

1. $f : \mathbb{R}^2 \rightarrow \mathbb{R} \wedge f \in C^2$

$$P_2(x, y) = 2x^2 - xy + 5x - y + 5 \text{ en } (-1, 1) = g(x, y)$$

$$P_2(x, y) = f(a, b) + f_x(a, b)(x+1) + f_y(a, b)(y-1) + \frac{f_{xx}(a, b)(x+1)^2}{2} + \frac{f_{yy}(a, b)(y-1)^2}{2} + f_{xy}(a, b)(x+1)(y-1)$$

$$\blacksquare g_x(x, y) = 4x - y + 5$$

$$\blacksquare g_y(x, y) = -x - 1$$

$$\blacksquare g_{xx}(x, y) = 4$$

$$\blacksquare g_{xy}(x, y) = -1$$

$$\blacksquare g_{yy}(x, y) = 0$$

$$\blacksquare f(-1, 1) = g(-1, 1) = 2 + 1 - 5 - 1 + 5 = 2$$

$$\blacksquare f_x(-1, 1) = g_x(-1, 1) = 0$$

$$\blacksquare f_{xx}(-1, 1) = g_{xx}(-1, 1) = 4$$

$$\blacksquare f_{xy}(-1, 1) = g_{xy}(-1, 1) = f_{yx}(-1, 1) = g_{yx}(-1, 1) \text{ Por } g, f \in C^2 = -1$$

$$\blacksquare f_{yy}(-1, 1) = g_{yy}(-1, 1) = -1$$

$\nabla f(-1, 1) = (0, 0)$ es punto critico

$\det(H_f) = -4 - (-1)(-1) = -5$ por el criterio del Hessiano pto silla

2. $\lim_{(x,y) \rightarrow (-1,1)} \frac{f(x,y)-2}{\|(x,y)-(-1,1)\|}$

$$f(x, y) = P_2(x, y) + R_2(x, y)$$

$$\frac{f(x,y)-2}{\|(x,y)-(-1,1)\|} = \frac{P_2(x,y)+R_2(x,y)-2}{\|(x,y)-(-1,1)\|} =$$

$$\frac{2x^2-xy+5x-y+3}{\|(x,y)-(-1,1)\|}$$

$$2 + 1 - 5 - 1 + 3$$