

1. a) $r : \mathbb{R} \rightarrow \mathbb{R}^3$

$$C : r(t) = (r_1(t), r_2(t), r_3(t)) = (x, y, z)$$

$$r_1 : \mathbb{R} \rightarrow \mathbb{R}$$

$$r_2 : \mathbb{R} \rightarrow \mathbb{R}$$

$$r_3 : \mathbb{R} \rightarrow \mathbb{R}$$

$$\begin{cases} x^2 + z^2 = 4 \\ y = 3z^2 \end{cases}$$

$$\blacksquare x^2 + z^2 = 4 \text{ es un círculo con centro en } (x, z) = (0, 0) \text{ con radio} = \sqrt{4} = 2$$

$$\Rightarrow \text{en polares } 2 \cos(t) + 2 \sin(t) = 1$$

$$\Rightarrow \begin{cases} x = 2 \cos(t) \\ z = 2 \sin(t) \end{cases} : \star$$

$$\blacksquare y = 3z^2 \xrightarrow{\star} y = 3(2 \sin(t))^2$$

$$\Rightarrow r(t) = \begin{cases} r_1(t) = 2 \cos(t) \\ r_2(t) = 3(2 \sin(t))^2 \\ r_3(t) = 2 \sin(t) \end{cases} : \star$$

$$\Rightarrow r(t) = (2 \cos(t), 3(2 \sin(t))^2, 2 \sin(t))$$

b) $\text{QvQ } \exists t_1, t_2 \in \mathbb{R}; r(t_1) = (2, 0, 0) \wedge r(t_2) = (0, 12, 2)$

$$\blacksquare r(t_1) = (2, 0, 0) \Leftrightarrow \begin{cases} 2 = 2 \cos(t_1) \\ 0 = 3(2 \sin(t_1))^2 \\ 0 = 2 \sin(t_1) \end{cases} \Leftrightarrow$$

$$2 = 2 \cos(t_1) \Leftrightarrow$$

$$1 = \cos(t_1) \Leftrightarrow$$

$$t_1 = \frac{\pi}{2}$$

$$\blacksquare r(t_2) = (0, 12, 2) \Leftrightarrow \begin{cases} 0 = 2 \cos(t_2) \\ 12 = 3(2 \sin(t_2))^2 \\ 2 = 2 \sin(t_2) \end{cases} \Leftrightarrow$$

$$0 = 2 \cos(t_2) \Leftrightarrow$$

$$0 = \cos(t_2) \Leftrightarrow$$

$$t_2 = 0$$

$$\blacksquare t_1 = \frac{\pi}{2}$$

$$\blacksquare t_2 = 0$$

c) $\blacksquare \vec{v} = (-2, 0, 0)$

$$\blacksquare \vec{u} = (0, -12, -2)$$

$$\blacksquare \text{Area}_p = |\vec{v} \times \vec{u}| = \left| \begin{vmatrix} i & j & k \\ -2 & 0 & 0 \\ 0 & -12 & -2 \end{vmatrix} \right| =$$

$$|0i - 4j + 24k| = \sqrt{16 + 576} = 24,33$$

2. a) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

Pruebo por curvas

$$\blacksquare \text{iterado } x = 0$$

$$\lim_{y \rightarrow 0} f(0, y)$$

$$\blacksquare \text{iterado } y = 0$$

$$\lim_{x \rightarrow 0} f(x, 0)$$

$$\blacksquare \text{rectas } y = mx$$

$$\lim_{x \rightarrow 0} f(x, mx)$$

$$\blacksquare \text{curvas } y = x^2$$

$$\lim_{x \rightarrow 0} f(x, x^2)$$

Intento demostrar por sandwich

$$\exists g(x, y) : \lim_{(x,y) \rightarrow (0,0)} g(x, y) = 0 \wedge 0 \leq |f(x, y)| \leq |g(x, y)|$$

Pruebo por definición

$$\exists \epsilon, \delta(\epsilon) > 0 : \|(x, y) - (0, 0)\| < \delta \Rightarrow |f(x, y)| < \epsilon$$

$$b) \lim_{(x,y) \rightarrow (0,0)} f(x, y)$$

Pruebo por curvas

- iterado $x = 0$
 $\lim_{y \rightarrow 0} f(0, y)$
- iterado $y = 0$
 $\lim_{x \rightarrow 0} f(x, 0)$
- rectas $y = mx$
 $\lim_{x \rightarrow 0} f(x, mx)$
- curvas $y = x^2$
 $\lim_{x \rightarrow 0} f(x, x^2)$

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Pruebo por definición

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$$3. f(x, y) =$$

Diferenciabilidad en a

f es diferenciable en $a \Leftrightarrow$

$$\exists L \in \mathbb{R} : \lim_{(x,y) \rightarrow (0,0)} \frac{f(x,y) - f(0,0) - \nabla f(0,0) \cdot (x-0, y-0)}{\|(x,y)\|} = L \wedge L = 0$$

Busco $\nabla f(0, 0)$

$$\nabla f(0, 0) = (f_x(0, 0), f_y(0, 0))$$

Por definición

- $f_x(0, 0) = \lim_{h \rightarrow 0} \frac{f(h, 0) - f(0, 0)}{h} =$
 $\lim_{h \rightarrow 0} f(h, 0) - f(0, 0) \cdot \frac{1}{h}$
- $f_y(0, 0) = \lim_{h \rightarrow 0} \frac{f(0, h) - f(0, 0)}{h} =$
 $f_y(0, 0) = \lim_{h \rightarrow 0} f(0, h) - f(0, 0) \cdot \frac{1}{h}$

$$4. F(s, t) = f(x(s), y(s))$$

$$z = \nabla F(0, 1) \cdot (s, t - 1) + F(0, 1) =$$

Busco $\nabla F(0, 1)$

$$\nabla F(s, t) = (F_s(s, t), F_t(s, t))$$

- $F_s(s, t) = f_x(x(s, t), y(s, t)) \cdot x_s(s, t) + f_y(x(s, t), y(s, t)) \cdot y_s(s, t) =$
- $F_t(s, t) = f_x(x(s, t), y(s, t)) \cdot x_t(s, t) + f_y(x(s, t), y(s, t)) \cdot y_t(s, t) =$