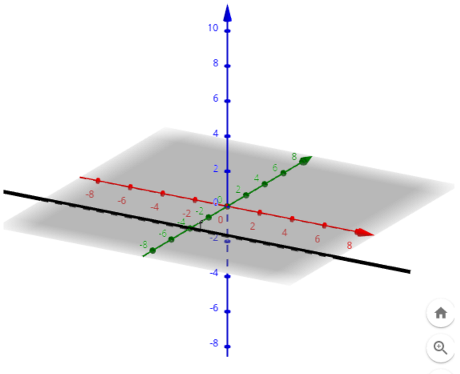
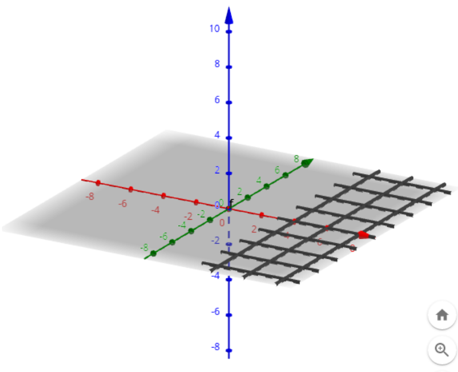
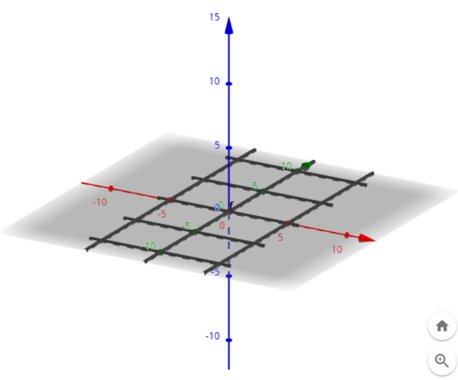
1. *a)* $y = -4$



