Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Ejercicio 1

a) $P_c \equiv result = 0 \wedge i = 0$
$Q_c \equiv (result = \sum_{i=0}^{|s|-1} i) \wedge_L (i = |s|)$
b) Al demostrar $\{I \wedge B\}$ ciclo $\{I\}$
c) $(I \wedge \neg B) \longrightarrow Q_c$
d) $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
e y f)

Demostración

$P_c \equiv result = 0 \wedge i = 0$
$Q_c \equiv (result = \sum_{k=0}^{|s|-1} k) \wedge_L (i = |s|)$
$I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv result = 0 \wedge i = 0 \longrightarrow 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
$i = 0 \longrightarrow 0 \leq i \leq |s| \equiv 0 \leq 0 \leq |s| \wedge |s| \geq 0 \equiv \text{true}$
$result = 0 \wedge i = 0 \longrightarrow result = \sum_{j=0}^{i-1} s[j] \equiv$
$0 = \sum_{j=0}^{-1} s[j] \wedge \sum_{j=0}^{-1} s[j] = 0$
$0 = \sum_{j=0}^{-1} s[j] \equiv 0 = 0 \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge \neg(i < |s|) \longrightarrow (result = \sum_{k=0}^{|s|-1} k) \wedge_L (i = |s|)$

- $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge \neg(i < |s|)$

  $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge (i \geq |s|) \equiv$

  $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge (i \geq |s|) \equiv$

  $(i = |s|) \wedge_L result = \sum_{j=0}^{|s|-1} s[j]$

$(i = |s|) \wedge_L \sum_{j=0}^{|s|-1} s[j] \longrightarrow (result = \sum_{k=0}^{|s|-1} k) \wedge_L (i = |s|)$

$\{I \wedge B\} ciclo \{I\}$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S1; S2$

- $\{I \wedge B\} ciclo \{I\} \equiv$
  $\{(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|)\} S1; S2; \{(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j])\} \equiv$

  - $wp(S1; S2;, I) \stackrel{Ax3}{\equiv} wp(S1, wp(S2, I))$
    $wp(S2, I) \equiv wp(i := i + 1, 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \stackrel{Ax1}{\equiv}$
    $0 \leq i + 1 \leq |s| \wedge_L result = \sum_{j=0}^{i} s[j]$
  - $wp(S1, wp(S2, I)) \equiv$
    $wp(result := result + s[i];, 0 \leq i + 1 \leq |s| \wedge_L result = \sum_{j=0}^{i} s[j]) \stackrel{Ax1}{\equiv}$
    $def(result := result + s[i]) \wedge_L 0 \leq i + 1 \leq |s| \wedge_L result + s[i] = \sum_{j=0}^{i} s[j] \equiv$
    $0 \leq i < |s| \wedge_L 0 \leq i + 1 \leq |s| \wedge_L result + s[i] = \sum_{j=0}^{i} s[j] \equiv$
    $0 \leq i < |s| \wedge_L (-1) \leq i \leq |s| - 1 \wedge_L result = \sum_{j=0}^{i} s[j] - s[i] \equiv$
    $0 \leq i < |s| \wedge_L (-1) \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
    $0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
  - Qvq $I \wedge B \longrightarrow wp(ciclo, I)$
    - $I \wedge B \equiv (0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \equiv$
      $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j])$
    - $wp(ciclo, I) \equiv 0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$

    $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \longrightarrow (0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \equiv$
    true

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $f_v \equiv |s| - i$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S1; S2$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$

  - $wp(ciclo, f_v < v_0)$
    $wp(ciclo, f_v < v_0) \equiv wp(S1; S2, |s| - i < v_0) \stackrel{Ax3}{\equiv}$
    - $wp(S1, wp(S2, (|s| - i) < v_0))$
    - $wp(S2, (|s| - i) < v_0)$
      $wp(i := i + 1, (|s| - i) < v_0) \stackrel{Ax1}{\equiv}$
      $def(i + 1) \wedge_L (|s| - (i + 1)) < v_0 \equiv$
      $(|s| - i - 1) < v_0$
    - $wp(S1, (|s| - i + 1) < v_0)$
      $wp(S1, (|s| - i + 1) < v_0) \equiv$
      $wp(result := result + s[i], (|s| - i - 1) < v_0) \stackrel{Ax1}{\equiv}$
      $def(result + s[i]) \wedge_L (|s| - i - 1) < v_0 \equiv$
      $0 \leq i < |s| \wedge_L (|s| - i - 1) < v_0$
  - Qvq $I \wedge B \wedge (v_0 = f_v) \longrightarrow wp(ciclo, f_v < v_0)$
    $I \wedge B \wedge (v_0 = f_v) \longrightarrow wp(ciclo, f_v < v_0) \equiv$
    $(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \wedge (v_0 = |s| - i) \longrightarrow 0 \leq i < |s| \wedge_L (|s| - i + 1) < v_0 \equiv$
    $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (v_0 = |s| - i) \longrightarrow 0 \leq i < |s| \wedge_L |s| - i < v_0 + 1 \equiv$
    true

