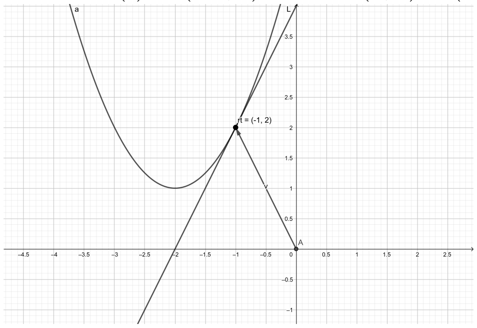
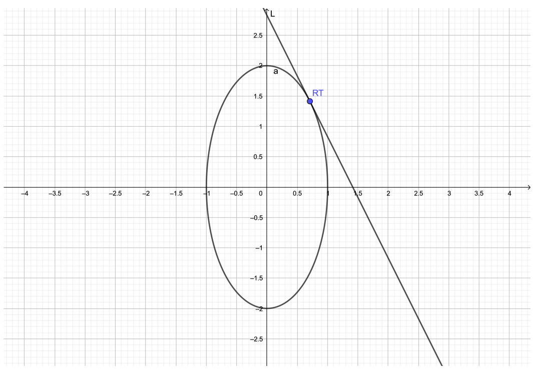
1. $a)$ $r(t) = (t-2, t^2+1) \wedge (-2 \leq t \leq 2) \wedge (t=1)$

    - $r'(t) = (1, 2t)$
    - $t = 1 \Rightarrow r(t) = (-1, 2) \wedge L = \lambda(1,2) + (-1,2)$

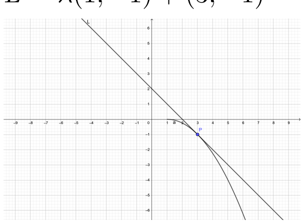
    $b)$ $r(t) = (\sin(t), 2\cos(t)) \wedge (0 \leq t \leq 2\pi) \wedge (t = \frac{\pi}{4})$

    - $r'(t) = (\cos(t), -2\sin(t))$
    - $t = \frac{\pi}{4} \Rightarrow r(t) = (\frac{\sqrt{2}}{2}, \sqrt{2}) \wedge L = \lambda(\frac{\sqrt{2}}{2}, -\sqrt{2}) + (\frac{\sqrt{2}}{2}, \sqrt{2})$

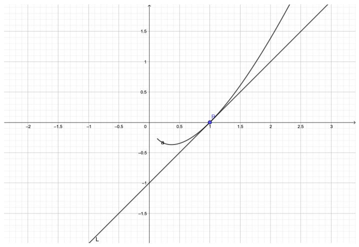
2. $a)$
    - $\begin{cases} x(t) = 1 + 2\sqrt{t} \\ y(t) = -t \\ 0 \leq t \leq 9 \end{cases}$
    - $P = (3, -1)$

    $r(t) = (1 + 2\sqrt{t}, -t) \Rightarrow$
    $r(t) = (3, -1) \Leftrightarrow$
    $1 + 2\sqrt{t} = 3 \wedge -t = -1 \Leftrightarrow$
    $t = 1$
    $r'(t) = (\frac{1}{t}, -1) \Rightarrow$
    $L = \lambda(1, -1) + (3, -1)$

    $b)$
    - $\begin{cases} x(t) = e^t \\ y(t) = te^t \\ -2 \leq t \leq 3 \end{cases}$
    - $P = (1, 0)$

