

# Ejercicio 11

## Correctitud

- $Pre \longrightarrow wp(\text{codigo previo al ciclo}, P_c)$
- $P_c \longrightarrow wp(\text{ciclo}, Q_c)$
- $Q_c \longrightarrow wp(\text{codigo posterior al ciclo}, Post)$

Por monotonía sabemos que  $Pre \longrightarrow wp(\text{programa completo}, Post)$

## Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$  ciclo  $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$  ciclo  $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

## Demostración

### Datos

- $Pre \equiv \text{true}$
- $Post \equiv r = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq r < |s| \wedge_L s[r] = e)$
- $P_c \equiv i = (|s| - 1) \wedge j = -1$
- $Q_c \equiv j = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e)$
- $I \equiv 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i + 1 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e)$
- $B \equiv i \geq 0$
- $S1 \equiv \text{if } s[i] = e \text{ then } j := i \text{ else skip fi}$
- $S2 \equiv i := i - 1$
- $\text{ciclo} \equiv S1; S2;$
- $f_v \equiv i$

$Pre \longrightarrow wp(\text{codigo previo al ciclo}, P_c)$

$wp(\text{codigo previo al ciclo}, P_c) \equiv$   
 $wp(i := |s| - 1; j := -1, i = (|s| - 1) \wedge j = -1) \stackrel{Ax3}{\equiv}$   
 $wp(i := |s| - 1, wp(j := -1, i = (|s| - 1) \wedge j = -1))$

- $wp(j := -1, i = (|s| - 1) \wedge j = -1) \stackrel{Ax1}{\equiv}$   
 $\{i = (|s| - 1) \wedge -1 = -1\}$

$wp(i := |s| - 1, i = (|s| - 1) \wedge -1 = -1) \stackrel{Ax1}{\equiv}$   
 $|s| - 1 = |s| - 1 \wedge -1 = -1 \equiv \text{true}$   
 $Pre \longrightarrow wp(\text{codigo previo al ciclo}, P_c) \equiv$   
 $\text{true} \longrightarrow \text{true} \equiv \text{true}$

$$Q_c \longrightarrow wp(\text{codigo posterior al ciclo}, Post)$$

$$\begin{aligned} & wp(\text{codigo posterior al ciclo}, Post) \equiv \\ & wp(r := j, r = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq r < |s| \wedge_L s[r] = e)) \equiv \\ & \{j = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e)\} \\ & Q_c \longrightarrow wp(\text{codigo posterior al ciclo}, Post) \equiv \\ & j = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \longrightarrow \\ & j = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge r \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \equiv \text{true} \end{aligned}$$

$$P_c \longrightarrow wp(\text{ciclo}, Q_c)$$

$$\begin{aligned} & \blacksquare P_c \longrightarrow I \ i = (|s| - 1) \wedge j = -1 \longrightarrow \\ & \quad 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i + 1 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e) \equiv \text{true} \\ & \blacksquare (I \wedge \neg B) \longrightarrow Q_c \\ & \quad 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i + 1 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e) \wedge i < 0 \longrightarrow \\ & \quad j = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \equiv \\ & \quad \text{false} \longrightarrow j = -1 \longrightarrow (\forall j : \mathbb{Z}) (0 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (0 \leq j < |s| \wedge_L s[j] = e) \equiv \text{true} \\ & \blacksquare \{I \wedge B\} \text{ ciclo } \{I\} \\ & \quad wp(i : i - 1, 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i + 1 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e)) \equiv \\ & \quad 0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e) \\ & \quad wp(\text{if } s[i] = e \text{ then } j := i \text{ else skip fi}, 0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow \\ & \quad (i - 1 < j < |s| \wedge_L s[j] = e)) \stackrel{Ax4}{\equiv} \\ & \quad 0 \leq i < |s| \wedge_L ( \\ & \quad (s[i] = e) \wedge (0 \leq i - 1 < |s| \wedge_L i = -1 \longrightarrow (\forall i : \mathbb{Z}) (i \leq i < |s| \longrightarrow_L s[i] \neq e) \wedge i \neq -1 \longrightarrow (i - 1 < i < |s| \wedge_L s[i] = e)) \vee \\ & \quad (s[i] \neq e) \wedge (0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e))) \\ & \quad 0 \leq i < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i + 1 \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i < j < |s| \wedge_L s[j] = e) \longrightarrow \\ & \quad 0 \leq i < |s| \wedge_L ( \\ & \quad (s[i] = e) \longrightarrow (i - 1 < i < |s| \wedge_L s[i] = e)) \vee \\ & \quad (s[i] \neq e) \wedge (0 \leq i - 1 < |s| \wedge_L j = -1 \longrightarrow (\forall j : \mathbb{Z}) (i \leq j < |s| \longrightarrow_L s[j] \neq e) \wedge j \neq -1 \longrightarrow (i - 1 < j < |s| \wedge_L s[j] = e))) \equiv \text{true} \\ & \blacksquare \{I \wedge B \wedge (v_0 = f_v)\} \text{ ciclo } \{f_v < v_0\} \\ & \quad wp(i : i - 1, i < v_0) \equiv i - 1 < v_0 \\ & \quad wp(\text{if } s[i] = e \text{ then } j := i \text{ else skip fi}, i - 1) \equiv 0 \leq i < |s| \wedge_L i - 1 < v_0 \\ & \quad I \wedge B \wedge v_0 = i \longrightarrow 0 \leq i < |s| \wedge_L i - 1 < v_0 \equiv \text{true} \\ & \blacksquare (I \wedge f_v \leq 0) \longrightarrow \neg B \\ & \quad \text{false} \longrightarrow i < 0 \equiv \text{true} \end{aligned}$$