Ejercicio 10

a)

```
i:= 0
while(i < s.size()) do
    if(s[i] = a)
        s[i] := b
    else
        skip
    endif;
    i:= i + 1;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv s = S_0 \wedge i = 0$
- $Q_c \equiv |s| = |S_0| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < |s|\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < |s|\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))$
- $I \equiv 0 \leq i \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))$
- $B \equiv i < |s|$
- $S1 \equiv$ if $s[i] = a$ then $s[i] := b$ else $skip$ fi
- $S2 \equiv i := i + 1$
- $ciclo \equiv S1; S2;$
- $f_v \equiv |s| - i$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv$
$s = S_0 \wedge i = 0 \longrightarrow 0 \leq i \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
$((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])) \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$

- $(I \wedge \neg B) \equiv$
  $0 \leq i \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])) \wedge i \geq |s| \longrightarrow$
  $|s| = |S_0| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < |s|\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < |s|\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$

$i = |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
$((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])) \longrightarrow$
$|s| = |S_0| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < |s|\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
$((\forall j : \mathbb{Z})\ (0 \leq j < |s|\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])) \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1; S2, I) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, I))$

- $wp(S2; I) \equiv$

  $wp(i := i + 1; , 0 \leq i \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))) \overset{Ax1}{\equiv}$

  $\{0 \leq i + 1 \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))\}$

- $wp(S1, wp(S2, I)) \equiv$

  $wp(\textsf{if}\ s[i] = a\ \textsf{then}\ s[i] := b\ \textsf{else}\ skip\ \textsf{fi}, 0 \leq i + 1 \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))) \overset{Ax4}{\equiv}$

  $(0 \leq i < 0) \wedge_L ((s[i] = a) \wedge wp(s[i] := b, 0 \leq i + 1 \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])))\vee$

  $(s[i] \neq a) \wedge wp(skip, 0 \leq i + 1 \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])))) \overset{Ax1, Ax2}{\equiv}$

  $(0 \leq i < 0) \wedge_L ((s[i] = a) \wedge (0 \leq i < 0) \wedge_L (0 \leq i + 1 \leq |setAt(s, i, b)| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a)$

  $\longrightarrow_L setAt(s, i, b)[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L setAt(s, i, b)[j] = S_0[j])))\vee$

  $(s[i] \neq a) \wedge 0 \leq i + 1 \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))) \equiv$

  $(0 \leq i < |s|) \wedge_L ($

  $(s[i] = a) \wedge ($

  $(\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] = a) \longrightarrow_L setAt(s, i, b)[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i + 1\ \wedge_L S_0[j] \neq a) \longrightarrow_L setAt(s, i, b)[j] = S_0[j])))\vee$

  $(s[i] \neq a) \wedge ($

  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))) \equiv$

Qvq $I \wedge B \longrightarrow wp(S1; S2, I)$

- $I \wedge B \equiv$

  $0 \leq i \leq |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])) \wedge i < |s| \equiv$

  $0 \leq i < |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$

  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))$

- $I \wedge B \longrightarrow wp(S1; S2, I) \equiv$

  $0 \leq i < |s| \wedge_L ((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j])) \longrightarrow$
  $(0 \leq i < |s|) \wedge_L ($
  $(s[i] = a) \wedge ($
  $(\forall j : \mathbb{Z})\ (0 \leq j < i+1\ \wedge_L S_0[j] = a) \longrightarrow_L setAt(s, i, b)[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < i+1\ \wedge_L S_0[j] \neq a) \longrightarrow_L setAt(s, i, b)[j] = S_0[j])))\vee$
  $(s[i] \neq a) \wedge ($
  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] = a) \longrightarrow_L s[j] = b))\wedge$
  $((\forall j : \mathbb{Z})\ (0 \leq j < i\ \wedge_L S_0[j] \neq a) \longrightarrow_L s[j] = S_0[j]))) \equiv \text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1; S2, f_v < v_0) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, |s| - i < v_0))$

- $wp(S2, i < v_0) \equiv$

  $wp(i := i+1, i < v_0)) \overset{Ax1}{\equiv}$

  $\{i + 1 < v_0)\}$

- $wp(S1, wp(S2, f_v < v_0)) \equiv$

  $wp(\textsf{if } s[i] = a \textsf{ then } s[i] := b \textsf{ else } skip \textsf{ fi}, i - 1 < v_0) \equiv$

  $\{0 \leq i < |s| \wedge_L i - 1 < v_0\}$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0)$

$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0) \equiv$

$(I \wedge B \wedge (v_0 = i) \longrightarrow$
$0 \leq i < |s| \wedge_L i - 1 < v_0 \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$I \wedge |s| - i \leq 0 \longrightarrow i \geq |s| \equiv \text{true}$