

## Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$  ciclo  $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$  ciclo  $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

## Ejercicio 1

- a)  $P_c \equiv result = 0 \wedge i = 0$
- $Q_c \equiv (result = \sum_{i=0}^{|s|-1} i) \wedge_L (i = |s|)$
- b) Al demostrar  $\{I \wedge B\}$  ciclo  $\{I\}$
- c)  $(I \wedge \neg B) \longrightarrow Q_c$
- d)  $\{I \wedge B \wedge (v_0 = f_v)\}$  ciclo  $\{f_v < v_0\}$
- e y f)

## Demostración

$$P_c \equiv result = 0 \wedge i = 0$$

$$Q_c \equiv (result = \sum_{k=0}^{|s|-1} k) \wedge_L (i = |s|)$$

$$I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$$

$$P_c \longrightarrow I$$

$$P_c \longrightarrow I \equiv result = 0 \wedge i = 0 \longrightarrow 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$$

$$i = 0 \longrightarrow 0 \leq i \leq |s| \equiv 0 \leq 0 \leq |s| \wedge |s| \geq 0 \equiv \text{true}$$

$$result = 0 \wedge i = 0 \longrightarrow result = \sum_{j=0}^{i-1} s[j] \equiv$$

$$0 = \sum_{j=0}^{-1} s[j] \wedge \sum_{j=0}^{-1} s[j] = 0$$

$$0 = \sum_{j=0}^{-1} s[j] \equiv 0 = 0 \equiv \text{true}$$

$$(I \wedge \neg B) \longrightarrow Q_c$$

$$0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge \neg(i < |s|) \longrightarrow (result = \sum_{k=0}^{|s|-1} k) \wedge_L (i = |s|)$$

- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge \neg(i < |s|)$
- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge (i \geq |s|) \equiv$
- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge (i \geq |s|) \equiv$
- $(i = |s|) \wedge_L result = \sum_{j=0}^{|s|-1} s[j]$

$$(i = |s|) \wedge_L \sum_{j=0}^{|s|-1} s[j] \longrightarrow (result = \sum_{k=0}^{|s|-1} k) \wedge_L (i = |s|)$$

$$\{I \wedge B\} \text{ciclo} \{I\}$$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S1; S2$

- $\{I \wedge B\} \text{ciclo}\{I\} \equiv$   
 $\{(0 \leq i \leq |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|)\} S1; S2; \{(0 \leq i \leq |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j])\} \equiv$ 
  - $wp(S1; S2; , I) \stackrel{Ax3}{\equiv} wp(S1, wp(S2, I))$   
 $wp(S2, I) \equiv wp(i := i + 1, 0 \leq i \leq |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \stackrel{Ax1}{\equiv}$   
 $0 \leq i + 1 \leq |s| \wedge_L \text{result} = \sum_{j=0}^i s[j]$
  - $wp(S1, wp(S2, I)) \equiv$   
 $wp(\text{result} := \text{result} + s[i]; , 0 \leq i + 1 \leq |s| \wedge_L \text{result} = \sum_{j=0}^i s[j]) \stackrel{Ax1}{\equiv}$   
 $\text{def}(\text{result} := \text{result} + s[i]) \wedge_L 0 \leq i + 1 \leq |s| \wedge_L \text{result} + s[i] = \sum_{j=0}^i s[j] \equiv$   
 $0 \leq i < |s| \wedge_L 0 \leq i + 1 \leq |s| \wedge_L \text{result} + s[i] = \sum_{j=0}^i s[j] \equiv$   
 $0 \leq i < |s| \wedge_L (-1) \leq i \leq |s| - 1 \wedge_L \text{result} = \sum_{j=0}^i s[j] - s[i] \equiv$   
 $0 \leq i < |s| \wedge_L (-1) \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]$   
 $0 \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]$
  - $\text{Qvq } I \wedge B \longrightarrow wp(\text{ciclo}, I)$ 
    - $I \wedge B \equiv (0 \leq i \leq |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \equiv$   
 $(0 \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j])$
    - $wp(\text{ciclo}, I) \equiv 0 \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]$   
 $(0 \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \longrightarrow (0 \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \equiv$   
 $\text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\} \text{ciclo} \{f_v < v_0\}$

