Plano tangente a superficies implícitas

$\Phi : \nabla F(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0$

Ejercios

1.
   - $2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$
   - $p = (3, 3, 5)$

   $F = 2(x-2)^2 + (y-1)^2 + (z-3)^2 \Rightarrow$
   $\nabla F(x, y, z) = (4(x-2), 2(y-1), 2(z-3))$
   $\nabla F(3, 3, 5) = (4, 4, 4) = N$
   $N \cdot (x - 3, y - 3, z - 5) = 0$
   $4x - 12 + 4y - 12 + 4z - 20 = 0 \Leftrightarrow$
   $\cancel{4}(x + y + z) = \cancel{4}$
   $x + y + z = 1$

2.
   - $y = x^2 - z^2$
   - $p = (4, 7, 4)$

   $F(x, y, z) = x^2 - y - z^2$
   $\nabla F(x, y, z) = (2x, -1, -2z)$
   $\nabla F(4, 7, 4) = (8, -1, -8)$
   $(8, -1, -8) \cdot (x - 4, y - 7, z - 4) = 0$
   $8x - 32 - y + 7 - 8z + 32 = 0$
   $8x - 8z + 7 = y$

3.
   - $xy + yz + zx = 5$
   - $p = (1, 2, 1)$

   $F(x, y, z) = xy + yz + zx$
   $\nabla F(x, y, z) = (y + z, x + z, x + y)$
   $\nabla F(x, y, z) = (3, 2, 3)$
   $(y + z, x + z, x + y) \cdot (x - 1, y - 2, z - 1) = 0$
   $3x - 3 + 2y - 4 + 3z - 3 =$
   $3x + 2y + 3z = 10$