

TPE - Reuniones Remotas

Grupo 6 - Alrescate.com

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proc esSeñal (in s: seq\langle \mathbb{Z} \rangle, in prof: \mathbb{Z}, in freq: \mathbb{Z}, out result: Bool) {
           Pre \{prof > 0 \land freq > 0\}
           Post \{result = true \leftrightarrow esSe\tilde{n}alAux(s, prof, freq)\}
}
     \texttt{pred esSe\~nalAux} \ (s: seq\langle \mathbb{Z}\rangle, \, prof \colon \mathbb{Z}, \, freq \colon \mathbb{Z}) \ \{
        |s| \geq 0 \land
           (frecuenciaEnRango(freq) \land_L)
           duraMasDeUnSegundo(s, freq)) \land
           profundidadCorrecta(s) \land
           ninguna Muestra Supera La Profundidad(s, prof)
     }
     pred frecuenciaEnRango (freq: \mathbb{Z}) \{ freq \in [8, 32] \}
     pred profundidadCorrecta (prof: \mathbb{Z}) \{ prof \in [8, 16, 32] \}
     \texttt{pred duraMasDeUnSegundo} \ (s: \ seq\langle \mathbb{Z}\rangle, \ freq: \mathbb{Z}) \ \{duraci\'onEnSegundos(s, freq) > 1\}
     pred ningunaMuestraSuperaLaProfundidad (s: seq\langle \mathbb{Z} \rangle, p: \mathbb{Z}) {
        (\forall i : \mathbb{Z}) \ 0 \le i < |s| \longrightarrow_L (-2)^{p-1} \le s[i] \le 2^{p-1} - 1
     fun duraciónEnSegundos (s: seq\langle \mathbb{Z}\rangle, freq: \mathbb{Z}) : \mathbb{Z} = |s| \ div \ (freq \cdot 1000);
```

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\begin{array}{l} \operatorname{proc\ seEnoj6?}\ (\operatorname{in\ s:\ se\~{n}al},\ \operatorname{in\ umbral:\ }\mathbb{Z},\ \operatorname{in\ prof:\ }\mathbb{Z},\ \operatorname{in\ freq:\ }\mathbb{Z},\ \operatorname{out\ result:\ Bool})\ \left\{ \begin{array}{l} \operatorname{Pre\ }\{umbral>0 \wedge esSe\~{n}alAux(s,prof,freq)\} \\ \operatorname{Post\ }\{ \\ result = true \leftrightarrow umbralEnRango(umbral,prof) \wedge \\ existeUnaSubsecuenciaQueSuperaUmbral(s,freq,umbral)\ \} \\ \end{array} \right\} \\ \operatorname{pred\ umbralEnRango}\ (umbral:\ \mathbb{Z},\ p:\ \mathbb{Z})\ \left\{ umbral \leq 2^{p-1}-1 \right\} \\ \operatorname{pred\ existeUnaSubsecuenciaQueSuperaUmbral}\ (s:\ se\~{n}al,\ freq:\ \mathbb{Z},\ umbral:\ \mathbb{Z})\ \left\{ (\exists d,h:\mathbb{Z})\ 0 \leq d,h < |s|+1 \ \wedge (h > (d+freq*1000*5)) \ \wedge_L\ (\\ (\forall i:\mathbb{Z})\ 0 \leq i < |subseq(s,d,h)| \ \longrightarrow_L\ abs(subseq(s,d,h)[i]) > umbral\ ) \\ \end{array} \right\} \\ \operatorname{fun\ abs\ }(x:\ \mathbb{Z}):\ \mathbb{Z}\ = \operatorname{if\ }x>0 \ \operatorname{then\ }x\ \operatorname{else\ }-x\ \operatorname{fi\ }; \end{array}
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\begin{aligned} & \text{proc esReunionVálida? (in r: } reunion, \text{ in } prof \colon \mathbb{Z}, \text{ in } freq \colon \mathbb{Z}, \text{ out } result \text{: Bool}) \text{ } \\ & \text{Pre } \{prof > 0 \land freq > 0\} \\ & \text{Post } \{result = \text{true} \leftrightarrow esReunionVálidaAux}(r, prof, freq)\} \end{aligned} \begin{cases} & \text{Pred esReunionVálidaAux}(r: reunion, prof \colon \mathbb{Z}, freq \colon \mathbb{Z}) \text{ } \{ \\ & |r| > 0 \land \\ & \text{contieneSeñalesValidas}(r, prof, freq) \land \\ & \text{lasLongitudesDeSeñalSonIguales}(r) \land \\ & \text{losHablantesDistintos}(r) \land \\ & \text{losHablantesEstanEnRangosDeOANMenos1}(r) \end{cases} \end{cases}  \begin{cases} & \text{pred contieneSeñalesValidas}(r: reunion, prof \colon \mathbb{Z}, freq \colon \mathbb{Z}) \text{ } \{ (\forall i \colon \mathbb{Z}) \text{ } 0 \leq i < |r| \longrightarrow_L esSeñalAux}(r[i]_0, prof, freq) \} \end{aligned} \end{cases}  \texttt{pred lasLongitudesDeSeñalSonIguales}(r: reunion) \text{ } \{ (\forall i, j \colon \mathbb{Z}) \text{ } 0 \leq i, j < |r| \land i \neq j \longrightarrow_L (|r[i]_0| = |r[j]_0|) \}  \texttt{pred todosHablantesDistintos}(r: reunion) \text{ } \{ (\forall i, j \colon \mathbb{Z}) \text{ } 0 \leq i, j < |r| \land i \neq j \longrightarrow_L (|r[i]_1 \neq r[j]_1) \}  \texttt{pred losHablantesEstanEnRangosDeOANMenos1}(r: reunion) \text{ } \{ (\forall i \colon \mathbb{Z}) \text{ } 0 \leq i < |r| \longrightarrow_L 0 \leq r[i]_1 < |r| \}
```

```
proc acelerar (inout r: reunion, in prof: \mathbb{Z}, in freq: \mathbb{Z}) {
           Pre {
               r_0 = r \wedge
                  esReuni\'onV\'alidaAux(r,prof,freq) \land
                  lasSe\~{n}alesDuranM\'{a}sDe2Segundos(r)
           Post {
               |r| = |r_0| \wedge_L
                  lasSe\~nalesTieneLaMitadDeMuestras(r, r_0) \land_L
                  losImpares(r, r_0)
}
     pred lasSeñalesDuranMásDe2Segundos (r: reunion, freq: \mathbb{Z}) {
        (|r| > 0 \land freq \neq 0) \land_L duraci\'onEnSegundos(r[0]_0, freq) > 2
     pred lasSeñalesTieneLaMitadDeMuestras (r: reunion, r_v: reunion) {
        (\forall i: \mathbb{Z}) \ 0 \leq i < |r| \ \longrightarrow_L \text{if} \ esPar(|r_v[i]_0|) \ \text{then} \ \text{else} \ \ \text{fi}|r[i]_0| = \frac{|r_v[i]_0|}{2} \ |r[i]_0| = \frac{|r_v[i]_0|-1}{2}
     \verb|pred losImpares| (r: reunion, r_v: reunion) | \{ |
        (\forall i: \mathbb{Z}) \ 0 \leq i < |r_v| \longrightarrow_L (
            (\exists j : \mathbb{Z}) \ 0 \le j < |r| \ \land_L (r_v[i]_1 = r[j]_1) \land_L (
               (\forall q: \mathbb{Z}) \stackrel{\circ}{0} \leq q < |r_v[i]_0| \land (\neg esPar(q)) \stackrel{\longrightarrow}{\longrightarrow}_L (r_v[i]_0[q] = r[j]_0[\frac{q-1}{2}])))
     }
```

```
\begin{array}{lll} \operatorname{proc\ ralentizar\ (inout\ r:\ reunion,\ in\ prof:\ \mathbb{Z},\ in\ freq:\ \mathbb{Z})\ \left\{ & \operatorname{Pre}\ \{esReuni\acute{on}V\acute{a}lidaAux(r,prof,freq)\land r_0=r\} \\ & \operatorname{Post}\ \left\{ & |r| = |r_0| \land_L \\ & (lasSe\~{n}alesTienenElDobleDeMuestras(r,r_0) \land_L \\ & promedioEntrePares(r,r_0)) \\ & \} \\ \end{array} \right\} \\ \\ \operatorname{pred\ lasSe\~{n}alesTienenElDobleDeMuestras\ (r:\ reunion,\ r_v:\ reunion)\ \left\{ & (\forall i:\ \mathbb{Z})\ 0 \leq i < |r_v| \ \longrightarrow_L \ (2 \cdot |r_v[i]_0|) = (|r[i]_0|+1) \\ \} \\ \\ \operatorname{pred\ promedioEntrePares\ (r:\ reunion,\ r_v:\ reunion)\ \left\{ & (\forall i:\ \mathbb{Z})\ 0 \leq i < |r| \ \longrightarrow_L \ (\\ & (\exists j:\ \mathbb{Z})\ 0 \leq i < |r| \ \longrightarrow_L \ (\\ & (\exists j:\ \mathbb{Z})\ 0 \leq q < |r[i]_0| \ \longrightarrow_L \\ & \text{if\ } esPar(q)\ \text{then\ } r[i]_0[q] = r_v[j]_0[\frac{q}{2}]\ \text{else\ } r[i]_0[q] = \frac{r_v[j]_0[\frac{q-1}{2}] + r_v[j]_0[\frac{q+1}{2}]}{2}\ \text{fi\ })\ ) \\ \\ \end{array} \right\} \\ \end{aligned}
