

## 1. Calculo en el primer octante

a)  $D_1$ 

$$\begin{cases} 0 \leq r \leq 1 \\ 0 \leq \theta \leq \frac{\pi}{4} \\ 0 \leq z \leq \sqrt{3} \end{cases}$$

$$\int_0^1 (\int_0^{\frac{\pi}{4}} (\int_0^{\sqrt{3}} r dz) d\theta) dr$$

$$\begin{aligned} \blacksquare \int_0^{\sqrt{3}} r dz &= \sqrt{3}r \\ \blacksquare \sqrt{3} \int_0^{\frac{\pi}{4}} r d\theta &= \sqrt{3} \frac{\pi}{4} r \\ \blacksquare \sqrt{3} \frac{\pi}{4} \int_0^1 r dr &= \sqrt{3} \frac{\pi}{4} \frac{1}{2} = \frac{\sqrt{3}\pi}{8} \end{aligned}$$

b)  $D_2$ 

$$\begin{cases} 0 \leq r \leq \sqrt{4-z^2} \\ 0 \leq \theta \leq \frac{\pi}{4} \\ \sqrt{3} \leq z \leq 2 \end{cases}$$

$$\int_{\sqrt{3}}^2 (\int_0^{\frac{\pi}{4}} (\int_0^{\sqrt{4-z^2}} r dr) d\theta) dz$$

$$\begin{aligned} \blacksquare \int_0^{\sqrt{4-z^2}} r dr &= \frac{\frac{4-z^2}{2}}{2} = 2 - \frac{z^2}{2} \\ \blacksquare \int_0^{\frac{\pi}{4}} 2 - \frac{z^2}{2} d\theta &= \frac{\pi}{2} - \frac{\pi z^2}{8} \\ \blacksquare \int_{\sqrt{3}}^2 \frac{\pi}{2} - \frac{\pi z^2}{8} dz &= \left. \frac{\pi}{2} z - \frac{\pi z^3}{24} \right|_{\sqrt{3}}^2 = \pi - \frac{\pi}{3} - \frac{\sqrt{3}\pi}{2} + \frac{\sqrt{3}\pi}{8} = \frac{2\pi}{3} - \frac{5\sqrt{3}\pi}{8} \end{aligned}$$

$D$  en el primer octante es  $D_1 + D_2$ , y el volumen total es  $8 \cdot D$

$$Vol = 8 \cdot \left( \frac{2\pi}{3} - \frac{\sqrt{3}\pi}{2} \right) = \frac{16\pi}{3} - 4\sqrt{3}\pi$$

$$2. \begin{cases} z = x^2 + y^2 \equiv z = r^2 \\ x^2 + y^2 + z^2 = 2 \equiv r^2 + z^2 = 2 \end{cases}$$

$$\begin{cases} z = x^2 + y^2 \\ x^2 + y^2 + x^2 + y^2 = 2 \equiv x^2 + y^2 = 1 \\ z = 1 \end{cases}$$

$$D = D_1 + D_2$$

a)  $D_1$ 

$$\begin{cases} 0 \leq z \leq \sqrt{2-r^2} \\ 0 \leq \theta \leq \frac{\pi}{4} \\ 0 \leq r \leq 1 \end{cases}$$

$$\int_0^1 (\int_0^{\frac{\pi}{4}} (\int_0^{\sqrt{2-r^2}} r dz) d\theta) dr$$

$$\left. \frac{\pi}{4} - \frac{1}{3} (2-r^2)^{\frac{3}{2}} \right|_0^1 =$$

$$\frac{\pi}{4} \left( -\frac{1}{3} \right)$$

b)  $D_2$ 

$$\begin{cases} 1 \leq z \leq \sqrt{2} \\ 0 \leq \theta \leq \frac{\pi}{4} \\ 0 \leq r \leq \sqrt{2-z^2} \end{cases}$$