Corrección de ciclos

Ejercicio 7 (Guía 5)

Especificación e implementación

proc copiarSecuencia (in s: $seq\langle\mathbb{Z}\rangle$, inout r: $seq\langle\mathbb{Z}\rangle$) {

Pre $\{|s| = |r| \wedge r = r_0\}$

Post $\{|s| = |r| \wedge_L (\forall j :\mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] = r[j])\}$

}

```
i := 0;
while (i < s.size()) do
  r[i]:=s[i];
  i:=i+1
endwhile
```

Especificación del ciclo

```
i := 0;
while (i < s.size()) do
  r[i]:=s[i];
  i:=i+1
endwhile
```

$$Q_c : i = |s| \wedge |s| = |r| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] = r[j])$$

$$P_c : i = 0 \wedge |s| = |r| \wedge r = r_0$$

$$I : 0 \leq i \leq |s| \wedge |r_0| = |r| \wedge |s| = |r| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j])$$

$$\wedge (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$$

$$f_v : n - i$$

$$\boxed{P_c \Rightarrow I}$$

$P_c : i = 0 \wedge |s| = |r| \wedge r = r_0$

$I : \boxed{0 \leq i \leq |s|} \wedge |r_0| = |r| \wedge |s| = |r| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j]) \wedge (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$

$$i = 0 \Rightarrow i \geq 0$$

$$P_c \Rightarrow I$$

$P_c : i = 0 \wedge |s| = |r| \wedge r = r_0$

$I : 0 \leq i \leq |s| \wedge \boxed{|r_0| = |r| \wedge |s| = |r|} \wedge_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j]) \wedge (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$

$$\boxed{P_c \Rightarrow I}$$

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$I : 0 \leq i \leq |s| \wedge |r_0| = |r| \wedge |s| = |r| \wedge_L \boxed{(\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j])} \wedge (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$

$$(\forall j : \mathbb{Z})(\underbrace{\underbrace{0 \leq j < 0}_{\text{False}} \rightarrow_L r[j] = s[j]}_{\text{True}})$$

$$\boxed{P_c \Rightarrow I}$$

$P_c : i = 0 \wedge |s| = |r| \wedge \boxed{r = r_0}$

$I : 0 \leq i \leq |s| \wedge |r_0| = |r| \wedge |s| = |r| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j]) \wedge \boxed{(\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])}$

$$(\forall j : \mathbb{Z})(0 \leq j < |r| \rightarrow_L r[j] = s[j])$$

$$\boxed{(I \wedge \neg B) \Rightarrow Q_c}$$

$I : 0 \leq i \leq |s| \wedge |r_0| = |r| \wedge |s| = |r| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j]) \wedge (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$

$\neg B : i \geq |s|$

$Q_c : \boxed{i = |s|} \wedge |s| = |r| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] = r[j])$

$$\left.\begin{array}{l} i \leq |s| \\ \\ i \geq |s| \end{array}\right\} \quad i = |s|$$

$$\boxed{(I \wedge \neg B) \Rightarrow Q_c}$$

$I : 0 \le i \le |s| \wedge |r_0| = |r| \wedge \boxed{|s| = |r|} \wedge_L (\forall j : \mathbb{Z})(0 \le j < i \rightarrow_L r[j] = s[j]) \wedge (\forall j : \mathbb{Z})(i \le j < |r| \rightarrow_L r[j] = r_0[j])$

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$I : 0 \leq i \leq |s| \wedge |r_0| = |r| \wedge |s| = |r| \wedge_L \boxed{(\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j])} \wedge (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$

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$$\{I \wedge B\}\ \textbf{ciclo}\ \{I\}$$

$$(I \wedge B) \rightarrow wp(ciclo, I)$$

```
i := 0;
while (i < s.size()) do
S1 r[i]:=s[i];
S2 i:=i+1
endwhile
```

$$\{I \land B\}\ \textbf{ciclo}\ \{I\}$$

```
i := 0;
while (i < s.size()) do
S1 r[i]:=s[i];
S2 i:=i+1
endwhile
```

$$wp(S1;S2,I) \stackrel{Ax3}{\equiv} wp(S1, wp(S2,I))$$

$$wp(S2,I) \stackrel{Ax1}{\equiv} def(i+1) \land_L 0 \leq i+1 \leq |s| \land |r_0| = |r| \land$$
$$|s| = |r| \land_L (\forall j:\mathbb{Z})(0 \leq j < i+1 \rightarrow_L r[j] = s[j]) \land (\forall j:\mathbb{Z})(i+1 \leq j < |r| \rightarrow_L r[j] = r_0[j])$$

$$\equiv 0 \leq i+1 \leq |s| \land |r_0| = |r| \land$$
$$|s| = |r| \land_L (\forall j:\mathbb{Z})(0 \leq j < i+1 \rightarrow_L r[j] = s[j]) \land (\forall j:\mathbb{Z})(i+1 \leq j < |r| \rightarrow_L r[j] = r_0[j])$$

