$f(x,y) = \sqrt[3]{x^3 + 8y^3}$

Diferenciabilidad en $(0,0)$

f es diferenciable en $(0,0) \Leftrightarrow$
$\exists L \in \Re : \lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x-0,y-0)}{\|(x,y)\|} = L \wedge L = 0$

Busco $\nabla f(0,0)$

$\nabla f(0,0) = (f_x(0,0), f_y(0,0))$

Por definición las derivadas parciales

- $f_x(0,0) = \lim_{h\to0} \frac{f(h,0)-f(0,0)}{h} =$
  $\lim_{h\to0} f(h,0) - f(0,0) \cdot \frac{1}{h}$
  $\lim_{h\to0} \sqrt[3]{h^3} - 0 \cdot \frac{1}{h}$
  $\lim_{h\to0} h \cdot \frac{1}{h} = 1$
- $f_y(0,0) = \lim_{h\to0} \frac{f(0,h)-f(0,0)}{h} =$
  $f_y(0,0) = \lim_{h\to0} f(0,h) - f(0,0) \cdot \frac{1}{h}$
  $f_y(0,0) = \sqrt[3]{8h^3} - 0 \cdot \frac{1}{h} = 2$

$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x,y)}{\|(x,y)\|} =$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}-0-(1,2)\cdot(x,y)}{\|(x,y)\|} =$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}-x-2y}{\|(x,y)\|} =$
$y = x$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}-x-2y}{\|(x,y)\|} =$
$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}}{\sqrt{x^2+y^2}} - \overset{\star}{\frac{x+2y}{\sqrt{x^2+y^2}}} =$
$\star : \lim_{(x,y)\to(0,0)} \frac{x+2y}{\sqrt{x^2+y^2}}$
Por $y = x$
$\lim_{x\to0} \frac{3x}{\sqrt{2x^2}} =$
$\lim_{x\to0} \frac{3x}{\sqrt{2}|x|}$
$\lim_{x\to0+} \frac{3x}{\sqrt{2}x} = 3$
$\lim_{x\to0-} \frac{3x}{\sqrt{2}(-x)} = -3$
$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} \frac{x+2y}{\sqrt{x^2+y^2}}$
$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x^3+8y^3}}{\sqrt{x^2+y^2}} - \overset{\star}{\frac{x+2y}{\sqrt{x^2+y^2}}}$
$\Rightarrow$ f no es diferenciable en $(0,0)$