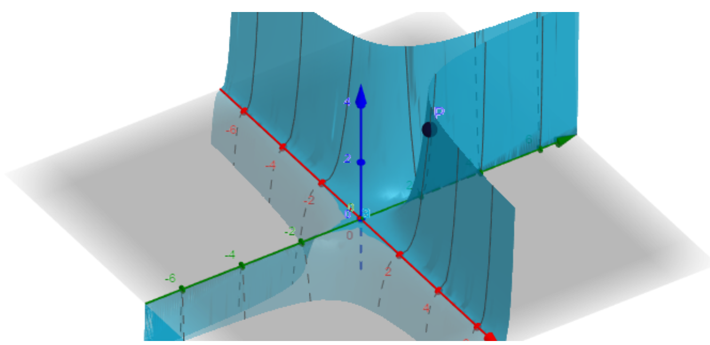
    $r(t) = (e^t, te^t) \Rightarrow$
    $r'(t) = (e^t, e^t + te^t)$
    $r(t) = (1, 0) \Leftrightarrow t = 0$
    $L = \lambda(1, 1) + (1, 0)$



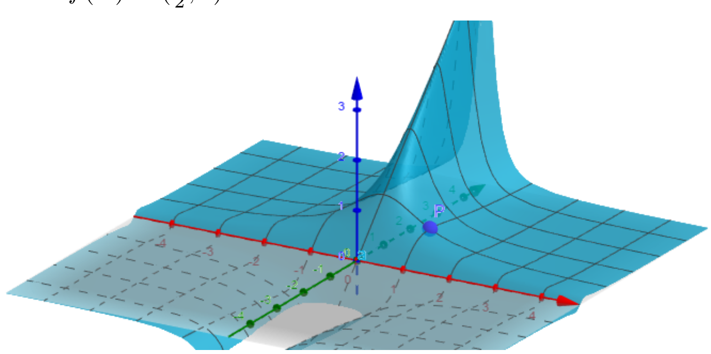
3. a) $r(t) = (te^{-t}, \tan(t), t^2 + t) \wedge t = 0$
   $r'(t) = (-e^{-t} + te^{-t}, \sec^2(t), 2t + 1)$
   $r(0) = (0, 0, 0)$
   $r'(0) = (-1, 1, 1)$
   $L = \lambda(-1, 1, 1)$

   b) $r(t) = (t^3 + 3t, t^2 + 1, 3t + 4) \wedge t = 0$
   $r'(t) = (3t^2 + 3, 2t, 3)$
   $r(0) = (0, 1, 4)$
   $r'(0) = (3, 0, 3)$
   $L = \lambda(3, 0, 3) + (0, 1, 4)$

4. a) $f(x, y) = x^2y^3 \wedge P = (2, 1)$
   - $f_x = 2y^3x$
   - $f_y = 3x^2y^2$
   - $\nabla f(P) = (4, 12)$



   b) $f(x, y) = \frac{y}{1+x^2y^2} \wedge P = (1, 1)$
   - $f_x = \frac{2xy^3}{(x^2y^2+1)^2}$
   - $f_y = \frac{1-x^2y^2}{(x^2y^2+1)^2}$
   - $\nabla f(P) = (\frac{1}{2}, 0)$



5. a) $f(x, y) = x^4 + 2xy + y^3x - 1$
   - $f_x = 4x^3 + 2y + y^3$

- $f_y = 2x + 3xy^2$

b) $f(x, y) = \sin(x)$
- $f_x = \cos(x)$
- $f_y = 0$

c) $f(x, y) = x^2 \sin^2(y)$
- $f_x = 2x \sin^2(y)$
- $f_y = 2x^2 \sin(y) \cos(y)$

d) $f(x, y) = xe^{x^2+y^2}$
- $f_x = e^{x^2+y^2} + x(2x)e^{x^2+y^2}$
- $f_y = x(2y)e^{x^2+y^2}$

e) $f(x, y, z) = ye^x + z$
- $f_x = ye^x$
- $f_y = e^x$
- $f_z = 1$

6. a) No existe ya que tiene picos

   b) Los limites no existen, ya que por izquirda dan -1 y por derecha 1

7. a) $f(x, y) = x^3y^5 + 2x^4y$
- $f_x = 3x^2y^5 + 8x^3y$
- $f_{xx} = 6xy^5 + 24x^2y$
- $f_{xy} = 15x^2y^4 + 8x^3$
- $f_y = x^35y^4 + 2x^4$
- $f_{yy} = 20x^3y^3$
- $f_{yx} = 15x^2y^4 + 8x^3$

