

1. $\int \int_D x dA, D, y = x, y = 0, x = 1$

a) Tipo 1

$$\int_0^1 (\int_0^x x dy) dx =$$

$$\begin{aligned} \blacksquare \int_0^x x dy &= \\ xy \Big|_0^x &= \\ x^2 & \end{aligned}$$

$$\begin{aligned} \blacksquare \int_0^1 x^2 dx &= \\ \frac{x^3}{3} \Big|_0^1 &= \\ \frac{1}{3} & \end{aligned}$$

b) Tipo 2

$$\int_0^1 (\int_y^1 x dx) dy$$

$$\begin{aligned} \blacksquare \int_y^1 x dx &= \\ \frac{x^2}{2} \Big|_y^1 &= \\ \frac{1}{2} - \frac{y^2}{2} & \end{aligned}$$

$$\begin{aligned} \blacksquare \int_0^1 \frac{1}{2} - \frac{y^2}{2} dy &= \\ \frac{y}{2} - \frac{y^3}{6} \Big|_0^1 &= \\ \frac{1}{2} - \frac{1}{6} = \frac{1}{3} & \end{aligned}$$

2. $\int \int_D xy dA, D, y = x^2, y = 3x$

a) Tipo 1

$$\int_0^3 (\int_{x^2}^{3x} xy dy) dx$$

$$\begin{aligned} \blacksquare \int_{x^2}^{3x} xy dy &= \\ \frac{xy^2}{2} \Big|_{x^2}^{3x} &= \\ \frac{9x^3}{2} - \frac{x^5}{2} & \end{aligned}$$

$$\begin{aligned} \blacksquare \int_0^3 \frac{9x^3}{2} - \frac{x^5}{2} dx &= \\ \frac{9x^4}{8} - \frac{x^6}{12} \Big|_0^3 &= \\ \frac{243}{8} & \end{aligned}$$

b) Tipo 2

$$\int_0^9 (\int_{\frac{y}{3}}^{\sqrt{y}} xy dx) dy$$

$$\begin{aligned} \blacksquare \int_{\frac{y}{3}}^{\sqrt{y}} xy dx &= \\ \frac{yx^2}{2} \Big|_{\frac{y}{3}}^{\sqrt{y}} &= \\ \frac{y^2}{2} - \frac{y^3}{18} & \end{aligned}$$

$$\begin{aligned} \blacksquare \int_0^9 \frac{y^2}{2} - \frac{y^3}{18} dy &= \\ \frac{y^3}{6} - \frac{y^4}{72} \Big|_0^9 &= \\ \frac{243}{2} - \frac{729}{8} = \frac{243}{8} & \end{aligned}$$