1. a) $r : \Re \to \Re^3$
$C : r(t) = (r_1(t), r_2(t), r_3(t)) = (x, y, z)$
$r_1 : \Re \to \Re$
$r_2 : \Re \to \Re$
$r_3 : \Re \to \Re$
$\begin{cases} y^2 + z^2 = 4 : \star \\ -x + y + z = 0 \end{cases}$
$\star$ es un circulo con centro en $(0, 0)$ y radio $\sqrt{4} = 2$
$\Rightarrow$ en polares $\star : 2\cos(t) + 2\sin(t) = 1$
$\Rightarrow$ Propongo $\begin{cases} y = 2\sin(t) \\ z = 2\cos(t) \end{cases} : \star'$
Se que $-x + y + z = 0 \Rightarrow$
Se que $x = y + z \overset{\star'}{\Rightarrow}$
$x = 2\sin(t) + 2\cos(t) \Rightarrow x = 2(\sin(t) + \cos(t)) \star''$
Por $\star' \wedge \star''$:
$\begin{cases} x = 2(\sin(t) + \cos(t)) \\ y = 2\sin(t) \\ z = 2\cos(t) \end{cases} = C$
$\Rightarrow C : r(t) = (2(\sin(t) + \cos(t), 2\sin(t), 2\cos(t)))$

b) $P \in C \Leftrightarrow$
$\exists k \in \Re : r(k) = (2, 2, 0)$
$\Leftrightarrow \begin{cases} 2 = 2(\sin(k) + \cos(k)) \\ 2 = 2\sin(k) : \triangle \\ 0 = 2\cos(k) : \pentagon \end{cases} = C$
Por $\pentagon : 2\cos(k) = 0 \Leftrightarrow$
$k \in \frac{1}{2\pi}, \frac{3}{2\pi}$
Por $\triangle \wedge \pentagon : 2 = 2\sin(k) \Leftrightarrow 1 = \sin(k)$
$1 = \sin(k) \wedge k \in \frac{1}{2\pi}, \frac{3}{2\pi} \Leftrightarrow \frac{1}{2\pi}$
$\Rightarrow r(1) = P \Rightarrow P \in C\square$
L es la recta tangente de C $\Leftrightarrow z = \lambda \cdot r'(0) + P$
Al ser r continua por ser funciones trigonometricas: $r'(t) = (2(\cos(t) - \sin(t)), 2\cos(t), -2\sin(t))$
$\Rightarrow r'(\frac{1}{2\pi}) = (-2, 0, -2)$
$\Rightarrow L : z = \lambda(-2, 0, -2) + (2, 2, 0)$

2. a) $\lim_{(x,y)\to(1,0)} f(x, y)$
$f(x, y) = \frac{(x-1)^2 y}{(x-1)^3 + y3}$

Pruebo por rectas

- recta $x = 1$
  $\lim_{y\to 0} \frac{0}{y3} = 0$
- recta $y = (x - 1)$
  $\lim_{x\to 1} \frac{\cancel{(x-1)^3}}{2\cancel{(x-1)^3}} = \frac{1}{2}$

Al dar distinto los limites se demuestra que $\nexists \lim_{(x,y)\to(1,0)} f(x, y)$

b) $\lim_{(x,y)\to(0,0)} f(x, y)$
$f(x, y) = \frac{x \sin(y^2)}{x^2 + y^2}$

Pruebo por curvas

- iterado $x = 0$
  $\lim_{y\to 0} f(0, y) =$
  $\lim_{y\to 0} \frac{0}{y^2} = 0$

Intento demostrar por sandwich

$\exists g(x,y) : \lim_{(x,y)\to(0,0)} g(x,y) = 0 \wedge 0 \leq |f(x,y)| \leq |g(x,y)|$
$|\frac{x\sin\left(y^2\right)}{x^2+y^2}| =$
$\frac{|x||\sin\left(y^2\right)|}{x^2+y^2} =$
$\frac{|x||\sin\left(y^2\right)|}{x^2+y^2} \overset{|\sin(k)|\leq|k|}{\leq}$
$\frac{|x|y^2}{x^2+y^2} =$
$|x| \cdot \frac{y^2}{x^2+y^2}$

