Ejercicio 9

a)

```
while(i >= (s.size() / 2)) do
    suma := suma + s[s.size()-1-i]
    i := i - 1;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \land \neg B) \longrightarrow Q_c$
- $\{I \land B\}$ ciclo $\{I\}$
- $\{I \land B \land (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \land f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv |s| \text{ mód } 2 = 0 \land i = |s| - 1 \land suma = 0$
- $Q_c \equiv |s| \text{ mód } 2 = 0 \land i = \frac{|s|}{2} - 1 \land_L suma = \sum_{j=0}^{\frac{|s|}{2}-1} s[j]$
- $I \equiv (|s| \text{ mód } 2 = 0 \land \frac{|s|}{2} - 1 \leq i \leq |s| - 1) \land_L suma = \sum_{j=0}^{|s|-i} s[j]$
- $B \equiv i \geq \frac{|s|}{2}$
- $S1 \equiv suma := suma + s[|s| - 1 - i];$
- $S2 \equiv i := i - 1;$
- $ciclo \equiv S1; S2;$
- $f_v \equiv i$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv$
$|s| \text{ mód } 2 = 0 \land i = |s| - 1 \land suma = 0 \longrightarrow (|s| \text{ mód } 2 = 0 \land \frac{|s|}{2} - 1 \leq i \leq |s| - 1) \land_L suma = \sum_{j=0}^{|s|-i} s[j] \equiv \text{true}$

$(I \land \neg B) \longrightarrow Q_c$

$(I \land \neg B) \longrightarrow Q_c \equiv$

- $(I \land \neg B) \equiv$

  $(|s| \text{ mód } 2 = 0 \land \frac{|s|}{2} - 1 \leq i \leq |s| - 1) \land_L suma = \sum_{j=0}^{|s|-i} s[j] \land \neg(i \geq \frac{|s|}{2}) \equiv$

  $(|s| \text{ mód } 2 = 0 \land \frac{|s|}{2} - 1 \leq i \leq |s| - 1) \land_L suma = \sum_{j=0}^{|s|-i} s[j] \land \frac{|s|}{2} > i \equiv$

  $(|s| \text{ mód } 2 = 0 \land \frac{|s|}{2} - 1 = i) \land_L suma = \sum_{j=0}^{|s|-i} s[j]$

$(I \land \neg B) \longrightarrow Q_c \equiv$
$(|s| \text{ mód } 2 = 0 \land \frac{|s|}{2} - 1 = i) \land_L suma = \sum_{j=0}^{|s|-i} s[j] \longrightarrow$
$|s| \text{ mód } 2 = 0 \land i = \frac{|s|}{2} - 1 \land_L suma = \sum_{j=0}^{\frac{|s|}{2}-1} s[j]$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1; S2, I) \stackrel{Ax3}{\equiv}$
$wp(S1, wp(S2, I))$

- $wp(S2; I) \equiv$

  $wp(i := i - 1; , (|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} - 1 \leq i \leq |s| - 1) \wedge_L suma = \sum_{j=0}^{|s|-i} s[j]) \stackrel{Ax1}{\equiv}$

  $\{(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} - 1 \leq i - 1 \leq |s| - 1) \wedge_L suma = \sum_{j=0}^{|s|-(i-1)+2} s[j]\}$

- $wp(S1, wp(S2, I)) \equiv$

  $wp(suma := suma + s[|s| - 1 - i]; , (|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} - 1 \leq i - 1 \leq |s| - 1) \wedge_L suma = \sum_{j=0}^{|s|-i+1} s[j]) \stackrel{Ax1}{\equiv}$

  $(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} - 1 \leq i - 1 \leq |s| - 1) \wedge_L suma + s[|s| - 1 - i] = \sum_{j=0}^{|s|-i+1} s[j]) \equiv$

  $(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} \leq i \leq |s|) \wedge_L suma + s[|s| - 1 - i] = \sum_{j=0}^{|s|-i+1} s[j])$

Qvq $I \wedge B \longrightarrow wp(S1; S2, I)$

- $I \wedge B \equiv$

  $(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} - 1 \leq i \leq |s| - 1) \wedge_L suma = \sum_{j=0}^{|s|-i} s[j] \wedge i \geq \frac{|s|}{2} \equiv$

  $(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} \leq i \leq |s| - 1) \wedge_L suma = \sum_{j=0}^{|s|-i} s[j]$

- $I \wedge B \longrightarrow wp(S1; S2, I) \equiv$

  $(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} \leq i \leq |s| - 1) \wedge_L suma = \sum_{j=0}^{|s|-i} s[j] \longrightarrow$

  $(|s| \text{ mód } 2 = 0 \wedge \frac{|s|}{2} \leq i \leq |s|) \wedge_L suma + s[|s| - 1 - i] = \sum_{j=0}^{|s|-i+1} s[j])$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1; S2, i < v_0) \stackrel{Ax3}{\equiv}$
$wp(S1, wp(S2, |s| - i < v_0))$

- $wp(S2, i < v_0) \equiv$

  $wp(i := i - 1, i < v_0)) \stackrel{Ax1}{\equiv}$
  $\{i - 1 < v_0)\}$

- $wp(S1, wp(S2, f_v < v_0)) \equiv$
  $wp(suma := suma + s[|s| - 1 - i], i - 1 < v_0) \equiv$
  $\{0 \leq |s| - 1 - i < |s| \wedge_L i - 1 < v_0\}$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0)$

$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0) \equiv$

$(I \wedge B \wedge (v_0 = i) \longrightarrow$
$0 \leq |s| - 1 - i < |s| \wedge_L i - 1 < v_0 \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$I \wedge i \leq 0 \longrightarrow i < \frac{|s|}{2} \equiv \text{true}$