```

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proc tonosDeVozElevados (inout r: reunion, in freq: \mathbb{Z}, in prof: \mathbb{Z}, out hablantes: seq\langle hablante \rangle) {
                                       Pre \{esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freg)\}
                                      Post {
                                                   |h| \geq |hs| \wedge_L
                                                               (siPertenecenAHablantesElPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, hablantes) \land (siPertenecenAHablantesElPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, hablantes)) \land (siPertenecenAHablantesElPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, hablantesElPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, hablantesElPromedioDeAmplitudEsMasGrand
                                                             losHablantesPertenecenALaReuni\'on(r, hablantes) \land
                                                             losHablantesNoSeRepiten(hablantes))
 }
                 \verb|pred| siPertenecenAHablantesElPromedioDeAmplitudEsMasGrandeOIgualQueElResto| (r: reunion, hs: seq\langle hablante \rangle) | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reu
                             (\forall i : \mathbb{Z}) \ 0 \leq i < |hs| \longrightarrow_L
                                       (r[i]_1 \in hs \land elPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, r[i]_0))
                                       (r[i]_1 \notin hs \land \neg elPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, r[i]_0))
                  }
                 pred elPromedioDeAmplitudEsMasGrandeOIgualQueElResto (r: reunion, s: se\tilde{n}al) {
                             (\forall i : \mathbb{Z}) \ 0 \le i < |r| \longrightarrow_L (tonoDeVoz(s) \ge tonoDeVoz(r[i]_0))
                  \texttt{fun tonoDeVoz} \ (s: se\~{n}al) : \mathbb{Z} \ = sumaDelValorAbsolutoDeAmplitudes(s) \ div \ |s| \ ;
                 fun sumaDelValorAbsolutoDeAmplitudes (s: se\tilde{n}al): \mathbb{Z} = \sum_{i=0}^{|s|} abs(s[i]);
                  pred losHablantesPertenecenALaReunión (r: reunion, hs: seq\langle hablante \rangle) {
                             (\forall i : \mathbb{Z}) \ 0 \le i < |hs| \longrightarrow_L ((\exists j : \mathbb{Z}) \ 0 \le j < |r| \land_L (hs[i] = r[j]_1))
                 pred losHablantesNoSeRepiten (r: reunion, hs: seq\langle hablante\rangle) {
                             (\forall i : \mathbb{Z}) \ 0 \le i < |hs| \longrightarrow_L (\#apariciones(hs, hs[i]) = 1)
```

```
