Ejercicio 4

a)

```
i:= 1;
result:= 1;
while(i < n) do
    i:= i + 1;
    if n mod i = 0 then result:= result + i else 0 fi;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv n \geq 1 \wedge i = 1 \wedge result = 0$
- $Q_c \equiv result = \sum_{j=1}^{n} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}$
- $I \equiv 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}$
- $B \equiv i < n$
- $S1 \equiv i := i + 1$
- $S2 \equiv \text{if } n \text{ mód } i = 0 \text{ then } result := result + i; \text{ else } skip; \text{ fi};$
- $ciclo \equiv S1; S2;$
- $f_v \equiv n - i$

$P_c \longrightarrow I$

$n \geq 1 \wedge i = 1 \wedge result = 0 \longrightarrow P_c \longrightarrow I \equiv 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}$

- $n \geq 1 \wedge i = 1 \wedge result = 1 \longrightarrow 1 \leq i \leq n \equiv \text{true}$
- $n \geq 1 \wedge i = 1 \wedge result = 1 \longrightarrow result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } skip; \text{ fi} \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$
$0 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \wedge i > n \longrightarrow result = \sum_{j=1}^{n} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \equiv$
$i = n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \longrightarrow result = \sum_{j=1}^{n} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1;S2,I) \overset{Ax3}{\equiv}$
$wp(S1,wp(S2,I))$

- $wp(S2;I) \equiv$

  $wp(\text{if } n \text{ mód } i = 0 \text{ then } result := result + i \text{ else } skip;\ \text{fi};, 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}) \overset{Ax4}{\equiv}$

  $\{def(n \text{ mód } i = 0) \wedge_L ((n \text{ mód } i = 0) \wedge wp(result := result + i, 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}) \vee$

  $(n \text{ mód } i \neq 0) \wedge wp(skip;, 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}))\} \overset{Ax1,Ax2}{\equiv}$

  $\{(i \neq 0) \wedge_L ((n \text{ mód } i = 0) \wedge 1 \leq i \leq n \wedge result + i = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \vee$

  $(n \text{ mód } i \neq 0) \wedge 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi})\} \equiv$

- $wp(S1,wp(S2,I)) \equiv$

  $wp(i := i + 1, (i \neq 0) \wedge_L ((n \text{ mód } i = 0) \wedge 1 \leq i \leq n \wedge result + i = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \vee$

  $(n \text{ mód } i \neq 0) \wedge 1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi})) \overset{Ax1}{\equiv}$

  $(i + 1 \neq 0) \wedge_L ((n \text{ mód } i + 1 = 0) \wedge 1 \leq i + 1 \leq n \wedge result + i + 1 = \sum_{j=1}^{i+1} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \vee$

  $(n \text{ mód } i + 1 \neq 0) \wedge 1 \leq i + 1 \leq n \wedge result = \sum_{j=1}^{i+1} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}))$

Qvq $I \wedge B \longrightarrow wp(S1;S2,I)$

- $I \wedge B \equiv$

  $1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \wedge i < n \equiv$

  $1 \leq i < n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}$

- $I \wedge B \longrightarrow wp(S1;S2,I) \equiv$

  $1 \leq i < n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \longrightarrow$

  $(i + 1 \neq 0) \wedge_L ((n \text{ mód } i + 1 = 0) \wedge 1 \leq i + 1 \leq n \wedge result + i + 1 = \sum_{j=1}^{i+1} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \vee$

  $(n \text{ mód } i + 1 \neq 0) \wedge 1 \leq i + 1 \leq n \wedge result = \sum_{j=1}^{i+1} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}))$

- $1 \leq i < n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \longrightarrow (i + 1 \neq 0) \equiv \text{true}$

- $1 \leq i < n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \longrightarrow$

  $((n \text{ mód } i + 1 = 0) \wedge (1 \leq i + 1 \leq n \wedge result + i + 1 = \sum_{j=1}^{i+1} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi}) \vee$

  $(n \text{ mód } i + 1 \neq 0) \wedge (1 \leq i + 1 \leq n \wedge result = \sum_{j=1}^{i+1} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi})) \equiv \text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1;S2,n - i < v_0) \overset{Ax3}{\equiv}$
$wp(S1,wp(S2,n - i < v_0)) \overset{Ax3}{\equiv}$

- $wp(S2,n - i < v_0) \equiv$

  $wp(\text{if } n \text{ mód } i = 0 \text{ then } result := result + i;\ \text{else } skip;\ \text{fi};, n - i - 1 < v_0)) \overset{Ax4}{\equiv}$
  $\{(i \neq 0) \wedge_L ((n \text{ mód } i = 0) \wedge (n - i - 1 < v_0) \vee$
  $(n \text{ mód } i \neq 0) \wedge (n - i - 1 < v_0))\}$

- $wp(S1,wp(S2,f_v < v_0)) \equiv$

  $wp(i := i + 1, (i \neq 0) \wedge_L ((n \text{ mód } i = 0) \wedge (n - i - 1 < v_0) \vee (n \text{ mód } i \neq 0) \wedge (n - i - 1 < v_0))) \overset{Ax1}{\equiv}$
  $\{(i + 1 \neq 0) \wedge_L ((n \text{ mód } i + 1 = 0) \wedge (n - i - 2 < v_0) \vee (n \text{ mód } i + 1 \neq 0) \wedge (n - i - 2 < v_0))\}$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1;S2,f_v < v_0)$
$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1;S2,f_v < v_0) \equiv$
$1 \leq i < n \wedge result = \sum_{j=1}^{i} \text{if } n \text{ mód } j = 0 \text{ then } j \text{ else } 0 \text{ fi} \wedge (v_0 = n - i) \longrightarrow$
$(i + 1 \neq 0) \wedge_L ((n \text{ mód } i + 1 = 0) \wedge (n - i - 2 < v_0) \vee (n \text{ mód } i + 1 \neq 0) \wedge (n - i - 2 < v_0))$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \textsf{if } n \text{ mód } j = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge n - i \leq 0 \longrightarrow i \geq n$
$1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \textsf{if } n \text{ mód } j = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge n \leq i \longrightarrow i \geq n$
$1 \leq i \leq n \wedge result = \sum_{j=1}^{i} \textsf{if } n \text{ mód } j = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \wedge n \leq i \longrightarrow i \geq n \leftrightarrow$
$n = i \wedge result = \sum_{j=1}^{i} \textsf{if } n \text{ mód } j = 0 \textsf{ then } j \textsf{ else } 0 \textsf{ fi} \longrightarrow i \geq n \equiv \text{true}$