Ejercicio 8

a)

```
i:= d;
while(i < s.size()) do
    s[i] := e;
    i := i + 1;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \land \neg B) \longrightarrow Q_c$
- $\{I \land B\}$ ciclo $\{I\}$
- $\{I \land B \land (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \land f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv d \geq 0 \land d < |s| \land s = S_0 \land i = d$
- $Q_c \equiv |s| = |S_0| \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < |s| \ \longrightarrow_L s[j] = e))$
- $I \equiv (d \leq i \leq |s| \land |s| = |S_0|) \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e))$
- $B \equiv i < |s|$
- $S1 \equiv s[i] := e$
- $S2 \equiv i := i + 1;$
- $ciclo \equiv S1; S2;$
- $f_v \equiv |s| - i$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv$
$d \geq 0 \land d < |s| \land s = S_0 \land i = d \longrightarrow$
$(d \leq i \leq |s| \land |s| = |S_0|) \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e)) \equiv \text{true}$

$(I \land \neg B) \longrightarrow Q_c$

$(I \land \neg B) \longrightarrow Q_c \equiv$

$(d \leq i \leq |s| \land |s| = |S_0|) \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e)) \land i \geq |s| \longrightarrow$
$|s| = |S_0| \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < |s| \ \longrightarrow_L s[j] = e)) \equiv$

$(d \leq i = |s| \land |s| = |S_0|) \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e)) \longrightarrow$
$|s| = |S_0| \land_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \land ((\forall j : \mathbb{Z})\ d \leq j < |s| \ \longrightarrow_L s[j] = e)) \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1; S2, I) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, I))$

- $wp(S2; I) \equiv$

  $wp(i := i+1, (d \leq i \leq |s| \wedge |s| = |S_0|) \wedge_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \wedge ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e))) \overset{Ax1}{\equiv}$
  $\{(d \leq i+1 \leq |s| \wedge |s| = |S_0|) \wedge_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \wedge ((\forall j : \mathbb{Z})\ d \leq j < i+1 \ \longrightarrow_L s[j] = e))\}$

- $wp(S1, wp(S2, I)) \equiv$
  $wp(s[i] := e;, (d \leq i+1 \leq |s| \wedge |s| = |S_0|) \wedge_L ($

  $((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \wedge ((\forall j : \mathbb{Z})\ d \leq j < i+1 \ \longrightarrow_L s[j] = e))) \overset{Ax1}{\equiv}$

  $\{(0 \leq i < |s|) \wedge_L ($
  $(d \leq i+1 \leq |setAt(s,i,e)| \wedge |setAt(s,i,e)| = |S_0|) \wedge_L ($
  $((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L setAt(s,i,e)[j] = S_0[j]) \wedge ((\forall j : \mathbb{Z})\ d \leq j < i+1 \ \longrightarrow_L setAt(s,i,e)[j] = e))\} \equiv$

  $\{(d \leq i < |s|) \wedge_L$
  $(|setAt(s,i,e)| = |S_0|) \wedge_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L setAt(s,i,e)[j] = S_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ d \leq j < i+1 \ \longrightarrow_L setAt(s,i,e)[j] = e))\}$

Qvq $I \wedge B \longrightarrow wp(S1; S2, I)$

- $I \wedge B \equiv$
  $(d \leq i < |s| \wedge |s| = |S_0|) \wedge_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \wedge ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e)) \equiv$
- $I \wedge B \longrightarrow wp(S1; S2, I) \equiv$
  $(d \leq i < |s| \wedge |s| = |S_0|) \wedge_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L s[j] = S_0[j]) \wedge ((\forall j : \mathbb{Z})\ d \leq j < i \ \longrightarrow_L s[j] = e)) \longrightarrow$
  $(d \leq i < |s|) \wedge_L$
  $(|setAt(s,i,e)| = |S_0|) \wedge_L (((\forall j : \mathbb{Z})\ 0 \leq j < d \ \longrightarrow_L setAt(s,i,e)[j] = S_0[j]) \wedge$
  $((\forall j : \mathbb{Z})\ d \leq j < i+1 \ \longrightarrow_L setAt(s,i,e)[j] = e)) \equiv \text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1; S2, |s| - i < v_0) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, |s| - i < v_0)) \overset{Ax3}{\equiv}$

- $wp(S2, n - i < v_0) \equiv$

  $wp(i := i+1, |s| - i < v_0)) \overset{Ax1}{\equiv}$
  $\{|s| - i - 1 < v_0)\}$

- $wp(S1, wp(S2, f_v < v_0)) \equiv$

  $wp(r[i] := s[i], |s| - i - 1 < v_0) \overset{Ax4}{\equiv}$
  $\{(0 \leq i < |s|) \wedge_L |s| - i - 1 < v_0\}$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0)$

$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0) \equiv$

$(I \wedge B \wedge (v_0 = |s| - i) \longrightarrow$
$(0 \leq i < |s|) \wedge_L |s| - i - 1 < v_0 \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$(I \wedge |s| - i \leq 0 \longrightarrow i \geq |s| \equiv \text{true}$