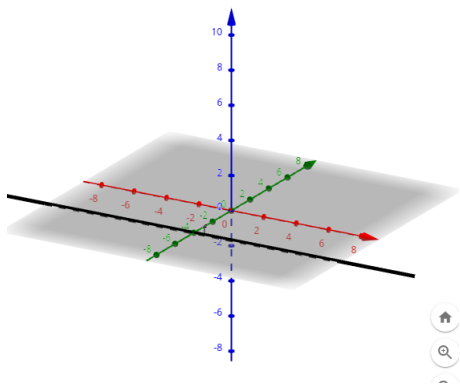
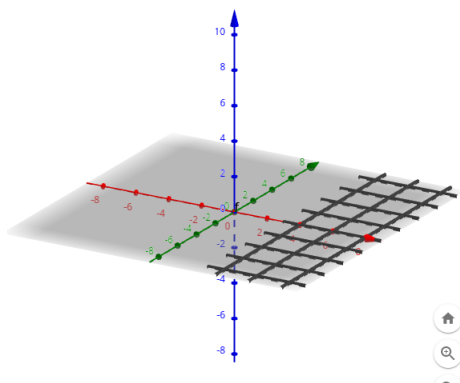


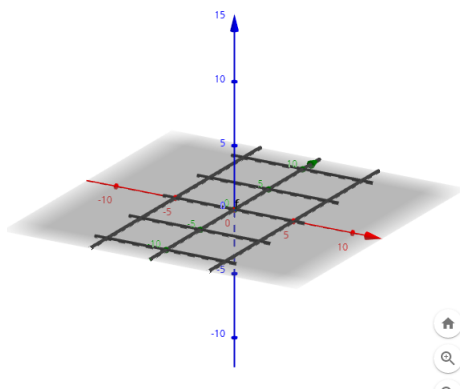
1. a) $y = -4$



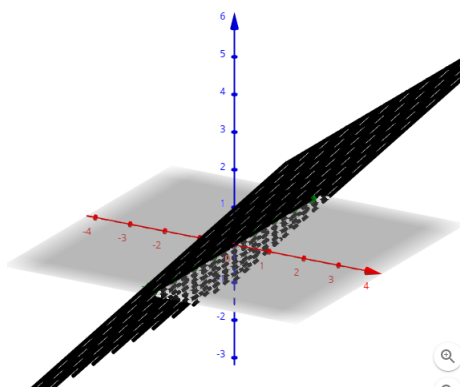
b) $b > 3$



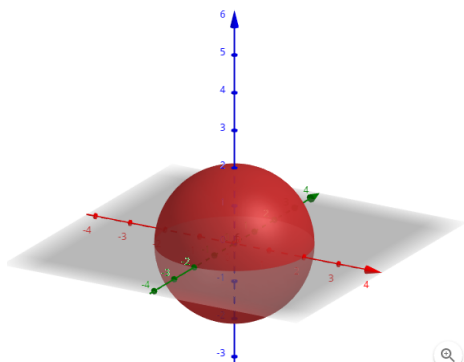
c) $0 \leq z \leq 6$



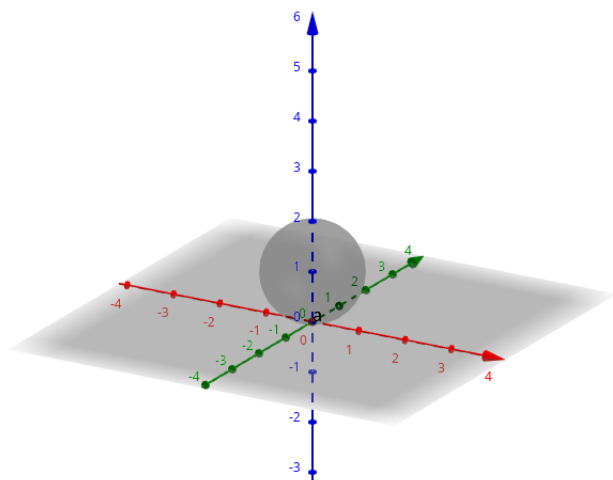
d) $x = z$



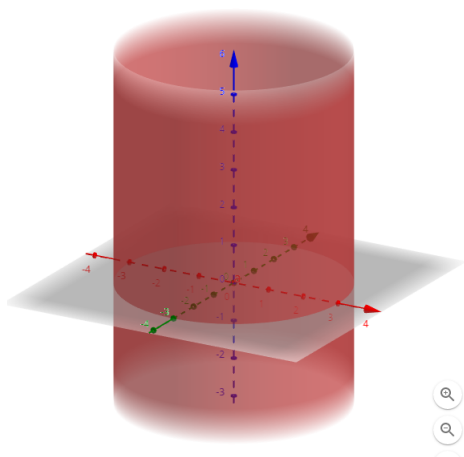
e) $x^2 + y^2 + z^2 \leq 4$



f) $x^2 + y^2 + z^2 > 2z \equiv x^2 + y^2 + (z - 1)^2 > 1$



g) $x^2 + y^2 \leq 9$



2. a) $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$

$P_0 = (a, b, c) \wedge b : (x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$ entonces

b es un círculo con centro en P_0 y radio r

$$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 6x + 4y - 2z + j = 11 + j$$

■ $-2ax = -6x \equiv a = 3$

$$\blacksquare -2by = 4y \equiv b = -2$$

$$\blacksquare -2cz = -2z \equiv c = 1$$

$$(x-3)^2 + (y+2)^2 + (z-1)^2 \equiv$$

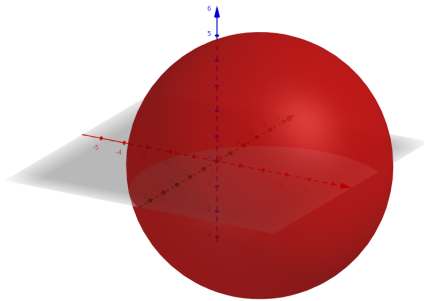
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) + (z^2 - 2z + 1) = 11 + j \equiv$$

