

$$\int \int_D \frac{1}{x^2+y^2+1} dA$$

$$D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 16, -x \leq y \leq x\}$$

$$\blacksquare \quad 1 \leq r \leq 4$$

$$\blacksquare \quad \frac{7}{8}\pi \leq \theta \leq \frac{9}{8}\pi$$

$$\int_1^4 \left(\int_{\frac{7}{8}\pi}^{\frac{9}{8}\pi} \frac{r}{r^2+1} d\theta \right) dr$$

$$\frac{r}{r^2+1} \left(\frac{9}{8}\pi - \frac{7}{8}\pi \right) =$$

$$\frac{r}{r^2+1} \left(\frac{1}{4}\pi \right) =$$

$$\frac{1}{4}\pi \int_1^4 \frac{r}{r^2+1} dr =$$

$$\int \frac{r}{r^2+1} dr$$

$$u = r^2 + 1, du = 2r dr, dr = \frac{du}{2r}$$

$$\int \frac{r}{r^2+1} dr = \int \frac{r}{u} \frac{du}{2r} =$$

$$\int \frac{1}{2u} du = \frac{1}{2\ln(u)} + C$$

$$\Rightarrow \int_1^4 \frac{r}{r^2+1} dr =$$

$$\frac{1}{2} \ln(r^2 + 1) \Big|_1^4 =$$

$$\frac{1}{2} (\ln(17) - \ln(2))$$

$$= \frac{\pi}{8} (\ln(17) - \ln(2))$$