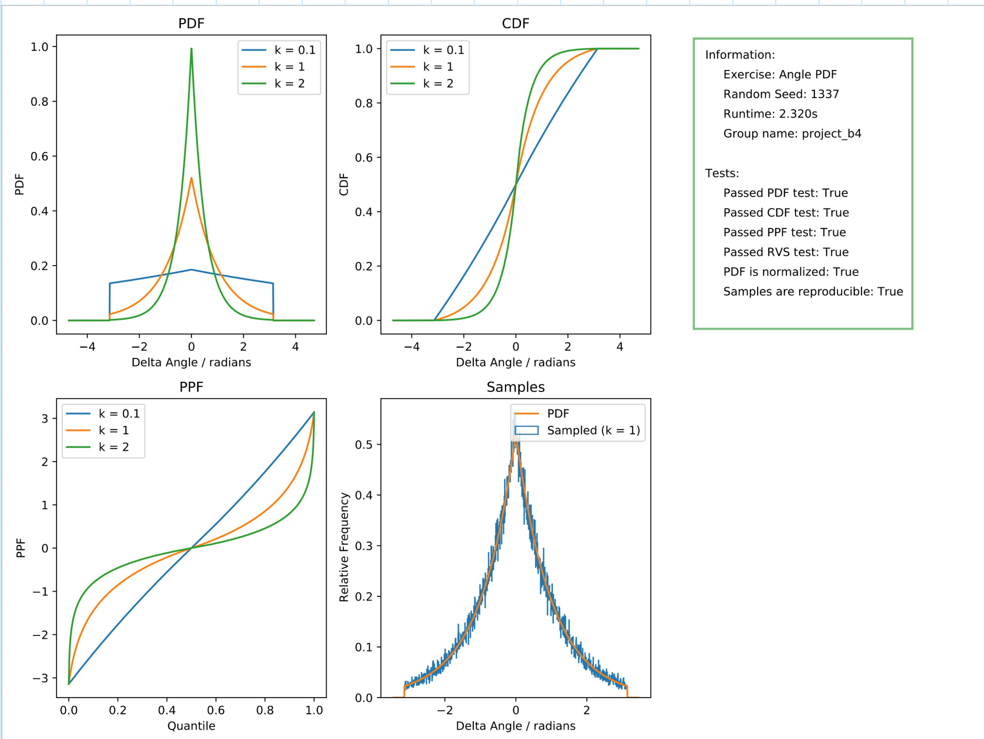
$$0 = \frac{1}{k}\ln\left(\frac{k}{N}u_0 + \exp(-\pi k)\right)$$

$$\Leftrightarrow\ 1 = \frac{k}{N}u_0 + \exp(-\pi k)$$

$$\Leftrightarrow\ u_0 = \frac{N}{k}\left(1 - \exp(-\pi k)\right)$$

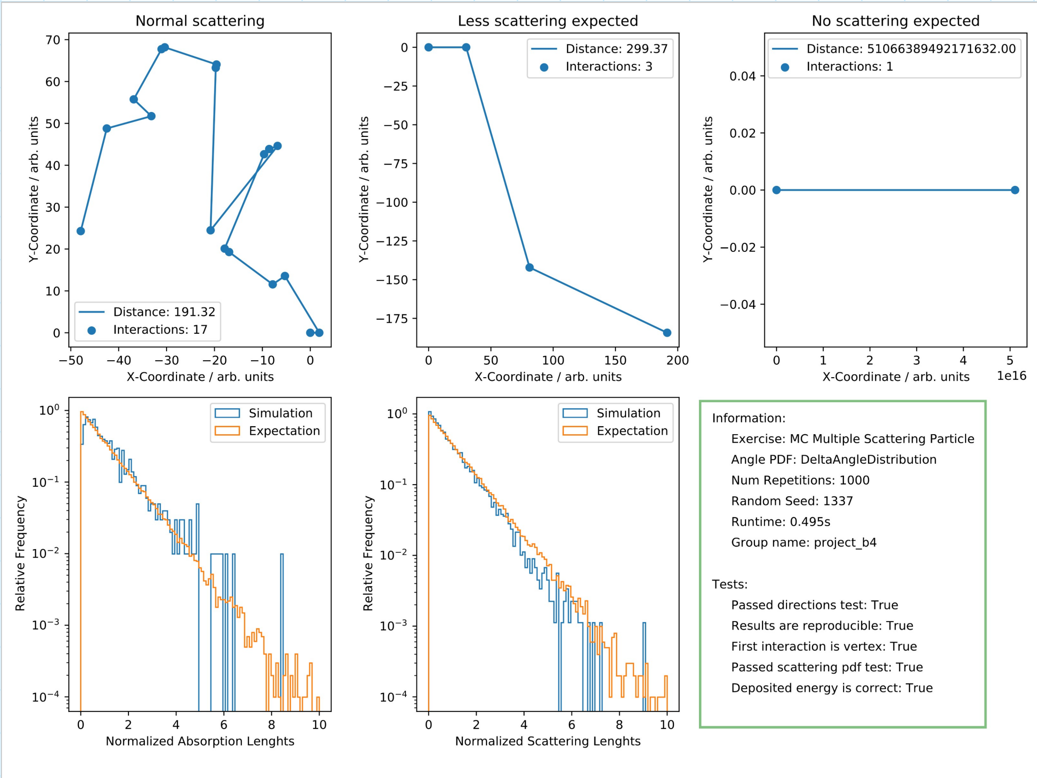
$$\Rightarrow\ \mathrm{PPF}(u) = \begin{cases} \frac{1}{k}\ln\left(\frac{k}{N}u + \exp(-\pi k)\right) & \text{für } 0 \le u \le u_0 \\ -\frac{1}{k}\ln\left(2 - \frac{k}{N}u - \exp(-\pi k)\right) & \text{für } u_0 < u \le 1 \end{cases}$$

exercise_a...



10)

exercise_...


Normal scattering
Distance: 191.32
Interactions: 17
X-Coordinate / arb. units
Y-Coordinate / arb. units
Less scattering expected
Distance: 299.37
Interactions: 3
No scattering expected
Distance: 51066389492171632.00
Interactions: 1
1e16
Simulation
Expectation
Relative Frequency
Normalized Absorption Lenghts
Normalized Scattering Lenghts
Information:
Exercise: MC Multiple Scattering Particle
Angle PDF: DeltaAngleDistribution
Num Repetitions: 1000
Random Seed: 1337
Runtime: 0.495s
Group name: project_b4
Tests:
Passed directions test: True
Results are reproducible: True
First interaction is vertex: True
Passed scattering pdf test: True
Deposited energy is correct: True