$$f(x,y) = \begin{cases} \frac{|x|y}{\sqrt{x^2+y^2}} & si(x,y) \neq (0,0) \\ 0 \quad si(x,y) = (0,0) \end{cases}$$

$\lim_{(x,y)\to(0,0)} f(x,y) = 0 \Rightarrow$

$\exists\delta(\epsilon) > 0 \ tq \ \|(x,y)\| < \delta \Rightarrow |f(x,y) - 0| < \epsilon$

$|\frac{|x|y}{\sqrt{x^2+y^2}}| =$

$\frac{|x||y|}{\sqrt{x^2+y^2}} =$

$\frac{|x||y|}{\|(x,y)\|} \leq$

$\frac{\|(x,y)\|^{\cancel{2}}}{\cancel{\|(x,y)\|}} < \delta < \epsilon$

$\Rightarrow f$ es continua en el origen $\blacksquare$

$\lim_{h\to 0} \frac{f(hv_1, h_v 2)}{h}$

$\lim_{h\to 0} \frac{1}{h} \cdot \frac{|hv_1|hv_2}{\sqrt{(hv_1)^2+(hv_2)^2}} =$

$\lim_{h\to 0} \frac{1}{h} \cdot \frac{|hv_1|hv_2}{\sqrt{h^2(v_1^2+v_2^2)}} =$

$\lim_{h\to 0} \frac{1}{h} \cdot \frac{|hv_1|hv_2}{\sqrt{h^2}\sqrt{v_1^2+v_2^2}} =$

$\lim_{h\to 0} \frac{1}{h} \cdot \frac{|hv_1|hv_2}{\sqrt{h^2}\|(v_1,v_2)\|} \overset{\|(v_1,v_2)\|=1}{=}$

$\lim_{h\to 0} \frac{1}{h} \cdot \frac{|hv_1|hv_2}{|h|} =$

$\lim_{h\to 0} \frac{1}{\cancel{h}} \cdot \frac{\cancel{|h|}|v_1|\cancel{h}v_2}{\cancel{|h|}} =$

$\lim_{h\to 0} |v_1|\, v_2 = |v_1|\, v_2$ $\blacksquare$

$f_x = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} =$

$f_x = \lim_{h\to 0} \frac{\frac{0}{|h|}}{h} = 0$

$f_y = \lim_{h\to 0} \frac{\frac{0}{|h|}}{h} = 0$

$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-f_x(x-0)-f_y(y-0)}{\|(x,y)\|} \overset{?}{=} 0$

$\lim_{(x,y)\to(0,0)} \frac{\frac{|x|y}{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} =$

$\lim_{(x,y)\to(0,0)} \frac{|x|y}{\|(x,y)^2\|} = 1$

$\lim_{(x,y)\to(0,0)} \frac{|x|y}{x^2+y^2}$

$y = mx$

$\lim_{(x,y)\to(0,0)} \frac{mx^2}{2x^2} = \frac{m}{2} \neq 0$

$\Rightarrow f$ no es diferenciable en el origen