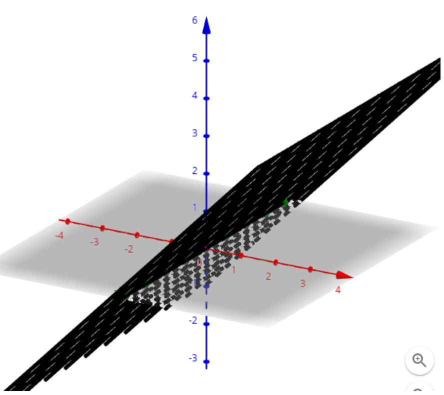
b) $b > 3$



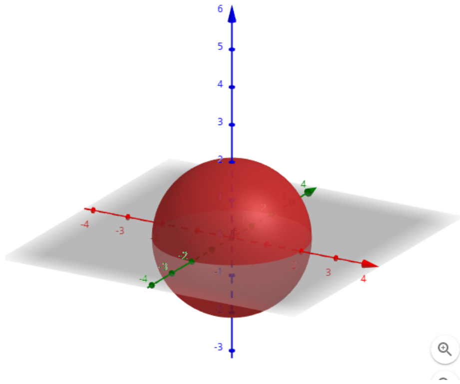
c) $0 \leq z \leq 6$



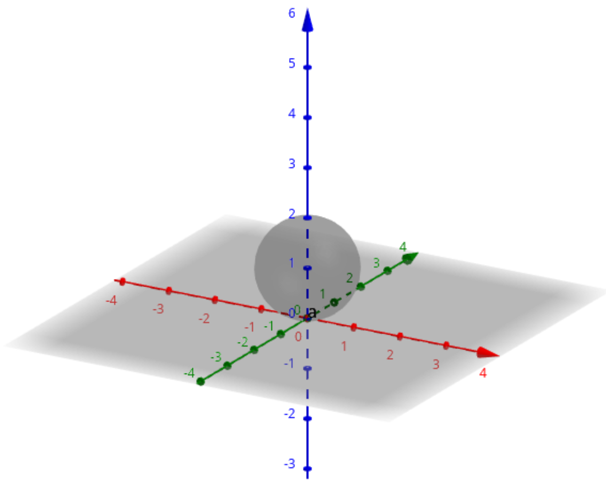
d) $x = z$



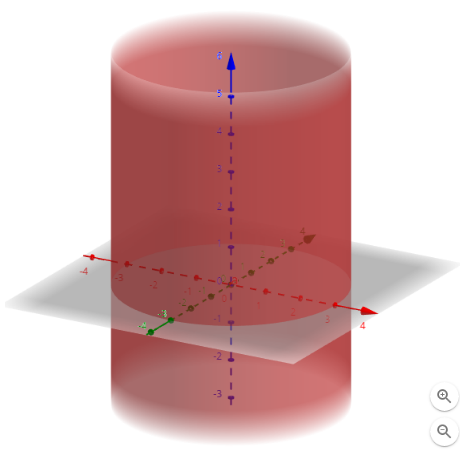
e) $x^2 + y^2 + z^2 \leq 4$



f) $x^2 + y^2 + z^2 > 2z \equiv x^2 + y^2 + (z-1)^2 > 1$



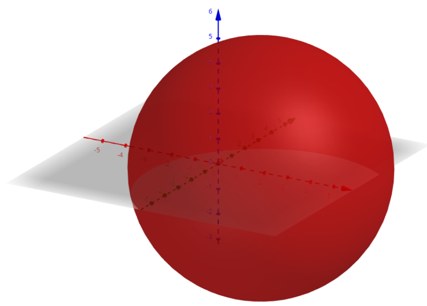
g) $x^2 + y^2 \leq 9$



2. a) $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$
   $P_0 = (a, b, c) \wedge b : (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ entonces
   b es un circulo con centro en P_0 y radio r
   $x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 6x + 4y - 2z + j = 11 + j$
   - $-2ax = -6x \equiv a = 3$

- $-2by = 4y \equiv b = -2$
- $-2cz = -2z \equiv c = 1$

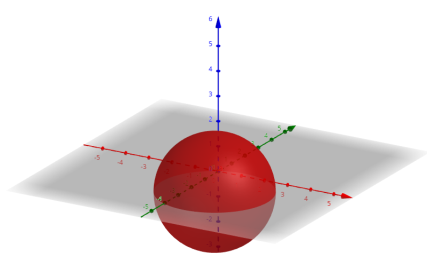
$(x-3)^2 + (y+2)^2 + (z-1)^2 \equiv$
$(x^2 - 6x + 9) + (y^2 + 4y + 4) + (z^2 - 2z + 1) = 11 + j \equiv$
$(x^2 + y^2 + z^2) - 6x + 4y - 2z + 9 + 4 + 1 = 11 + j \equiv$
$(x^2 + y^2 + z^2) - 6x + 4y - 2z + 14 = 11 + j \equiv$
$(x^2 + y^2 + z^2) - 6x + 4y - 2z = 25$
Es un circulo con centro en $(3, -2, 1)$ y radio 5

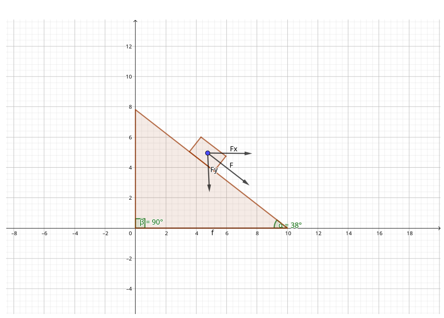


b) $4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$
$4(x^2 + y^2 + z^2 - 2x + 4y) = 1 \equiv$
$x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$
$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$
$x^2 + y^2 + z^2 - 2ax + a^2 - 2by + b^2 - 2cz + c^2 = x^2 + y^2 + z^2 - 2x + 4y = \frac{1}{4}$

- $-2ax = -2x \equiv a = 1$
- $-2by = 4y \equiv b = -2$
- $-2cz = 0 \equiv c = 0$

$(x-1)^2 + (y+2)^2 + (z)^2 \equiv$
$x^2 + -2x + 1 + y^2 + 4y + 4 + z^2 \equiv$
$(x^2 + y^2 + z^2 - 2x + 4y) + 5 = \frac{1}{4} + 5 \equiv$
$x^2 + y^2 + z^2 - 2x + 4y + 5 = \frac{21}{4}$
$(x-1)^2 + (y+2)^2 + (z)^2 = \frac{21}{4}$
Es un circulo con centro en $(1, -2, 0)$ y radio $\frac{\sqrt{21}}{2}$



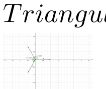
3. 

