

$$1. \int \int_D y^2 dA, D = \{(x, y) \in \mathbb{R}^2 : -1 \leq y \leq 1, -y - 2 \leq x \leq y\}$$

D es una region tipo 2 y por fubini

$$\int_{-1}^1 (\int_{-y-2}^y y^2 dx) dy$$

$$\begin{aligned} \blacksquare \int_{-y-2}^y y^2 dx &= \\ xy^2 \Big|_{-y-2}^y &= \\ y^3 - (-y-2)y^2 &= \\ y^3 + y^3 + 2y^2 &= \\ 2y^3 + 2y^2 & \\ \blacksquare \int_{-1}^1 2y^3 + 2y^2 dy &= \\ \frac{y^4}{2} + \frac{2y^3}{3} \Big|_{-1}^1 &= \\ \frac{1}{2} + \frac{2}{3} - \frac{1}{2} + \frac{2}{3} &= \\ \frac{4}{3} & \end{aligned}$$

$$2. \int \int_D \frac{y}{x^5+1} dA, D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$$

D es una region tipo 1 y por fubini

$$= \int_0^1 (\int_0^{x^2} \frac{y}{x^5+1} dy) dx$$

$$\begin{aligned} \blacksquare \int_0^{x^2} \frac{y}{x^5+1} dy &\stackrel{Barrow}{=} \\ \frac{y^2}{2x^5+2} \Big|_0^{x^2} &= \\ \frac{x^4}{2x^5+2} & \\ \blacksquare \int_0^1 \frac{x^4}{2x^5+2} dx &= \\ \int \frac{x^4}{2x^5+2} dx &= \\ u = x^5, du = 5x^4 dx, dx = \frac{du}{5x^4} & \\ \int \frac{1}{10u+10} du &= \\ \frac{\ln(u+1)}{10} &= \\ \int_0^1 \frac{\ln(x^5+1)}{10} &\stackrel{barrow}{=} \\ \frac{\ln(2)}{10} - \frac{\ln(1)}{10} &= \frac{\ln(2)}{10} \end{aligned}$$