1. Polinomio de Taylor

$k \geq 1 \in \mathbb{Z} \wedge f : \mathbb{R} \to \mathbb{R}$ diferenciable k veces en el punto $a \in \mathbb{R}$

$\Rightarrow h_k : \mathbb{R} \to \mathbb{R}\ tq$

- $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... + \frac{f^{(k)}(a)}{k!} + h_k(x)(x-a)^k$
- $lim_{x \to a} h_k(x) = 0$

a)
- $f(x) = \frac{1}{1-x}$
- orden 5
- $x_0 = 0$

- $f(x) = (1-x)^{-1}$
- $f'(x) = -1(1-x)^{-2}$
- $f''(x) = 2(1-x)^{-3}$
- $f^{(3)}(x) = -6(1-x)^{-4}$
- $f^{(4)}(x) = 24(1-x)^{-5}$
- $f^{(5)}(x) = -120(1-x)^{-6}$

$\Rightarrow P_5(x) = 1 - 1x + \frac{2x^2}{2!} - \frac{6x^3}{3!} + \frac{24x^4}{4!} - \frac{120x^5}{5!}$

b)
- $f(x) = \sin(x)$
- orden 4
- $x_0 = 0$

- $f(x) = \sin(x)$
- $f'(x) = \cos(x)$
- $f''(x) = -\sin(x)$
- $f^{(3)}(x) = -\cos(x)$
- $f^{(4)}(x) = \sin(x)$

$\Rightarrow P_4(x) = x - \frac{x^3}{3!}$

c)
- $f(x) = \sin(x)$
- orden 5
- $x_0 = 0$

- $f(x) = \sin(x) \Rightarrow f(0) = 0$
- $f'(x) = \cos(x) \Rightarrow f'(0) = 1$
- $f''(x) = -\sin(x) \Rightarrow f''(0) = 0$
- $f^{(3)}(x) = -\cos(x) \Rightarrow f^{(3)}(0) = -1$
- $f^{(4)}(x) = \sin(x) \Rightarrow f^{(4)}(0) = 0$
- $f^{(5)}(x) = \cos(x) \Rightarrow f^{(5)}(0) = 1$

$\Rightarrow P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

d)
- $f(x) = \cos(x)$
- orden 5
- $x_0 = 0$

- $f(x) = \cos(x) \Rightarrow f(0) = 1$
- $f'(x) = -\sin(x) \Rightarrow f'(0) = 0$
- $f''(x) = -\cos(x) \Rightarrow f''(0) = -1$
- $f^{(3)}(x) = \sin(x) \Rightarrow f^{(3)}(0) = 0$
- $f^{(4)}(x) = \cos(x) \Rightarrow f^{(4)}(0) = 1$
- $f^{(5)}(x) = -\sin(x) \Rightarrow f^{(5)}(0) = 0$

$\Rightarrow P_5(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

e)
- $f(x) = \ln(x)$
- orden 4
- $x_0 = 1$

- $f(x) = \ln(x) \Rightarrow f(1) = 0$
- $f'(x) = x^{-1} \Rightarrow f'(1) = 1$
- $f''(x) = -x^{-2} \Rightarrow f''(1) = -1$
- $f^{(3)}(x) = 2x^{-3} \Rightarrow f^{(3)}(1) = 2$
- $f^{(4)}(x) = -6x^{-4} \Rightarrow f^{(4)}(1) = 6$

$\Rightarrow P_4(x) = 0 + (x-1) - \frac{(x-1)^2}{2!} + 2\frac{(x-1)^3}{3!} - 6\frac{(x-1)^4}{4!}$

f)
- $f(x) = \sqrt{x}$
- orden 3
- $x_0 = 4$

- $f(x) = \sqrt{x} \Rightarrow f(4) = 2$
- $f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(4) = \frac{1}{2}$
- $f''(x) = -\frac{1}{4x^{\frac{3}{2}}} \Rightarrow f''(4) = -\frac{1}{4(4^{\frac{3}{2}})}$
- $f^{(3)}(x) = \frac{3}{8x^{\frac{5}{2}}} \Rightarrow f^{(3)}(4) = \frac{3}{8(4^{\frac{5}{2}})}$

