

1. a) $r : \mathbb{R} \rightarrow \mathbb{R}^3$
 $C : r(t) = (r_1(t), r_2(t), r_3(t)) = (x, y, z)$
 $r_1 : \mathbb{R} \rightarrow \mathbb{R}$
 $r_2 : \mathbb{R} \rightarrow \mathbb{R}$
 $r_3 : \mathbb{R} \rightarrow \mathbb{R}$

$$\begin{cases} z = x + y \\ y = x \end{cases}$$

b)

2. a) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

Pruebo por curvas

- iterado $x = 0$
 $\lim_{y \rightarrow 0} f(0, y)$
- iterado $y = 0$
 $\lim_{x \rightarrow 0} f(x, 0)$
- rectas $y = mx$
 $\lim_{x \rightarrow 0} f(x, mx)$
- curvas $y = x^2$
 $\lim_{x \rightarrow 0} f(x, x^2)$

Intento demostrar por sandwich

$$\exists g(x, y) : \lim_{(x,y) \rightarrow (0,0)} g(x, y) = 0 \wedge 0 \leq |f(x, y)| \leq |g(x, y)|$$

Pruebo por definición

$$\exists \epsilon, \delta(\epsilon) > 0 : \|(x, y) - (0, 0)\| < \delta \Rightarrow |f(x, y)| < \epsilon$$

- b) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

Pruebo por curvas

- iterado $x = 0$
 $\lim_{y \rightarrow 0} f(0, y)$
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3. $f(x, y) =$

Diferenciabilidad en a

f es diferenciable en $a \Leftrightarrow$

$$\exists L \in \Re : \lim_{(x,y) \rightarrow (0,0)} \frac{f(x,y) - f(0,0) - \nabla f(0,0) \cdot (x-0, y-0)}{\|(x,y)\|} = L \wedge L = 0$$

Busco $\nabla f(0,0)$

$$\nabla f(0,0) = (f_x(0,0), f_y(0,0))$$

Por definición

$$\begin{aligned} \blacksquare f_x(0,0) &= \lim_{h \rightarrow 0} \frac{f(h,0) - f(0,0)}{h} = \\ &= \lim_{h \rightarrow 0} f(h,0) - f(0,0) \cdot \frac{1}{h} \\ \blacksquare f_y(0,0) &= \lim_{h \rightarrow 0} \frac{f(0,h) - f(0,0)}{h} = \\ &= \lim_{h \rightarrow 0} f(0,h) - f(0,0) \cdot \frac{1}{h} \end{aligned}$$

4. $F(s,t) = f(x(s), y(s))$

$$z = \nabla F(0,1) \cdot (s, t-1) + F(0,1) =$$

Busco $\nabla F(0,1)$

$$\nabla F(s,t) = (F_s(s,t), F_t(s,t))$$

$$\begin{aligned} \blacksquare F_s(s,t) &= f_x(x(s,t), y(s,t)) \cdot x_s(s,t) + f_y(x(s,t), y(s,t)) \cdot y_s(s,t) = \\ \blacksquare F_t(s,t) &= f_x(x(s,t), y(s,t)) \cdot x_t(s,t) + f_y(x(s,t), y(s,t)) \cdot y_t(s,t) = \end{aligned}$$