Polinomio de Taylor

$k \geq 1 \in \mathbb{Z} \wedge f : \mathbb{R} \to \mathbb{R}$ diferenciable k veces en el punto $a \in \mathbb{R}$
$\Rightarrow h_k : \mathbb{R} \to \mathbb{R}\ tq$

- $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... + \frac{f^{(k)}(a)}{k!} + h_k(x)(x-a)^k$
- $lim_{x \to a} h_k(x) = 0$

1. - $f(x) = \frac{1}{1-x}$
   - orden 5
   - $x_0 = 0$

   - $f(x) = (1-x)^{-1}$
   - $f'(x) = -1(1-x)^{-2}$
   - $f''(x) = 2(1-x)^{-3}$
   - $f^{(3)}(x) = -6(1-x)^{-4}$
   - $f^{(4)}(x) = 24(1-x)^{-5}$
   - $f^{(5)}(x) = -120(1-x)^{-6}$

   $\Rightarrow P_5(x) = 1 - 1x + \frac{2x^2}{2!} - \frac{6x^3}{3!} + \frac{24x^4}{4!} - \frac{120x^5}{5!}$
2. - $f(x) = \sin(x)$
   - orden 4
   - $x_0 = 0$

   - $f(x) = \sin(x)$
   - $f'(x) = \cos(x)$
   - $f''(x) = -\sin(x)$
   - $f^{(3)}(x) = -\cos(x)$
   - $f^{(4)}(x) = \sin(x)$

   $\Rightarrow P_4(x) = x - \frac{x^3}{3!}$
3. - $f(x) = \sin(x)$
   - orden 5
   - $x_0 = 0$

   - $f(x) = \sin(x) \Rightarrow f(0) = 0$
   - $f'(x) = \cos(x) \Rightarrow f'(0) = 1$
   - $f''(x) = -\sin(x) \Rightarrow f''(0) = 0$
   - $f^{(3)}(x) = -\cos(x) \Rightarrow f^{(3)}(0) = -1$
   - $f^{(4)}(x) = \sin(x) \Rightarrow f^{(4)}(0) = 0$
   - $f^{(5)}(x) = \cos(x) \Rightarrow f^{(5)}(0) = 1$

   $\Rightarrow P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$
4. - $f(x) = \cos(x)$
   - orden 5
   - $x_0 = 0$

- $f(x) = \cos(x) \Rightarrow f(0) = 1$
- $f'(x) = -\sin(x) \Rightarrow f'(0) = 0$
- $f''(x) = -\cos(x) \Rightarrow f''(0) = -1$
- $f^{(3)}(x) = \sin(x) \Rightarrow f^{(3)}(0) = 0$
- $f^{(4)}(x) = \cos(x) \Rightarrow f^{(4)}(0) = 1$
- $f^{(5)}(x) = -\sin(x) \Rightarrow f^{(5)}(0) = 0$

$\Rightarrow P_5(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

5.
- $f(x) = \ln(x)$
- orden 4
- $x_0 = 1$

- $f(x) = \ln(x) \Rightarrow f(1) = 0$
- $f'(x) = x^{-1} \Rightarrow f'(1) = 1$
- $f''(x) = -x^{-2} \Rightarrow f''(1) = -1$
- $f^{(3)}(x) = 2x^{-3} \Rightarrow f^{(3)}(1) = 2$
- $f^{(4)}(x) = -6x^{-4} \Rightarrow f^{(4)}(1) = 6$

$\Rightarrow P_4(x) = 0 + (x-1) - \frac{(x-1)^2}{2!} + 2\frac{(x-1)^3}{3!} - 6\frac{(x-1)^4}{4!}$

6.
- $f(x) = \sqrt{x}$
- orden 3
- $x_0 = 4$

- $f(x) = \sqrt{x} \Rightarrow f(4) = 2$
- $f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(4) = \frac{1}{2}$
- $f''(x) = -\frac{1}{4x^{\frac{3}{2}}} \Rightarrow f''(4) = -\frac{1}{4(4^{\frac{3}{2}})}$
- $f^{(3)}(x) = \frac{3}{8x^{\frac{5}{2}}} \Rightarrow f^{(3)}(4) = \frac{3}{8(4^{\frac{5}{2}})}$

$\Rightarrow P_3(x) = 2 + \frac{1}{2}(x-4) - \frac{1}{4(4^{\frac{3}{2}})}\frac{(x-1)^2}{2!} + \frac{3}{8(4^{\frac{5}{2}})}\frac{(x-1)^3}{3!}$

7.
- $f(x) = e^x$
- orden 5
- $x_0 = 0$

- $f(x) = e^x \Rightarrow f(0) = 1$
- $f'(x) = e^x \Rightarrow f'(0) = 1$
- $f''(x) = e^x \Rightarrow f''(0) = 1$
- $f^{(4)}(x) = e^x \Rightarrow f^{(3)}(0) = 1$
- $f^{(5)}(x) = e^x \Rightarrow f^{(3)}(0) = 1$

$\Rightarrow P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$

8.
- $f(x) = (1+x)^6$
- orden 6
- $x_0 = 0$

- $f(x) = (1+x)^6 \Rightarrow f(0) = 1$
- $f'(x) = 6(1+x)^5 \Rightarrow f'(0) = 6$
- $f''(x) = 30(1+x)^4 \Rightarrow f''(0) = 30$

- $f^{(3)}(x) = 120(1+x)^3 \Rightarrow f^{(3)}(0) = 120$
- $f^{(4)}(x) = 360(1+x)^2 \Rightarrow f^{(3)}(0) = 360$
- $f^{(5)}(x) = 720(1+x) \Rightarrow f^{(3)}(0) = 720$
- $f^{(6)}(x) = 720 \Rightarrow f^{(3)}(0) = 720$

$\Rightarrow P_6(x) = 1 + 6x + 30\frac{x^2}{2!} + 120\frac{x^3}{3!} + 360\frac{x^4}{4!} + 720\frac{x^5}{5!} + 720\frac{x^6}{6!} =$
$1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$