

$$\begin{aligned}
1. \int_2^{+\infty} \frac{dx}{x \ln^2(x)} &= \\
-\frac{1}{\ln(x)} \Big|_{+\infty}^2 &= \\
\lim_{x \rightarrow +\infty} -\frac{1}{\ln(2)} + \frac{1}{\ln(x)} &= -\frac{1}{\ln(2)}
\end{aligned}$$

$$\begin{aligned}
2. \int_0^1 \frac{dx}{\sqrt{1-x^2}} & \\
\begin{aligned}
&\blacksquare x = \sin(u) \Rightarrow \arcsin(x) = u \\
&\blacksquare dx = \cos(u) du
\end{aligned} & \\
\Rightarrow \int_0^1 \frac{dx}{\sqrt{1-x^2}} &= \\
\int_0^1 \frac{\cos(u) du}{\sqrt{1-\sin^2(u)}} &\stackrel{\sin^2(u)+\cos^2(u)=1}{=} \\
\int_0^1 \frac{\cancel{\cos(u)} du}{\sqrt{\cancel{\cos^2(u)}}} &= \\
\int_0^1 1 du = u \Big|_0^1 &= \arcsin(x) \Big|_0^1 = \arcsin(0) - \arcsin(1) = -\arcsin(1)
\end{aligned}$$

$$\begin{aligned}
3. \int_0^{+\infty} \frac{\arctan(x)}{1+x^2} dx & \\
\begin{aligned}
&\blacksquare u = \arctan(x) \\
&\blacksquare du = \frac{1}{1+x^2} dx
\end{aligned} & \\
\Rightarrow \int_0^{+\infty} u du &= \\
\frac{\arctan(x)^2}{2} \Big|_0^{+\infty} &= \\
\lim_{x \rightarrow \infty} \frac{\arctan(x)^2}{2} &= \frac{\pi^2}{8}
\end{aligned}$$

$$\begin{aligned}
4. \int_{-\infty}^{+\infty} \frac{dx}{1+x^2} & \\
= \arctan(x) \Big|_{-\infty}^{+\infty} &= -\pi
\end{aligned}$$

$$\begin{aligned}
5. \int_{-\infty}^{+\infty} \frac{dx}{x^2+4x+9} &= \\
\int_{-\infty}^{+\infty} \frac{dx}{(x+2)^2+5} &= \\
\int_{-\infty}^{+\infty} \frac{dx}{(x+2)^2+\sqrt{5}^2} & \\
\begin{aligned}
&\blacksquare u = x - 2 \\
&\blacksquare du = dx
\end{aligned} & \\
\int_{-\infty}^{+\infty} \frac{du}{u^2+\sqrt{5}^2} &= \\
\frac{1}{\sqrt{5}} \arctan\left(\frac{x}{\sqrt{5}}\right) \Big|_{-\infty}^{+\infty} &= \frac{\pi}{\sqrt{5}}
\end{aligned}$$

$$6. \int_0^{+\infty} \frac{x}{\sqrt{1+x^5}} dx$$

$$\begin{aligned}
7. \int_{-1}^3 \frac{dx}{(1-x)^3} & \\
\begin{aligned}
&\blacksquare u = 1 - x \\
&\blacksquare du = -dx
\end{aligned} &
\end{aligned}$$

$$\begin{aligned}
&\int_{-1}^3 \frac{-dx}{u^3} \\
&\int_{-1}^3 \frac{-dx}{u^3} \\
&\frac{1}{2}(1-x)^{-2} \Big|_{-1}^3 \\
&\frac{1}{2(1-3)^2} - \frac{1}{2(2)^2} = \text{Converge}
\end{aligned}$$

$$8. \int_{-\infty}^{+\infty} \sin(2x) dx =$$

$$\left. \frac{-\cos(2x)}{2} \right|_{-\infty}^{+\infty} =$$

*diverge*

$$9. \int_0^4 \frac{x}{x^2-4} dx =$$

$$\int_0^4 \frac{x}{(x-2)(x+2)} dx =$$

$$\blacksquare u = x^2 - 4$$

$$\blacksquare du = 2x dx \Rightarrow dx = \frac{du}{2x}$$

$$\int_0^4 \cancel{u} \frac{du}{\cancel{2}} =$$

$$\left. \frac{\ln(x^2-4)}{2} \right|_0^4 =$$

$$\frac{\ln(0)}{2} - \frac{\ln(-4)}{2} = \textit{Diverge}$$