

1. $\int \int \int_E (9 - x^2 - y^2) dV, E: x^2 + y^2 + z^2 \leq 9 \wedge z \leq 0$

E describe en coordenadas esfericas

$$\begin{cases} x = \rho \cos(\theta) \sin(\phi) & 0 \leq \rho \leq 3 \\ y = \rho \sin(\theta) \sin(\phi) & \theta \in [0, 2\pi] \\ z = \rho \cos(\phi) & \phi \in [\frac{\pi}{2}, \pi] \end{cases}$$

$$f(x, y) = 9 - x^2 - y^2$$

$$T(\rho \cos(\theta) \sin(\phi), \rho \sin(\theta) \sin(\phi), \rho \cos(\phi)) = 9 - (\rho \cos(\theta) \sin(\phi))^2 - (\rho \sin(\theta) \sin(\phi))^2 =$$

$$9 - \rho^2 \sin^2(\phi) (\cos^2(\theta) + \sin^2(\theta)) =$$

$$9 - \rho^2 \sin^2(\phi)$$

$$\int_0^3 (\int_0^{2\pi} (\int_{\frac{\pi}{2}}^{\pi} 9 - x^2 - y^2 d\phi) d\theta) d\rho =$$

$$\int_0^3 (\int_0^{2\pi} (\int_{\frac{\pi}{2}}^{\pi} (9 - \rho^2 \sin^2(\phi)) \rho^2 \sin(\phi) d\phi) d\theta) d\rho =$$

$$\int_0^3 (\int_0^{2\pi} (\int_{\frac{\pi}{2}}^{\pi} 9\rho^2 \sin(\phi) - \rho^4 \sin^3(\phi) d\phi) d\theta) d\rho =$$

$$\begin{aligned} & \int_{\frac{\pi}{2}}^{\pi} 9\rho^2 \sin(\phi) - \rho^4 \sin^3(\phi) d\phi \\ & \int \sin^3(\phi) d\phi = \\ & \int (1 - \cos^2(\phi)) \sin(\phi) d\phi = \\ & -\cos(\phi) + \frac{\cos^3(\phi)}{3} + C \\ & \int_{\frac{\pi}{2}}^{\pi} 9\rho^2 \sin(\phi) - \rho^4 \sin^3(\phi) d\phi \\ & 9\rho^2 \cos(\phi) + \cos(\phi) + \frac{\cos^3(\phi)}{3} \Big|_{\frac{\pi}{2}}^{\pi} = \\ & 9\rho^2 + 1 + \frac{1}{3} = 9\rho^2 + \frac{4}{3} \end{aligned}$$

$$\begin{aligned} & \int_0^{2\pi} 9\rho^2 + \frac{4}{3} d\theta = \\ & 18\pi\rho^2 + \frac{8\pi}{3} \end{aligned}$$

$$\begin{aligned} & \int_0^3 18\pi\rho^2 + \frac{8\pi}{3} d\rho = \\ & 9\pi\rho + \frac{8\pi\rho}{3} \Big|_0^3 = \\ & 27\pi + 8\pi = 35\pi \end{aligned}$$

$\int \int \int_E x e^{x^2+y^2+z^2} dV, E: x^2 + y^2 + z^2 \leq 1$ en el primer octante

$$\begin{cases} x = \rho \cos(\theta) \sin(\phi) & 0 \leq \rho \leq 1 \\ y = \rho \sin(\theta) \sin(\phi) & 0 \leq \theta \leq \frac{\pi}{2} \\ z = \rho \cos(\phi) & 0 \leq \phi \leq \frac{\pi}{2} \end{cases}$$

$$\int_0^1 (\int_0^{\frac{\pi}{2}} (\int_0^{\frac{\pi}{2}} \rho \cos(\theta) \sin^2(\phi) e^{\rho^2} \rho^2 d\phi) d\theta) d\rho$$

$$\begin{aligned} & \int_0^{\frac{\pi}{2}} \rho \cos(\theta) \sin^2(\phi) e^{\rho^2} \rho^2 d\phi = \\ & \rho \cos(\theta) (1 - \sin(\phi)) e^{\rho^2} \rho^2 \Big|_0^{\frac{\pi}{2}} = \\ & \frac{\pi}{4} \cos(\theta) e^{\rho^2} \rho^3 \end{aligned}$$

$$\begin{aligned} & \frac{\pi}{4} e^{\rho^2} \rho^3 \int_0^{\frac{\pi}{2}} \cos(\theta) d\theta = \\ & \frac{\pi}{4} e^{\rho^2} \rho^3 - \sin(\theta) \Big|_0^{\frac{\pi}{2}} = \\ & -\frac{\pi}{4} e^{\rho^2} \rho^3 \end{aligned}$$

$$\begin{aligned} & -\frac{\pi}{4} \int_0^1 e^{\rho^2} \rho^3 d\rho = \\ & -\frac{\pi}{4} \cdot \frac{1}{2} = \\ & -\frac{\pi}{8} \end{aligned}$$