Blatt 6

Freitag, 4. Juni 2021 13:15

Aufgabe 12:

Population 0:

(1;1), (2;1), (1.5;2), (2;2), (2;3), (3,3)

Population 1:

(1.5;1), (2.5;1), (3.5;1), (2.5,2), (3.5;2), (4.5;2)

a) Mittelwerte:

$$\vec{\mu}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} \vec{x}_{i;j}$$

$$N_0 = N_1 = 6$$

$$\vec{\mu}_0 = \frac{1}{6}\begin{pmatrix} 11.5 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{11.5}{6} \\ 2 \end{pmatrix}$$

$$\vec{\mu}_1 = \frac{1}{6}\begin{pmatrix} 18 \\ 9 \end{pmatrix} = \begin{pmatrix} 3 \\ \frac{3}{2} \end{pmatrix}$$

Streumatrizen:

$$S_j = \sum_{i=1}^{N_j} (\vec{x}_{j;i} - \vec{\mu}_j)(\vec{x}_{j;i} - \vec{\mu}_j)^T$$

$$S_0 = \begin{pmatrix} -\frac{11}{12} \\ -1 \end{pmatrix}\begin{pmatrix} -\frac{11}{12} & -1 \end{pmatrix} + \begin{pmatrix} \frac{1}{12} \\ -1 \end{pmatrix}\begin{pmatrix} \frac{1}{12} & -1 \end{pmatrix} + \begin{pmatrix} -\frac{5}{12} \\ 0 \end{pmatrix}\begin{pmatrix} -\frac{5}{12} & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{1}{12} \\ 0 \end{pmatrix}\begin{pmatrix} \frac{1}{12} & 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{12} \\ 1 \end{pmatrix}\begin{pmatrix} \frac{1}{12} & 1 \end{pmatrix} + \begin{pmatrix} \frac{13}{12} \\ 1 \end{pmatrix}\begin{pmatrix} \frac{13}{12} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{121}{144} & \frac{11}{12} \\ \frac{11}{12} & 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{144} & -\frac{1}{12} \\ -\frac{1}{12} & 1 \end{pmatrix} + \begin{pmatrix} \frac{25}{144} & 0 \\ 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{1}{144} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{144} & \frac{1}{12} \\ \frac{1}{12} & 1 \end{pmatrix} + \begin{pmatrix} \frac{169}{144} & \frac{13}{12} \\ \frac{13}{12} & 1 \end{pmatrix}$$

$$S_0 = \begin{pmatrix} \frac{53}{24} & 2 \\ 2 & 4 \end{pmatrix}$$

$$S_1 = \begin{pmatrix} -\frac{3}{2} \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$+ \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{9}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix} + \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} + \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} + \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} + \begin{pmatrix} \frac{9}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

$$S_1 = \begin{pmatrix} \frac{11}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

$$S_W = S_0 + S_1 = \begin{pmatrix} \frac{53}{24} & 2 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} \frac{11}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{185}{24} & \frac{7}{2} \\ \frac{7}{2} & \frac{11}{2} \end{pmatrix}$$

$$S_B = (\vec{\mu}_0 - \vec{\mu}_1)(\vec{\mu}_0 - \vec{\mu}_1)^T$$

$$\vec{\mu}_0 = \begin{pmatrix} \frac{11.5}{6} \\ 2 \end{pmatrix} \qquad \vec{\mu}_1 = \begin{pmatrix} 3 \\ \frac{3}{2} \end{pmatrix}$$

$$S_B = \begin{pmatrix} -\frac{13}{12} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{13}{12} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{169}{144} & -\frac{13}{24} \\ -\frac{13}{24} & \frac{1}{4} \end{pmatrix}$$

b) Wie lautet $\vec{\lambda}$?

$$\vec{\lambda}^* = S_W^{-1}(\vec{\mu}_0 - \vec{\mu}_1)$$

Inverse von 2×2 Matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$S_W = \begin{pmatrix} \frac{185}{24} & \frac{7}{2} \\ \frac{7}{2} & \frac{11}{2} \end{pmatrix} \qquad \vec{\mu}_0 = \begin{pmatrix} \frac{11.5}{6} \\ 2 \end{pmatrix} \qquad \vec{\mu}_1 = \begin{pmatrix} 3 \\ \frac{3}{2} \end{pmatrix}$$

