$f(x,y) = \begin{cases} \frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2} + 2 & si(x,y) \neq (0,0) \\ a \quad si(x,y) = (0,0) \end{cases} \Rightarrow$

$h(x,y) = \frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}$

QvQ $\lim_{(x,y)\to(0,0)} h(x,y) = 0$

$\exists g(x,y) : \lim_{(x,y)\to(0,0)} g(x,y) = 0 \wedge 0 \leq |h(x,y)| \leq |g(x,y)|$

$|f(x,y)| = |\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}| =$

$\frac{|x^2y^2-\sin\left(x^4\right)|}{x^2+\frac{1}{3}y^2} \leq$

$\frac{x^2y^2+|\sin\left(x^4\right)|}{x^2+\frac{1}{3}y^2} \overset{|\sin(k)|\leq|k|}{\leq}$

$\frac{x^2y^2+x^4}{x^2+\frac{1}{3}y^2} =$

$\frac{x^2}{x^2+\frac{1}{3}y^2}y^2 + x^4 \overset{\frac{x^2}{x^2+\frac{1}{3}y^2}\leq 1}{\leq}$

$y^2 + x^4 \overset{(x,y)\to 0}{\to} 0$

$\Rightarrow \lim_{(x,y)\to(0,0)} h(x,y) = 0$

$\Rightarrow f(x,y)$ es continua en todo $\Re^2 \Leftrightarrow a = 2$ ya que es un conciente de polinomios y trigonometricas continuas donde el denominador se anula en el $(x,y) = (0,0)$

$f(x,y)$ es diferencibale en el $(0,0) \Leftrightarrow$

$\exists L: \ lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\boldsymbol{\nabla} f(0,0)\cdot(x,y)}{\|(x,y)\|} = L \wedge L = 0$

$\boldsymbol{\nabla} f(x,y) = (f_x(x,y), f_y(x,y))$

- $f_x(0,0)$ $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$

  $f_x(0,0) = \lim_{h\to 0} \frac{\frac{-\sin\left(h^4\right)}{h^2}+2-2}{h} =$

  $f_x(0,0) = \lim_{h\to 0} \frac{-\sin\left(h^4\right)}{h^2} \cdot \frac{1}{h} =$

  $f_x(0,0) = \lim_{h\to 0} \frac{-\sin\left(h^4\right)}{h^3} \overset{"\cdot\frac{h}{h}"}{=}$

  $f_x(0,0) = \lim_{h\to 0} \frac{-h\sin\left(h^4\right)}{h^4} \overset{"\cdot\frac{h}{h}"}{=}$

  $f_x(0,0) = \lim_{h\to 0} \frac{-h\sin\left(h^4\right)}{h^4} \overset{"lim_{k\to 0}\frac{\sin(k)}{k}=1"}{=}$

  $f_x(0,0) = \lim_{h\to 0} 1\cdot(-h) = 0$

- $f_y(0,0)$

  $f_y(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} =$

  $\lim_{h\to 0} \frac{\frac{0}{\frac{1}{3}h^2}+2-2}{h} =$

  $\lim_{h\to 0} \frac{0}{h} = 0$

$\Rightarrow \boldsymbol{\nabla} f(0,0) = (0,0)$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\boldsymbol{\nabla} f(0,0)\cdot(x,y)}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-(0,0)\cdot(x,y)}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}+2-2}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2} \cdot \frac{1}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2} \cdot \frac{1}{\|(x,y)\|} =$

$|\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2} \cdot \frac{1}{\|(x,y)\|}| =$

$\frac{|x^2y^2-\sin\left(x^4\right)|}{x^2+\frac{1}{3}y^2} \cdot \frac{1}{\|(x,y)\|} \overset{\text{"}des.\ triang\text{"}}{\leq}$

$\frac{x^2y^2+|\sin\left(x^4\right)|}{x^2+\frac{1}{3}y^2} \cdot \frac{1}{\|(x,y)\|} \overset{|\sin(k)|\leq|k|}{\leq}$

$\frac{x^2(y^2+x^2)}{x^2+\frac{1}{3}y^2} \cdot \frac{1}{\|(x,y)\|} =$

$\frac{x^2}{x^2+\frac{1}{3}y^2} \cdot (y^2+x^2) \cdot \frac{1}{\|(x,y)\|} \overset{\frac{x^2}{x^2+\frac{1}{3}y^2}\leq 1}{\leq}$

$\frac{y^2+x^2}{\|(x,y)\|} \overset{\frac{x^2}{x^2+\frac{1}{3}y^2}\leq 1}{\leq}$

$\frac{y^2+x^2}{\|(x,y)\|} =$

$\frac{\|(x,y)\|^{\cancel{2}}}{\cancel{\|(x,y)\|}} =$

$\|(x,y)\| \wedge \|(x,y)\| \overset{(x,y)\to(0,0)}{\to} 0$

$\Rightarrow$ entonces f es diferencibale en (0,0)