

TPE - Reuniones Remotas

Grupo 6 - Alrescate.com

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proc esSeñal (in s: seq\langle \mathbb{Z} \rangle, in prof: \mathbb{Z}, in freq: \mathbb{Z}, out result: \mathsf{Bool}) {
           Pre \{prof > 0 \land freq > 0\}
           Post \{result = true \leftrightarrow esSe\tilde{n}alAux(s, prof, freq)\}
}
     pred esSeñalAux (s: seq\langle \mathbb{Z} \rangle, prof: \mathbb{Z}, freq: \mathbb{Z}) {
        |s| \ge 0 \land
           frecuenciaEnRango(freq) \land
           profundidadCorrecta(s) \land
           duraMasDeUnSegundo(s, freq) \land
           ninguna Muestra Supera La Profundidad(s, prof)
     }
     pred frecuenciaEnRango (freq: \mathbb{Z}) {freq \in [8,32]}
     pred profundidadCorrecta (prof: \mathbb{Z}) \{ prof \in [8, 16, 32] \}
     \texttt{pred duraMasDeUnSegundo} \ (s: \ seq\langle \mathbb{Z}\rangle, \ freq: \mathbb{Z}) \ \{duraci\'onEnSegundos(s, freq) > 1\}
     pred ningunaMuestraSuperaLaProfundidad (s: seq\langle \mathbb{Z} \rangle, p: \mathbb{Z}) {
        (\forall i : \mathbb{Z}) \ 0 \le i < |s| \longrightarrow_L (-2)^{p-1} \le s[i] \le 2^{p-1} - 1
     fun duración
EnSegundos (s: seq\langle\mathbb{Z}\rangle, freq:\mathbb{Z}) : \mathbb{Z}=\frac{|s|}{(freq\cdot 1000)} ;
```

```
\begin{array}{l} \operatorname{proc\ seEnoj6?}\ (\operatorname{in\ s:\ se\~{n}al},\ \operatorname{in\ umbral:\ }\mathbb{Z},\ \operatorname{in\ prof:\ }\mathbb{Z},\ \operatorname{in\ freq:\ }\mathbb{Z},\ \operatorname{out\ result:\ Bool})\ \left\{ \begin{array}{l} \operatorname{Pre\ }\{umbral>0 \wedge esSe\~{n}alAux(s,prof,freq)\} \\ \operatorname{Post\ }\{ \\ result = true \leftrightarrow umbralEnRango(umbral,prof) \wedge \\ existeUnaSubsecuenciaQueSuperaUmbral(s,freq,umbral)\ \} \\ \end{array} \right\} \\ \operatorname{pred\ umbralEnRango}\ (umbral:\ \mathbb{Z},\ p:\ \mathbb{Z})\ \left\{ umbral \leq 2^{p-1}-1 \right\} \\ \operatorname{pred\ existeUnaSubsecuenciaQueSuperaUmbral}\ (s:\ se\~{n}al,\ freq:\ \mathbb{Z},\ umbral:\ \mathbb{Z})\ \left\{ (\exists d,h:\mathbb{Z})\ 0 \leq d,h < |s|+1 \ \wedge (h > (d+freq*1000*5)) \wedge_L \left( \\ (\forall i:\mathbb{Z})\ 0 \leq i < |subseq(s,d,h)| \ \longrightarrow_L \ abs(subseq(s,d,h)[i]) > umbral\ ) \\ \end{array} \right\} \\ \operatorname{fun\ abs\ }(x:\ \mathbb{Z}):\ \mathbb{Z}\ = \operatorname{if\ }x>0 \ \operatorname{then\ }x \ \operatorname{else\ }-x \ \operatorname{fi\ }; \end{array}
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 \begin{array}{l} \operatorname{proc \ esReunionValida? \ (in \ r: reunion, \ in \ prof: \ \mathbb{Z}, \ in \ freq: \ \mathbb{Z}, \ out \ result: \ Bool) \ \left\{ \\ \operatorname{Pre} \ \left\{ prof > 0 \land freq > 0 \right\} \\ \operatorname{Post} \ \left\{ result = \operatorname{true} \leftrightarrow \operatorname{esReunionValidaAux}(r, \operatorname{prof}, \operatorname{freq}) \right\} \\ \end{array} \right\} \\ \operatorname{Post} \ \left\{ result = \operatorname{true} \leftrightarrow \operatorname{esReunionValidaAux}(r, \operatorname{prof}, \operatorname{freq}) \right\} \\ \operatorname{pred} \ \operatorname{esReunionValidaAux}(r: \operatorname{reunion}, \operatorname{prof}: \ \mathbb{Z}, \operatorname{freq}: \ \mathbb{Z}) \ \left\{ \\ \operatorname{contieneSe\~nalesValidaS}(r, \operatorname{prof}, \operatorname{freq}) \land \\ \operatorname{lasLongitudesDeSe\~nalSonIguales}(r) \land \\ \operatorname{losHablantesEstanEnRangosDe0ANMenos1}(r) \\ \end{array} \right\} \\ \operatorname{pred} \ \operatorname{contieneSe\~nalesValidaS}(r: \operatorname{reunion}, \operatorname{prof}: \ \mathbb{Z}, \operatorname{freq}: \ \mathbb{Z}) \ \left\{ (\forall i: \ \mathbb{Z}) \ 0 \leq i < |r| \ \longrightarrow_L \operatorname{esSe\~nalAux}(r[i]_0, \operatorname{prof}, \operatorname{freq}) \right\} \\ \operatorname{pred} \ \operatorname{lasLongitudesDeSe\~nalSonIguales}(r: \operatorname{reunion}) \ \left\{ (\forall i: \ \mathbb{Z}) \ 0 \leq i, j < |r| \land i \neq j \ \longrightarrow_L (|r[i]_0| = |r[j]_0|) \right\} \\ \operatorname{pred} \ \operatorname{losHablantesEstanEnRangosDe0ANMenos1}(r: \operatorname{reunion}) \ \left\{ (\forall i: \ \mathbb{Z}) \ 0 \leq i < |r| \ \longrightarrow_L (r[i]_1 \neq r[j]_1) \right\} \\ \operatorname{pred} \ \operatorname{losHablantesEstanEnRangosDe0ANMenos1}(r: \operatorname{reunion}) \ \left\{ (\forall i: \ \mathbb{Z}) \ 0 \leq i < |r| \ \longrightarrow_L 0 \leq r[i]_1 < |r| \right\} \\ \end{array}
```

```
proc acelerar (inout r: reunion, in prof: \mathbb{Z}, in freq: \mathbb{Z}) {
           Pre {
               esReuni\'onV\'alidaAux(r,prof,freq) \land_L
                  lasSe\~{n}alesDuranM\'{a}sDe2Segundos(r) \land
                  r_0 = r
           Post {
              |r| = |r_0| \wedge_L
                  lasSe\~nalesTieneLaMitadDeMuestras(r, r_0) \land_L
                  losImpares(r, r_0)
}
     \texttt{pred lasSe\~nalesDuranM\'asDe2Segundos} \ (r: \ reunion, \ freq: \ \mathbb{Z}) \ \{duraci\'onEnSegundos(r[0]_0, freq) > 2\}
     pred lasSeñalesTieneLaMitadDeMuestras (r: reunion, r_v: reunion) {
        (\forall i: \mathbb{Z}) \ 0 \leq i < |r| \longrightarrow_L \text{ if } esPar(|r_v[i]_0|) \text{ then else } \operatorname{fi}|r[i]_0| = \frac{|r_v[i]_0|}{2} \ |r[i]_0| = \frac{|r_v[i]_0|-1}{2}
     pred losImpares (r: reunion, r_v: reunion) {
        (\forall i : \mathbb{Z}) \ 0 \le i < |r_v| \longrightarrow_L (
           (\exists j: \mathbb{Z}) \ 0 \le j < |r| \ \wedge_L (r_v[i]_1 = r[j]_1) \wedge_L (
              (\forall q: \mathbb{Z}) \ 0 \leq q < |r_v[i]_0| \land (\neg esPar(q)) \xrightarrow{\longrightarrow}_L (r_v[i]_0[q] = r[j]_0[\frac{q-1}{2}])))
     }
```

```