$$(x^2 + y^2 + z^2) - 6x + 4y - 2z + 9 + 4 + 1 = 11 + j \equiv$$

$$(x^2 + y^2 + z^2) - 6x + 4y - 2z + 14 = 11 + j \equiv$$

$$(x^2 + y^2 + z^2) - 6x + 4y - 2z = 25$$

Es un círculo con centro en $(3, -2, 1)$ y radio 5



$$b) 4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$$

$$4(x^2 + y^2 + z^2 - 2x + 4y) = 1 \equiv$$

$$x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$$

$$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$$

$$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$$

$$\blacksquare -2ax = -2x \equiv a = 1$$

$$\blacksquare -2by = 4y \equiv b = -2$$

$$\blacksquare -2cz = 0 \equiv c = 0$$

$$(x-1)^2 + (y+2)^2 + (z)^2 \equiv$$

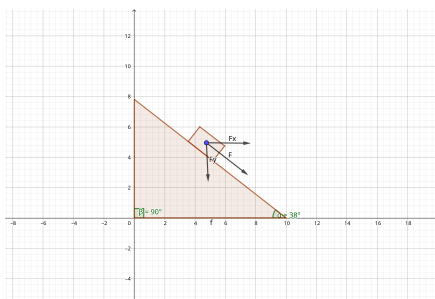
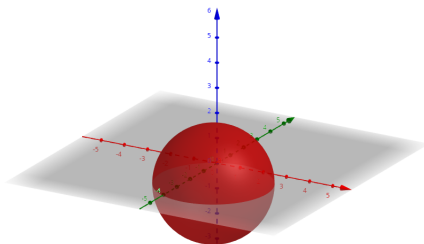
$$x^2 + -2x + 1 + y^2 + 4y + 4 + z^2 \equiv$$

$$(x^2 + y^2 + z^2 - 2x + 4y) + 5 = \frac{1}{4} + 5 \equiv$$

$$x^2 + y^2 + z^2 - 2x + 4y + 5 = \frac{21}{4}$$

$$(x-1)^2 + (y+2)^2 + (z)^2 = \frac{21}{4}$$

Es un círculo con centro en $(1, -2, 0)$ y radio $\frac{\sqrt{21}}{2}$



3.

