

1. $\lim_{(x,y) \rightarrow (1,0)} f(x,y)$

$$f(x,y) = \frac{(x-1)^2 y}{(x-1)^3 + y^3}$$

Pruebo por rectas

- recta $x = 1$

$$\lim_{y \rightarrow 0} \frac{0}{y^3} = 0$$

- recta $y = (x-1)$

$$\lim_{x \rightarrow 1} \frac{\cancel{(x-1)}^3}{2 \cancel{(x-1)}^3} = \frac{1}{2}$$

Al dar distinto los limites se demuestra que $\nexists \lim_{(x,y) \rightarrow (1,0)} f(x,y)$

2. $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$

$$f(x,y) = \frac{x \sin(y^2)}{x^2 + y^2}$$

Pruebo por curvas

- iterado $x = 0$

$$\lim_{y \rightarrow 0} f(0,y) =$$

$$\lim_{y \rightarrow 0} \frac{0}{y^2} = 0$$

Intento demostrar por sandwich

$$\exists g(x,y) : \lim_{(x,y) \rightarrow (0,0)} g(x,y) = 0 \wedge 0 \leq |f(x,y)| \leq |g(x,y)|$$

$$\left| \frac{x \sin(y^2)}{x^2 + y^2} \right| =$$

$$\frac{|x| |\sin(y^2)|}{x^2 + y^2} =$$

$$\frac{|x| |\sin(y^2)|}{x^2 + y^2} \leq \frac{|\sin(k)| \leq |k|}{\leq}$$

$$\frac{|x| y^2}{x^2 + y^2} =$$

$$|x| \cdot \frac{y^2}{x^2 + y^2}$$

$$x^2 \geq 0 \Rightarrow x^2 + y^2 \geq y^2 \Rightarrow 1 \geq \frac{y^2}{x^2 + y^2} : \star$$

$$|x| \cdot \frac{y^2}{x^2 + y^2} \stackrel{\star}{\leq}$$

$$|x| \cdot 1 \xrightarrow{(x,y) \rightarrow (0,0)} = 0$$

$$\Rightarrow 0 \leq |f(x,y)| \leq |x|$$

$$\Rightarrow \text{por sandwich } \lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$$