\begin{array}{ll} \operatorname{proc\ ralentizar\ (inout\ r:\ reunion,\ in\ prof:\ \mathbb{Z},\ in\ freq:\ \mathbb{Z})\ \left\{ \\ &\operatorname{Pre}\ \left\{ esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq)\land r_0=r\right\} \\ &\operatorname{Post}\ \left\{ \\ &|r|=|r_0|\land_L \\ &lasSe\~{n}alesTienenElDobleDeMuestras(r,r_0)\land_L \\ &promedioEntrePares(r,r_0) \\ &promedioEntrePares(r,r_0) \\ \right\} \\ \left\} \\ &\operatorname{pred\ lasSe\~{n}alesTienenElDobleDeMuestras\ (r:\ reunion,\ r_v:\ reunion)\ \left\{ \\ &(\forall i:\ \mathbb{Z})\ 0\leq i<|r_v|\longrightarrow_L (2\cdot|r_v[i]_0|)=(|r[i]_0|+1) \\ \right\} \\ &\operatorname{pred\ promedioEntrePares\ (r:\ reunion,\ r_v:\ reunion)\ \left\{ \\ &(\forall i:\ \mathbb{Z})\ 0\leq i<|r|\longrightarrow_L (\\ &(\exists j:\ \mathbb{Z})\ 0\leq j<|r_v|\land_L (\ r[i]_1=r_v[j]_1\ )\land_L (\\ &(\forall q:\ \mathbb{Z})\ 0\leq q<|r[i]_0|\longrightarrow_L \\ &\operatorname{if\ }esPar(q)\ \operatorname{then\ }\ r[i]_0[q]=r_v[j]_0[\frac{q}{2}]\ \operatorname{else\ }\ r[i]_0[q]=\frac{r_v[j]_0[\frac{q-1}{2}]+r_v[j]_0[\frac{q+1}{2}]}{2}\ \operatorname{fi\ })\ ) \\ &\operatorname{\}} \end{array}
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```
proc tonosDeVozElevados (inout r: reunion, in freq: \mathbb{Z}, in prof: \mathbb{Z}, out hablantes: seq\langle hablante \rangle) {
                          Pre \{esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freg)\}
                         Post {
                                 |h| \geq |hs| \wedge_L
                                         losHablantesPertenecenALaReuni\'on(r, hablantes) \land
                                        losHablantesNoSeRepiten(hablantes)
 }
           \verb|pred| siPertenecenAHablantesElPromedioDeAmplitudEsMasGrandeOIgualQueElResto| (r: reunion, hs: seq\langle hablante \rangle) | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reunion, hs: seq\langle hablante \rangle) \} | \{ (r: reu
                   (\forall i : \mathbb{Z}) \ 0 \leq i < |hs| \longrightarrow_L
                          (r[i]_1 \in hs \land elPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, r[i]_0))
                          (r[i]_1 \notin hs \land \neg elPromedioDeAmplitudEsMasGrandeOIgualQueElResto(r, r[i]_0))
            }
           pred elPromedioDeAmplitudEsMasGrandeOIgualQueElResto (r: reunion, s: se\tilde{n}al) {
                   (\forall i : \mathbb{Z}) \ 0 \le i < |r| \longrightarrow_L (tonoDeVoz(s) \ge tonoDeVoz(r[i]_0))
            fun tonoDeVoz (s: se\tilde{n}al): \mathbb{Z} = sumaDelValorAbsolutoDeAmplitudes(s) div |s|;
           fun sumaDelValorAbsolutoDeAmplitudes (s: se\tilde{n}al): \mathbb{Z} = \sum_{i=0}^{|s|} abs(s[i]);
            pred losHablantesPertenecenALaReunión (r: reunion, hs: seq\langle hablante \rangle) {
                   (\forall i : \mathbb{Z}) \ 0 \le i < |hs| \longrightarrow_L ((\exists j : \mathbb{Z}) \ 0 \le j < |r| \land_L (hs[i] = r[j]_1))
           pred losHablantesNoSeRepiten (r. reunion, hs. seq\langle hablante\rangle) {
                   (\forall i : \mathbb{Z}) \ 0 \le i < |hs| \longrightarrow_L (\#apariciones(hs, hs[i]) = 1)
```