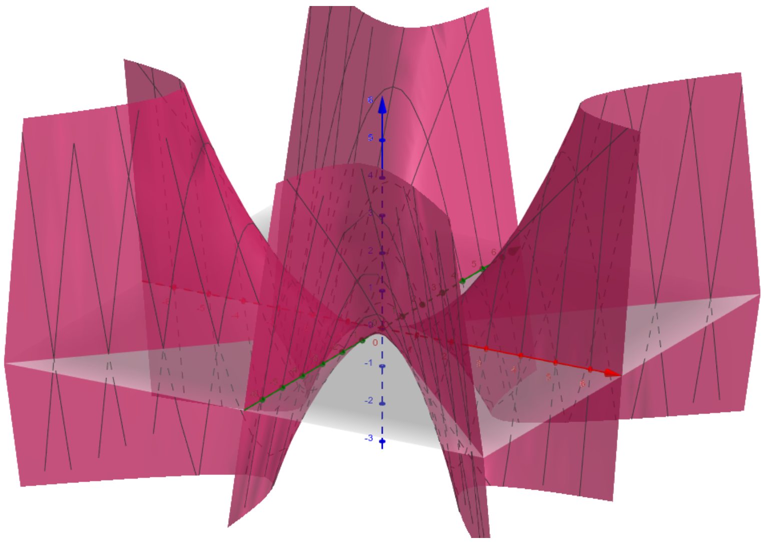
b) $f(x, y) = \sin^2(x + y)$
- $f_x = 2\sin(x + y)\cos(x + y)$
- $f_{xx} = -2\sin^2(x + y) + 2\cos^2(x + y)$
- $f_{xy} = -2\sin^2(x + y) + 2\cos^2(x + y)$
- $f_y = 2\sin(x + y)\cos(x + y)$
- $f_{yy} = -2\sin^2(x + y) + 2\cos^2(x + y)$
- $f_{yx} = -2\sin^2(x + y) + 2\cos^2(x + y)$

c) $f(x, y) = \sqrt{x^2 + y^2}$
- $f_x = \frac{x}{\sqrt{x^2+y^2}}$
- $f_{xx} = \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}$
- $f_{xy} = \frac{xy}{(x^2+y^2)^{\frac{3}{2}}}$
- $f_y = \frac{y}{\sqrt{x^2+y^2}}$
- $f_{yy} = \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}$
- $f_{yx} = \frac{xy}{(x^2+y^2)^{\frac{3}{2}}}$

d) $f(x, y) = \frac{xy}{x-y}$
- $f_x = \frac{y^2}{(x-y)^2}$
- $f_{xx} = \frac{-2y^2}{(x-y)^3}$
- $f_{xy} = \frac{-2yx}{(x-y)^3}$
- $f_y = \frac{x^2}{(x-y)^2}$

- $f_{yy} = \frac{2x^2}{(x-y)^3}$
- $f_{yx} = \frac{-2yx}{(x-y)^3}$

8. $f(x,y) = \begin{cases} \frac{x^3y-xy^3}{x^2+y^2} & si\ (x,y) \neq (0,0) \\ 0 \quad si\ (x,y) = (0,0) \end{cases}$

   a) 

   b)
   - $f_x(x,y) = \frac{y(x^4+4x^2y^2-y^4)}{(x^2+y^2)^2}$
   - $f_y(x,y) = \frac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}$