$\Rightarrow P_3(x) = 2 + \frac{1}{2}(x-4) - \frac{1}{4(4^{\frac{3}{2}})}\frac{(x-1)^2}{2!} + \frac{3}{8(4^{\frac{5}{2}})}\frac{(x-1)^3}{3!}$

g)
- $f(x) = e^x$
- orden 5
- $x_0 = 0$

- $f(x) = e^x \Rightarrow f(0) = 1$
- $f'(x) = e^x \Rightarrow f'(0) = 1$
- $f''(x) = e^x \Rightarrow f''(0) = 1$
- $f^{(4)}(x) = e^x \Rightarrow f^{(3)}(0) = 1$
- $f^{(5)}(x) = e^x \Rightarrow f^{(3)}(0) = 1$

$\Rightarrow P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$

h)
- $f(x) = (1+x)^6$
- orden 6
- $x_0 = 0$

- $f(x) = (1+x)^6 \Rightarrow f(0) = 1$
- $f'(x) = 6(1+x)^5 \Rightarrow f'(0) = 6$
- $f''(x) = 30(1+x)^4 \Rightarrow f''(0) = 30$
- $f^{(3)}(x) = 120(1+x)^3 \Rightarrow f^{(3)}(0) = 120$
- $f^{(4)}(x) = 360(1+x)^2 \Rightarrow f^{(3)}(0) = 360$
- $f^{(5)}(x) = 720(1+x) \Rightarrow f^{(3)}(0) = 720$
- $f^{(6)}(x) = 720 \Rightarrow f^{(3)}(0) = 720$

$\Rightarrow P_6(x) = 1 + 6x + 30\frac{x^2}{2!} + 120\frac{x^3}{3!} + 360\frac{x^4}{4!} + 720\frac{x^5}{5!} + 720\frac{x^6}{6!} =$
$1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$

2. *a*)
- $f(x) = ln(x+1)^2$
- orden 3
- $x_0 = 0$

- $f(x) = \ln(x+1)^2 \Rightarrow f(0) = 0$

- $f'(x) = 2\ln(x+1)\frac{1}{x+1} \Rightarrow f'(0) = 0$
- $f''(x) = \frac{2}{(x+1)^2} - \frac{2\ln(x+1)}{(x+1)^2} \Rightarrow f''(0) = 2$
- $f'''(x) = \frac{4\ln(x+1)-6}{(x+1)^3} \Rightarrow f'''(0) = -6$

$\Rightarrow P_3(x) = 2\frac{x^2}{2!} - 6\frac{x^3}{3!} = x^2 - x^3$

b)
- $g(x) = e^{x+2}$
- orden 3
- $x_0 = 0$

- $g(x) = e^{x+2} \Rightarrow g(0) = e^2$
- $g'(x) = e^{x+2} \Rightarrow g'(0) = e^2$
- $g''(x) = e^{x+2} \Rightarrow g''(0) = e^2$
- $g'''(x) = e^{x+2} \Rightarrow g'''(0) = e^2$

$\Rightarrow P_3(x) = e^2(1 + x + \frac{x^2}{2} + \frac{x^3}{6})$

c)
- $p(x) = x^4 - 5x^3 + 5x^2 + x + 2$
- potencias de $x - 2$

- $p(x) = x^4 - 5x^3 + 5x^2 + x + 2$
- orden 4
- $x_0 = 2$

- $p(x) = x^4 - 5x^3 + 5x^2 + x + 2 \Rightarrow p(2) = 16 - 5 \cdot 8 + 5 \cdot 4 + 2 + 2 = 0$
- $p'(x) = 4x^3 - 15x^2 + 10x + 1 \Rightarrow p'(2) = -7$
- $p''(x) = 12x^2 - 30x + 10 \Rightarrow p''(2) = -2$
- $p'''(x) = 24x - 30 \Rightarrow p'''(2) = 18$
- $p''''(x) = 24 \Rightarrow p'''(2) = 24$

$\Rightarrow P_4(x) = -7(x-2) - 2\frac{(x-2)^2}{2!} + 18\frac{(x-2)^3}{3!} + 24\frac{(x-2)^4}{4!} = -7(x-2) - (x-2)^2 + 3(x-2)^3 + (x-2)^4$

d)
- $g(x) = \sqrt{x}$
- potencias de $x - 1$
- orden 3
- $x_0 = 1$

- $g(x) = \sqrt{x} \Rightarrow g(1) = 1$
- $g'(x) = \frac{1}{2\sqrt{x}} \Rightarrow g'(1) = \frac{1}{2}$
- $g''(x) = \frac{-1}{4x^{\frac{3}{2}}} \Rightarrow g''(1) = -\frac{1}{4}$
- $g'''(x) = \frac{3}{8x^{\frac{5}{2}}} \Rightarrow g'''(1) = \frac{3}{8}$

