

$$\blacksquare \quad z = f(x, y)$$

$$\blacksquare \quad f \in C^2$$

$$\blacksquare \quad x = r^2 + s^2$$

$$\blacksquare \quad y = 2rs$$

$$\frac{\partial^2 z}{\partial r \partial s} = \frac{\partial}{\partial r} \left( \frac{\partial z}{\partial s} \right)$$

$$\frac{\partial z}{\partial s} = f_x \cdot 2s + f_y \cdot 2r$$

$$\frac{\partial(f_x \cdot 2s)}{\partial r} = f_{xx} \cdot 2r + f_{xy} \cdot 2s$$

$$\frac{\partial(f_y \cdot 2r)}{\partial r} = (f_{yx} \cdot 2r + f_{yy} \cdot 2s)2r + 2f_y$$

$$\frac{\partial^2 z}{\partial r \partial s} = f_{xx} \cdot 2r + f_{xy} \cdot 2s + (f_{yx} \cdot 2r + f_{yy} \cdot 2s)2r + 2f_y$$