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\begin{array}{l} \operatorname{proc \ ordenar \ (inout \ r: reunion, \ in \ freq: \mathbb{Z}, \ in \ prof: \mathbb{Z}) \ \{} \\ \operatorname{Pre} \ \{esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq) \land r_0 = r\} \\ \operatorname{Post} \ \{ \\ esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq) \land \\ ordenadaDeMayorAMenorPorTonoDeVoz(r) \land \\ esUnaPermutaci\acute{o}n(r_0,r) \\ \} \\ \} \\ \operatorname{pred} \ \operatorname{ordenadaDeMayorAMenorPorTonoDeVoz \ (r: reunion) \ \{} \\ (\forall i: \mathbb{Z}) \ 1 \leq i < |r| \ \longrightarrow_L \ tonoDeVoz(r[i-1]_0) \geq tonoDeVoz(r[i]_0) \\ \} \\ \\ \operatorname{pred} \ \operatorname{esUnaPermutaci\acute{o}n \ (x: reunion, \ y: reunion) \ \{} \\ |x| = |y| \land_L \\ (\forall i: \mathbb{Z}) \ 0 \leq i < |x| \ \longrightarrow_L \ (} \\ (\exists j: \mathbb{Z}) \ 0 \leq j < |y| \ \land_L \ (x[i]_1 = y[j]_1 \land x[i]_0 = y[j]_0 \ ) \ ) \\ \} \\ \end{array}
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proc silencios (in s: se\~nal, in freq: \mathbb{Z}, in prof: \mathbb{Z}, out intervalos: seq\langle intervalo \rangle) {
          Pre \{esSe\tilde{n}alAux(s,prof,freq) \land (umbral > 0)\}
          Post {
             noHayIntervalosRepetidos(intervalos) \land
                (\forall i : \mathbb{Z}) \ 0 \leq i < |intervalos| \longrightarrow_L (esSilencio(s, umbral, freq, intervalos[i]))
}
    pred esSilencio (s: senal, umbral: Z, freq: Z, in: intervalo) {
       (finEsMayorQueInicio(in_0, in_1) \land estaDentroDeLaSe\~nal(s, in_0, in_1)) \land_L
          esAlMenosUnDecimoDeSegundo(freq, in_0, in_1) \land
          entreIndicesNoPasaCiertoUmbral(s, umbral, in_0, in_1) \land
          losAdyacentesSuperanElUmbral(s, umbral, in_0, in_1)
    }
    pred noHayIntervalosRepetidos (ins: seg\langle intervalo \rangle) \{(\forall i : \mathbb{Z}) \ 0 \le i < |ins| \longrightarrow_L (\#apariciones(ins, e) = 1)\}
    pred finEsMayorQueInicio (inicio: \mathbb{Z}, fin: \mathbb{Z}) {fin > inicio}
    pred estaDentroDeLaSeñal (s: se\tilde{n}al, inicio: \mathbb{Z}, fin: \mathbb{Z},) \{(inicio \geq 0) \land (fin < |s|)\}
    \texttt{pred esAlMenosUnDecimoDeSegundo} \ (freq: \ \mathbb{Z}, \ inicio: \ \mathbb{Z}, \ fin: \ \mathbb{Z},) \ \{(fin-inicio+1) \geq (frecuencia*100)\}
    pred losAdyacentesSuperanElUmbral (s: se\tilde{n}al, umbral: \mathbb{Z}, inicio: \mathbb{Z}, fin: \mathbb{Z}) {
          (inicio = 0) \lor ((inicio - 1 \ge 0) \land_L (s[inicio - 1] \ge umbral))
          (\widehat{fin} = |s| - 1) \lor ((fin + 1 < |s|) \land_L (s[fin + 1] \ge umbral))
    }
    pred entreIndicesNoPasaCiertoUmbral (s: se\~nal, umbral: \mathbb{Z}, inicio: \mathbb{Z}, fin: \mathbb{Z}) {
       (\forall i : \mathbb{Z}) \ inicio \leq i < fin + 1 \longrightarrow_L (abs(s[i]) < umbral)
```