$\Rightarrow P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{4}\frac{(x-1)^2}{2!} + \frac{3}{8}\frac{(x-1)^3}{3!} = 1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16} =$

3. *a)* $f(x) = \frac{1}{1-x}$

   $P_n(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + .. + x^n$

   b) $f(x) = \cos(x)$

   $P_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}..$

   c) $f(x) = \sin(x)$

   $P_n(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}..$

   d) $f(x) = e^{2x}$

   $P_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + .. + \frac{x^n}{n!}$

   e) $f(x) = \frac{1}{(1-x)^2}$

   $P_n(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + .. + (n+1)x^n$

   f) $f(x) = ln(1+x)$

   $P_n(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + .. + (-1)^{n+1}\frac{x^n}{n}$

4.
- $p(x) = (x-2)^5 + 3(x-2)^4 + 3(x-2)^2 - 8$
- f de orden 5 en $x_0 = 2$

$f^{(3)}(2) = 0$

$\frac{f^{(4)}(2)}{4!} = 3 \Rightarrow f^{(4)}(2) = 3 \cdot 4!$

No se puede conocer $f^{(6)}(2)$

Si p fuera de orden 7 $\Rightarrow f^{(6)}(2) = 0$

5. $p_2(x) = -2 + 3(x-2) - 3(x-2)^2$

$q_2(x) = 5 + 12(x-2)^2$

$t(x) = (-2+3(x-2)-3(x-2)^2)(-2+3(x-2)-3(x-2)^2) - (-3(x-2)^2)(12(x-2)^2) - (12(x-2)^2)(3(x-2))$

$s(x) = \frac{(-2+3(x-2)-3(x-2)^2)}{(-2+3(x-2)-3(x-2)^2)}$

6.
a) $R_4(x) = e^c \frac{x^5}{5!}$
b) $R_5(x) = \frac{720}{(1-c)^7} \frac{x^6}{6!}$
c) $R_5(x) = -\sin(c) \frac{x^6}{6!}$
d) $R_6(x) = -\cos(c) \frac{x^7}{7!}$
e) $R_6(x) = -\frac{-6}{c^2} \frac{(x-1)^4}{4!}$

7.
a) $p_2(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{3}{8(1+c)^{\frac{5}{2}}} \frac{x^3}{3!}$
b) $R_2(\frac{2}{10}) = \frac{3}{8(1+c)^{\frac{5}{2}}} \frac{(\frac{2}{10})^3}{3!} =$

$\frac{\cancel{3}}{8(1+c)^{5/2}} \frac{8}{10^3(\cancelto{2}{3!})} =$

$\frac{1}{8(1+c)^{5/2}} \frac{\cancelto{4}{8}}{10^3(\cancel{2})} =$

$\frac{1}{\cancelto{2}{8}(1+c)^{5/2}} \frac{\cancelto{1}{4}}{10^3} =$

$\frac{1}{10^3(1+c)^{5/2}}$

8.
a) $f(x) = \cos(x) \wedge p_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!}$

$f(x) - p_4(x) = E_4(x) = -\sin(c)\frac{x^4}{5!}$

$|-\sin(c)\frac{x^4}{5!}| \le 5 \cdot 10^{-5}$

$|-\sin(c)\frac{x^4}{5!}| \le$

$|\sin(c)| \frac{x^4}{5!} \overset{|\sin(x)| \le x}{\le}$

$|c| \frac{x^4}{5!} \overset{|c| \le |x-x_0|}{\le}$

$|x| \frac{x^4}{5!} =$

$\frac{|x|^5}{5!} \le 5 \cdot 10^{-5} \Leftrightarrow$

$|x| \le \sqrt[5]{5! \cdot 5 \cdot 10^{-5}}$

b) $f(x) = \sin(x) \wedge p_1(x) = x$

$f(x) - p_2(x) = E_1(x) = \frac{-\sin(c)}{2} x^2$

$\frac{|\sin(c)|}{2} \le \frac{|c| x^2}{2} \le$

$\frac{|x| x^2}{2} \le$

$\frac{|x|^3}{2} \le 10^{-3} \Leftrightarrow$

$|x| \le \sqrt[3]{2 \cdot 10^{-3}}$

9. **Polinomio de taylor - $\mathbb{R}^2$**

$P_2(x,y)$ de $f = f(x,y)$ en $p = (a,b)$ con $f \in C^2$ en D (disco centrado en p)

