$$f(x,y) = \begin{cases} \frac{x^3e^y+3x^2y}{x^2+y^2} & si\ (x,y) \neq (0,0) \\ 0 \quad si\ (x,y) = (0,0) \end{cases}$$

Diferenciabilidad en $(0,0)$

f es diferenciable en $(0,0) \Leftrightarrow$
$\exists L \in \Re : \lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x-0,y-0)}{\|(x,y)\|} = L \wedge L = 0$

Busco $\nabla f(0,0)$

$\nabla f(0,0) = (f_x(0,0), f_y(0,0))$

Por definición

- $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} =$
  $\lim_{h\to 0} f(h,0) - f(0,0) \cdot \frac{1}{h} =$
  $\lim_{h\to 0} \frac{\cancel{h^3}}{\cancel{h^3}} = 1$
- $f_y(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} =$
  $f_y(0,0) = \lim_{h\to 0} f(0,h) - f(0,0) \cdot \frac{1}{h}$
  $f_y(0,0) = \lim_{h\to 0} \frac{0}{h^2} \cdot \frac{1}{h} = 0$

$\nabla f(0,0) = (1,0)$
$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x-0,y-0)}{\|(x,y)\|} =$
$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{\frac{x^3e^y+3x^2y}{x^2+y^2}-0-(1,0)\cdot(x-0,y-0)}{\|(x,y)\|} =$
$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{\frac{x^3e^y+3x^2y}{x^2+y^2}-x}{\|(x,y)\|} =$
$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^3e^y+3x^2y}{x^2+y^2} \frac{1}{\sqrt{x^2+y^2}} - \frac{x}{\sqrt{x^2+y^2}} =$
$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^3e^y+3x^2y}{x^2+y^2} \frac{1}{\sqrt{x^2+y^2}} - \lim_{(x,y)\to\overset{\star}{(0,0)}} \frac{x}{\sqrt{x^2+y^2}} =$

- $\star : \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$
  Por la recta $y = x$
  $\lim_{x\to 0} \frac{x}{\sqrt{x^2+x^2}} =$
  $\lim_{x\to 0} \frac{x}{\sqrt{2}|x|} =$
  $\lim_{x\to 0^+} \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$
  $\lim_{x\to 0^-} \frac{x}{\sqrt{2}(-x)} = -\frac{1}{\sqrt{2}}$
  $\Rightarrow \nexists \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$

$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^3e^y+3x^2y}{x^2+y^2} \frac{1}{\sqrt{x^2+y^2}} - \lim_{(x,y)\to\overset{\star}{(0,0)}} \frac{x}{\sqrt{x^2+y^2}} \wedge \nexists \star$
$\Rightarrow$ Por algebra del limites no $\nexists$ la resta de limites por lo tanto $\nexists L$
$\Rightarrow f(x,y)$ no es diferenciable en el $(0,0)$