- $\alpha = 38^\circ$
- $F = 50N$
 $F_x = 50N \cdot \cos 38^\circ = 39,40N$
 $F_y = 50N \cdot \sin 38^\circ = -30,78N$

4.
 - $F_1 = 10N$
 - $F_2 = 12N$
 - $\alpha = 45^\circ$
 - $\theta = 30^\circ$
 - $F_{1x} = F_1 \cdot \cos \alpha = 10N \cdot \cos(45^\circ) = -7,07N$
 - $F_{1y} = F_1 \cdot \sin \alpha = 10N \cdot \sin(45^\circ) = 7,07N$
 - $F_{2x} = F_2 \cdot \cos \theta = 12N \cdot \cos(30^\circ) = 10,39N$
 - $F_{2y} = F_2 \cdot \sin \theta = 12N \cdot \sin(30^\circ) = 6N$
 - $F_{rx} = F_{1x} + F_{2x} = -7,07N + 10,39N = 3,32N$
 - $F_{ry} = F_{1y} + F_{2y} = 7,07N + 6N = 13,07N$
 - $F_r = \sqrt{(F_{rx})^2 + (F_{ry})^2} = \sqrt{(3,32N)^2 + (13,07N)^2} = 13,48N$

5. $u \in \mathbb{R}^2 \wedge \|u\| = 1$

- *Triangulo*



- $u \cdot v \stackrel{\|u\|=\|v\|}{=} \|u\|^2 * \cos(\alpha) = -\frac{1}{2}$
- $u \cdot w \stackrel{\|u\|=\|w\|}{=} \|u\|^2 * \cos(\alpha) = -\frac{1}{2}$

- *Cuadrado*

- $u \cdot v \stackrel{\|u\|=\|v\|}{=} \|u\|^2 * \cos(90^\circ) = 0$
- $u \cdot w \stackrel{\|u\|=\|w\|}{=} \|u\|^2 * \cos(45^\circ) = \frac{\sqrt{2}}{2}$

6. a) $u = (3, -4), v = (5, 0)$
 $P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(3, -4) \cdot (5, 0)}{\|(3, -4)\|^2} \cdot (3, -4) =$
 $\frac{15}{25} \cdot (3, -4) = (\frac{9}{5}, -\frac{12}{5})$
- b) $u = (1, 2), v = (-4, 1)$
 $P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(1, 2) \cdot (-4, 1)}{\|(1, 2)\|^2} \cdot (1, 2) =$
 $-\frac{2}{5} \cdot (1, 2) = (-\frac{2}{5}, -\frac{4}{5})$
- c) $u = (3, 6, 2), v = (1, 2, 3)$
 $P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(3, 6, 2) \cdot (1, 2, 3)}{\|(3, 6, 2)\|^2} \cdot (3, 6, 2) =$
 $\frac{3}{7} \cdot (3, 6, 2) = (\frac{9}{7}, \frac{18}{7}, \frac{6}{7})$

7. u, v vectores QvQ $o_u(v) = v - p_u(v)$ es ortogonal a u

$o_u(v)$ es ortogonal a $u \Leftrightarrow o_u(v) \cdot u = 0$

$$o_u(v) = v - \frac{u \cdot v}{\|u\|^2} \cdot u$$

$$\text{QvQ } o_u(v) \cdot u = 0$$

$$(v - \frac{u \cdot v}{\|u\|^2} \cdot u) \cdot u \equiv u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot u \cdot u \stackrel{u \cdot u = \|u\|^2}{=} u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot \|u\|^2 \stackrel{u \neq 0}{=} u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot \|u\|^2 \equiv u \cdot v - u \cdot v = 0 \quad \blacksquare$$

8. u, v vectores $u \neq 0 \wedge v \neq 0$ QvC $p_u(v) = p_v(u)$

$$v = \lambda u \rightarrow p_u(v) = v \wedge v = \theta u \rightarrow p_v(u) = u$$

$$p_u(v) = p_v(u) \equiv \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{u \cdot v}{\|v\|^2} \cdot v \stackrel{u \cdot v \neq 0}{\equiv} \frac{\cancel{u \cdot v}}{\|u\|^2} \cdot u = \frac{\cancel{u \cdot v}}{\|v\|^2} \cdot v \equiv$$

$$\frac{u}{\|u\|^2} = \frac{v}{\|v\|^2} \cdot v \leftrightarrow u = \lambda \cdot v \wedge v = \theta u \wedge \lambda, \theta \in \Re \leftrightarrow u = v$$

$$\text{Si } u \cdot v = 0 \rightarrow \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{u \cdot v}{\|v\|^2} \cdot v = 0$$

$$p_u(v) = p_v(u) \leftrightarrow u \cdot v = 0 \vee u = v \quad \blacksquare$$

9. $W = 4m \cdot 20N \cdot \cos(50^\circ) = 51,42J$

10. $W = (Q - P) \cdot F \equiv (6, 2, 12) \cdot (8, -6, 9) = 144J$