Hessiano de f en p : $H_f(p) = \begin{vmatrix} f_{xx}(p) & f_{xy}(p) \\ f_{yx}(p) & f_{yy}(p) \end{vmatrix}$

$\Rightarrow P_2(x,y) = f(p) + \boldsymbol{\nabla} f(p) \cdot (x-a, y-b) + \frac{1}{2} \begin{vmatrix} x-a & y-b \end{vmatrix} \cdot H_f(p) \cdot \begin{vmatrix} x-a \\ y-b \end{vmatrix}$

Polinomio de taylor - $\mathbb{R}^2$

$P_2(x,y,z)$ de $f = f(x,y,z)$ en $p = (a,b,c)$ con $f \in C^2$ en D (disco centrado en p)

Hessiano de f en p : $H_f(p) = \begin{vmatrix} f_{xx}(p) & f_{xy}(p) & f_{xz}(p) \\ f_{yx}(p) & f_{yy}(p) & f_{yz}(p) \\ f_{zx}(p) & f_{zy}(p) & f_{zz}(p) \end{vmatrix}$

$\Rightarrow P_2(x,y) = f(p) + \boldsymbol{\nabla} f(p) \cdot (x-a, y-b) + \frac{1}{2} \begin{vmatrix} x-a & y-b & z-c \end{vmatrix} \cdot H_f(p) \cdot \begin{vmatrix} x-a \\ y-b \\ z-c \end{vmatrix}$

a)
- $f(x,y) = (x+y)^2$
- $p = (0,0)$

- $f_x(x,y) = 2x + 2y$
- $f_y(x,y) = 2x + 2y$
- $f_{xx}(x,y) = 2$
- $f_{xy}(x,y) = 2$
- $f_{yx}(x,y) = 2$
- $f_{yy}(x,y) = 2$

$P_2(x,y) = \boldsymbol{\nabla} f(p) \cdot (x,y) + \frac{1}{2} \begin{vmatrix} x & y \end{vmatrix} H_f(p) \begin{vmatrix} y \\ x \end{vmatrix} =$

$\frac{1}{2} \begin{vmatrix} x & y \end{vmatrix} \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} \begin{vmatrix} y \\ x \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 2x+2y & 2x+2y \end{vmatrix} \begin{vmatrix} y \\ x \end{vmatrix} = \frac{1}{2}(2xy + 2y^2 + 2x^2 + 2yx) =$

$y^2 + x^2 + 2xy \Rightarrow$

$P_1(x,y) = 0$

$P_1(x,y) = x^2 + y^2 + 2xy$

b)
- $f(x,y) = e^{x+y}$
- $p = (0,0)$
- $f_x(x,y) = e^{x+y}$
- $f_y(x,y) = e^{x+y}$
- $f_{xx}(x,y) = e^{x+y}$
- $f_{xy}(x,y) = e^{x+y}$
- $f_{yx}(x,y) = e^{x+y}$
- $f_{yy}(x,y) = e^{x+y}$

$P_2(x,y) = f(0,0) + \boldsymbol{\nabla} f(0,0) \cdot (x,y) + \frac{1}{2} \begin{vmatrix} x & y \end{vmatrix} H_f(0,0) \begin{vmatrix} y \\ x \end{vmatrix} =$

$1 + (1,1) \cdot (x,y) + \frac{1}{2} \begin{vmatrix} x & y \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} y \\ x \end{vmatrix} =$

$1 + x + y + \frac{1}{2} \begin{vmatrix} x+y & x+y \end{vmatrix} \begin{vmatrix} y \\ x \end{vmatrix} =$

$1 + x + y + \frac{1}{2}(y(x+y) + x(x+y)) =$

$1 + x + y + \frac{1}{2}(yx + y^2 + x^2 + yx) =$

$1 + x + y + \frac{1}{2}(2yx + y^2 + x^2) =$

$1 + x + y + yx + \frac{y^2}{2} + \frac{x^2}{2} \Rightarrow$

$P_1(x,y) = 1 + x + y$

$P_2(x,y) = 1 + x + y + xy + \frac{x^2}{2} + \frac{y^2}{2}$

c)

