

$$f(x, y) = \begin{cases} \frac{(x-1)^3 \cos(y)}{(x-1)^2 + y^2} & \text{si } (x, y) \neq (1, 0) \\ a & \text{si } (x, y) = (1, 0) \end{cases}$$

Veamos el límite de $f(x, y)$ en $(1, 0)$

Pruebo por curvas

- $x = 1 \Rightarrow \lim_{y \rightarrow 0} \frac{0 \cdot \cos(y)}{(y^2)} = 0$
- $y = 0 \Rightarrow \lim_{x \rightarrow 1} \frac{(x-1)^3}{(x-1)^2} = 0$
- $y = (x-1) \Rightarrow \lim_{x \rightarrow 1} \frac{(x-1)^{\cancel{3}} \cos(x-1)}{2(x-1)^2} = 0$

Pruebo por definición

$$\exists \delta : \|(x-1, y)\| < \delta \Rightarrow |f(x, y)| < \epsilon$$

$$\left| \frac{(x-1)^3 \cos(y)}{(x-1)^2 + y^2} \right| \leq$$

$$\frac{(x-1)^3 |\cos(y)|}{(x-1)^2 + y^2} \leq$$

$$\frac{(x-1)^3 |\cos(y)|}{(x-1)^2 + y^2} \stackrel{\delta \leq 1}{\leq}$$

$$\frac{(x-1)^3}{(x-1)^2 + y^2} \leq$$

$$\frac{\|(x-1, y)\|^{\cancel{3}}}{\|(x-1, y)\|^2} = \delta \Rightarrow \delta = \min(1, \epsilon)$$