$$\begin{cases} x = \rho\cos(\theta)\sin(\phi) & 0 \le \rho \le 2 \\ y = \rho\sin(\theta)\sin(\phi) & 0 \le \theta \le \frac{\pi}{2} \\ z = \rho\cos(\phi) & \frac{\pi}{4} \le \phi \le \frac{\pi}{2} \end{cases}$$

$\int_0^2 (\int_0^{\frac{\pi}{2}} (\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \rho^2 \sin(\phi) d\phi) d\theta) d\rho$

- $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \rho^2 \sin(\phi) d\phi =$
  $\rho^2 - \cos(\phi)\big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$
  $\rho^2(-\cos(\frac{\pi}{2}) + \cos(\frac{\pi}{4})) =$
  $\rho^2 \frac{\sqrt{2}}{2}$
- $\int_0^{\frac{\pi}{2}} \rho^2 \frac{\sqrt{2}}{2} d\theta =$
  $\frac{\pi\sqrt{2}}{4}\rho^2$
- $\frac{\pi\sqrt{2}}{4} \int_0^2 -\rho^2 d\rho =$
  $\frac{\pi\sqrt{2}}{4} \left.\frac{-\rho^3}{3}\right|_0^2 =$
  $\frac{\pi\sqrt{2}}{4} \frac{-8}{3} = -\frac{2\pi\sqrt{2}}{3}$