- $f(x,y) = \frac{1}{x^2+y^2+1} = (x^2+y^2+1)^{-1}$
- $p = (0,0)$
- $f_x(x,y) = \frac{-2}{(x^2+y^2+1)^2}$
- $f_y(x,y) = \frac{-2}{(x^2+y^2+1)^2}$
- $f_{xx}(x,y) = \frac{6x^2-2(y^2+1)}{(x^2+y^2+1)^3}$
- $f_{xy}(x,y) = \frac{8xy}{(x^2+y^2+1)^3}$
- $f_{yx}(x,y) = \frac{8xy}{(x^2+y^2+1)^3}$
- $f_{yy}(x,y) = \frac{2(x^2-3y^2+1)}{(x^2+y^2+1)^3}$
- $f_x(0,0) = -2$
- $f_y(0,0) = -2$
- $f_{xx}(0,0) = -2$
- $f_{xy}(0,0) = 0$
- $f_{yx}(0,0) = 0$
- $f_{yy}(0,0) = 2$

$P_2(x,y) = 1 - 2x - 2y + \frac{1}{2}(-2x^2 + 2x^2) =$
$1 - 2x - 2y - x^2 + x^2$
$P_1(x,y) = 1 - 2x - 2y$

d)

- $f(x,y) = (x + xy + 2y)$
- $p = (1,1)$
- $f_x(x,y) = 1 + y$
- $f_y(x,y) = x + 2$
- $f_{xx}(x,y) = 0$
- $f_{xy}(x,y) = 1$
- $f_{yx}(x,y) = 1$
- $f_{yy}(x,y) = 0$
- $f_x(1,1) = 2$
- $f_y(1,1) = 3$
- $f_{xx}(1,1) = 0$
- $f_{xy}(1,1) = 1$
- $f_{yx}(1,1) = 1$
- $f_{yy}(1,1) = 0$

$P_2(x,y) = f(1,1) \cdot \boldsymbol{\nabla} f(1,1) \cdot (x-1, y-1) + \frac{1}{2}\begin{vmatrix} x-1 & y-1 \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} x-1 \\ y-1 \end{vmatrix} =$

$4 + 2(x-1) + 3(y-1) + \frac{1}{2} \cdot \begin{vmatrix} y-1 & x-1 \end{vmatrix} \cdot \begin{vmatrix} x-1 \\ y-1 \end{vmatrix} =$

$4 + 2(x-1) + 3(y-1) + \frac{1}{2} \cdot (y-1)(x-1) + (x-1)(y-1) =$
$4 + 2(x-1) + 3(y-1) + \frac{1}{\cancel{2}} \cdot \cancel{2}(y-1)(x-1) =$
$4 + 2(x-1) + 3(y-1) + (y-1)(x-1) \Rightarrow$
$P_1(x,y) = 4 + 2(x-1) + 3(y-1)$
$P_2(x,y) = 4 + 2(x-1) + 3(y-1) + (x-1)(y-1)$

e)

- $f(x,y) = e^{(x-1)^2}\cos(y)$
- $p = (1,0)$
- $f_x(x,y) = 2e^{(x-1)^2}(x-1)\cos(y)$
- $f_y(x,y) = -e^{(x-1)^2}\sin(y)$
- $f_{xx}(x,y) = 2e^{(x-1)^2}(2x^2-4x+3)\cos(y)$
- $f_{xy}(x,y) = -2e^{(x-1)^2}(x-1)\sin(y)$
- $f_{yx}(x,y) = -2e^{(x-1)^2}(x-1)\sin(y)$
- $f_{yy}(x,y) = -e^{(x-1)^2}\cos(y)$

- $f_x(1,0) = 0$
- $f_y(1,0) = 0$
- $f_{xx}(1,0) = 1$
- $f_{xy}(1,0) = 0$
- $f_{yx}(1,0) = 0$
- $f_{yy}(1,0) = -1$

