Pregunta 4

- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$

Demostración

Datos

- $Q_c \equiv |s| = |s_0| \wedge_L ((\forall i : \mathbb{Z})\ 1 \leq i < |s| \wedge_L s_0[i-1] < s_0[i] \longrightarrow_L s[i-1] = s_0[i]) \wedge_L$
  $((\forall i : \mathbb{Z})\ 1 \leq i < |s|\ \wedge_L s_0[i-1] \geq s_0[i] \longrightarrow_L s[i-1] = s_0[i-1])$
- $I \equiv 1 \leq i \leq |s| \wedge |s| = |s_0| \wedge_L$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i-1\ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i-1\ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1])$
  $((\forall j : \mathbb{Z})\ i-1 \leq j < |s|\ \longrightarrow_L s[j] = s_0[j])$
- $B \equiv i < |s|$
- $S1 \equiv$ if $s[i-1] < s[i]$ then $s[i-1] := s[i];$ else $skip;$ fi;
- $S2 \equiv i := i + 1$
- $ciclo \equiv S1; S2;$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1; S2, I) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, I))$

- $wp(S2; I) \equiv$

  $wp(i := i + 1; , I) \overset{Ax1}{\equiv}$
  $\{def(i+1) \wedge_L I^i_{i+1})\} \equiv$
  $\{\text{true} \wedge_L 1 \leq i+1 \leq |s| \wedge |s| = |s_0| \wedge_L$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i\ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i\ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$
  $((\forall j : \mathbb{Z})\ i \leq j < |s|\ \longrightarrow_L s[j] = s_0[j])\} \equiv$

  $\{0 \leq i < |s| \wedge |s| = |s_0| \wedge_L$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i\ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i\ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$
  $((\forall j : \mathbb{Z})\ i \leq j < |s|\ \longrightarrow_L s[j] = s_0[j])\}$

- $wp(S1, wp(S2, I)) \equiv$

  $wp($if $s[i-1] < s[i]$ then $s[i-1] := s[i];$ else $skip;$ fi$;, I^i_{i+1}) \overset{Ax4}{\equiv}$

  $\{def(s[i-1] < s[i]) \wedge_L ((s[i-1] < s[i] \wedge_L wp(s[i-1] := s[i], I^i_{i+1}) \vee (s[i-1] \geq s[i] \wedge_L wp(skip; , I^i_{i+1}))))\}$

  - $def(s[i-1] < s[i]) \equiv 0 \leq i-1 < |s| \equiv 1 \leq i \leq |s|$
  - $wp(skip; , I^i_{i+1}) \overset{Ax3}{\equiv} I^i_{i+1}$
  - $wp(s[i-1] := s[i], I^i_{i+1}) \overset{Ax2}{\equiv} wp(s := setAt(s, i-1, s[i]), I^i_{i+1}) \equiv$

    $(((def(s) \wedge def(i-1)) \wedge_L 0 \leq i-1 < |s|) \wedge def(s[i])) \wedge_L (I^i_{i+1})^s_{setAt(s,i-1,s[i])} \equiv$

    $1 \leq i \leq |s| \wedge_L (I^i_{i+1})^s_{setAt(s,i-1,s[i])}$

- $wp(S1, wp(S2, I)) \equiv$

  $\Big\{1 \leq i \leq |s| \wedge_L \Big((s[i-1] < s[i] \wedge_L 1 \leq i \leq |s| \wedge_L {I^i_{i+1}}^s_{setAt(s,i-1,s[i])}) \vee (s[i-1] \geq s[i] \wedge_L I^i_{i+1})\Big)\Big\} \equiv$

  $\Big\{\Big((1 \leq i \leq |s| \wedge_L s[i-1] < s[i] \wedge_L {I^i_{i+1}}^s_{setAt(s,i-1,s[i])}) \vee (1 \leq i \leq |s| \wedge_L s[i-1] \geq s[i] \wedge_L I^i_{i+1})\Big)\Big\}$

  - $1 \leq i \leq |s| \wedge_L s[i-1] \geq s[i] \wedge_L I^i_{i+1} \equiv$

    $1 \leq i \leq |s| \wedge_L s[i-1] \geq s[i] \wedge_L 0 \leq i < |s| \wedge |s| = |s_0| \wedge_L$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$
    $((\forall j : \mathbb{Z})\ i \leq j < |s| \longrightarrow_L s[j] = s_0[j]) \equiv$

    $1 \leq i < |s| \wedge_L s[i-1] \geq s[i] \wedge |s| = |s_0| \wedge_L$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$
    $((\forall j : \mathbb{Z})\ i \leq j < |s| \longrightarrow_L s[j] = s_0[j])$

  - $1 \leq i \leq |s| \wedge_L s[i-1] < s[i] \wedge_L {I^i_{i+1}}^s_{setAt(s,i-1,s[i])} \equiv$