\begin{array}{l} \operatorname{proc \ ordenar \ (inout \ r: reunion, \ in \ freq: \mathbb{Z}, \ in \ prof: \mathbb{Z}) \ \{} \\ \operatorname{Pre} \ \{esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq) \land r_0 = r\} \\ \operatorname{Post} \ \{ \\ esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq) \land \\ ordenadaDeMayorAMenorPorTonoDeVoz(r) \land \\ esUnaPermutaci\acute{o}n(r_0,r) \\ \} \\ \} \\ \operatorname{pred} \ \operatorname{ordenadaDeMayorAMenorPorTonoDeVoz} \ (r: reunion) \ \{ \\ (\forall i: \mathbb{Z}) \ 1 \leq i < |r| \ \longrightarrow_L \ tonoDeVoz(r[i-1]_0) \geq tonoDeVoz(r[i]_0) \\ \} \\ \\ \operatorname{pred} \ \operatorname{esUnaPermutaci\acute{o}n} \ (x: reunion, \ y: reunion) \ \{ \\ |x| = |y| \land_L \\ (\forall i: \mathbb{Z}) \ 0 \leq i < |x| \ \longrightarrow_L ( \\ (\exists j: \mathbb{Z}) \ 0 \leq j < |y| \ \land_L ( \ x[i]_1 = y[j]_1 \land x[i]_0 = y[j]_0 ) ) \\ \} \\ \end{array}
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proc silencios (in s: se\~nal, in freq: \mathbb{Z}, in prof: \mathbb{Z}, out intervalos: seq\langle intervalo \rangle) {
          Pre \{esSe\tilde{n}alAux(s,prof,freq) \land (umbral > 0)\}
          Post {
             noHayIntervalosRepetidos(intervalos) \land
                (\forall i : \mathbb{Z}) \ 0 \leq i < |intervalos| \longrightarrow_L (esSilencio(s, umbral, freq, intervalos[i]))
}
    pred esSilencio (s: senal, umbral: Z, freq: Z, in: intervalo) {
       (finEsMayorQueInicio(in_0, in_1) \land estaDentroDeLaSe\~nal(s, in_0, in_1)) \land_L
          (esAlMenosUnDecimoDeSegundo(freq, in_0, in_1) \land
          entreIndicesNoPasaCiertoUmbral(s, umbral, in_0, in_1) \land
          losAdyacentesSuperanElUmbral(s, umbral, in_0, in_1))
    }
    pred noHayIntervalosRepetidos (ins: seg\langle intervalo \rangle) \{(\forall i : \mathbb{Z}) \ 0 \le i < |ins| \longrightarrow_L (\#apariciones(ins, i) = 1)\}
    pred finEsMayorQueInicio (inicio: \mathbb{Z}, fin: \mathbb{Z}) {fin > inicio}
    pred estaDentroDeLaSeñal (s: se\tilde{n}al, inicio: \mathbb{Z}, fin: \mathbb{Z},) \{(inicio \geq 0) \land (fin < |s|)\}
    \texttt{pred esAlMenosUnDecimoDeSegundo} \ (freq: \ \mathbb{Z}, \ inicio: \ \mathbb{Z}, \ fin: \ \mathbb{Z},) \ \{(fin-inicio+1) \geq (frecuencia*100)\}
    pred losAdyacentesSuperanElUmbral (s: se\tilde{n}al, umbral: \mathbb{Z}, inicio: \mathbb{Z}, fin: \mathbb{Z}) {
          (inicio = 0) \lor ((inicio - 1 \ge 0) \land_L (s[inicio - 1] \ge umbral))
          (\widehat{fin} = |s| - 1) \ \lor \ (\ (\ fin + 1 < |s|\ ) \ \land_L \ (\ s[fin + 1] \ge umbral\ )\ )
    }
    pred entreIndicesNoPasaCiertoUmbral (s: se\~nal, umbral: \mathbb{Z}, inicio: \mathbb{Z}, fin: \mathbb{Z}) {
       (\forall i : \mathbb{Z}) \ inicio \leq i < fin + 1 \longrightarrow_L (abs(s[i]) < umbral)
