1. • 
$$z = x^2y^3$$

• 
$$x = s\cos(t)$$

• 
$$y = s\sin(t)$$

a) 
$$\frac{\partial z}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s} =$$

$$f_x(x(s), y(s)) \cdot \cos(t) + f_y(x(s), y(s)) \cdot \sin(t) =$$

$$2xy^3 \big|_{(x(s), y(s))} \cdot \cos(t) + 3x^2y^2 \big|_{(x(s), y(s))} \cdot \sin(t) =$$

$$2s\cos(t)(s\sin(t))^3 \cdot \cos(t) + 3(s\cos(t))^2(s\sin(t))^2 + \sin(t)$$

b) 
$$\frac{\partial z}{\partial t} \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = f_x(x(t), y(t)) \cdot (-s\sin(t)) + f_y(x(t), y(t)) \cdot s\cos(t) = 2s\cos(t)(s\sin(t))^3 \cdot (-s\sin(t)) + 3(s\cos(t))^2(s\sin(t))^2 \cdot s\cos(t) = 0$$

2. • 
$$z = \sin(x)\cos(y)$$

$$x = st^2$$

$$y = s^2 t$$

a) 
$$\frac{\partial z}{\partial s} \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = \cos(st^2)\cos(s^2t) \cdot t^2 + \cos(st^2)(-\sin(s^2t)) \cdot 2st$$

b) 
$$\frac{\partial z}{\partial t} \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = \cos(st^2)\cos(s^2t) \cdot 2st + \cos(st^2)(-\sin(s^2t)) \cdot s^2$$

3. • 
$$z = e^{x+2y}$$

• 
$$x = \frac{s}{t}$$

• 
$$y = \frac{t}{s}$$

a) 
$$\frac{\partial z}{\partial s} \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = e^{\frac{s}{t} + \frac{2t}{s}} \cdot \frac{1}{t} 2e^{\frac{s}{t} + \frac{2t}{s}} \cdot \frac{-t}{s^2}$$

b) 
$$\frac{\partial z}{\partial t} \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = e^{\frac{s}{t} + \frac{2t}{s}} \cdot \frac{-s}{t^2} 2e^{\frac{s}{t} + \frac{2t}{s}} \cdot \frac{1}{s}$$