Ejercicio 7

a)

```
i:= 0;
while(i < s.size()) do
    r[i] := s[i];
    i := i + 1;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv |s| = |r| \wedge r = r_0 \wedge i = 0$
- $Q_c \equiv |s| = |r| \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < |s| \ \longrightarrow_L s[j] = r[j]$
- $I \equiv ((0 \leq i \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = s[j]$
- $B \equiv i < |s|$
- $S1 \equiv r[i] := s[i]$
- $S2 \equiv i := i + 1;$
- $ciclo \equiv S1; S2;$
- $f_v \equiv |s| - i$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv$
$|s| = |r| \wedge r = r_0 \wedge i = 0 \longrightarrow ((0 \leq i \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[j] = s[j] \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$

$((0 \leq i \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[i] = s[i] \wedge i \geq |s| \longrightarrow$
$|s| = |r| \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < |s| \ \longrightarrow_L s[j] = r[j] \equiv$

$(i = |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i \ \longrightarrow_L r[i] = s[i] \longrightarrow$
$|s| = |r| \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < |s| \ \longrightarrow_L s[j] = r[j] \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1;S2,I) \overset{Ax3}{\equiv}$
$wp(S1,wp(S2,I))$

- $wp(S2;I) \equiv$

  $wp(i := i+1, ((0 \leq i \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i\ \longrightarrow_L r[j] = s[j]) \overset{Ax1}{\equiv}$
  $\{((0 \leq i+1 \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i+1\ \longrightarrow_L r[j] = s[j]\}$

- $wp(S1, wp(S2,I)) \equiv$

  $wp(r[i] := s[i];, ((0 \leq i+1 \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i+1\ \longrightarrow_L r[j] = s[j]) \overset{Ax1}{\equiv}$

  $\{(0 \leq i < |s| \wedge |r| = |s|) \wedge_L ($
  $((0 \leq i+1 \leq |s|) \wedge (|s| = |setAt(r,i,s[i])|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i+1\ \longrightarrow_L setAt(r,i,s[i])[j] = s[j])$

Qvq $I \wedge B \longrightarrow wp(S1;S2,I)$

- $I \wedge B \equiv$

  $((0 \leq i \leq |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i\ \longrightarrow_L r[j] = s[j] \wedge i < |s| \equiv$
  $((0 \leq i < |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i\ \longrightarrow_L r[j] = s[j]$

- $I \wedge B \longrightarrow wp(S1;S2,I) \equiv$

  $((0 \leq i < |s|) \wedge (|s| = |r|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i\ \longrightarrow_L r[j] = s[j] \longrightarrow$
  $((0 \leq i+1 \leq |s|) \wedge (|s| = |setAt(r,i,s[i])|)) \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < i+1\ \longrightarrow_L setAt(r,i,s[i])[j] = s[j]) \equiv \text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1;S2,|s| - i < v_0) \overset{Ax3}{\equiv}$
$wp(S1,wp(S2,|s| - i < v_0)) \overset{Ax3}{\equiv}$

- $wp(S2, n - i < v_0) \equiv$

  $wp(i := i+1, |s| - i < v_0)) \overset{Ax1}{\equiv}$
  $\{|s| - i - 1 < v_0)\}$

- $wp(S1, wp(S2, f_v < v_0)) \equiv$

  $wp(r[i] := s[i], |s| - i - 1 < v_0) \overset{Ax4}{\equiv}$
  $\{(0 \leq i < |s| \wedge |r| = |s|) \wedge_L |s| - i - 1 < v_0\}$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1;S2,f_v < v_0)$

$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1;S2,f_v < v_0) \equiv$

$(I \wedge B \wedge (v_0 = |s| - i) \longrightarrow$
$((0 \leq i < |s| \wedge |r| = |s|) \wedge_L |s| - i - 1 < v_0 \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$(I \wedge |s| - i \leq 0 \longrightarrow i \geq |s| \equiv \text{true}$