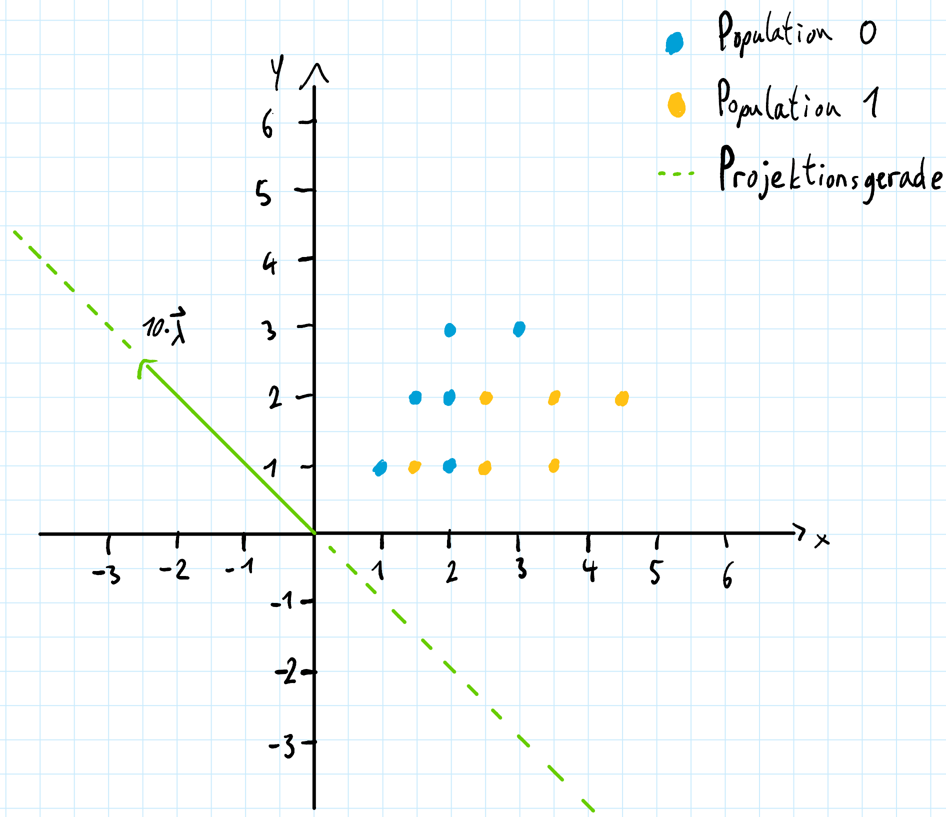
$$S_W^{-1} = \frac{1}{\frac{185}{24} \cdot \frac{11}{2} - \frac{7}{2} \cdot \frac{7}{2}} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix}$$

$$= \frac{48}{1447} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix}$$

$$\vec{\lambda} = \frac{48}{1447} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix} \begin{pmatrix} -\frac{13}{12} \\ \frac{1}{2} \end{pmatrix}$$

$$= \frac{48}{1447} \begin{pmatrix} -\frac{185}{24} \\ \frac{367}{48} \end{pmatrix} = \frac{1}{1447} \begin{pmatrix} -370 \\ 367 \end{pmatrix} \approx \begin{pmatrix} -0.2557 \\ +0.2536 \end{pmatrix}$$

c)



d) Projektion: $x'_{j,i} = \vec{\lambda} \cdot \vec{x}_{j,i}$

$$\vec{\lambda} = \frac{1}{1447} \begin{pmatrix} -370 \\ 367 \end{pmatrix}$$

Population 0:

$(1;1), (2;1), (1.5;2), (2;2), (2;3), (3,3)$

Population 1:

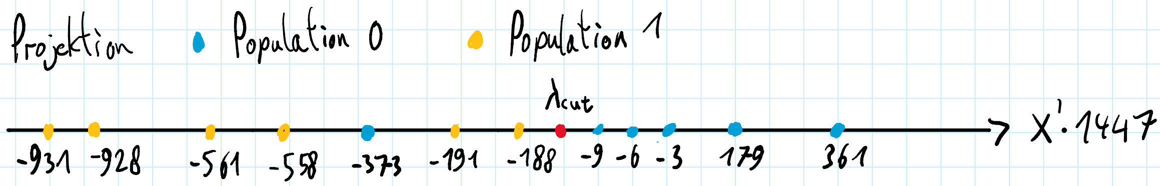
$(1.5;1), (2.5;1), (3.5;1), (2.5,2), (3.5;2), (4.5;2)$

$x'_{0,1} = -\frac{3}{1447}$ $\quad x'_{0,2} = -\frac{373}{1447}$ $\quad x'_{0,3} = \frac{179}{1447}$

$x'_{0,4} = -\frac{6}{1447}$ $\quad x'_{0,5} = \frac{361}{1447}$ $\quad x'_{0,6} = -\frac{9}{1447}$

$x'_{1,1} = -\frac{188}{1447}$ $\quad x'_{1,2} = -\frac{558}{1447}$ $\quad x'_{1,3} = -\frac{928}{1447}$

$x'_{1,4} = -\frac{191}{1447}$ $\quad x'_{1,5} = -\frac{561}{1447}$ $\quad x'_{1,6} = -\frac{931}{1447}$



e) $\lambda_{cut} = -\frac{98,5}{1447}$

$$\text{Effizienz} = \frac{tp}{tp+fn} = \frac{6}{6+0} = 1$$

$$\text{Reinheit} = \frac{tp}{tp+fp} = \frac{6}{6+1} = \frac{6}{7} = 0.857$$

Den Parameter haben wir so gewählt, dass möglichst viele Punkte richtig klassifiziert wurden.