$$\boxed{\{I \wedge B\} \textbf{ ciclo } \{I\}}$$

```
i := 0;
while (i < s.size()) do
S1  r[i]:=s[i];
S2  i:=i+1
endwhile
```

$$wp(S1; S2, I) \stackrel{Ax3}{\equiv} wp(S1, wp(S2, I))$$

```
S1  r := setAt(r, i, s[i])
```

$$wp(S1; S2, I) \stackrel{Ax3}{\equiv} wp(r := setAt(r, i, s[i]), wp(S2, I))$$

$$\stackrel{Ax1}{\equiv} def(setAt(r, i, s[i]) \wedge_L wp(S2, I)^{r}_{setAt(r,i,s[i])}$$

$$\{I \wedge B\} \textbf{ ciclo } \{I\}$$

$$wp(S1;S2,I) \equiv \overbrace{0 \le i < |r| \wedge 0 \le i < |s|}^{def(setAt(r,i,s[i])} \wedge_L 0 \le i+1 \le |s| \wedge |r_0| = \boxed{|setAt(r,i,s[i])|} \wedge$$
$$|s| = \boxed{|setAt(r,i,s[i])|} \wedge_L (\forall j : \mathbb{Z})(0 \le j < i+1 \rightarrow_L \boxed{setAt(r,i,s[i])}[j] = s[j]) \wedge$$
$$(\forall j : \mathbb{Z})(i+1 \le j < |r| \rightarrow_L \boxed{setAt(r,i,s[i])}[j] = r_0[j])$$

$$|setAt(r,i,s[i])| = |r|$$

$$wp(S1;S2,I) \equiv 0 \le i < |r| \wedge 0 \le i < |s| \wedge_L 0 \le i+1 \le |s| \wedge \boxed{|r_0| = |r|} \wedge$$
$$\boxed{|s| = |r|} \wedge_L (\forall j : \mathbb{Z})(0 \le j < i+1 \rightarrow_L setAt(r,i,s[i])[j] = s[j]) \wedge$$
$$(\forall j : \mathbb{Z})(i+1 \le j < |r| \rightarrow_L setAt(r,i,s[i])[j] = r_0[j])$$

$$\{I \land B\}\ \textbf{ciclo}\ \{I\}$$

$$\begin{aligned} wp(S1;S2,I) \equiv\ & 0 \leq i < |r| \land 0 \leq i < |s| \land_L 0 \leq i+1 \leq |s| \land |r_0| = \boxed{|setAt(r,i,s[i])|} \land \\ & |s| = \boxed{|setAt(r,i,s[i])|} \land_L (\forall j : \mathbb{Z})(0 \leq j < i+1 \rightarrow_L \boxed{setAt(r,i,s[i])}[j] = s[j]) \land \\ & (\forall j : \mathbb{Z})(i+1 \leq j < |r| \rightarrow_L \boxed{setAt(r,i,s[i])}[j] = r_0[j]) \end{aligned}$$

$$setAt(r,i,s[i])[j] \begin{cases} s[i] & \text{Si i = j} \\ r[j] & \text{Si no} \end{cases}$$

$$\begin{aligned} wp(S1;S2,I) \equiv\ & 0 \leq i < |r| \land 0 \leq i < |s| \land_L 0 \leq i+1 \leq |s| \land |r_0| = |r| \land \\ & |s| = |r| \land_L (\forall j : \mathbb{Z})(0 \leq j < i+1 \rightarrow_L setAt(r,i,s[i])[j] = s[j]) \land \\ & (\forall j : \mathbb{Z})(i+1 \leq j < |r| \rightarrow_L \boxed{r[j] = r_0[j]}) \end{aligned}$$

$$\{I \land B\}\ \textbf{ciclo}\ \{I\}$$

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$$\begin{aligned} wp(S1; S2, I) \equiv\ & 0 \leq i < |r| \land 0 \leq i < |s| \land_L 0 \leq i + 1 \leq |s| \land |r_0| = \boxed{|setAt(r, i, s[i])|} \land \\ & |s| = \boxed{|setAt(r, i, s[i])|} \land_L (\forall j : \mathbb{Z})(0 \leq j < i + 1 \rightarrow_L \boxed{setAt(r, i, s[i])}[j] = s[j]) \land \\ & (\forall j : \mathbb{Z})(i + 1 \leq j < |r| \rightarrow_L \boxed{setAt(r, i, s[i])}[j] = r_0[j]) \end{aligned}$$

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$I : 0 \leq i \leq |s| \land |r_0| = |r| \land |s| = |r| \land_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j]) \land (\forall j : \mathbb{Z})(i \leq j < |r| \rightarrow_L r[j] = r_0[j])$

$B : i < |s|$

$$\begin{aligned} wp(S1; S2, I) \equiv\ & \boxed{0 \leq i < |s|} \land_L 0 \leq i + 1 \leq |s| \land |r_0| = |r| \land \\ & |s| = |r| \land_L (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L r[j] = s[j]) \\ & (\forall j : \mathbb{Z})(i + 1 \leq j < |r| \rightarrow_L r[j] = r_0[j]) \end{aligned}$$

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