Ejercicio 6

a)

```
i:= 0;
j:= 1;
result:= 0;
while(j < s.size()) do
    if (s[j] > s[i])
        i:= j;
    else
        skip;
    endif
j:= j + 1;
endwhile
```

Teorema del invariante

- $P_c \longrightarrow I$
- $(I \wedge \neg B) \longrightarrow Q_c$
- $\{I \wedge B\}$ ciclo $\{I\}$
- $\{I \wedge B \wedge (v_0 = f_v)\}$ ciclo $\{f_v < v_0\}$
- $(I \wedge f_v \leq 0) \longrightarrow \neg B$

Demostración

Datos

- $P_c \equiv |s| \geq 1 \wedge i = 0 \wedge j = 1$
- $Q_c \equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < |s| \ \longrightarrow_L s[j] \leq s[i]$
- $I \equiv (0 \leq i < |s| \wedge 0 \leq j \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j \ \longrightarrow_L s[k] \leq s[i]$
- $B \equiv j < |s|$
- $S1 \equiv \textsf{if } s[j] > s[i] \textsf{ then } i := j;\ \textsf{else } skip;\ \textsf{fi}$
- $S2 \equiv j := j + 1;$
- $ciclo \equiv S1; S2;$
- $f_v \equiv |s| - j$

$P_c \longrightarrow I$

$P_c \longrightarrow I \equiv$
$|s| \geq 1 \wedge i = 0 \wedge j = 1 \longrightarrow (0 \leq i < |s| \wedge 0 \leq j \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j \ \longrightarrow_L s[k] \leq s[i] \equiv \text{true}$

$(I \wedge \neg B) \longrightarrow Q_c$

$(I \wedge \neg B) \longrightarrow Q_c \equiv$
$(0 \leq i < |s| \wedge 0 \leq j \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j \ \longrightarrow_L s[k] \leq s[i] \wedge j \geq |s| \longrightarrow$
$0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < |s| \ \longrightarrow_L s[j] \leq s[i] \equiv$
$(0 \leq i < |s| \wedge j = |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j \ \longrightarrow_L s[k] \leq s[i] \longrightarrow$
$0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})\ 0 \leq j < |s| \ \longrightarrow_L s[j] \leq s[i] \equiv \text{true}$

$\{I \wedge B\}$ **ciclo** $\{I\}$

$wp(S1; S2, I) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, I))$

- $wp(S2; I) \equiv$

  $wp(j := j+1, (0 \leq i < |s| \wedge 0 \leq j \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j\ \longrightarrow_L s[k] \leq s[i]) \overset{Ax1}{\equiv}$
  $\{(0 \leq i < |s| \wedge 0 \leq j+1 \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[i]\}$

- $wp(S1, wp(S2, I)) \equiv$

  $wp(\text{if } s[j] > s[i] \text{ then } i := j;\ \text{else } skip;\ \text{fi}, (0 \leq i < |s| \wedge 0 \leq j+1 \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[i]) \overset{Ax4}{\equiv}$

  $\{(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L ($
  $((s[j] > s[i]) \wedge (0 \leq j < |s| \wedge 0 \leq j+1 \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[j]) \vee$
  $((s[j] \leq s[i]) \wedge ((0 \leq i < |s| \wedge 0 \leq j+1 \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[i])))\} \equiv$

  $\{(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L ($
  $((s[j] > s[i]) \wedge (0 \leq j < |s| \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[j]) \vee$
  $((s[j] \leq s[i]) \wedge ((0 \leq i < |s| \wedge 0 \leq j+1 \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[i])))\}$

Qvq $I \wedge B \longrightarrow wp(S1; S2, I)$

- $I \wedge B \equiv$
  $(0 \leq i < |s| \wedge 0 \leq j \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j\ \longrightarrow_L s[k] \leq s[i] \wedge j < |s| \equiv$
  $(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j\ \longrightarrow_L s[k] \leq s[i]$

- $I \wedge B \longrightarrow wp(S1; S2, I) \equiv$
  $(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j\ \longrightarrow_L s[k] \leq s[i] \longrightarrow$
  $(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L ($
  $((s[j] > s[i]) \wedge (0 \leq j < |s| \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[j]) \vee$
  $((s[j] \leq s[i]) \wedge ((0 \leq i < |s| \wedge 0 \leq j+1 \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j+1\ \longrightarrow_L s[k] \leq s[i]))) \equiv \text{true}$

$\{I \wedge B \wedge (v_0 = f_v)\}$ **ciclo** $\{f_v < v_0\}$

$wp(S1; S2, |s| - j < v_0) \overset{Ax3}{\equiv}$
$wp(S1, wp(S2, |s| - j < v_0)) \overset{Ax3}{\equiv}$

- $wp(S2, n - i < v_0) \equiv$

  $wp(j := j+1, |s| - j < v_0)) \overset{Ax1}{\equiv}$
  $\{|s| - j - 1 < v_0)\}$

- $wp(S1, wp(S2, f_v < v_0)) \equiv$

  $wp(\text{if } s[j] > s[i] \text{ then } i := j;\ \text{else } skip;\ \text{fi}, |s| - j - 1 < v_0) \overset{Ax4}{\equiv}$
  $\{(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L ($
  $(s[j] > s[i] \wedge |s| - j - 1 < v_0) \vee$
  $(s[j] \geq s[i] \wedge |s| - j - 1 < v_0)$

Qvq $(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0)$

$(I \wedge B \wedge (v_0 = f_v)) \longrightarrow wp(S1; S2, f_v < v_0) \equiv$

$(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j\ \longrightarrow_L s[k] \leq s[i] \wedge (v_0 = |s| - j) \longrightarrow$
$(0 \leq i < |s| \wedge 0 \leq j < |s|) \wedge_L |s| - j - 1 < v_0 \equiv \text{true}$

$(I \wedge f_v \leq 0) \longrightarrow \neg B$

$(I \wedge f_v \leq 0) \longrightarrow \neg B \equiv$
$(0 \leq i < |s| \wedge 0 \leq j \leq |s|) \wedge_L (\forall k : \mathbb{Z})\ 0 \leq k < j \ \longrightarrow_L s[k] \leq s[i] \wedge |s| - j \leq 0 \longrightarrow j \geq |s|$
$\equiv$ true