- $\alpha = 38°$
- $F = 50N$
  $F_x = 50N \cdot \cos 38° = 39{,}40N$
  $F_y = 50N \cdot \sin 38° = -30{,}78N$

4.
- $F_1 = 10N$
- $F_2 = 12N$
- $\alpha = 45°$
- $\theta = 30°$
- $F_{1x} = F_1 \cdot \cos\alpha = 10N \cdot \cos(45°) = -7{,}07N$
- $F_{1y} = F_1 \cdot \sin\alpha = 10N \cdot \sin(45°) = 7{,}07N$
- $F_{2x} = F_2 \cdot \cos\theta = 12N \cdot \cos(30°) = 10{,}39N$
- $F_{2y} = F_2 \cdot \sin\theta = 12N \cdot \sin(30°) = 6N$
- $F_{rx} = F_{1x} + F_{2x} = -7{,}07N + 10{,}39N = 3{,}32N$
- $F_{ry} = F_{1y} + F_{2y} = 7{,}07N + 6N = 13{,}07N$
- $F_r = \sqrt{(F_{rx})^2 + (F_{ry})^2} = \sqrt{(3{,}32N)^2 + (13{,}07N)^2} = 13{,}48N$

5. $u \in \Re^2 \wedge \|u\| = 1$

- $Triangulo$

  - $u \cdot v \overset{\|u\|=\|v\|}{\equiv} \|u\|^2 * \cos(\alpha) = -\frac{1}{2}$
  - $u \cdot w \overset{\|u\|=\|w\|}{\equiv} \|u\|^2 * \cos(\alpha) = -\frac{1}{2}$
- $Cuadrado$
  - $u \cdot v \overset{\|u\|=\|v\|}{\equiv} \|u\|^2 * \cos(90°) = 0$
  - $u \cdot w \overset{\|u\|=\|w\|}{\equiv} \|u\|^2 * \cos(45°) = \frac{\sqrt{2}}{2}$

6.
a) $u = (3, -4), v = (5, 0)$
$P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(3,-4)\cdot(5,0)}{\|(3,-4)\|^2} \cdot (3, -4) =$
$\frac{15}{25} \cdot (3, -4) = (\frac{9}{5}, -\frac{12}{5})$

b) $u = (1, 2), v = (-4, 1)$
$P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(1,2)\cdot(-4,1)}{\|(1,2)\|^2} \cdot (1, 2) =$
$-\frac{2}{5} \cdot (1, 2) = (-\frac{2}{5}, -\frac{4}{5})$

c) $u = (3, 6, 2), v = (1, 2, 3)$
$P_u(v) = \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{(3,6,2)\cdot(1,2,3)}{\|(3,6,2)\|^2} \cdot (3, 6, 2) =$
$\frac{3}{7} \cdot (3, 6, 2) = (\frac{9}{7}, \frac{18}{7}, \frac{6}{7})$

7. u, v vectores QvQ $o_u(v) = v - p_u(v)$ es ortogonal a u
$o_u(v)$ es ortogonal a $u \leftrightarrow o_u(v) \cdot u = 0$
$o_u(v) = v - \frac{u \cdot v}{\|u\|^2} \cdot u$
QvQ $o_u(v) \cdot u = 0$
$(v - \frac{u \cdot v}{\|u\|^2} \cdot u) \cdot u \equiv u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot u \cdot u \overset{u \cdot u = \|u\|^2}{\equiv} u \cdot v - \frac{u \cdot v}{\|u\|^2} \cdot \|u\|^2 \overset{u \neq 0}{\equiv}$
$u \cdot v - \frac{u \cdot v}{\cancel{\|u\|^2}} \cdot \cancel{\|u\|^2} \equiv u \cdot v - u \cdot v = 0 \quad \blacksquare$

