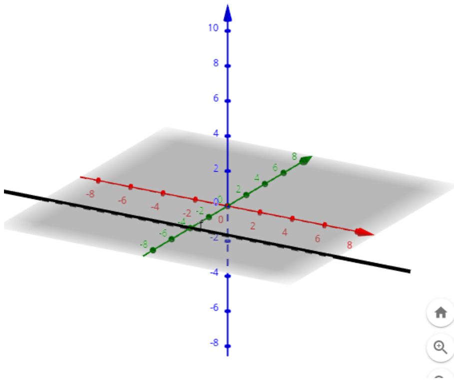
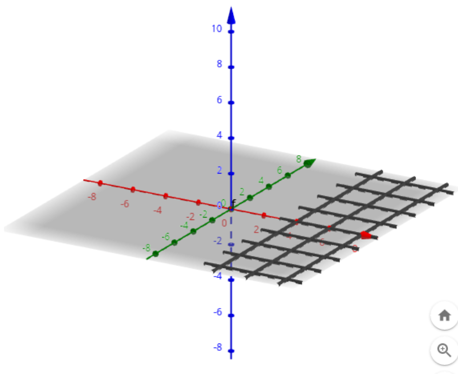
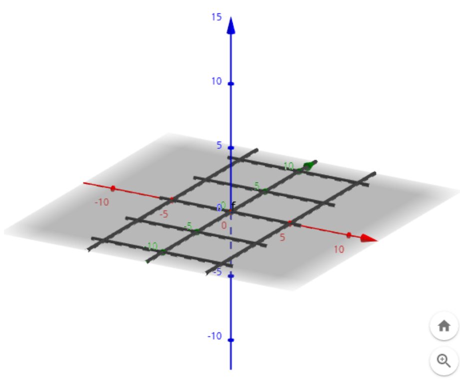
1. *a)* $y = -4$



