Ejercicio 14

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv |r| = |a| + |b| \wedge r = R_0 \wedge i := 0$
- $Q_c \equiv |r| = |R_0| \wedge (\forall j : \mathbb{Z})\ 0 \leq j < |a| \ \longrightarrow_L r[j] = a[j]$
- $I \equiv (0 \leq i \leq |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = a[j])$
- $B \equiv i < |a|$
- $S1 \equiv r[i] = a[i]$
- $S2 \equiv i := i + 1$
- $ciclo \equiv S1; S2$
- $f_v \equiv |a| - i$

$P_c \longrightarrow I$

$|r| = |a| + |b| \wedge r = R_0 \wedge i := 0 \longrightarrow (0 \leq i \leq |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = a[j]) \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(0 \leq i \leq |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = a[j]) \wedge i \geq |a| \longrightarrow$
$|r| = |R_0| \wedge (\forall j : \mathbb{Z})\ 0 \leq j < |a| \ \longrightarrow_L r[j] = a[j] \equiv$

$(i = |a|) \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = a[j]) \longrightarrow$
$|r| = |R_0| \wedge (\forall j : \mathbb{Z})\ 0 \leq j < |a| \ \longrightarrow_L r[j] = a[j] \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

- $\{I \wedge B\}$ ciclo $\{I\}$

  $wp(S1; S2; , I) \overset{Ax3}{\equiv} wp(S1, wp(S2, I))$

  - $wp(S2, I) \equiv wp(i := i + 1, I)$
    $\{(0 \leq i + 1 \leq |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i + 1 \ \longrightarrow_L r[j] = a[j])\}$
  - $wp(S2, wp(S2, I)) \equiv wp(r[i] = a[i], wp(S3, I)) \equiv$
    $\{(0 \leq i < |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i + 1 \ \longrightarrow_L setAt(r, i, a[i])[j] = a[j])\}$

$(0 \leq i \leq |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = a[j]) \wedge i < |a| \equiv$

$(0 \leq i < |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = a[j]) \longrightarrow$
$(0 \leq i < |a| \wedge |r| = |R_0| \wedge |r| = |a| + |b|) \wedge_L ((\forall j : \mathbb{Z})\ 0 \leq j < i + 1 \ \longrightarrow_L setAt(r, i, a[i])[j] = a[j])$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(i : i + 1, |a| - i < v_0) \equiv |a| - i - 1 < v_0$

$wp(r[i] = a[i], |a| - i - 1 < v_0) \equiv 0 \leq i < |a| \wedge |a| - i - 1 < v_0$

$I \wedge B \wedge v_0 = |a| - i \longrightarrow 0 \leq i < |a| \wedge |a| - i - 1 < v_0 \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$I \wedge |a| - i \leq 0 \longrightarrow i < |a| \equiv \text{true}$