1. $f(x,y) = x^2 + y^2 - 2x$

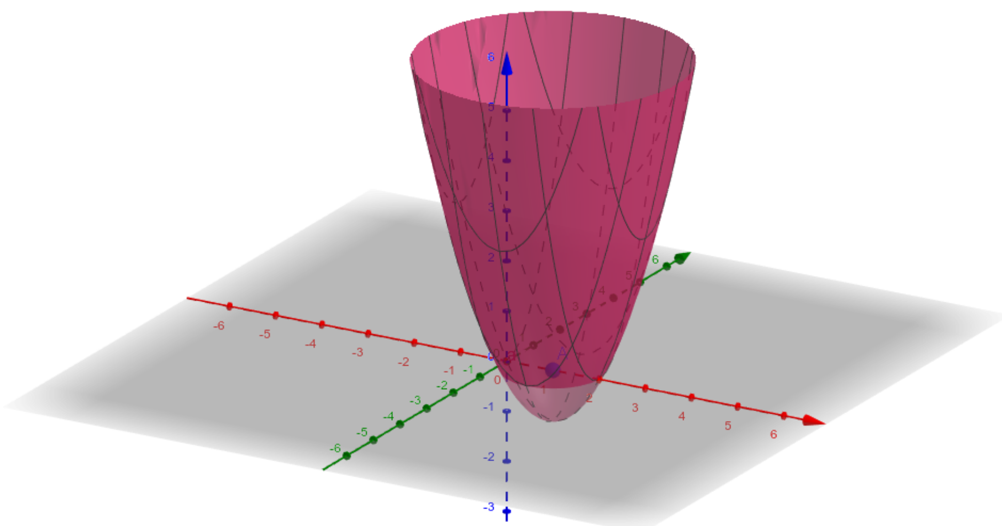
   - $f_x(x,y) = 2x - 2$
   - $f_y(x,y) = 2y$

   $\boldsymbol{\nabla} f(x,y) = (0,0) \Leftrightarrow$

   $\left\{ \begin{array}{l} 2x - 2 = 0 \\ 2y = 0 \end{array} \right. \Leftrightarrow (x,y) = (1,0)$

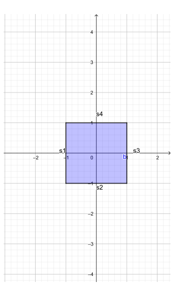
   $H_f(x,y) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \wedge det(H_f(x,y)) = 4$

   Por el criterio es un minimo local



2. $f(x,y) = x^2 + y^2 + x^2y + 4 \wedge D = \{(x,y) \in \mathbb{R}^2 : |x| \ \leq 1 \wedge |y| \ \leq 1\}$

   D compactor y $f(x,y)$ continua $\Rightarrow$ f(x,y) alcanza su maximo y minimo en D por waiertrass



Interior de D

- $f_x(x,y) = 2x + 2xy$
- $f_y(x,y) = 2y + x^2$

$\boldsymbol{\nabla} f(x,y) = (0,0) \Leftrightarrow$

$\left\{ \begin{array}{l} 2x + 2xy = 0 \Rightarrow 2x + x^3 = 0 \Leftrightarrow x(2 + x^2) = 0 \Leftrightarrow x = 0 \vee x = \pm\sqrt{2} \\ 2y + x^2 = 0 \Rightarrow y = \frac{x^2}{2} \end{array} \right. \quad \Rightarrow$

Ptos criticos $= (0,0), \overset{\notin D}{\cancel{(\sqrt{2},1), (-\sqrt{2},1)}}$

Borde de D

S1

$f(-1, y) = y^2 + y + 5 = g(y) \wedge y \in [-1, 1]$
$g'(y) = 2y + 1 = 0 \Leftrightarrow (x, y) = (-1, -\frac{1}{2})$

S2

$f(x, -1) = \cancel{x^2} + 1 \cancel{-x^2} + 4 = 5 = h(x) \wedge x \in [-1, 1]$
$h'(x) = 0 \Rightarrow (x, -1) \wedge x \in [-1, 1]$