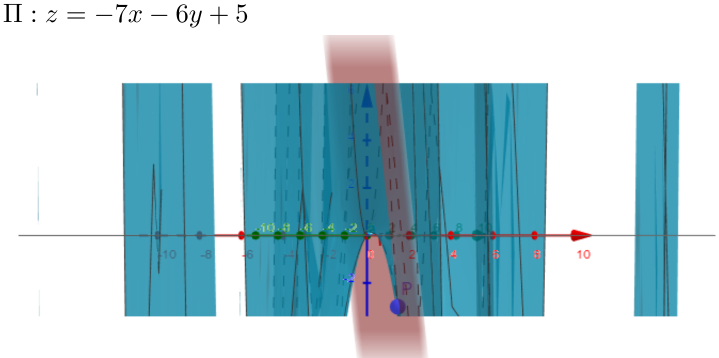
   c)
   - $f_x(0,0) = \text{lím}_{h\to 0} \frac{0}{h^2} = 0$
   - $f_y(0,0) = \text{lím}_{h\to 0} \frac{0}{h^2} = 0$

   d)
   - $f_xy(0,0) = \text{lím}_{h\to 0} \frac{-y^5)}{y^4} = -1$
   - $f_yx(0,0) = \text{lím}_{h\to 0} \frac{x^5}{x^4} = 1$

   e) No se contradice porque no son continuas

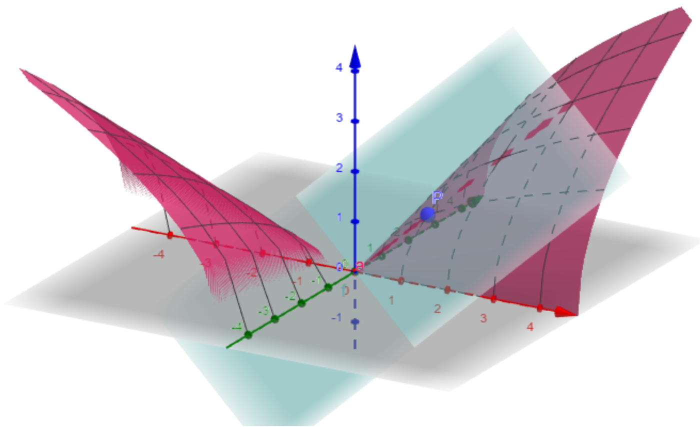
9. a)
   - $z = f(x,y) = 3y^2 - 2x^2 + x$
   - $P = (2,-1,-3)$
   - $f_x(x,y) = -4x + 1$
     Que al ser un polinomio es continua en todo R
   - $f_y(x,y) = 6y$
     Que al ser un polinomio es continua en todo R

   $\Rightarrow z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) =$
   $z = -7(x-2) - 6(y+1) - 3 =$
   $z = -7x - 6y + 14 - 6 - 3$
   $\Pi : z = -7x - 6y + 5$

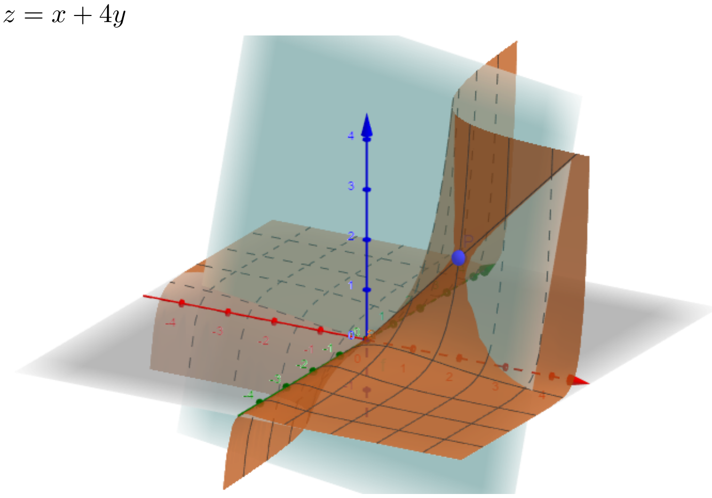
b)
- $z = \sqrt{xy}$
- $P = (1, 1, 1)$
- $f_x(x, y) = ((xy)^{\frac{1}{2}})' = \frac{1}{2} \cdot (xy)^{-\frac{1}{2}} = \frac{1}{2\sqrt{xy}}$
  Es continua en (1,1,1) ya que no se anula el denominador
- $f_x(x, y) = ((xy)^{\frac{1}{2}})' = \frac{1}{2} \cdot (xy)^{-\frac{1}{2}} = \frac{1}{2\sqrt{xy}}$
  Es continua en (1,1,1) ya que no se anula el denominador