8. u, v vectores $u \neq 0 \wedge v \neq 0$ QvC $p_u(v) = p_v(u)$

   $v = \lambda u \rightarrow p_u(v) = v \wedge v = \theta u \rightarrow p_v(u) = u$

   $p_u(v) = p_v(u) \equiv \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{u \cdot v}{\|v\|^2} \cdot v \overset{u \cdot v \neq 0}{\equiv} \frac{\cancel{u \cdot v}}{\|u\|^2} \cdot u = \frac{\cancel{u \cdot v}}{\|v\|^2} \cdot v \equiv$

   $\frac{u}{\|u\|^2} = \frac{v}{\|v\|^2} \cdot v \leftrightarrow u = \lambda \cdot v \wedge v = \theta u \wedge \lambda, \theta \in \Re \leftrightarrow u = v$

   Si $u \cdot v = 0 \rightarrow \frac{u \cdot v}{\|u\|^2} \cdot u = \frac{u \cdot v}{\|v\|^2} \cdot v = 0$

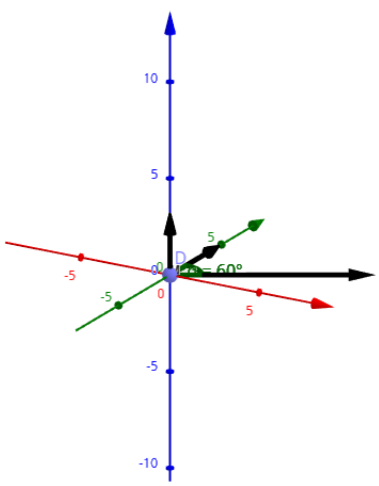
   $p_u(v) = p_v(u) \leftrightarrow u \cdot v = 0 \vee u = v \quad \blacksquare$

9. $W = 4m \cdot 20N \cdot \cos(50°) = 51,42J$

10. $W = (Q - P) \cdot F \equiv (6, 2, 12) \cdot (8, -6, 9) = 144J$

11. *a*)
    - $\|u\| = 5$
    - $\|v\| = 10$
    - $\alpha = 60°$

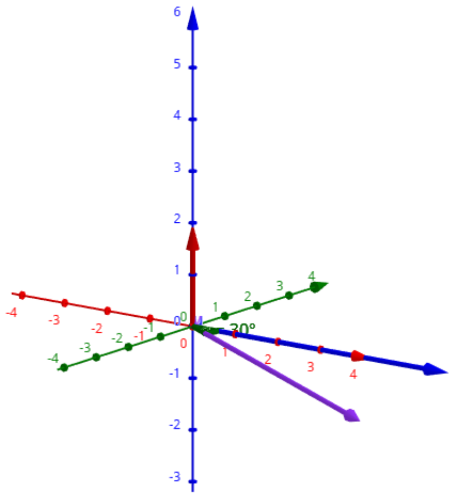
    1) Sentido $u \times v$



    2) $\|u \times v\| = \|a\| \cdot \|b\| \cdot \sin(\alpha) \equiv 5 \cdot 10 \cdot \sin(60°) \approx 43{,}30$

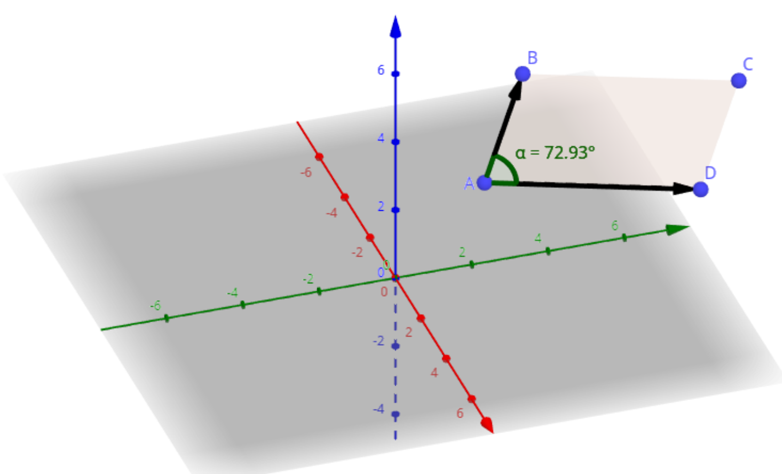
    b)
    - $\|u\| = 6$
    - $\|v\| = 8$
    - $\alpha = 30°$

