

1.  $\int \int x \cos(y) dA, Dy = 0, y = x^2, x = 1$

Tipo 1

$$\int_0^1 (\int_0^{x^2} x \cos(y) dy) dx$$

$$\begin{aligned} \blacksquare \int_0^{x^2} x \cos(y) dy &= \\ x \sin(y) \Big|_0^{x^2} &= \\ x \sin(x^2) & \\ \blacksquare \int_0^1 x \sin(x^2) dx & \\ u = x^2, du = 2x dx, dx = \frac{du}{2x} \Rightarrow & \\ x \sin(x^2) dx \equiv x \sin(u) \frac{du}{2x} = & \\ \sin(u) du \cdot \frac{1}{2} = & \\ \int \sin(u) du = -\cos(u) + C \Rightarrow & \\ \frac{-\cos(x^2)}{2} \Big|_0^1 = & \\ \frac{-\cos(1)}{2} + \frac{1}{2} & \end{aligned}$$

2.  $\int \int y^2 dA, D$  triangulo con vertices  $(0, 1), (1, 2), (4, 1)$

Parto D en dos triangulos rectangulos

$$\begin{aligned} \blacksquare D_1, 0 \leq x \leq 1 \wedge 1 \leq y \leq x+1 \\ \blacksquare D_1, 1 \leq y \leq 2 \wedge 1 \leq x \leq -3y+7 \end{aligned}$$

a)  $D_1$  tipo 1

$$\int_0^1 (\int_1^{x+1} y^2 dy) dx =$$

$$\begin{aligned} \blacksquare \int_1^{x+1} y^2 dy &= \\ \frac{y^3}{3} \Big|_1^{x+1} &= \\ \frac{(x+1)^3}{3} - \frac{1}{3} & \\ \blacksquare \int_0^1 \left( \frac{(x+1)^3}{3} - \frac{1}{3} \right) dx &= \\ \frac{(x+1)^4}{12} - \frac{x}{3} \Big|_0^1 &= \\ \frac{2^4}{12} - \frac{1}{3} - \frac{1}{12} &= \\ \frac{11}{12} & \end{aligned}$$

b)  $D_2$  tipo 2

$$\int_1^2 (\int_1^{-3y+7} y^2 dx) dy =$$

$$\begin{aligned} \blacksquare \int_1^{-3y+7} y^2 dx &= \\ xy^2 \Big|_1^{-3y+7} &= \\ (-3y+7)y^2 - y^2 &= \\ -3y^3 + 6y^2 & \\ \blacksquare \int_1^2 -3y^3 + 6y^2 dy &= \\ \frac{-3y^4}{4} + 2y^3 \Big|_1^2 &= \\ \frac{-3(2^4)}{4} + 2^4 + \frac{3}{4} - 2 &= \\ \frac{11}{4} & \end{aligned}$$

$$\Rightarrow A_{D_1} + A_{D_2} = \frac{11}{12} + \frac{11}{4} = \frac{11}{3}$$

3.  $\int \int_D xy^2 dA, D : x = 0 \wedge x = \sqrt{1-y^2}$

Tipo 2

$$\int_{-1}^1 (\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy^2 dy) dx$$

$$\begin{aligned} & \blacksquare \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy^2 dy \\ & \quad \frac{xy^3}{3} \bigg|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} = \\ & \quad \frac{2x(\sqrt{1-x^2})^3}{3} \\ & \blacksquare \int_{-1}^2 \end{aligned}$$