Ejercicio 5

a)

```
j:= 1;
result:= 0;
while(j < s.size()) do
    if j mod 2 = 1 then result:= result + s[j]; else skip; fi;
    j:= j + 1;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv j = 1 \wedge result = 0$
- $Q_c \equiv result = \sum_{i=0}^{|s|-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi}$
- $I \equiv 0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi}$
- $B \equiv j < |s|$
- $S1 \equiv \textsf{if } j \bmod 2 = 1 \textsf{ then } result := result + s[j]; \textsf{ else } skip; \textsf{ fi}$
- $S2 \equiv j := j + 1;$
- $ciclo \equiv S1; S2;$
- $f_v \equiv |s| - j$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv$
$j = 1 \wedge result = 0 \longrightarrow 0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi}$

- $j = 1 \wedge result = 0 \longrightarrow 0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi} \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$
$0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi} \wedge j \geq |s| \longrightarrow result = \sum_{i=0}^{|s|-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi} \equiv$
$j = |s| \wedge_L result = \sum_{i=0}^{j-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi} \longrightarrow result = \sum_{i=0}^{|s|-1} \textsf{if } i \bmod 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi} \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1; S2, I) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, I))$

- $wp(S2; I) \equiv$

  $wp(j := j+1, 0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}) \overset{Ax4}{\equiv}$

  $\{0 \leq j+1 \leq |s| \wedge_L result = \sum_{i=0}^{j+1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}\}$

- $wp(S1, wp(S2, I)) \equiv$

  $wp(\text{if } j \text{ mód } 2 = 1 \text{ then } result := result + s[j]; \text{ else } skip; \text{ fi}, 0 \leq j+1 \leq |s| \wedge_L result = \sum_{i=0}^{j} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}) \equiv$

  $\{((j \text{ mód } 2 = 1) \wedge wp(result := result + s[j], 0 \leq j+1 \leq |s| \wedge_L result = \sum_{i=0}^{j} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}) \vee$

  $(j \text{ mód } 2 \neq 1) \wedge wp(skip, 0 \leq j+2 \leq |s| \wedge_L result = \sum_{i=0}^{j} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}))\} \equiv$

  $\{((j \text{ mód } 2 = 1) \wedge 0 \leq j < |s| \wedge_L (0 \leq j+2 \leq |s| \wedge_L result + s[j] = \sum_{i=0}^{j} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}) \vee$

  $(j \text{ mód } 2 \neq 1) \wedge (0 \leq j+1 \leq |s| \wedge_L result = \sum_{i=0}^{j+1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}))\} \equiv$

  $\{((j \text{ mód } 2 = 1) \wedge (0 \leq j < |s| \wedge_L result + s[j] = \sum_{i=0}^{j+1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}) \vee$

  $(j \text{ mód } 2 \neq 1) \wedge (0 \leq j+1 \leq |s| \wedge_L result = \sum_{i=0}^{j+1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}))\}$

Qvq $I \wedge B \longrightarrow wp(S1; S2, I)$

- $I \wedge B \equiv$

  $0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi} \wedge j < |s| \equiv$

  $0 \leq j < |s| \wedge_L result = \sum_{i=0}^{j-1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi} \equiv$

- $I \wedge B \longrightarrow wp(S1; S2, I) \equiv$

  $0 \leq j < |s| \wedge_L result = \sum_{i=0}^{j-1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi} \longrightarrow$

  $((j \text{ mód } 2 = 1) \wedge (0 \leq j < |s| \wedge_L result + s[j] = \sum_{i=0}^{j} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi}) \vee$

  $(j \text{ mód } 2 \neq 1) \wedge (0 \leq j+1 \leq |s| \wedge_L result = \sum_{i=0}^{j} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi})) \equiv \text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1; S2, |s| - j < v_0) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, |s| - j < v_0)) \overset{Ax3}{\equiv}$

- $wp(S2, n - i < v_0) \equiv$

  $wp(j := j+1, |s| - j < v_0)) \overset{Ax1}{\equiv}$
  $\{|s| - j - 1 < v_0)\}$

- $wp(S1, wp(S2, f_v < v_0)) \equiv$

  $wp(\text{if } j \text{ mód } 2 = 1 \text{ then } result := result + s[j]; \text{ else } skip; \text{ fi}, |s| - j - 1 < v_0) \overset{Ax4}{\equiv}$
  $\{(j \text{ mód } 2 = 1) \wedge (0 \leq j < |s| \wedge_L |s| - j - 1 < v_0) \vee$
  $(j \text{ mód } 2 \neq 1) \wedge |s| - j - 1 < v_0\}$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0)$

$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0) \equiv$

$0 \leq j < |s| \wedge_L result = \sum_{i=0}^{j-1} \text{if } i \text{ mód } 2 = 1 \text{ then } s[i] \text{ else } 0 \text{ fi} \wedge (v_0 = |s| - j) \longrightarrow$
$((j \text{ mód } 2 = 1) \wedge (0 \leq j < |s| \wedge_L |s| - j - 1 < v_0)) \vee ((j \text{ mód } 2 \neq 1) \wedge ()|s| - j - 1 < v_0)) \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$0 \leq j \leq |s| \wedge_L result = \sum_{i=0}^{j-1} \textsf{if } i \text{ mód } 2 = 1 \textsf{ then } s[i] \textsf{ else } 0 \textsf{ fi} \wedge |s| - j \leq 0 \longrightarrow j \geq |s|$
$\equiv \text{true}$