Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Ejercicio 2

Demostración

- $P_c \equiv n \geq 0 \wedge result = 0 \wedge i = 0$
- $Q_c \equiv result = \sum_{j=0}^{n-1} \textsf{if } j \bmod 2 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$
- $B \equiv i < n$
- $I \equiv 0 \leq i \leq n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$
- $f_v \equiv n - i$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv n \geq 0 \wedge result = 0 \wedge i = 0 \longrightarrow 0 \leq i \leq n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$

- $n \geq 0 \wedge result = 0 \wedge i = 0 \longrightarrow 0 \leq i \leq n+1 \equiv \text{true}$
- $n \geq 0 \wedge result = 0 \wedge i = 0 \longrightarrow i \bmod 2 = 0 \equiv \text{true}$
- $n \geq 0 \wedge result = 0 \wedge i = 0 \longrightarrow result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$

$0 \leq i \leq n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge \neg(i < n) \longrightarrow$
$result = \sum_{j=0}^{n-1} \textsf{if } j mod 2 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv$

$0 \leq i \leq n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge (i \geq n) \longrightarrow$
$result = \sum_{j=0}^{n-1} \textsf{if } j \bmod 2 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv$

$0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \longrightarrow$
$result = \sum_{j=0}^{n-1} \textsf{if } j \bmod 2 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$

- $0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$
  $0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv$
  $0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = \sum_{j=0}^{n} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv$
  $0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = (\sum_{j=0}^{i} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}) + \textsf{if } n \bmod 2 = 0 \textsf{ then } n \textsf{ else } 0 \textsf{ fi} \equiv$
  $0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = (\sum_{j=0}^{i} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi})$

$0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = (\sum_{j=0}^{i} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}) \longrightarrow$
$result = \sum_{j=0}^{n-1} \textsf{if } j \bmod 2 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv$

- $0 \leq i = n+1 \longrightarrow$(n-1) = i

$0 \leq i = n+1 \wedge i \bmod 2 = 0 \wedge result = (\sum_{j=0}^{i} \textsf{if } j \bmod 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}) \longrightarrow$
$result = \sum_{j=0}^{i} \textsf{if } j \bmod 2 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \equiv$
true

$\{I \wedge B\} ciclo \{I\}$

- $P_c \equiv n \geq 0 \wedge result = 0 \wedge i = 0$
- $Q_c \equiv result = \sum_{j=0}^{n-1}$ if j mód 2 then j else 0 fi
- $B \equiv i < n$
- $I \equiv 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1}$ if j mód $2 = 0$ then j else 0 fi
- $S1 : result := result + i;$
- $S2 : i := i + 2;$
- $ciclo \equiv S1; S2$
- $f_v \equiv n - i$

$\{I \wedge B\} ciclo \{I\} \equiv$

$wp(ciclo, I) \equiv wp(S1; S2, I) \stackrel{Ax3}{\equiv} wp(S1, wp(S2, I))$

- wp(S2, I)

  $wp(i := i+2;, 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1}$ if j mód $2 = 0$ then j else 0 fi) $\stackrel{Ax1}{\equiv}$

  $\{def(i+2) \wedge_L 0 \leq i+2 \leq n+1 \wedge i+2 \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i+1}$ if j mód $2 = 0$ then j else 0 fi$\} \equiv$

  $\{0 \leq i+2 \leq n+1 \wedge i+2 \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i+1}$ if j mód $2 = 0$ then j else 0 fi$\} \equiv$

  $\{0 \leq i+2 \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i+1}$ if j mód $2 = 0$ then j else 0 fi$\} \equiv$

  $\{-2 \leq i \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i+1}$ if j mód $2 = 0$ then j else 0 fi$\} \equiv$

  $\{-2 \leq i \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result = (\sum_{j=0}^{i}$ if j mód $2 = 0$ then j else 0 fi$) +$ if $i+1$ mód $2 = 0$ then $i+1$ else 0 fi$)\} \equiv$

  $\{-2 \leq i \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result = (\sum_{j=0}^{i}$ if j mód $2 = 0$ then j else 0 fi$)\}$

- wp(S2, wp(S1, I))

  $wp(S2, wp(S1, I)) \equiv$

  $wp(result := result + i, \{-2 \leq i \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result = (\sum_{j=0}^{i}$ if j mód $2 = 0$ then j else 0 fi$)\}) \stackrel{Ax1}{\equiv}$

  $\{def(result + i) \wedge_L -2 \leq i \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result + i = (\sum_{j=0}^{i}$ if j mód $2 = 0$ then j else 0 fi$)\} \equiv$

  $\{\leq i-2 \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result + i = (\sum_{j=0}^{i}$ if j mód $2 = 0$ then j else 0 fi$)\}$

Qvq $(I \wedge B) \longrightarrow wp(ciclo, I)$

- $(I \wedge B)$

  $(I \wedge B) \equiv 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1}$ if j mód $2 = 0$ then j else 0 fi $\wedge i < n \equiv$

  $(I \wedge B) \equiv 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1}$ if j mód $2 = 0$ then j else 0 fi $\equiv$

  $(I \wedge B) \equiv 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-2}$ if j mód $2 = 0$ then j else 0 fi

$0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-2}$ if j mód $2 = 0$ then j else 0 fi $\longrightarrow$
$-2 \leq i \leq n-1 \wedge i \text{ mód } 2 = 0 \wedge result + i = (\sum_{j=0}^{i}$ if j mód $2 = 0$ then j else 0 fi$)$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

- $I \equiv 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \text{ mód } 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$
- $B \equiv i < n$
- $f_v \equiv n - i$
- $S1 : result := result + i;$
- $S2 : i := i + 2;$
- $ciclo \equiv S1; S2$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
  - $wp(ciclo, f_v < v_0)$
    $wp(ciclo, f_v < v_0) \equiv wp(S1; S2, n - i < v_0) \overset{Ax3}{\equiv}$
    $wp(S1, wp(S2, n - i < v_0))$
    - wp(S2, $n - i < v_0$)
      $wp(S2, n - i < v_0) \equiv wp(i = i + 2, n - i < v_0) \overset{Ax1}{\equiv}$
      $def(i + 2) \wedge_L n - i - 2 < v_0 \equiv$
      $n - i - 2 < v_0$

    $wp(S1, wp(S2, n - i - 2 < v_0)) \equiv$
    $wp(result := result + 1, n - i - 2 < v_0) \overset{Ax1}{\equiv}$
    $\{def(result + 1) \wedge_L n - i - 2 < v_0\} \equiv$
    $\{n - i - 2 < v_0\}$
- Qvq $I \wedge B \wedge (v_0 = f_v) \longrightarrow n - i - 2 < v_0$
  $(I \wedge B \wedge v_0 = f_v) \equiv$
  $0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-2} \textsf{if } j \text{ mód } 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge v_0 = n - i$
  $0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-2} \textsf{if } j \text{ mód } 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge v_0 = n - i \longrightarrow n - i - 2 < v_0 \equiv$
  true

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

- $I \equiv 0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \text{ mód } 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi}$
- $B \equiv (i < n)$
- $f_v \equiv n - i$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$
  $(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
  $0 \leq i \leq n+1 \wedge i \text{ mód } 2 = 0 \wedge result = \sum_{j=0}^{i-1} \textsf{if } j \text{ mód } 2 = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge n - i \leq 0 \longrightarrow (i \geq n) \equiv$
  true