$\Rightarrow P_2(x,y) = 1 + \frac{x^2}{2} - \frac{y^2}{2}$
$\Rightarrow P_1(x,y) = 1$

f)
- $f(x,y) = e^x \sin(xy)$
- $p = (2, \frac{\pi}{4})$
- $f_x(x,y) = e^x(\sin(xy) + \cos(xy))$
- $f_y(x,y) = e^x \cos(xy)$
- $f_{xx}(x,y) = e^x(2y\cos(xy) - (y^2-1)\sin(xy))$
- $f_{xy}(x,y) = e^x((x+1)\cos(xy) - xy\sin(xy))$
- $f_{yx}(x,y) = e^x((x+1)\cos(xy) - xy\sin(xy))$
- $f_{yy}(x,y) = -e^x \sin(x,y)$
- $f(2, \frac{\pi}{4}) = e^2$
- $f_x(2, \frac{\pi}{4}) = e^2$
- $f_y(2, \frac{\pi}{4}) = 0$
- $f_{xx}(2, \frac{\pi}{4}) = 1$
- $f_{xy}(2, \frac{\pi}{4}) = -e^2\frac{\pi}{2}$
- $f_{yx}(2, \frac{\pi}{4}) = -e^2\frac{\pi}{2}$
- $f_{yy}(2, \frac{\pi}{4}) = -e^2$

$P_1(x,y) = e^2 + e^2(x-2)$

$P_2(x,y) = e^2 + e^2(x-2) + \frac{(x-2)^2}{2} - \frac{e^2(y-\frac{\pi}{4})^2}{2} - e^2\frac{\pi}{2}(x-2)(y-\frac{\pi}{4})$

g)
- $f(x,y) = ln(1+xy)$
- $p = (2,3)$
- $f_x(x,y) = \frac{y}{1+xy}$
- $f_y(x,y) = \frac{x}{1+xy}$
- $f_{xx}(x,y) = \frac{y^2}{(1+xy)^2}$
- $f_{xy}(x,y) = \frac{1}{(1+xy)^2}$
- $f_{yx}(x,y) = \frac{1}{(1+xy)^2}$
- $f_{yy}(x,y) = \frac{x^2}{(1+xy)^2}$
- $f(2,3) = \ln(7)$
- $f_x(2,3) = \frac{2}{7}$
- $f_y(2,3) = \frac{3}{7}$
- $f_{xx}(2,3) = -\frac{9}{49}$
- $f_{xy}(2,3) = \frac{1}{49}$
- $f_{yx}(2,3) = \frac{1}{49}$
- $f_{yy}(2,3) = -\frac{4}{49}$

$P_1(x,y) = \ln(7) + \frac{2(x-2)}{7} + \frac{3(y-2)}{7}$

$P_2(x,y) = \ln(7) + \frac{2(x-2)}{7} + \frac{3(y-2)}{7} - \frac{9(x-2)^2}{2\cdot 49} - \frac{4(y-3)^2}{2\cdot 49} + \frac{(x-2)(y-3)}{49}$

h)
- $f(x,y) = x + \sqrt{y} + \sqrt[3]{z}$
- $p = (2,3,4)$
- $f_x(x,y,z) = 1$
- $f_y(x,y,z) = \frac{1}{2\sqrt{y}}$
- $f_z(x,y,z) = \frac{1}{3z^{\frac{2}{3}}}$
- $f_{xx}(x,y,z) = 0$

- $f_{xy}(x,y,z) = 0$
- $f_{xz}(x,y,z) = 0$
- $f_{yx}(x,y,z) = 0$
- $f_{yy}(x,y,z) = -\frac{1}{4y^{\frac{3}{2}}}$
- $f_{yz}(x,y,z) = 0$
- $f_{zx}(x,y,z) = 0$
- $f_{zy}(x,y,z) = 0$
- $f_{zz}(x,y,z) = -\frac{2}{9z^{\frac{5}{3}}}$
- $f(2,3,4) = 2 + \sqrt{3} + \sqrt[3]{4}$
- $f_x(2,3,4) = 1$
- $f_y(2,3,4) = \frac{1}{2\sqrt{3}}$
- $f_z(2,3,4) = \frac{1}{3\cdot 4^{\frac{2}{3}}}$
- $f_{xx}(2,3,4) = 0$
- $f_{xy}(2,3,4) = 0$
- $f_{xz}(2,3,4) = 0$
- $f_{yx}(2,3,4) = 0$
- $f_{yy}(2,3,4) = -\frac{1}{4\cdot 3^{\frac{3}{2}}}$
- $f_{yz}(2,3,4) = 0$
- $f_{zx}(2,3,4) = 0$
- $f_{zy}(2,3,4) = 0$
- $f_{zz}(2,3,4) = -\frac{2}{9\cdot 4^{\frac{5}{3}}}$

$P_1(x,y,z) = 2 + \sqrt{3} + \sqrt[3]{4} + (x-2) + \frac{1}{2\sqrt{3}}(y-3) + \frac{1}{3\cdot 4^{\frac{2}{3}}}(z-4)$

