

1. $f(x, y, z) = \sqrt{|xyz|}$ No existen las derivadas parciales en el origen

$$2. f(x, y) = \begin{cases} \frac{x^4 - y^4}{x^2 + y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$f_x(0, 0) = \lim_{x \rightarrow 0} \frac{f(h, 0) - f(0, 0)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\frac{h^4}{h^2}}{h} =$$

$$\lim_{h \rightarrow 0} \frac{h^2}{h^3} = 0$$

$$f_y(0, 0) = \lim_{h \rightarrow 0} \frac{f(0, h) - f(0, 0)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\frac{-h^4}{h^2}}{h} =$$

$$\lim_{h \rightarrow 0} \frac{-h^2}{h^3} = 0$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{\frac{x^4 - y^4}{x^2 + y^2}}{\|x^2 + y^2\|} \stackrel{?}{=} 0$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{\frac{x^4 - y^4}{\|x^2 + y^2\|^2}}{\|x^2 + y^2\|} =$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x^4 - y^4}{\|x^2 + y^2\|^3}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{|x^4 - y^4|}{\|x^2 + y^2\|^3} \stackrel{DesTrian}{\leq}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x^4 + y^4}{\|x^2 + y^2\|^3} \leq$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{2\|x^2 + y^2\|^2}{\|x^2 + y^2\|^3} \leq$$

$$2\lambda < \epsilon \Rightarrow \lambda \leq \frac{\epsilon}{2}$$

$$\Pi : z = 0$$

$$3. f(x, y) = \begin{cases} \frac{x^2}{x^2 + y^2} \cdot \sin\left(\frac{1}{\sqrt{x^2 + y^2}}\right) & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$f_x(0, 0) = \lim_{h \rightarrow 0} \frac{f(h, 0) - f(0, 0)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\frac{h^2}{h^2} \cdot \sin\left(\frac{1}{\sqrt{h^2}}\right)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\sin\left(\frac{1}{\sqrt{h^2}}\right)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\sin\left(\frac{1}{\sqrt{h^2}}\right)}{h} = \nexists$$

\Rightarrow No es diferenciable en el origen