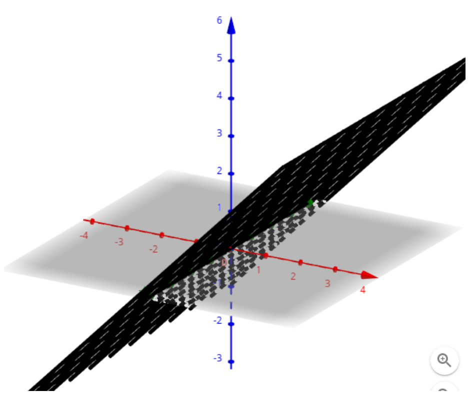
b) $b > 3$



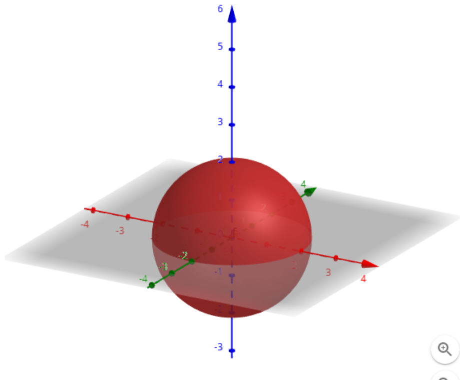
c) $0 \leq z \leq 6$



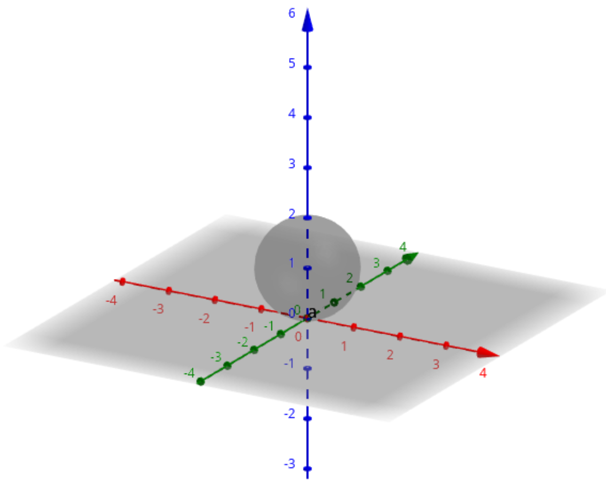
d) $x = z$



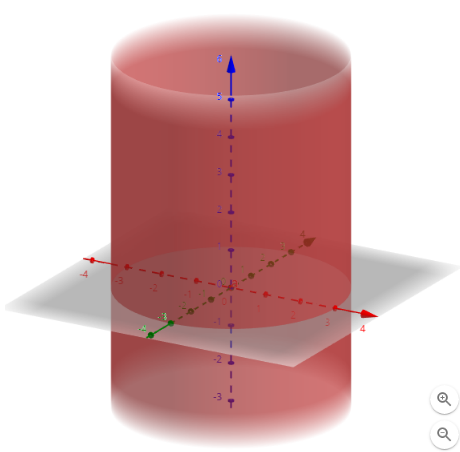
e) $x^2 + y^2 + z^2 \leq 4$



f) $x^2 + y^2 + z^2 > 2z \equiv x^2 + y^2 + (z-1)^2 > 1$



g) $x^2 + y^2 \leq 9$



2. *a)* $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$

   $P_0 = (a, b, c) \wedge b : (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ entonces

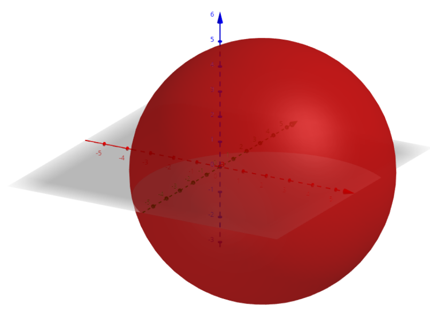
   b es un circulo con centro en P_0 y radio r

   $x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 6x + 4y - 2z + j = 11 + j$

   - $-2ax = -6x \equiv a = 3$

- $-2by = 4y \equiv b = -2$
- $-2cz = -2z \equiv c = 1$

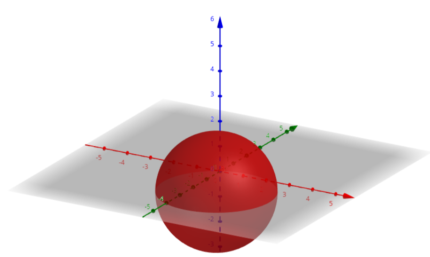
$(x-3)^2 + (y+2)^2 + (z-1)^2 \equiv$
$(x^2 - 6x + 9) + (y^2 + 4y + 4) + (z^2 - 2z + 1) = 11 + j \equiv$
$(x^2 + y^2 + z^2) - 6x + 4y - 2z + 9 + 4 + 1 = 11 + j \equiv$
$(x^2 + y^2 + z^2) - 6x + 4y - 2z + 14 = 11 + j \equiv$
$(x^2 + y^2 + z^2) - 6x + 4y - 2z = 25$
Es un circulo con centro en $(3, -2, 1)$ y radio 5

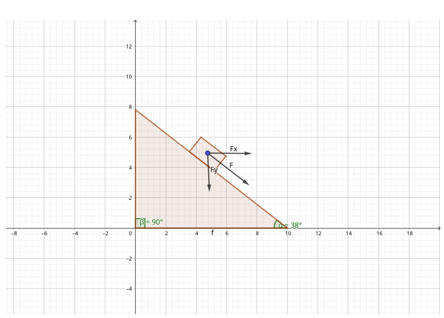


b) $4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$
$4(x^2 + y^2 + z^2 - 2x + 4y) = 1 \equiv$
$x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$
$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$
$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$

- $-2ax = -2x \equiv a = 1$
- $-2by = 4y \equiv b = -2$
- $-2cz = 0 \equiv c = 0$