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 \begin{array}{l} \operatorname{proc\ hablantesSuperpuestos\ }(\operatorname{in\ }r: reunion, \operatorname{in\ }prof\colon \mathbb{Z}, \operatorname{in\ } mbral\colon \mathbb{Z}, \operatorname{out\ }result\colon \operatorname{Bool}) \ \ \\ \operatorname{Pre\ } \{esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq)\} \\ \operatorname{Post\ } \{result = \operatorname{true} \leftrightarrow \neg noHayHablantesSuperpuestos(r,freq,umbral)\} \\ \\ \operatorname{pred\ } \operatorname{noHayHablantesSuperpuestos\ }(r: reunion,freq\colon \mathbb{Z}, umbral\colon \mathbb{Z}) \ \ \\ (\forall i,j:\mathbb{Z}) \ 0 \leq i,j < |r| \ \land \ (i \neq j) \ \longrightarrow_L \\ (\forall k,l:\mathbb{Z}) \ 0 \leq k,l < |r[i]_0| \ \land \ k < l \ \longrightarrow_L \\ \neg haySilencio(subseq(r[i]_0,k,l),umbral,freq) \ \longrightarrow_L esSilencio(r[j]_0,umbral,freq,(k,l)) \ \ \\ \operatorname{pred\ } \operatorname{haySilencio\ }(s: se\~{n}al, umbral\colon \mathbb{Z}, freq\colon \mathbb{Z}) \ \{(\exists i,j:\mathbb{Z}) \ (0 \leq i,j < |s| \land (i < j)) \ \land_L \ esSilencio(s, umbral,(i,j)) \ \ \} \\ \end{array}
```

```
proc reconstruir (in s: se\tilde{n}al, in prof: \mathbb{Z}, in freq: \mathbb{Z}, out result: se\tilde{n}al) {
                          Pre \{esSe\~{n}alAux(s,prof,freq) \land tieneAlMenos2MuestrasDistintasDeCero(s)\}
                         Post {
                                 |s| = |result| \wedge_L (
                                        enDondeNoSeaCeroDebenCoincidir(s, result) \land
                                        enDondeEsCeroDebeSerElPromedioDeSusVecinosNoNulos(s, result))
 }
           pred tieneAlMenos2MuestrasDistintasDeCero (s: señal) \{(|s| - \#apariciones(s, 0)) \ge 2\}
           pred enDondeNoSeaCeroDebenCoincidir (original: se\tilde{n}al, reconstruida: se\tilde{n}al) {
                   (\forall i : \mathbb{Z}) \ 0 \le i < |original| \longrightarrow_L ((original[i] \ne 0) \longrightarrow (original[i] = reconstruida[i]))
           pred enDondeEsCeroDebeSerElPromedioDeSusVecinosNoNulos (original: se\~nal, reconstruida: se\~nal) {
                   (\forall i : \mathbb{Z}) \ 0 \leq i < |original| \longrightarrow_L ((original|i| = 0) \longrightarrow
                          esUnPromedioDeSusVecinosNoNulosMasCercanos(original, reconstruida, i))
           pred esUnPromedioDeSusVecinosNoNulosMasCercanos (original:se\tilde{n}al, reconstruida:se\tilde{n}al, i: \mathbb{Z}) {
                   (\exists j, k, l : \mathbb{Z}) \ 0 \le j, k, l < |original| \land (dist(j, i) < dist(i, k) \land dist(j, i) < dist(i, l)) \land_L (i, l) \land_L (i, 
                          (s[j] \neq 0 \land s[k] \neq 0 \land s[l] \neq 0) \land
                          ( (\forall m : \mathbb{Z}) \ 0 \leq m < |original| \land m \notin [i, j, k, l] \longrightarrow_L
                                        original[m] \neq 0 \longrightarrow
                                        (dist(i,m) \ge dist(i,j) \lor dist(i,m) \ge dist(i,k) \lor dist(i,m) \ge dist(i,l)) \land
                          (reconstruida[i] \in [promedio(original[j], original[k]), promedio(original[j], original[l])]))
           }
           fun dist (x: \mathbb{Z}, y: \mathbb{Z}) : \mathbb{Z} = abs(x - y);
           fun promedio (a: \mathbb{Z}, b: \mathbb{Z}): \mathbb{Z} = (a+b) \ div \ 2;
```