Ejercicio 11

Correctitud

- $Pre \longrightarrow wp(\text{codigo previo al ciclo}, P_c)$
- $P_c \longrightarrow wp(ciclo, Q_c)$
- $Q_c \longrightarrow wp(\text{codigo posterior al ciclo}, Post)$

Por monotonía sabemos que Pre $\longrightarrow$wp(programa completo, Post)

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $Pre \equiv \text{true}$
- $Post \equiv r = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq r < |s| \wedge_L s[r] = e)$
- $P_c \equiv i = (|s| - 1) \wedge j = -1$
- $Q_c \equiv j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e)$
- $I \equiv 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i + 1 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e)$
- $B \equiv i \geq 0$
- $S1 \equiv \textsf{if } s[i] = e \textsf{ then } j := i \textsf{ else } skip \textsf{ fi}$
- $S2 \equiv i := i - 1$
- $ciclo \equiv S1; S2;$
- $f_v \equiv i$

$Pre \longrightarrow wp($**codigo previo al ciclo**$, P_c)$

$wp(\text{codigo previo al ciclo}, P_c) \equiv$
$wp(i := |s| - 1; j := -1, i = (|s| - 1) \wedge j = -1) \overset{Ax3}{\equiv}$
$wp(i := |s| - 1, wp(j := -1, i = (|s| - 1) \wedge j = -1)$

- $wp(j := -1, i = (|s| - 1) \wedge j = -1) \overset{Ax1}{\equiv}$
  $\{i = (|s| - 1) \wedge -1 = -1\}$

$wp(i := |s| - 1, i = (|s| - 1) \wedge -1 = -1) \overset{Ax1}{\equiv}$
$|s| - 1 = |s| - 1 \wedge -1 = -1 \equiv \text{true}$
$Pre \longrightarrow wp(\text{codigo previo al ciclo}, P_c) \equiv$
$\text{true} \longrightarrow \text{true} \equiv \text{true}$

$Q_c \longrightarrow wp($**codigo posterior al ciclo**$, Post)$

$wp(\text{codigo posterior al ciclo}, Post) \equiv$
$wp(r := j, r = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq r < |s| \wedge_L s[r] = e) \equiv$
$\{j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e\}$
$Q_c \longrightarrow wp(\text{codigo posterior al ciclo}, Post \equiv$
$j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \longrightarrow$
$j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e \equiv \text{true}$

$P_c \longrightarrow wp(ciclo, Q_c)$

- $P_c \longrightarrow I\ i = (|s| - 1) \wedge j = -1 \longrightarrow$

  $0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i + 1 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e) \equiv \text{true}$

- $(I \wedge \neg B) \longrightarrow Q_c$

  $0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i + 1 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e) \wedge i < 0 \longrightarrow$

  $j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \equiv$

  $\text{false} \longrightarrow j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (0 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \equiv \text{true}$

- $\{I \wedge B\}$ ciclo $\{I\}$

  $wp(i : i - 1, 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i + 1 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e)) \equiv$

  $0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e)$

  $wp(\textsf{if}\ s[i] = e\ \textsf{then}\ j := i\ \textsf{else}\ skip\ \textsf{fi}, 0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e)) \overset{Ax4}{\equiv}$

  $0 \leq i < |s| \wedge_L ($

  $(s[i] = e) \wedge (0 \leq i - 1 < |s| \wedge_L i = -1 \longrightarrow (\forall i : \mathbb{Z})\ (i \leq i < |s| \ \longrightarrow_L s[i] \neq e) \wedge i \neq -1 \longrightarrow (i - 1 < i < |s| \wedge_L s[i] = e)) \vee$

  $(s[i] \neq e) \wedge (0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e)))$

  $0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i + 1 \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e) \longrightarrow$

  $0 \leq i < |s| \wedge_L ($

  $(s[i] = e) \longrightarrow (i - 1 < i < |s| \wedge_L s[i] = e)) \vee$

  $(s[i] \neq e) \wedge (0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z})\ (i \leq j < |s| \ \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e))) \equiv \text{true}$

- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$

  $wp(i : i - 1, i < v_0) \equiv i - 1 < v_0$

  $wp(\textsf{if}\ s[i] = e\ \textsf{then}\ j := i\ \textsf{else}\ skip\ \textsf{fi}, i - 1) \equiv 0 \leq i < |s| \wedge_L i - 1 < v_0$

  $I \wedge B \wedge v_0 = i \longrightarrow 0 \leq i < |s| \wedge_L i - 1 < v_0 \equiv \text{true}$

- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

  $I \wedge i \leq 0 \longrightarrow i < 0 \equiv \text{true}$