$P_2(x,y,z) = 2 + \sqrt{3} + \sqrt[3]{4} + (x-2) + \frac{1}{2\sqrt{3}}(y-3) + \frac{1}{3\cdot 4^{\frac{2}{3}}}(z-4) - \frac{(y-3)^2}{2\cdot 4\cdot 3^{\frac{3}{2}}} - \frac{2\cdot(z-4)^2}{2\cdot 9\cdot 4^{\frac{5}{3}}}$

10.
- $f : \mathbb{R}^2 \to \mathbb{R}$
- $f(x,y) = xe^y$

a) $P_1(x,y) = 1 + (x-1) + y$

b) $P_1(0,98, 0{,}02) = 1 - 0{,}02 + 0{,}02 = 1$

11.
- $f : \mathbb{R}^2 \to \mathbb{R}$
- $f(x,y) = e^{x^2 - y^2}$

a) $P_1(x,y) = 1 + 2(x-1) + 2(y-1)$

b) $\frac{4}{10} = (1 + \frac{1}{10})^2 - (1 - \frac{1}{10})^2$

$P_1(1+\frac{1}{10}, 1+\frac{1}{10}) = 1 + 2(1 + \frac{1}{10} - 1) + 2(1 + \frac{1}{10} - 1) =$

$1 + \cancel{2}(\frac{1}{\cancelto{5}{10}}) + \cancel{2}(\frac{1}{\cancelto{5}{10}}) =$

$1 + \frac{2}{5} = \frac{7}{5}$

12. $P_2(x,y) = xy + R_2(x,y)$

13.
- $f : \mathbb{R}^2 \to \mathbb{R}^2$
- $f(x,y) = (x+1, 2y - e^x)$
- $g : \mathbb{R}^2 \to \mathbb{R} \in C^1$
- $P_2(x,y) deg \circ f$ en $(0,0)$

- $P_2(x,y) = 4 + 3x - 2y - x^2 + 5xy$
- $\boldsymbol{\nabla} g(1,-1)$

$h = g \circ f = g(f(x,y)) = g(x+1, 2y - e^x)$

$h(0,0) = g(f(0,0)) \overset{f(0,0=(1,-1))}{=} g(1,-1)$

- $h_x(0,0) = P_{2x}(0,0)$
  $P_{2x}(x,y) = 3 - 2x + 5y \Rightarrow P_{2x}(0,0) = 3$
- $h_y(0,0) = P_{2y}(0,0)$
  $P_{2y}(x,y) = -2 + 5y \Rightarrow P_{2y}(0,0) = -2$

$h_x(0,0) = g_x(1,-1) \cdot 1 + g_y(1,-1) \cdot (-1)$
$h_y(0,0) = g_x(1,-1) \cdot 0 + g_y(1,-1) \cdot 2$
$h_y(0,0) = -2 = 2g_y(1,-1) \Rightarrow g_y(1,-1) = -1$
$h_x(0,0) = 3 = g_x(1,-1) + 1 \Rightarrow g_x(1,-1) = 2$
$\Rightarrow \boldsymbol{\nabla} g(1,-1) = (-1,2)$

14. $P_2(x,y) = 1 - x^2 - y^2 + R_2(x,y)$
    $\lim_{(x,y)\to(0,0)} \frac{f(x,y)+x^2+y^2-1}{x^2+y^2} \equiv$
    $\lim_{(x,y)\to(0,0)} \frac{P_2(x,y)+x^2+y^2-1}{x^2+y^2} \equiv$
    $\lim_{(x,y)\to(0,0)} \frac{\cancel{1-x^2-y^2}+R_2(x,y)\cancel{+x^2+y^2-1}}{x^2+y^2} \equiv$
    $\lim_{(x,y)\to(0,0)} \frac{R_2(x,y)}{x^2+y^2} \equiv$
    $\lim_{(x,y)\to(0,0)} \frac{R_2(x,y)}{\|(x,y)\|^2} = 0$

15. *a)* $\lim_{(x,y)\to(1,1)} \frac{f(x,y)}{\|(x,y)-(1,1)\|} \equiv$
    $\lim_{(x,y)\to(1,1)} \frac{p(x,y)+R_3(x,y)}{\|(x,y)-(1,1)\|} \equiv$

    b) $\lim_{(x,y)\to(1,1)} \frac{f(x,y)}{\|(x,y)-(1,1)\|} \equiv$