- $I \equiv 0 \leq i \leq |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $f_v \equiv |s| - i$
- $S1 : \text{result} := \text{result} + s[i];$
- $S2 : i := i + 1;$
- $\text{ciclo} \equiv S1; S2$
- $\{I \wedge B \wedge (v_0 = f_v)\} \text{ciclo} \{f_v < v_0\}$ 
  - $wp(\text{ciclo}, f_v < v_0)$   
 $wp(\text{ciclo}, f_v < v_0) \equiv wp(S1; S2, |s| - i < v_0) \stackrel{Ax3}{\equiv}$ 
    - $wp(S1, wp(S2, (|s| - i) < v_0))$
    - $wp(S2, (|s| - i) < v_0)$   
 $wp(i := i + 1, (|s| - i) < v_0) \stackrel{Ax1}{\equiv}$   
 $\text{def}(i + 1) \wedge_L (|s| - (i + 1)) < v_0 \equiv$   
 $(|s| - i - 1) < v_0$
    - $wp(S1, (|s| - i + 1) < v_0)$   
 $wp(S1, (|s| - i + 1) < v_0) \equiv$   
 $wp(\text{result} := \text{result} + s[i], (|s| - i - 1) < v_0) \stackrel{Ax1}{\equiv}$   
 $\text{def}(\text{result} + s[i]) \wedge_L (|s| - i - 1) < v_0 \equiv$   
 $0 \leq i < |s| \wedge_L (|s| - i - 1) < v_0$
  - $\text{Qvq } I \wedge B \wedge (v_0 = f_v) \longrightarrow wp(\text{ciclo}, f_v < v_0)$   
 $I \wedge B \wedge (v_0 = f_v) \longrightarrow wp(\text{ciclo}, f_v < v_0) \equiv$   
 $(0 \leq i \leq |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \wedge (v_0 = |s| - i) \longrightarrow 0 \leq i < |s| \wedge_L (|s| - i + 1) < v_0 \equiv$   
 $(0 \leq i < |s| \wedge_L \text{result} = \sum_{j=0}^{i-1} s[j]) \wedge (v_0 = |s| - i) \longrightarrow 0 \leq i < |s| \wedge_L |s| - i < v_0 + 1 \equiv$   
 $\text{true}$

$$(I \wedge f_v \leq 0) \longrightarrow \neg B$$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $f_v \equiv |s| - i$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge |s| - i \leq 0 \longrightarrow \neg(i < |s|) \equiv$
- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge |s| - i \leq 0 \longrightarrow i \geq |s| \equiv$
- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge |s| \leq i \longrightarrow i \geq |s| \equiv \text{true}$

## Cuerpo del ciclo invertido

$$\{I \wedge B\}ciclo\{I\}$$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S2; S1$
- $\{I \wedge B\}ciclo\{I\} \equiv$
- $\{(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|)\} S2; S1; \{(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j])\} \equiv$
- $wp(S2; S1; , I) \stackrel{Ax3}{\equiv} wp(S2, wp(S1, I))$
- $wp(S1, I) \equiv wp(result := result + s[i], 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \stackrel{Ax1}{\equiv}$
- $def(result + s[i]) \wedge_L 0 \leq i \leq |s| \wedge_L result + s[i] = \sum_{j=0}^{i-1} s[j] \equiv$
- $0 \leq i < |s| \wedge_L 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^i s[j])$
- $wp(S2, wp(S1, I)) \equiv$
- $wp(i := i + 1; , 0 \leq i < |s| \wedge_L 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^i s[j]) \stackrel{Ax1}{\equiv}$
- $0 \leq i < |s| \wedge_L 0 \leq i + 1 \leq |s| \wedge_L result = \sum_{j=0}^{i+1} s[j] \equiv$
- $0 \leq i < |s| \wedge_L 0 < i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j] \equiv$
- $0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j] \equiv$
- $Qvq I \wedge B \longrightarrow wp(ciclo, I)$
- $I \wedge B \equiv (0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \equiv$
- $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j])$
- $wp(ciclo, I) \equiv 0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j]$
- $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \longrightarrow 0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j] \equiv$
- true

$\{I \wedge B \wedge (v_0 = f_v)\} \text{ ciclo } \{f_v < v_0\}$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $f_v \equiv |s| - i$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S2; S1$
- $\{I \wedge B \wedge (v_0 = f_v)\} \text{ ciclo } \{f_v < v_0\}$

- $wp(ciclo, f_v < v_0)$

$$wp(ciclo, f_v < v_0) \equiv wp(S2; S1, |s| - i < v_0) \stackrel{Ax3}{\equiv}$$

$$\circ wp(S2, wp(S1, (|s| - i) < v_0))$$

$$\circ wp(S1, (|s| - i) < v_0)$$

$$wp(result := result + s[i], (|s| - i) < v_0) \stackrel{Ax1}{\equiv}$$

$$def(result + s[i]) \wedge_L (|s| - i) < v_0 \equiv$$

$$0 \leq i < |s| \wedge_L (|s| - i) < v_0$$

$$\circ wp(S2, 0 \leq i < |s| \wedge_L (|s| - i) < v_0)$$

$$wp(S2, 0 \leq i < |s| \wedge_L (|s| - i) < v_0) \equiv$$

$$wp(i := i + 1, 0 \leq i < |s| \wedge_L (|s| - i) < v_0) \stackrel{Ax1}{\equiv}$$

$$0 \leq i + 1 < |s| \wedge_L (|s| - i - 1) < v_0 \equiv$$

$$0 \leq i + 1 < |s| \wedge_L (|s| - i - 1) < v_0$$

- $\text{Qvq } I \wedge B \wedge (v_0 = f_v) \longrightarrow wp(ciclo, f_v < v_0)$

$$I \wedge B \wedge (v_0 = f_v) \longrightarrow wp(ciclo, f_v < v_0) \equiv$$

$$(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \wedge (v_0 = |s| - i) \longrightarrow 0 \leq i + 1 < |s| \wedge_L (|s| - i - 1) < v_0 \equiv$$

$$(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (v_0 = |s| - i) \longrightarrow 0 \leq i + 1 < |s| \wedge_L (|s| - i - 1) < v_0 \equiv$$

true