    1) Sentido $u \times v$



2) $\|u \times v\| = \|a\| \cdot \|b\| \cdot \sin(\alpha) \equiv 6 \cdot 8 \cdot \sin(30°) = 24$

12.
- $A = (1, 2, 3)$
- $B = (1, 3, 6)$
- $C = (3, 8, 6)$
- $D = (3, 7, 3)$
- $\overrightarrow{u} = B - A = (0, 1, 3)$
- $\overrightarrow{v} = C - A = (2, 6, 3)$



$Area = \|\overrightarrow{u} \times \overrightarrow{v}\| = \|(0, 1, 3) \times (2, 6, 3)\|$

$$\begin{vmatrix} i & j & k \\ 0 & 1 & 3 \\ 2 & 6 & 3 \end{vmatrix}$$

$= (1 \cdot 3) - (6 \cdot 3)) \cdot \widehat{i} - (0 \cdot 3) - (2 \cdot 3)) \cdot \widehat{j} + (0 \cdot 6) - (2 \cdot 1)) \cdot \widehat{k} = (-15, 6, -2)$

$\|(-15,6,-2)\| = \sqrt{(-15)^2 + 6^2 + (-2)^2} = \sqrt{225 + 36 + 4} = \sqrt{265}$

13.
- $\|t\| = \|r \times F\| = \|r\| \cdot \|F\| \cdot \sin(\theta)$
- $\|t\| = \|0{,}18cm\| \cdot \|60N\| \cdot \sin(80°) \approx 10{,}64$

14.
- $u, v, w \in \Re^3 \wedge A \in \Re^{3\times 3} \wedge A = \begin{vmatrix} u \\ v \\ w \end{vmatrix}$ $QvQ\ u \cdot (v \times w) = det(A)$

$$u \cdot (v \times w) = u \cdot det(\begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}) = u \cdot (((w_2 \cdot v_3) - (v_2 \cdot w_3)) \cdot i - ((v_1 \cdot w_3) - (w_1 \cdot v_3)) \cdot j + ((v_2 \cdot w_3) - (w_2 \cdot v_3)) \cdot k) =$$

$$((w_2 \cdot v_3) - (v_2 \cdot w_3)) \cdot u_1 - ((v_1 \cdot w_3) - (w_1 \cdot v_3)) \cdot u_2 + ((v_2 \cdot w_3) - (w_2 \cdot v_3)) \cdot u_3 = det(\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}) \blacksquare$$

15.
- $AB = (2, 1, 1)$
- $AC = (1, -1, 2)$
- $AD = (0, -2, 3)$

$$Area = \|AB \cdot (AC \times AD)\| = \|det(\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & -2 & 3 \end{vmatrix})\| = |2 \cdot (-3 - (-4)) - 3 + (-2) = 2 \cdot (7) - 3 - 2| = 9$$

16.
- $A = (1, 3, 2)$
- $B = (3, -1, 6)$
- $C = (5, 2, 0)$
- $D = (3, 6, -4)$

$a = \overrightarrow{AB} \wedge b = \overrightarrow{AC} \Rightarrow a = (2, -4, 4) \wedge b = (4, -1, -2)$

$$n = a \times b = det(\begin{vmatrix} i & j & k \\ 2 & -4 & 4 \\ 4 & -1 & -2 \end{vmatrix}) =$$

$12\widehat{i} + 20\widehat{j} + 14\widehat{k} \overset{\times\frac{1}{2}}{\equiv} 6\widehat{i} + 10\widehat{j} + 7\widehat{k}$

