1. $f(x,y) = x^2 + xy + y^2 + y$

- $f_x(x,y) = 2x + y$
  $= 0 \Leftrightarrow y = -2x$
- $f_y(x,y) = x + 2y + 1$
  $= 0 \Leftrightarrow x = -2y - 1$

$\Leftrightarrow x = \frac{1}{3} \wedge y = -\frac{2}{3}$

$H_f(\frac{1}{3}, -\frac{2}{3}) = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \Rightarrow det(H_f(\frac{1}{3}, -\frac{2}{3})) = 2$

$(\frac{1}{3}, -\frac{2}{3})$ es mínimo local de f

2. $f(x,y) = (x-y)(1-xy) = x - yx^2 - y + xy^2$

- $f_x(x,y) = 1 - 2xy + y^2$
  $= 0 \Leftrightarrow 0 = 1 + y(-2x + y)$
  $\Leftrightarrow -1 = y(-2x + y) \Leftrightarrow (x,y) \in \{(1,1), (-1,-1)\}$
- $f_y(x,y) = -1 - x^2 + 2xy$
  $= 0 \Leftrightarrow 0 = -1 + x(-x + 2y)$

$H_f(x,y) = \begin{vmatrix} -2y & -2x + 2y \\ -2x + 2y & 2x \end{vmatrix}$

Ptos criticos

- $(-1,-1)$
  $H_f(-1,-1) = -4 \Rightarrow$ es punto silla
- $(1,1)$
  $H_f(1,1) = -4 \Rightarrow$ es punto silla

3. $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$

- $f_x(x,y) = 6xy - 12x$
  $6x(y-2) = 0 \Leftrightarrow x = 0 \vee y = 2$
- $f_y(x,y) = 3y^2 + 3x^2 - 12y$
  $(-2,2), (2,2), (0,0), (0,4)$

$H_f(x,y) = \begin{vmatrix} 6y - 12 & 6x \\ 6x & 6y - 12 \end{vmatrix}$

Ptos criticos

- $(0,0)$
  $H_f(0,0) = -24$ pto silla
- $(0,2)$
  $H_f(0,2) = 0 - 0$ maximo
- $(2,2)$
  $H_f(2,2) = 0 - 12^2$ pto silla
- $(-2,2)$
  $H_f(-2,2) = 24^2 - 12^2$ maximo
- $(0,4)$
  $H_f(0,4) = (4 \cdot 12)^2 - 0$ minimo

4. $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$

- $f_x(x,y) = y + \frac{-1}{x^2}$
- $f_y(x,y) = x + \frac{-1}{y^2}$

$$H_f(x,y) = \begin{vmatrix} \frac{2}{x^3} & 1 \\ 1 & \frac{2}{y^3} \end{vmatrix}$$

Ptos criticos

- $(1,-1)$
  $H_f(1,-1) = -4 - 1$ pto silla
- $(1,1)$
  $H_f(1,1) = 4 - 1$ minimo
- $(-1,1)$
  $H_f(-1,1) = -4 - 1$ pto silla

5. $f(x,y) = y\cos(x)$

  - $f_x(x,y) = -y\sin(x)$
  - $f_y(x,y) = \cos(x)$

  $$H_f(x,y) = \begin{vmatrix} -y\cos(x) & \sin(x) - \sin(x) & 1 \end{vmatrix}$$

  Ptos criticos

  - $(0,0)$
    $H_f(0,0) = 0 - 0$ pto silla
  - $(\frac{\pi}{2}, 0)$
    $H_f(\frac{\pi}{2}, 0) = 0 + 1$ pto silla

6. $f(x,y) = e^y(y^2 - x^2) = e^y y^2 - e^y x^2$

  - $f_x(x,y) = -2xe^y$
    $x = 0$
  - $f_y(x,y) = e^y(y^2 + 2y) - e^y x^2$
    $y = 0 \vee y = -2$

  $$H_f(x,y) = \begin{vmatrix} -2e^y & -2xe^y \\ -2xe^y & e^y(-x^2 + y^2 + 4y + 2) \end{vmatrix}$$

  ptos criticos

  - $(0,0)$
    $H_f(0,0) = -2(2) - 0$ pto silla
  - $(0,-2)$
    $H_f(0,0) = -4e^{-2}$ pto silla