$x^2 \geq 0 \Rightarrow x^2+y^2 \geq y^2 \Rightarrow 1 \geq \frac{y^2}{x^2+y^2} : \star$

$|x| \cdot \frac{y^2}{x^2+y^2} \overset{\star}{\leq}$
$|x| \cdot 1 \overset{(x,y)\to(0,0)}{\to} = 0$
$\Rightarrow 0 \leq |f(x,y)| \leq |x|$
$\Rightarrow$ por sandwich $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

3. $f(x,y) = \sqrt[3]{x^3+8y^3}$

Diferenciabilidad en $(0,0)$

f es diferenciable en $(0,0) \Leftrightarrow$
$\exists L \in \Re : \lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x-0,y-0)}{\|(x,y)\|} = L \wedge L = 0$

Busco $\nabla f(0,0)$

$\nabla f(0,0) = (f_x(0,0), f_y(0,0))$

Por definición las derivadas parciales

- $f_x(0,0) = \lim_{h\to0} \frac{f(h,0)-f(0,0)}{h} =$
  $\lim_{h\to0} f(h,0) - f(0,0) \cdot \frac{1}{h}$
  $\lim_{h\to0} \sqrt[3]{h^3} - 0 \cdot \frac{1}{h}$
  $\lim_{h\to0} h \cdot \frac{1}{h} = 1$
- $f_y(0,0) = \lim_{h\to0} \frac{f(0,h)-f(0,0)}{h} =$
  $f_y(0,0) = \lim_{h\to0} f(0,h) - f(0,0) \cdot \frac{1}{h}$
  $f_y(0,0) = \sqrt[3]{8h^3} - 0 \cdot \frac{1}{h} = 2$

$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x,y)}{\|(x,y)\|} =$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}-0-(1,2)\cdot(x,y)}{\|(x,y)\|} =$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}-x-2y}{\|(x,y)\|} =$
$y = x$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}-x-2y}{\|(x,y)\|} =$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}}{\sqrt{x^2+y^2}} - \overset{\star}{\frac{x+2y}{\sqrt{x^2+y^2}}} =$
$\star : \lim_{(x,y)\to(0,0)} \frac{x+2y}{\sqrt{x^2+y^2}}$
Por $y = x$

$\lim_{x \to 0} \frac{3x}{\sqrt{2x^2}} =$

$\lim_{x \to 0} \frac{3x}{\sqrt{2}|x|}$

$\lim_{x \to 0+} \frac{3x}{\sqrt{2}x} = 3$

$\lim_{x \to 0-} \frac{3x}{\sqrt{2}(-x)} = -3$

$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} \frac{x+2y}{\sqrt{x^2+y^2}}$

$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}}{\sqrt{x^2+y^2}} - \frac{\overset{\star}{x+2y}}{\sqrt{x^2+y^2}}$

$\Rightarrow$ f no es diferenciable en $(0,0)$

4. $f(1,4) = z(1,4) = 3(1) - 4 + 7 = 6$

   $f_x(1,4) = z_x(1,4) = 3$

   $f_y(1,4) = z_y(1,4) = -1$

   $\begin{cases} x(s,t) = \cos(s)t^2 \\ y(s,y) = (s+2t)^2 \end{cases}$

   $\begin{cases} x_s(s,t) = -\sin(s)t^2 \\ x_t(s,t) = 2\cos(s)t \\ y_s(s,t) = 2(s+2t) \\ y_t(s,t) = 4(s+2t) \end{cases}$

   $F(s,t) = f(x(s,t), y(s,t))$

   a) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s} =$
      $f_x(1,4) \cdot x_s(0,-1) + f_y(1,4) \cdot y_s(0,-1) =$
      $3 \cdot 0 + (-1) \cdot 4 = -4$

   b) $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial t}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$
      $f_x(1,4) \cdot x_t(0,-1) + f_y(1,4) \cdot y_t(0,-1) =$
      $3 \cdot (-2) + (-1) \cdot -8 = -6 + 8 = 2$

   $z = \nabla F(0,-1) \cdot (s, t+1) + F(0,-1) =$

$\nabla F(0,-1)$ ya lo calculamos antes

$\nabla F(s,t) = (-4, 2)$

$\Rightarrow z = (-4,2) \cdot (s, t+1) + f(1,4)$

$\Rightarrow z = -4s + 2(t+1) + 6$