    $1 \leq i \leq |s| \wedge_L s[i-1] < setAt(s, i-1, s[i])[i] \wedge_L$
    $0 \leq i < |s| \wedge |setAt(s, i-1, s[i])| = |s_0| \wedge_L$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j]) \wedge$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j-1]) \wedge$
    $((\forall j : \mathbb{Z})\ i \leq j < |setAt(s, i-1, s[i])| \longrightarrow_L s[j] = s_0[j]) \equiv$

    $1 \leq i < |s| \wedge_L s[i-1] < s[i] \wedge_L |s| = |s_0| \wedge_L$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j]) \wedge$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j-1]) \wedge$
    $((\forall j : \mathbb{Z})\ i \leq j < |setAt(s, i-1, s[i])| \longrightarrow_L s[j] = s_0[j])$

- $wp(S1, wp(S2, I)) \equiv$

  $\Big\{\Big((1 \leq i \leq |s| \wedge_L s[i-1] < s[i] \wedge_L {I^i_{i+1}}^s_{setAt(s,i-1,s[i])}) \vee (1 \leq i \leq |s| \wedge_L s[i-1] \geq s[i] \wedge_L I^i_{i+1})\Big)\Big\}$

  $\Big\{\Big(\ (1 \leq i < |s| \wedge_L s[i-1] < s[i] \wedge_L |s| = |s_0| \wedge_L$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j-1]) \wedge$
  $((\forall j : \mathbb{Z})\ i \leq j < |setAt(s, i-1, s[i])| \longrightarrow_L setAt(s, i-1, s[i])[j] = s_0[j])\big)$
  $\vee$
  $\big(1 \leq i < |s| \wedge_L s[i-1] \geq s[i] \wedge |s| = |s_0| \wedge_L$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ 0 \leq j < i \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$
  $((\forall j : \mathbb{Z})\ i \leq j < |s| \longrightarrow_L s[j] = s_0[j]))\ \Big)\Big\}$

- QvQ $I \wedge B \longrightarrow wp(ciclo, I)$

  - $I \wedge B \equiv$
    $1 \leq i \leq |s| \wedge |s| = |s_0| \wedge_L$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i-1 \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
    $((\forall j : \mathbb{Z})\ 0 \leq j < i-1 \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1])$

$((\forall j : \mathbb{Z})\ i-1 \leq j < |s| \ \longrightarrow_L s[j] = s_0[j]) \wedge i < |s| \equiv$

$1 \leq i < |s| \wedge |s| = |s_0| \wedge_L$
$((\forall j : \mathbb{Z})\ 0 \leq j < i-1 \ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$
$((\forall j : \mathbb{Z})\ 0 \leq j < i-1 \ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1])$
$((\forall j : \mathbb{Z})\ i-1 \leq j < |s| \ \longrightarrow_L s[j] = s_0[j])$

- QvQ $I \wedge B \longrightarrow wp(ciclo, I) \equiv$

  $1 \leq i < |s| \wedge |s| = |s_0| \wedge_L$

  $((\forall j : \mathbb{Z})\ 0 \leq j < i-1 \ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$

  $((\forall j : \mathbb{Z})\ 0 \leq j < i-1 \ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$

  $((\forall j : \mathbb{Z})\ i-1 \leq j < |s| \ \longrightarrow_L s[j] = s_0[j])$

  $\longrightarrow$

  $\Big\{\Big(\big(1 \leq i < |s| \wedge_L s[i-1] < s[i] \ \wedge_L |s| = |s_0| \wedge_L$

  $((\forall j : \mathbb{Z})\ 0 \leq j < i \ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j]) \wedge$

  $((\forall j : \mathbb{Z})\ 0 \leq j < i \ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L setAt(s, i-1, s[i])[j-1] = s_0[j-1]) \wedge$

  $((\forall j : \mathbb{Z})\ i \leq j < |setAt(s, i-1, s[i])| \ \longrightarrow_L setAt(s, i-1, s[i])[j] = s_0[j])\big)$

  $\vee$

  $\big(1 \leq i < |s| \ \wedge_L s[i-1] \geq s[i] \wedge |s| = |s_0| \wedge_L$

  $((\forall j : \mathbb{Z})\ 0 \leq j < i \ \wedge_L s_0[j-1] < s_0[j] \longrightarrow_L s[j-1] = s_0[j]) \wedge$

  $((\forall j : \mathbb{Z})\ 0 \leq j < i \ \wedge_L s_0[j-1] \geq s_0[j] \longrightarrow_L s[j-1] = s_0[j-1]) \wedge$

  $((\forall j : \mathbb{Z})\ i \leq j < |s| \ \longrightarrow_L s[j] = s_0[j])\big) \Big)\Big\}$

- $j = i - 1$