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $f_v \equiv |s| - i$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$
  $(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
  $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge\ |s| - i \leq 0 \longrightarrow \neg(i < |s|) \equiv$
  $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge\ |s| - i \leq 0 \longrightarrow i \geq |s| \equiv$
  $0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j] \wedge\ |s| \leq i \longrightarrow i \geq |s| \equiv \text{true}$

Cuerpo del ciclo invertido

$\{I \wedge B\}ciclo\{I\}$

- $I \equiv 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S2; S1$
- $\{I \wedge B\}ciclo\{I\} \equiv$
  $\{(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|)\}S2; S1; \{(0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j])\} \equiv$

  - $wp(S2; S1; , I) \stackrel{Ax3}{\equiv} wp(S2, wp(S1, I))$
    $wp(S1, I) \equiv wp(result := result + s[i], 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \stackrel{Ax1}{\equiv}$
    $def(result + s[i]) \wedge_L 0 \leq i \leq |s| \wedge_L result + s[i] = \sum_{j=0}^{i-1} s[j]) \equiv$
    $0 \leq i < |s| \wedge_L 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i} s[j])$
  - $wp(S2, wp(S1, I)) \equiv$
    $wp(i := i + 1; , 0 \leq i < |s| \wedge_L 0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i} s[j])) \stackrel{Ax1}{\equiv}$
    $0 \leq i < |s| \wedge_L 0 \leq i + 1 \leq |s| \wedge_L result = \sum_{j=0}^{i+1} s[j]) \equiv$
    $0 \leq i < |s| \wedge_L 0 < i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j]) \equiv$
    $0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j]) \equiv$
  - Qvq $I \wedge B \longrightarrow wp(ciclo, I)$
    - $I \wedge B \equiv (0 \leq i \leq |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \wedge (i < |s|) \equiv$
      $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j])$
    - $wp(ciclo, I) \equiv 0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j])$

    $(0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i-1} s[j]) \longrightarrow 0 \leq i < |s| \wedge_L result = \sum_{j=0}^{i+1} s[j]) \equiv$
    true

$\{I \land B \land (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

- $I \equiv 0 \leq i \leq |s| \land_L result = \sum_{j=0}^{i-1} s[j]$
- $B \equiv (i < |s|)$
- $f_v \equiv |s| - i$
- $S1 : result := result + s[i];$
- $S2 : i := i + 1;$
- $ciclo \equiv S2; S1$
- $\{I \land B \land (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
  - $wp(ciclo, f_v < v_0)$
    $wp(ciclo, f_v < v_0) \equiv wp(S2; S1, |s| - i < v_0) \overset{Ax3}{\equiv}$
    - $wp(S2, wp(S1, (|s| - i) < v_0))$
    - $wp(S1, (|s| - i) < v_0)$
      $wp(result := resutl + s[i], (|s| - i) < v_0) \overset{Ax1}{\equiv}$
      $def(resutl + s[i]) \land_L (|s| - i) < v_0 \equiv$
      $0 \leq i < |s| \land_L (|s| - i) < v_0$
    - $wp(S2, 0 \leq i < |s| \land_L (|s| - i) < v_0)$
      $wp(S2, 0 \leq i < |s| \land_L (|s| - i) < v_0) \equiv$
      $wp(i := i + 1, 0 \leq i < |s| \land_L (|s| - i) < v_0) \overset{Ax1}{\equiv}$
      $0 \leq i + 1 < |s| \land_L (|s| - i - 1) < v_0 \equiv$
      $0 \leq i + 1 < |s| \land_L (|s| - i - 1) < v_0$
  - Qvq $I \land B \land (v_0 = f_v) \longrightarrow wp(ciclo, f_v < v_0)$
    $I \land B \land (v_0 = f_v) \longrightarrow wp(ciclo, f_v < v_0) \equiv$
    $(0 \leq i \leq |s| \land_L result = \sum_{j=0}^{i-1} s[j]) \land (i < |s|) \land (v_0 = |s| - i) \longrightarrow 0 \leq i + 1 < |s| \land_L (|s| - i - 1) < v_0 \equiv$
    $(0 \leq i < |s| \land_L result = \sum_{j=0}^{i-1} s[j]) \land (v_0 = |s| - i) \longrightarrow 0 \leq i + 1 < |s| \land_L (|s| - i - 1) < v_0 \equiv$
    true