$\Rightarrow z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) =$
$z = \frac{1}{2}(x - 1) + \frac{1}{2}(y - 1) + 1$
$z = \frac{x}{2} + \frac{y}{2}$



c)
- $z = xe^{xy}$
- $P = (2, 0, 2)$
- $f_x(x, y) = e^{xy} + xye^{xy}$
  Es continua en todo $\Re^2$
- $f_y(x, y) = x^2e^{xy}$
  Es continua en todo $\Re^2$

$\Rightarrow z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) =$
$z = x - 2 + 4y - 2$
$z = x + 4y$



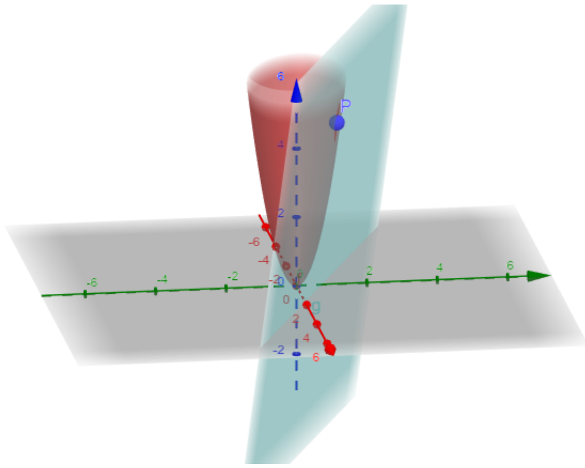
10.
- $z = f(x, y) = x^2 + xy + 3y^2$
- $P = (1, 1, 5)$
- $f_x(x, y) = 2x + y$ Continua en todo $\Re^2$
- $f_y(x, y) = x + 6y$ Continua en todo $\Re^2$
- $f_x(1, 1) = 3$

- $f_y(1,1) = 7$

$\Rightarrow z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) =$

$z = 3(x-1) + 7(y-1) + 5 = 3x + 7y - 3 - 7 + 5$

$z = 3x + 7y - 5$



11. Definición

$f : \Re^2 \to \Re$ es diferenciable en $(a,b) \in \Re^2 \Leftrightarrow$

a) $\exists (f_x(a,b) \wedge f_y(a,b))$

b) $\lim_{(x,y)\to(a,b)} \frac{f(x,y)-f(a,b)-f_x(a,b)(x-a)-f_y(a,b)(y-b)}{\|(x,y)-(a,b)\|} = 0$

Ejericios

a)
- $f(x,y) = 1 + x\ln(xy-5)$
- $P = (2,3)$
- $f_x(x,y) = \ln(xy-5) + \frac{xy}{xy-5}$
- $f_y(x,y) = \frac{x^2}{xy-5}$
- $f(2,3) = 1$
- $f_x(2,3) = 6$
- $f_y(2,3) = 4$
- $\lim_{(x,y)\to(2,3)} \frac{1+x\ln(xy-5)-1-6(x-2)-4(y-3)}{\|(x-2,y-3)\|} \stackrel{?}{=} 0$

$\lim_{(x,y)\to(2,3)} \frac{\cancel{1}+x\ln(xy-5)\cancel{-1}-6(x-2)-4(y-3)}{\|(x-2,y-3)\|} =$

