Regla de la cadena

$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$

Ejercicios

1. - $f(x,y) = x^2 + y^2 + xy$
   - $r(t) = (\sin(t), e^t)$

   $f : \Re^2 \to \Re$

   $r : \Re \to \Re^2$

   $z = (f \circ r) = f(r(t)) \Rightarrow$

   $z : \Re \to \Re$

   $z = f(r(t)) = f(\sin(t), e^t) = \sin(t)^2 + e^{2t} + e^t \sin(t)$

   $\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial \sin(t)}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial e^t}{\partial t} =$

   $f_x(r(t)) \cdot \frac{\partial \sin(t)}{\partial t} + f_y(r(t)) \cdot \frac{\partial e^t}{\partial t} =$

   $2x + y\big|_{r(t)} \cdot \cos(t) + \ 2y + x\big|_{r(t)} \cdot e^t =$

   $(2\sin(t) + e^t) \cdot \cos(t) + (2e^t + \sin(t)) \cdot e^t =$

   $2\sin(t)\cos(t) + \cos(t)e^t + 2e^{2t} + e^t \sin(t)$

   $(\sin(t)^2 + e^{2t} + e^t \sin(t))' =$

   $2\sin(t)\cos(t) + 2e^{2t} + e^t \sin(t) + e^t \cos(t)$

2. - $f(x,y) = \cos(x + 4y)$
   - $r(t) = (5t^4, \frac{1}{t})$

   $f : \Re^2 \to \Re$

   $r : \Re \to \Re^2$

   $z = (f \circ r)(t)$

   $z = \Re \to \Re$

   $\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial 5t^4}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial \frac{1}{t}}{\partial t} =$

   $f_x(r(t)) \cdot 20t^3 + f_y(r(t)) \cdot \frac{-1}{t^2} =$

   $-\sin(x + 4y)\big|_{r(t)} \cdot 20t^3 + \ (-4)\sin(x + 4y)\big|_{r(t)} \cdot \frac{-1}{t^2} =$

   $-\sin\left(5t^4 + \frac{4}{t}\right) \cdot 20t^3 + (-4)\sin\left(5t^4 + \frac{4}{t}\right) \cdot \frac{-1}{t^2} =$

   $-\sin\left(5t^4 + \frac{4}{t}\right) \cdot 20t^3 + \frac{4\sin\left(5t^4 + \frac{4}{t}\right)}{t^2} =$

3. - $f(x,y) = \sqrt{1 + x^2 + y^2}$
   - $r(t) = (\ln(t), \cos(t))$

   $f : \Re^2 \to \Re$

   $r : \Re \to \Re^2$

   $z = (f \circ r)(t)$

   $z : \Re \to \Re$

   $\frac{\partial z}{\partial t} = f_x(r(t)) \cdot \frac{1}{x} + f_y(r(t)) \cdot (-\sin(t))$

   $\left.\frac{x}{\sqrt{1+x^2+y^2}}\right|_{r(t)} \cdot \frac{1}{x} + \ \left.\frac{y}{\sqrt{1+x^2+y^2}}\right|_{r(t)} \cdot (-\sin(t))$

   $\frac{ln(t)}{\sqrt{1+ln(t)^2+\cos(t)^2}} \cdot \frac{1}{x} + \ \frac{\cos(t)}{\sqrt{1+ln(t)^2+\cos(t)^2}} \cdot (-\sin(t))$