

1. $\int \int_D 1dV, D = \{0 \leq r \leq 2 \wedge 0 \leq \theta \leq 2\pi \wedge 0 \leq z \leq r\}$
2. Bajo el paraboloide $z = 18 - 2x^2 - 2y^2$ y arriba del plano xy
 $0 = 18 - (2(x^2 + y^2)) \equiv$
 $2(x^2 + y^2) = 18 \equiv$
 $x^2 + y^2 = 9$ Circunferencia de radio 3 centrada en el origen

Al sustituir por polares D es de tipo 1

En el plano xy la sombra que se proyecta del paraboloide es $r \cdot \theta : (r, \theta) \in \mathbb{R}^3 : 0 \leq r \leq 3 \wedge 0 \leq \theta \leq 2\pi \Rightarrow$

- $0 \leq r \leq 3$
- $0 \leq \theta \leq 2\pi$
- $0 \leq z \leq 18 - 2(x^2 + y^2) \stackrel{\text{polares}}{\equiv} 0 \leq z \leq 18 - 2r^2$

$$\int_0^3 (\int_0^{2\pi} (\int_0^{18-2r^2} 1dz) d\theta) dr$$

- $\int_0^{18-2r^2} r dz =$
 $rz \Big|_0^{18-2r^2} =$
 $r(18 - 2r^2) - 0 =$
 $18r - 2r^3$
- $\int_0^{2\pi} 18r - 2r^3 d\theta =$
 $18r\theta - 2r^3\theta \Big|_0^{2\pi} =$
 $18r\theta -$
 $18r\pi - 4r^3\pi$
- $\int_0^3 18r\pi - 4r^3\pi dr =$
 $9r^2\pi - r^4\pi \Big|_0^3 =$
 $81\pi - 81\pi =$

$$\int_0^3 (\int_0^{2\pi} 18r - 2r^3 d\theta) dr$$

- $\int_0^{2\pi} 18r - 2r^3 d\theta =$
 $2(\int_0^{2\pi} 9r - r^3 d\theta) =$
 $2(9r\theta - \theta r^3 \Big|_0^{2\pi}) =$
 $2(18r\pi - 2\pi r^3) =$
 $4(9r\pi - \pi r^3) =$
- $4(\int_0^3 9r\pi - \pi r^3 dr) =$
 $4(\frac{9\pi}{2}r^2 - \frac{\pi}{4}r^4 \Big|_0^3) =$
 $4(\frac{81\pi}{2} - \frac{81\pi}{4}) =$
 $2 \cdot 81\pi - 81\pi = 81\pi$

3. Encerrado por el hiperboloide $-x^2 - y^2 + z^2 = 1$ y el plano $z = 2$

- $z^2 - 1 = x^2 + y^2 \stackrel{z=2}{\equiv} 3 = x^2 + y^2$ Circunferencia de radio $\sqrt{3}$ centrado en $(0, 0)$
- $z = 2$

$$\int_0^{\sqrt{3}} (\int_0^{2\pi} 1d\theta) dr$$

- $\int_0^{2\pi} r d\theta =$
 $r\theta \Big|_0^{2\pi} =$
 $2r\pi$
- $\int_0^{\sqrt{3}} 2r\pi dr =$
 $r^2\pi \Big|_0^{\sqrt{3}} =$
 3π

$$4. \text{ 1er octante} = \int_2^4 \left(\int_0^{\frac{\pi}{2}} \left(\int_r^{\sqrt{16-r^2}} r dz \right) d\theta \right) dr$$

$$\begin{aligned}
 & \blacksquare \int_r^{\sqrt{16-r^2}} r dz = \\
 & \quad rz \Big|_r^{\sqrt{16-r^2}} = \\
 & \quad r\sqrt{16-r^2} - r^2 \\
 & \blacksquare \int_0^{\frac{\pi}{2}} r\sqrt{16-r^2} - r^2 d\theta = \\
 & \quad r\sqrt{16-r^2}\theta - r^2\theta \Big|_0^{\frac{\pi}{2}} = \\
 & \quad \frac{r\sqrt{16-r^2}\pi}{2} - \frac{r^2\pi}{2} \\
 & \blacksquare \int_2^4 \frac{r\sqrt{16-r^2}\pi}{2} - \frac{r^2\pi}{2} dr = \\
 & \quad \bullet \int \frac{r\sqrt{16-r^2}\pi}{2} dr = \frac{\pi}{2} \int r(16-r^2)^{\frac{1}{2}} dr = \\
 & \quad \frac{\pi(16-r^2)^{\frac{3}{2}}}{\frac{3}{2}} + C \\
 & \quad \left(\frac{(16-r^2)^{\frac{3}{2}}}{3} \right)_r = \frac{1}{3} \cdot \frac{3(16-r^2)^{\frac{1}{2}} 2r}{2} \\
 & \quad \bullet \int \frac{r^2\pi}{2} dr = \frac{r^3\pi}{6} + C \\
 & \quad \frac{\pi(16-r^2)^{\frac{3}{2}}}{6} - \frac{r^3\pi}{6} \Big|_2^4 = \\
 & \quad \frac{\pi(16-4^2)^{\frac{3}{2}}}{6} - \frac{4^3\pi}{6} - \left(\frac{\pi(16-2^2)^{\frac{3}{2}}}{6} - \frac{2^3\pi}{6} \right) = \\
 & \quad -\frac{32\pi}{3} - \frac{12\frac{3}{2}\pi}{6} + \frac{4\pi}{3} = \star \Rightarrow \\
 & \quad V = 8 \cdot \star
 \end{aligned}$$