S3

$f(1, y) = 1 + y^2 + y + 4 = h(x)$

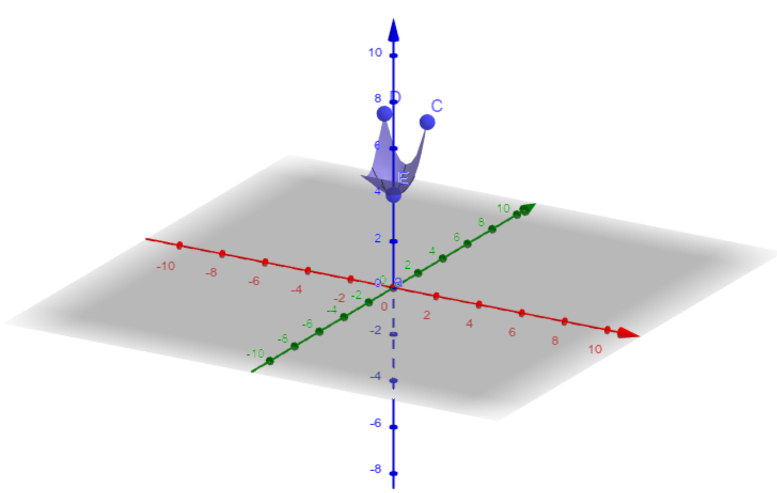
S4

$f(x, 1) = 2x^2 + 5 = i(x) \wedge x \in [-1, 1]$
$i'(x) = 4x = 0 \Leftrightarrow (x, y) = (0, 1)$

Ptos criticos

- $(0, 0) \Rightarrow f(0, 0) = 4$
- $(-1, -\frac{1}{2}) \Rightarrow f(-1, -\frac{1}{2}) = 1 + \frac{1}{4} + \frac{1}{2} + 4 = 5 + \frac{3}{4}$
- $(x, -1) \Rightarrow f(x, -1) = 5$
- $(0, 1) \Rightarrow f(0, 1) = 5$
- $(1, 1) \Rightarrow f(1, 1) = 7$
- $(-1, 1) \Rightarrow f(-1, 1) = 7$
- $(-1, -1) \Rightarrow f(-1, -1) = 5$
- $(1, -1) \Rightarrow f(-1, -1) = 5$

- $\max f = 7$ en los puntos $(1, 1) \wedge (-1, 1)$
- $\max f = 4$ en el punto $(0, 0)$



3. $f(x, y) = x^4 + y^4 - 4xy + 2 \wedge D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3 \wedge 0 \leq y \leq 5\}$

Interior de D

- $f_x(x,y) = 4x^3 - 4y$
- $f_y(x,y) = 4y^3 - 4x$

$\nabla f(x,y) = (0,0) \Leftrightarrow$

$$\left\{ \begin{array}{l} 4x^3 - 4y = 0 \Rightarrow 4y^9 - 4y = 4y(y^8 - 1) = 0 \Leftrightarrow y \in \{0, 1, -1\} \\ 4y^3 - 4x = 0 \Rightarrow x = y^3 \end{array} \right. \Rightarrow$$

