

$$1. E = x^2 + y^2 = 16 \wedge z = -5 \wedge z = 4$$

$$x^2 + y^2 = 16 \equiv T(r, \theta) = (r \cos(\theta), r \sin(\theta)), 0 \leq \theta \leq 2\pi$$

$$\wedge -5 \leq z \leq 4$$

$$E' = \begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ z = z \end{cases} \quad \wedge$$

$$\begin{cases} 0 \leq \theta \leq 2\pi \\ -4 \leq r \leq 4 \\ -5 \leq z \leq 4 \end{cases}$$

$$\int \int \int_E \sqrt{x^2 + y^2} dV(x, y) = \int \int \int_{E'} r^2 dr$$

$$\int_{-4}^4 (\int_0^{2\pi} (\int_{-5}^4 r^2 dz) d\theta) dr$$

$$\begin{aligned} \blacksquare \int_{-5}^4 r^2 dz &= \\ \left. \frac{r^3}{3} \right|_{-5}^4 &= \\ \frac{4^3}{3} + \frac{5^3}{3} &= 63 \end{aligned}$$

$$2. E = z = x^2 + y^2 \wedge z = 4$$

$$E' = \begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ z = z \end{cases} \quad \wedge$$

$$\begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq 2 \\ r^2 \leq z \leq 4 \end{cases}$$

$$\int_0^2 (\int_0^{2\pi} (\int_{r^2}^4 z r dz) d\theta) dr$$

$$\begin{aligned} \blacksquare \int_{r^2}^4 z r dz &= \\ \left. r \frac{z^2}{2} \right|_{r^2}^4 &= \\ r(8 - \frac{r^4}{2}) &= \\ 8r - \frac{r^5}{2} & \\ \blacksquare \int_0^{2\pi} 8r - \frac{r^5}{2} d\theta &= \\ 16r\pi - 8r^5\pi &= 8\pi(2r - r^5) \\ \blacksquare 8\pi \cdot \int_0^2 2r - r^5 dr &= \\ 8\pi \left(r^2 - \frac{r^6}{6} \right) \bigg|_0^2 &= \\ 8\pi(4 - \frac{32}{3}) & \end{aligned}$$

$$3. E = x^2 + y^2 = 1 \wedge z = 0 \wedge z^2 = 4x^2 + 4y^2$$

$$E' = \begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ z = z \end{cases} \quad \wedge$$

$$\begin{cases} 0 \leq \theta \leq 2\pi \\ \frac{z}{2} \leq r \leq 1 \\ 0 \leq z \leq 2 \end{cases}$$

$$\int_0^2 (\int_0^{2\pi} (\int_{\frac{z}{2}}^1 r^3 \cos^2(\theta) dr) d\theta) dz$$

$$\begin{aligned} \blacksquare \int_{\frac{z}{2}}^1 r^3 \cos^2(\theta) dr &= \\ \cos^2(\theta) \left. \frac{r^4}{4} \right|_{\frac{z}{2}}^1 &= \\ \cos^2(\theta) (\frac{1}{4} - \frac{z^4}{64}) &= \end{aligned}$$

$$\begin{aligned}
 & \blacksquare \left(-\frac{z^4}{64} + \frac{1}{4}\right) \int_0^{2\pi} \cos^2(\theta) d\theta = \\
 & \left(-\frac{z^4}{64} + \frac{1}{4}\right) -2 \cos(\theta) \sin(\theta) \Big|_0^{2\pi} = \\
 & -\frac{z^4}{64} + \frac{1}{4} \\
 & \blacksquare \int_0^2 -\frac{z^4}{64} + \frac{1}{4} dz = \\
 & -\frac{z^5}{320} + \frac{z}{4} \Big|_0^2 = \\
 & -\frac{1}{10} + \frac{1}{2} = \frac{2}{5}
 \end{aligned}$$