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 \begin{array}{l} \operatorname{proc\ hablantesSuperpuestos\ }(\operatorname{in\ }r: reunion, \operatorname{in\ }prof\colon \mathbb{Z}, \operatorname{in\ } mbral\colon \mathbb{Z}, \operatorname{out\ }result\colon \operatorname{Bool}) \ \ \\ \operatorname{Pre\ } \{esReuni\acute{o}nV\acute{a}lidaAux(r,prof,freq)\} \\ \operatorname{Post\ } \{result = \operatorname{true} \leftrightarrow \neg noHayHablantesSuperpuestos(r,freq,umbral)\} \ \\ \operatorname{pred\ } \operatorname{noHayHablantesSuperpuestos\ }(r: reunion,freq\colon \mathbb{Z}, umbral\colon \mathbb{Z}) \ \ \\ (\forall i,j:\mathbb{Z}) \ 0 \leq i,j < |r| \land (i \neq j) \longrightarrow_L \\ (\forall k,l:\mathbb{Z}) \ 0 \leq k,l < |r[i]_0| \ \land k < l \longrightarrow_L \\ \neg haySilencio(subseq(r[i]_0,k,l),umbral,freq) \longrightarrow_L esSilencio(r[j]_0,umbral,freq,(k,l)) \ \ \\ \operatorname{pred\ } \operatorname{haySilencio\ }(s: se\~{n}al,umbral\colon \mathbb{Z},freq\colon \mathbb{Z}) \ \{(\exists i,j:\mathbb{Z}) \ (0 \leq i,j < |s| \land (i < j)) \land_L esSilencio(s,umbral,(i,j)) \ \ \} \ \end{array}
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```
proc reconstruir (in s: se\tilde{n}al, in prof: \mathbb{Z}, in freq: \mathbb{Z}, out result: se\tilde{n}al) {
                          Pre \{esSe\~{n}alAux(s,prof,freq) \land tieneAlMenos2MuestrasDistintasDeCero(s)\}
                         Post {
                                 |s| = |result| \wedge_L (
                                        enDondeNoSeaCeroDebenCoincidir(s, result) \land_L
                                        enDondeEsCeroDebeSerElPromedioDeSusVecinosNoNulos(s, result))
 }
           pred tieneAlMenos2MuestrasDistintasDeCero (s: señal) \{(|s| - \#apariciones(s, 0)) \ge 2\}
           pred enDondeNoSeaCeroDebenCoincidir (original: se\tilde{n}al, reconstruida: se\tilde{n}al) {
                   (\forall i : \mathbb{Z}) \ 0 \le i < |original| \longrightarrow_L ((original[i] \ne 0) \longrightarrow (original[i] = reconstruida[i]))
           pred enDondeEsCeroDebeSerElPromedioDeSusVecinosNoNulos (original: se\~nal, reconstruida: se\~nal) {
                   (\forall i : \mathbb{Z}) \ 0 \leq i < |original| \longrightarrow_L ((original|i| = 0) \longrightarrow
                          esUnPromedioDeSusVecinosNoNulosMasCercanos(original, reconstruida, i))
           pred esUnPromedioDeSusVecinosNoNulosMasCercanos (original:se\tilde{n}al, reconstruida:se\tilde{n}al, i: \mathbb{Z}) {
                   (\exists j, k, l : \mathbb{Z}) \ 0 \le j, k, l < |original| \land (dist(j, i) < dist(i, k) \land dist(j, i) < dist(i, l)) \land_L (i, l) \land_L (i, 
                          (s[j] \neq 0 \land s[k] \neq 0 \land s[l] \neq 0) \land
                          ( (\forall m : \mathbb{Z}) \ 0 \leq m < |original| \land m \notin [i, j, k, l] \longrightarrow_L
                                        original[m] \neq 0 \longrightarrow
                                        (dist(i,m) \ge dist(i,j) \lor dist(i,m) \ge dist(i,k) \lor dist(i,m) \ge dist(i,l)) \land
                          (reconstruida[i] \in [promedio(original[j], original[k]), promedio(original[j], original[l])]))
           }
           fun dist (x: \mathbb{Z}, y: \mathbb{Z}) : \mathbb{Z} = abs(x - y);
           fun promedio (a: \mathbb{Z}, b: \mathbb{Z}): \mathbb{Z} = (a+b) \ div \ 2;
```