$\lim_{(x,y)\to(2,3)} \frac{x\ln(xy-5)-6(x-2)-4(y-3)}{\|(x-2,y-3)\|} =$

$\lim_{(x,y)\to(2,3)} \frac{x\ln(xy-5)-2(3(x-2)+2(y-3))}{\|(x-2,y-3)\|} =$

Pruebo por curvas

- $y = x + 1$
  $\lim_{x\to 2} \frac{x\ln(x(x+1)-5)-6(x-2)-4((x+1)-3)}{\|(x-2,x-2)\|} =$
- No se

b)
- $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & si\ (x,y) \neq (0,0) \\ 0 & si\ (x,y) = (0,0) \end{cases}$
- $P = (0,0)$

No es diferenciable en el origen

12.
- $f(2,5) = 6$
- $f_x(2,5) = 1$
- $f_y(2,5) = -1$
- $f(2{,}2, 4{,}9) \stackrel{?}{=}$

$\Pi : z = f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5) =$

$\Pi : z = 6 + x - 2 - y + 5 =$

$\Pi : z = 6 + x - y + 3$

$\Pi : z = 6 + 2{,}2 - 4{,}9 + 3 = 6{,}3$

Rta: $f(2{,}2, 4{,}9) \approx 6{,}3$

13. *a)* $f(x,y,z) = \sqrt{|xyz|}$ No existen las derivadas parciales en el origen

b) $f(x,y) = \begin{cases} \frac{x^4-y^4}{x^2+y^2} & si\ (x,y) \neq (0,0) \\ 0 & si\ (x,y) = (0,0) \end{cases}$

$f_x(0,0) = \lim_{x\to} \frac{f(h,0)-f(0,0)}{h} =$

$\lim_{h\to 0} \frac{\frac{h^4}{h^2}}{h} =$

$\lim_{h\to 0} \frac{h^{\cancel{4}}}{\cancel{h^3}} = 0$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} =$

$\lim_{h\to 0} \frac{\frac{-h^4}{h^2}}{h} =$

$\lim_{h\to 0} \frac{-h^{\cancel{4}}}{\cancel{h^3}} = 0$

$\lim_{(x,y)\to(0,0)} \frac{\frac{x^4-y^4}{x^2+y^2}}{\|x^2+y^2\|} \stackrel{?}{=} 0$

$\lim_{(x,y)\to(0,0)} \frac{\frac{x^4-y^4}{\|x^2+y^2\|^2}}{\|x^2+y^2\|} =$

$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{\|x^2+y^2\|^3}$

$\lim_{(x,y)\to(0,0)} \frac{|x^4-y^4|}{\|x^2+y^2\|^3} \stackrel{DesTrian}{\leq}$

$\lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{\|x^2+y^2\|^3} \leq$

$\lim_{(x,y)\to(0,0)} \frac{2\|x^2+y^2\|^{\cancel{4}}}{\cancel{\|x^2+y^2\|^3}} \leq$

$2\lambda < \epsilon \Rightarrow \lambda \leq \frac{\epsilon}{2}$

$\Pi : z = 0$

c) $f(x,y) = \begin{cases} \frac{x^2}{x^2+y^2} \cdot \sin\left(\frac{1}{\sqrt{x^2+y^2}}\right) & si\ (x,y) \neq (0,0) \\ 0 & si\ (x,y) = (0,0) \end{cases}$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} =$

$\lim_{h\to 0} \frac{\frac{\cancel{h^2}}{\cancel{h^2}} \cdot \sin\left(\frac{1}{\sqrt{h^2}}\right)}{h} =$

$\lim_{h\to 0} \frac{\sin\left(\frac{1}{\sqrt{h^2}}\right)}{h} =$

$\lim_{h\to 0} \frac{\sin\left(\frac{1}{\sqrt{h^2}}\right)}{h} = \nexists$

$\Rightarrow$ No es diferenciable en el origen

14.

15.

16.

17.

18.

19.

20.

21.