

$$1. \lim_{(x,y) \rightarrow (7,2)} x^2 + y^2 - xy$$

$$7^2 + 2^2 - (7 \cdot 2) = 39$$

$$2. \lim_{(x,y) \rightarrow (0,1)} xe^{xy}$$

$$0 \cdot 1 = 0$$

$$3. \lim_{(x,y) \rightarrow (2,1)} \frac{4-xy}{x^2+3y^2}$$

$$\frac{4-2}{4+3(1)} = \frac{2}{7}$$

$$4. \lim_{(x,y) \rightarrow (0,0)} = \frac{x-y}{x+y}$$

$$\lim_{x \rightarrow 0} f(x, 0) = 1$$

$$\lim_{y \rightarrow 0} f(0, y) = -1$$

$$\Rightarrow \text{Por curvas} \nexists L$$

$$5. \lim_{(x,y) \rightarrow (0,0)} = \frac{x^4-4y^2}{x^2+2y^2}$$

$$\lim_{x \rightarrow 0} f(x, 0) = 0$$

$$\lim_{y \rightarrow 0} f(0, y) = -2$$

$$\Rightarrow \text{Por curvas} \nexists L$$

$$6. \lim_{(x,y) \rightarrow (0,0)} = \frac{xy}{x^2+y^2}$$

$$\lim_{x \rightarrow 0} f(x, 0) = 0$$

$$\lim_{y \rightarrow 0} f(0, y) = 0$$

$$\lim_{y \rightarrow 0} f(x, x) = \frac{1}{2}$$

$$\Rightarrow \text{Por curvas} \nexists L$$

$$7. \lim_{(x,y) \rightarrow (1,0)} = \frac{xy-y}{(x-1)^2+y^2}$$

$$\lim_{x \rightarrow 1} f(x, 0) = 0$$

$$\lim_{y \rightarrow 0} f(0, y) = 0$$

$$\lim_{x \rightarrow 1} f(x, x) = \frac{x(x-1)}{(x-1)^2+x^2} = 0$$

$$\lim_{x \rightarrow 1} f(x, (x-1)) = \frac{(x-1)^2}{2(x-1)^2} = \frac{1}{2}$$

$$\Rightarrow \text{Por curvas} \nexists L$$

$$8. \lim_{(x,y) \rightarrow (0,0)} = \frac{xy}{\sqrt{x^2+y^2}}$$

$$\lim_{x \rightarrow 0} f(x, 0) = 0$$

$$\lim_{y \rightarrow 0} f(0, y) = 0$$

Sospecho que  $L = 0$ , lo pruebo por definición

$$\|(x, y)\| < \delta \Rightarrow \left| \frac{xy}{\sqrt{x^2+y^2}} \right| < \epsilon$$

$$\blacksquare \quad |x| \leq \|(x, y)\| < \delta$$

$$\blacksquare \quad |y| \leq \|(x, y)\| < \delta$$

$$\left| \frac{xy}{\sqrt{x^2+y^2}} \right| =$$

$$\frac{|y||y|}{\sqrt{x^2+y^2}} =$$

$$\frac{|y||y|}{\sqrt{x^2+y^2}} =$$

$$\frac{|y||y|}{\|(x, y)\|} \leq$$

$$\frac{\|(x, y)\| \|(x, y)\|}{\|(x, y)\|} =$$

$$\|(x, y)\| \leq \delta \Rightarrow \delta = \epsilon \blacksquare$$

$$9. \lim_{(x,y) \rightarrow (0,0)} = \frac{x^4 - y^4}{x^2 + y^2}$$

$$\frac{x^4 - y^4}{x^2 + y^2} =$$

$$\frac{(x^2 - y^2)(x^2 + y^2)}{x^2 + y^2} =$$

$$x^2 + y^2$$

$$(x, y) \rightarrow (0, 0) \Rightarrow x^2 + y^2 \rightarrow 0 \quad \blacksquare$$

$$10. \lim_{(x,y) \rightarrow (0,0)} = \frac{y^2 \sin^2(x)}{x^4 + 2y^4}$$

$$\lim_{x \rightarrow 0} f(x, 0) = 0$$

$$\lim_{y \rightarrow 0} f(0, y) = 0$$

$$\lim_{x \rightarrow 0} f(x, x) = \lim_{x \rightarrow 0} \frac{x^2 \sin^2(x)}{x^4 + 2x^4} =$$

$$\lim_{x \rightarrow 0} \frac{x^2 \sin^2(x)}{3x^4} =$$

$$\lim_{x \rightarrow 0} \frac{\cancel{x^2}}{3\cancel{x^2}} \cdot \overset{\rightarrow 1}{\frac{\sin(x)}{x}} \cdot \overset{\rightarrow 1}{\frac{\sin(x)}{x}} = \frac{1}{3}$$

$$\Rightarrow \nexists L$$

$$11. \lim_{(x,y) \rightarrow (0,3)} \frac{x^2(y-3)^2 e^x}{x^2 + (y-3)^2}$$

$$\blacksquare \lim_{x \rightarrow 0} f(x, 0) = \lim_{x \rightarrow 0} \frac{x^2(-3)^2 e^x}{x^2 + (-3)^2} = 0$$

$$\blacksquare \lim_{y \rightarrow 3} f(0, y) = \lim_{y \rightarrow 3} \frac{0}{(y-3)^2} = 0$$

$$\blacksquare \lim_{y \rightarrow 3} f(y-3, y) = \lim_{y \rightarrow 3} \frac{\cancel{(y-3)}^2 (y-3)^2 e^{y-3}}{2\cancel{(y-3)}^2} =$$

$$\lim_{y \rightarrow 3} \frac{\overset{\rightarrow 0}{(y-3)^2} \overset{\rightarrow 1}{e^{y-3}}}{2} = 0$$

Sospecho que  $L = 0$  pruebo por definici3n

$$\|(x, y-3)\| < \delta \Rightarrow \left| \frac{x^2(y-3)^2 e^x}{x^2 + (y-3)^2} \right| < \epsilon$$

$$\blacksquare |x| \leq \|(x, y-3)\| < \delta$$

$$\blacksquare |y-3| \leq \|(x, y-3)\| < \delta$$

$$\left| \frac{x^2(y-3)^2 e^x}{x^2 + (y-3)^2} \right| =$$

$$\frac{|x^2(y-3)^2 e^x|}{x^2 + (y-3)^2} =$$

$$\frac{x^2(y-3)^2 |e^x|}{x^2 + (y-3)^2} \overset{x \rightarrow 0 \Rightarrow e^x \rightarrow 1}{=} \frac{x^2(y-3)^2}{x^2 + (y-3)^2} \leq$$

$$\frac{x^2(y-3)^2}{x^2 + (y-3)^2} \leq$$

$$\frac{\|(x, \cancel{y-3})\|^2 \|(x, y-3)\|^2}{\|(x, \cancel{y-3})\|^2} \overset{\delta < 1}{\leq}$$

$$\delta = \epsilon \Rightarrow$$

$$\delta = \min(1, \epsilon) \quad \blacksquare$$