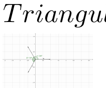
$(x-1)^2 + (y+2)^2 + (z)^2 \equiv$
$x^2 + -2x + 1 + y^2 + 4y + 4 + z^2 \equiv$
$(x^2 + y^2 + z^2 - 2x + 4y) + 5 = \frac{1}{4} + 5 \equiv$
$x^2 + y^2 + z^2 - 2x + 4y + 5 = \frac{21}{4}$
$(x-1)^2 + (y+2)^2 + (z)^2 = \frac{21}{4}$
Es un circulo con centro en $(1, -2, 0)$ y radio $\frac{\sqrt{21}}{2}$



3. 

- $\alpha = 38°$
- $F = 50N$
  $F_x = 50N \cdot \cos 38° = 39{,}40N$
  $F_y = 50N \cdot \sin 38° = -30{,}78N$

4. - $F_1 = 10N$
   - $F_2 = 12N$
   - $\alpha = 45°$
   - $\theta = 30°$
   - $F_{1x} = F_1 \cdot \cos\alpha = 10N \cdot \cos(45°) = -7{,}07N$
   - $F_{1y} = F_1 \cdot \sin\alpha = 10N \cdot \sin(45°) = 7{,}07N$
   - $F_{2x} = F_2 \cdot \cos\theta = 12N \cdot \cos(30°) = 10{,}39N$
   - $F_{2y} = F_2 \cdot \sin\theta = 12N \cdot \sin(30°) = 6N$
   - $F_{rx} = F_{1x} + F_{2x} = -7{,}07N + 10{,}39N = 3{,}32N$
   - $F_{ry} = F_{1y} + F_{2y} = 7{,}07N + 6N = 13{,}07N$
   - $F_r = \sqrt{(F_{rx})^2 + (F_{ry})^2} = \sqrt{(3{,}32N)^2 + (13{,}07N)^2} = 13{,}48N$

5. $u \in \Re^2 \wedge \|u\| = 1$
   - $Triangulo$
     
     - $u \cdot v \overset{\|u\|=\|v\|}{\equiv} \|u\|^2 * \cos(\alpha) = -\frac{1}{2}$
     - $u \cdot w \overset{\|u\|=\|w\|}{\equiv} \|u\|^2 * \cos(\alpha) = -\frac{1}{2}$
   - $Cuadrado$
     - $u \cdot v \overset{\|u\|=\|v\|}{\equiv} \|u\|^2 * \cos(90°) = 0$
     - $u \cdot w \overset{\|u\|=\|w\|}{\equiv} \|u\|^2 * \cos(45°) = \frac{\sqrt{2}}{2}$

6. a) $u = (3,-4), v = (5,0)$
   $P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(3,-4)\cdot(5,0)}{\|(3,-4)\|^2} \cdot (3,-4) =$
   $\frac{15}{25} \cdot (3,-4) = (\frac{9}{5}, -\frac{12}{5})$
   b) $u = (1,2), v = (-4,1)$
   $P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(1,2)\cdot(-4,1)}{\|(1,2)\|^2} \cdot (1,2) =$
   $-\frac{2}{5} \cdot (1,2) = (-\frac{2}{5}, -\frac{4}{5})$
   c) $u = (3,6,2), v = (1,2,3)$
   $P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(3,6,2)\cdot(1,2,3)}{\|(3,6,2)\|^2} \cdot (3,6,2) =$
   $\frac{3}{7} \cdot (3,6,2) = (\frac{9}{7}, \frac{18}{7}, \frac{6}{7})$

