

1. $\int \int_D x^2 y dA$, D : mitad superior del disco con centro en el origen y radio 5

- $x = r \cos(\theta)$
- $y = r \sin(\theta)$
- $0 \leq r \leq 5$
- $0 \leq \theta \leq \pi$

\Rightarrow

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$T(r, \theta) = (r \cos(\theta), r \sin(\theta))$$

$$\int \int_D f(x, y) dA(x, y) = \int \int_D f(T(r, \theta)) |JT(u, v)| dA(u, v)$$

- $f(T(r, \theta)) = r^2 \cos^2(\theta) r \sin(\theta) = r^3 \cos^2(\theta) \sin(\theta)$

$$JT(u, v) = |\cos(\theta) - r \sin(\theta) \sin(\theta) r \cos(\theta)| = r \cos^2(\theta) + r \sin^2(\theta) = r$$

$$\int_0^\pi \left(\int_0^5 r^3 \cos^2(\theta) \sin(\theta) dr \right) d\theta =$$

- $\int_0^5 r^3 \cos^2(\theta) \sin(\theta) dr =$
 $\frac{r^4 \cos^2(\theta) \sin(\theta)}{4} \Big|_0^5 =$
 $\frac{625 \cos^2(\theta) \sin(\theta)}{4}$
- $\frac{625}{4} \int_0^\pi \cos^2(\theta) \sin(\theta) d\theta =$
 $\frac{625}{4} - \frac{1}{3} \cos^3(\theta) \Big|_0^\pi =$
 $\frac{625}{4} \cdot \frac{2}{3}$

2. $\int \int_D (2x - y) dA$, D : region del primer cuadrante encerrada por la circunferencia $x^2 + y^2 = 4$, $x = 0$ e $y = x$

$$D = \{(r, \theta) \in \mathbb{R}^2 : 0 \leq r \leq 2 \wedge \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}\}$$

$$T(r, \theta) = (r \cos(\theta), r \sin(\theta))$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\int_0^2 f(T(r, \theta)) \cdot JT(r, \theta) dr \right) d\theta =$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\int_0^2 (2r \cos(\theta) - r \sin(\theta)) \cdot r dr \right) d\theta =$$

- $\int_0^2 (2r \cos(\theta) - r \sin(\theta)) \cdot r dr =$
 $\int_0^2 (2r^2 \cos(\theta) - r^2 \sin(\theta)) dr =$
 $\int_0^2 r^2 (2 \cos(\theta) - \sin(\theta)) dr =$
 $(2 \cos(\theta) - \sin(\theta)) \int_0^2 r^2 dr =$
 $(2 \cos(\theta) - \sin(\theta)) \left(\frac{r^3}{3} \Big|_0^2 \right) =$
 $(2 \cos(\theta) - \sin(\theta)) \frac{8}{3} =$
 $(2 \cos(\theta) - \sin(\theta)) \frac{8}{3} =$
- $\frac{8}{3} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 \cos(\theta) - \sin(\theta)) d\theta =$
 $\frac{8}{3} - 2 \sin(\theta) - \cos(\theta) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$
 $\frac{8}{3} \cdot 2 \left(1 - \frac{1}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}}$

3. $\int \int_D \sin(x^2 + y^2) dA$, D : region del primer cuadrante encerrada por la circunferencia con centro en el origen y radios 1 y 3

$$\int_0^{\frac{\pi}{2}} \left(\int_1^3 \sin(r^2) r dr \right) d\theta =$$

- $\int_1^3 \sin(r^2) r dr =$
 $-\frac{1}{2} \cos(r^2) \Big|_1^3 =$
 $-\frac{1}{2} \cos(9) + \frac{1}{2} \cos(1) =$
 $\frac{1}{2} (\cos(1) - \cos(9))$

$$\begin{aligned} & \blacksquare \frac{1}{2}(\cos(1) - \cos(9)) \int_0^{\frac{\pi}{2}} 1 d\theta = \\ & \frac{1}{2}(\cos(1) - \cos(9))\left(\frac{\pi}{2}\right) \end{aligned}$$

4. $\int \int_D e^{-x^2-y^2} dA$, D : region acotada por las semicircunferencias $x = \sqrt{4-y^2}$ y el eje y

$$\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \left(\int_0^2 e^{-r^2} r dr \right) d\theta =$$

$$\begin{aligned} & \blacksquare \int_0^2 e^{-r^2} r dr = \\ & \left. -e^{-r^2} \right|_0^2 = \\ & -e^{-4} - 1 \end{aligned}$$

$$\begin{aligned} & \blacksquare \int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} -e^{-4} - 1 d\theta = \\ & (-e^{-4} - 1) \int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} 1 d\theta = \\ & (-e^{-4} - 1) \left(\frac{5\pi}{2} - \frac{3\pi}{2} \right) = \\ & (-e^{-4} - 1)\pi \end{aligned}$$