$\Pi = 6 \cdot (x - 1) + 10 \cdot (y - 3) + 7 \cdot (z - 2) =$

$6x - 6 + 10y - 30 + 7z - 14 =$

$6x + 10y + 7z - 50 = 0 \equiv$

$6x + 10y + 7z = 50 \Rightarrow$

$\Pi : 6x + 10y + 7z = 50$

$QvQ\ A, B, C, D \in \Pi$

- $\Pi : 6(1) + 10(3) + 7(2) \overset{?}{=} 50$ ✓
- $\Pi : 6(3) + 10(-1) + 7(6) \overset{?}{=} 50$ ✓
- $\Pi : 6(5) + 10(2) + 7(0) \overset{?}{=} 50$ ✓
- $\Pi : 6(3) + 10(6) + 7(-4) \overset{?}{=} 50$ ✓

17. *a)*
   - $A = (1,3,1)$
   - $B = (2,1,1)$
   - $C = (3,4,1)$
   - $a = \overrightarrow{AB} = (1,-2,0)$
   - $b = \overrightarrow{AC} = (2,1,0)$
   - Ecuación parametrica $= (x,y,z) = \overrightarrow{OP_0} + \alpha \cdot \overrightarrow{u} + \beta \cdot \overrightarrow{v} \Rightarrow$
     $\Pi : (x,y,z) = (1,3,1) + \alpha \cdot (1,-2,0) + \beta \cdot (2,1,0), \alpha, \beta \in \Re$

   QvQ $A, B, C \in \Pi$
   - $(1,3,1) = (1,3,1) + 0 \cdot (1,-2,0) + 0 \cdot (2,1,0)$
   - $(2,1,1) = (1,3,1) + \alpha \cdot (1,-2,0) + \beta \cdot (2,1,0)$
     - $2 = 1 + \alpha + \beta(2) \overset{\star}{\Rightarrow} 2 = 1 + \alpha + (\alpha(2) - 2)(2) \Rightarrow 2 = 1 + \alpha + \alpha(4) - 4 \Rightarrow 5 = \alpha(5) \Rightarrow \star' : \alpha = 1$
     - $1 = 3 + \alpha(-2) + \beta \Rightarrow \star : \beta = \alpha(2) - 2 \overset{\star'}{\Rightarrow} \beta = 0$
     - $1 = 1$
   - $(3,4,1) = (1,3,1) + 0 \cdot (1,-2,0) + 1 \cdot (2,1,0)$

   b)
   - $n = a \times b = det(\begin{vmatrix} i & j & k \\ 1 & -2 & 0 \\ 2 & 1 & 0 \end{vmatrix}) = 0\widehat{i} - 0\widehat{j} + 5\widehat{k} = (0,0,5)$
   - $\Pi : (0,0,5) \cdot (x-1, y-3, z-1) = 0$

18. *a)*
   - $L_1 = t(1,-1,2) + (1,1,0)$
   - $L_2 = t(-1,1,0) + (2,0,2)$

   $\exists t_1, t_2 \in \Re : (t_1 + 1, -t_1 + 1, 2t_1) = (-t_2 + 2, t_2, 2)$
   - $t_1 + 1 = -t_2 + 2$
   - $-t_1 + 1 = t_2 \overset{\star}{\Rightarrow} 0 = t_2$
   - $2t_1 = 2 \Rightarrow \star : t_1 = 1$

   $P = (2,0,2) : P \in L_1 \wedge P \in L_2$

   $\Rightarrow$ La intersección entre L_1 y L_2 es P ■

   b)
   - $P = (1,1,2) \in L_2 \wedge$ $P_2 = (1,1,0) \in L_1 \wedge$ $P_3(3,-1,4) \in L_1$
   - $a = \overrightarrow{PP_2} = (0,0,-2)$
   - $b = \overrightarrow{PP_3} = (2,-2,2)$

   $a \times b = det(\begin{vmatrix} i & j & k \\ 0 & 0 & -2 \\ 2 & -2 & 2 \end{vmatrix}) = -4\widehat{i} - 4\widehat{j} - 0\widehat{k}$

   $\Pi : (-4,-4,0) \cdot (x-1, y-1, z) = 0 \equiv \Pi : -4x + 4 - 4y + 4 = 0 \equiv \Pi : -4x - 4y = -8 \; \Pi : x + y = 2$

19. *a)* Plano que corta los ejes en a

   b) Plano que corta los ejes x e y en z depende de a

   c) Planos que giran en torno al eje x formando un cilindro de radio 1, segun el valor de a