7. u, v vectores QvQ $o_u(v) = v - p_u(v)$ es ortogonal a u
   $o_u(v)$ es ortogonal a $u \leftrightarrow o_u(v) \cdot u = 0$
   $o_u(v) = v - \frac{u \cdot v}{\|u\|^2} \cdot u$
   QvQ $o_u(v) \cdot u = 0$
   $(v - \frac{u \cdot v}{\|u\|^2} \cdot u) \cdot u \equiv u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot u \cdot u \overset{u \cdot u = \|u\|^2}{\equiv} u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot \|u\|^2 \overset{u \neq 0}{\equiv}$
   $u \cdot v - \frac{u \cdot v}{\cancel{\|u\|^2}} \cdot \cancel{\|u\|^2} \equiv u \cdot v - u \cdot v = 0 \quad \blacksquare$

8. u, v vectores $u \neq 0 \wedge v \neq 0$ QvC $p_u(v) = p_v(u)$

$v = \lambda u \rightarrow p_u(v) = v \wedge v = \theta u \rightarrow p_v(u) = u$

$p_u(v) = p_v(u) \equiv \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{u \cdot v}{\|v\|^2} \cdot v \overset{u \cdot v \neq 0}{\equiv} \frac{\cancel{u \cdot v}}{\|u\|^2} \cdot u = \frac{\cancel{u \cdot v}}{\|v\|^2} \cdot v \equiv$

$\frac{u}{\|u\|^2} = \frac{v}{\|v\|^2} \cdot v \leftrightarrow u = \lambda \cdot v \wedge v = \theta u \wedge \lambda, \theta \in \Re \leftrightarrow u = v$

Si $u \cdot v = 0 \rightarrow \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{u \cdot v}{\|v\|^2} \cdot v = 0$

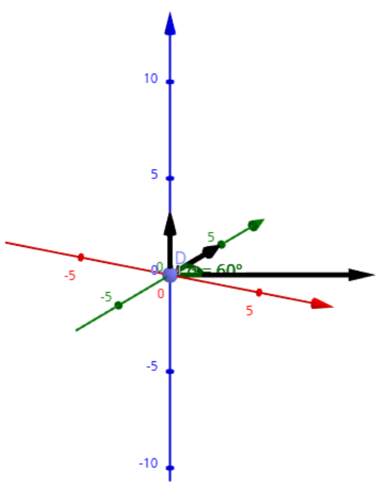
$p_u(v) = p_v(u) \leftrightarrow u \cdot v = 0 \vee u = v \quad \blacksquare$

9. $W = 4m \cdot 20N \cdot \cos(50°) = 51,42J$

10. $W = (Q - P) \cdot F \equiv (6, 2, 12) \cdot (8, -6, 9) = 144J$

11. *a*)
    - $\|u\| = 5$
    - $\|v\| = 10$
    - $\alpha = 60°$

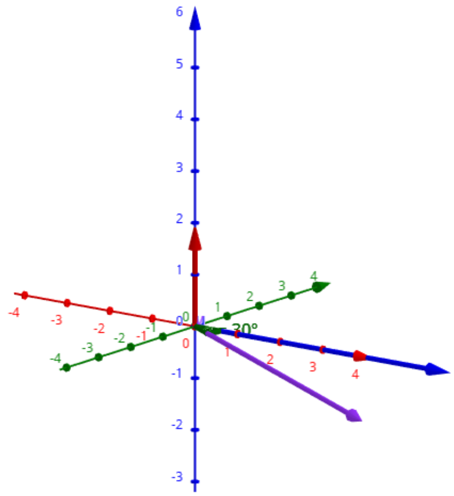
    1) Sentido $u \times v$



    2) $\|u \times v\| = \|a\| \cdot \|b\| \cdot \sin(\alpha) \equiv 5 \cdot 10 \cdot \sin(60°) \approx 43{,}30$

    b)
    - $\|u\| = 6$
    - $\|v\| = 8$
    - $\alpha = 30°$

    1) Sentido $u \times v$



2) $\|u \times v\| = \|a\| \cdot \|b\| \cdot \sin(\alpha) \equiv 6 \cdot 8 \cdot \sin(30°) = 24$