(d) Berechnen Sie A_0 und δ und deren Fehler und Korrelation aus a_1 und a_2.

$$f(\Psi) = A_0 \cos(\Psi + \delta) \overset{!}{=} a_1 \cos(\Psi) + a_2 \sin(\Psi)$$

mit $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$$\Rightarrow A_0 \cos(\Psi + \delta) = A_0 \cos(\Psi)\cos(\delta) - A_0 \sin(\Psi)\sin(\delta)$$

$$\overset{!}{=} a_1 \cos(\Psi) + a_2 \sin(\Psi) \quad \text{Koeffizientenvergleich}$$

$$\Rightarrow A_0 \cos\delta = a_1 \quad \text{und} \quad -A_0 \sin\delta = a_2$$

$$\Leftrightarrow A_0 = \frac{a_1}{\cos\delta} \quad \text{einsetzen}$$

$$\Rightarrow -\frac{a_1}{\cos\delta} \sin\delta = a_2$$

$$\Leftrightarrow \delta = \arctan\left(-\frac{a_2}{a_1}\right)$$

$$\Rightarrow A_0 = \frac{a_1}{\cos\left(\arctan\left(-\frac{a_2}{a_1}\right)\right)}$$

- Fehler und Korrelation:

$$\vec{y} = \vec{g}(\vec{x}) \Rightarrow Var[\vec{y}] = J \cdot Var[\vec{x}] \cdot J^T$$

mit $J \mathrel{\hat{=}}$ Jacobi-Matrix von $\vec{g}(\vec{x})$

hier $\vec{y} = \begin{pmatrix} A_0 \\ \delta \end{pmatrix}$ $\quad \vec{x} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ $\quad \vec{g}(\vec{x}) = \begin{pmatrix} \frac{a_1}{\cos\left(\arctan\left(-\frac{a_2}{a_1}\right)\right)} \\ \arctan\left(-\frac{a_2}{a_1}\right) \end{pmatrix}$

$$J = \begin{pmatrix} \frac{\partial g_1}{\partial a_1} & \frac{\partial g_1}{\partial a_2} \\ \frac{\partial g_2}{\partial a_1} & \frac{\partial g_2}{\partial a_2} \end{pmatrix}$$

$$\frac{\partial g_1}{\partial a_1} = \frac{\partial}{\partial a_1}\left(\frac{a_1}{\cos\left(\arctan\left(-\frac{a_2}{a_1}\right)\right)}\right) = \left(\left(\frac{a_2}{a_1}\right)^2 + 1\right)^{-\frac{1}{2}}$$

$$\frac{\partial g_2}{\partial a_1} = \frac{a_2}{a_1^2 + a_2^2}$$

$$\frac{\partial g_1}{\partial a_2} = \frac{a_2}{a_1}\left(\left(\frac{a_2}{a_1}\right)^2 + 1\right)^{-\frac{1}{2}}$$

$$\frac{\partial g_2}{\partial a_2} = -\frac{a_1}{a_1^2 + a_2^2}$$

$$J = \begin{pmatrix} \left(\left(\frac{a_2}{a_1}\right)^2 + 1\right)^{-\frac{1}{2}} & \frac{a_2}{a_1}\left(\left(\frac{a_2}{a_1}\right)^2 + 1\right)^{-\frac{1}{2}} \\ \frac{a_2}{a_1^2 + a_2^2} & -\frac{a_1}{a_1^2 + a_2^2} \end{pmatrix}$$

$$Var\left[\begin{pmatrix} A_0 \\ \delta \end{pmatrix}\right] = J \cdot Var\left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\right] \cdot J^T$$