Ptos criticos $(0,0), (1,1), \overset{\notin D}{(1,-1)}$

Borde de D

S1

$f(0,y) = y^4 + 2 = g(y) \wedge 0 \leq y \leq 5$

$g'(y) = 4y^3, (0,0)$

S2

$f(x,0) = x^4 + 2 \Rightarrow (0,0)$

S3

$f(3,y) = 3^4 + y^4 - 12y + 2 = h(y)$

$h'(y) = 4y^3 - 12 = 0 \Leftrightarrow y = \sqrt[3]{3}$

$\Rightarrow (3, \sqrt[3]{3})$

S3

$f(x,5) = x^4 + 5^4 - 20x + 2 = i(x)$

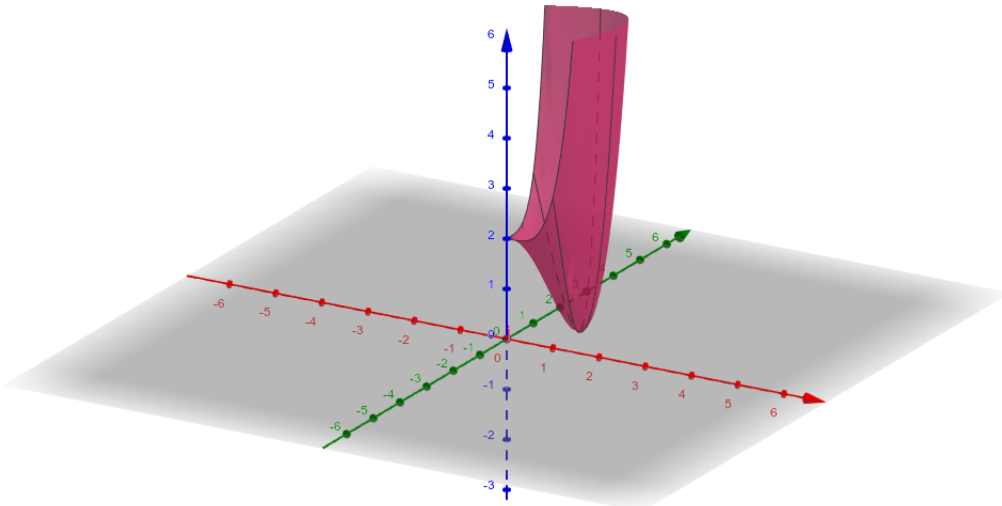
$i'(x) = 4x^3 - 20 = 0 \Leftrightarrow x = \sqrt[3]{5}$

$\Rightarrow (\sqrt[3]{5}, 5)$

Ptos criticos

- $(0,0) \Rightarrow f(0,0) = 2$
- $(1,1) \Rightarrow f(1,1) = 0$
- $(3, \sqrt[3]{3}) \Rightarrow f(3, \sqrt[3]{3}) = 70{,}02$
- $(\sqrt[3]{5}, 5) \Rightarrow f(\sqrt[3]{5}, 5) = 601{,}35$
- $(0,5) \Rightarrow f(0,5) = 627$
- $(3,5) \Rightarrow f(3,5) = 648$
- $(3,0) \Rightarrow f(3,0) = 83$

f alcanza su maximo en $(3,5)$ y su minimo en $(1,1)$



4. $f(x,y) = xy^2 \wedge D = \{(x,y) \in \mathbb{R}^2 : x \geq 0 \wedge y \geq 0 \wedge x^2 + y^2 \leq 3\}$

Interior de D

- $f_x(x,y) = y^2$
- $f_y(x,y) = 2xy$

$\boldsymbol{\nabla} f(x,y) = (0,0) \Leftrightarrow$
$\begin{cases} y^2 = 0 \\ 2xy = 0 \end{cases}$

Borde de D

$x = 0$

$f(0,y) = 0$

$y = 0$

$f(x,0) = 0$

$x = \sqrt{3}\cos(t), y = \sqrt{3}\sin(t)$

$f(\sqrt{3}\cos(t), \sqrt{3}\sin(t)) = \sqrt{3}\cos(t)3\sin^2(t) = h(t)$
$h'(t) = 3\sqrt{3}(-\sin(t)\sin^2(t) + 2\sin(t)\cos^2(t)) \Rightarrow$
$-\sin(t)\sin^2(t) + 2\sin(t)\cos^2(t) = 0 \Leftrightarrow$
$t \neq 0 \Rightarrow 2\cos^2(t) = \sin^2(t) \Leftrightarrow$
$2 = \frac{\sin^(t)}{\cos^(t)} \Leftrightarrow$
$2 = (\frac{\sin(t)}{\cos(t)})^2 \Leftrightarrow$
$2 = \tan^2(t) \Leftrightarrow$
$\sqrt{2} = \tan(t) \Leftrightarrow t = \arctan(\sqrt{2})$
$\sqrt{0{,}88}$
0,955316618

Pto criticos

- $P_1 = (0,0) \Rightarrow f(P_1) = 0$ min
- $P_2 = (\sqrt{3},0) \Rightarrow f(P_2) = 0$ min
- $P_3 = (0,\sqrt{3}) \Rightarrow f(P_3) = 0$ min
- $P_4 = (\sqrt{3}\cos\left(\arctan\left(\sqrt{2}\right)\right), \sqrt{3}\sin\left(\arctan\left(\sqrt{2}\right)\right)) \Rightarrow f(P_5) = 2$ max