1. a)

$r : \Re \to \Re^3$
$r(t) = (r_1(t), r_2(t), r_3(t)) = (x, y, z)$
$r_1 : \Re \to \Re$
$r_2 : \Re \to \Re$
$r_3 : \Re \to \Re$

$$\begin{cases} x^2 + y^2 - z = 0 \Rightarrow z = x^2 + y^2 \\ x^2 - 4x + y^2 + z = 0 \Rightarrow z = 4x - x^2 - y^2 \end{cases} \Rightarrow$$

$$\begin{cases} z = x^2 + y^2 : \star \\ x^2 - 4x + y^2 + z = 0 \overset{\star}{\Rightarrow} x^2 + y^2 = 4x - x^2 - y^2 : \star' \end{cases}$$

$\star' : x^2 + y^2 = 4x - x^2 - y^2 \Rightarrow$
$2x^2 - 4x + 2y^2 = 0 \Rightarrow$
$x^2 - 2x + y^2 = 0 \Rightarrow$
$(x-1)^2 - 1 + y^2 = 0 \Rightarrow$
$(x-1)^2 + y^2 = 1$ Que en $\Re^2$ es un circulo de radio 1 con centro en $(1, 0) \Rightarrow$

$$\begin{cases} x - 1 = \cos(t) \Rightarrow x = \cos(t) + 1 = r_1(t) \\ y = \sin(t) = r_2(t) \\ z = x^2 + y^2 \Rightarrow z = (\cos(t) + 1)^2 + \sin(t)^2 = r_3(t) \end{cases} \Rightarrow$$

$r(t) = (\cos(t) + 1, \sin(t), (\cos(t) + 1)^2 + \sin(t)^2) = C$

b) QvQ $P = (1 - \frac{\sqrt(2)}{2}, \frac{\sqrt{2}}{2}, 2 - \sqrt{2}) \in C \Leftrightarrow$
$\exists k \in \Re : r(k) = P \Leftrightarrow$

$$\begin{cases} \cos(k) + 1 = 1 - \frac{\sqrt(2)}{2} \\ \sin(k) = \frac{\sqrt{2}}{2} \\ (\cos(k) + 1)^2 + \sin(k)^2 = 2 - \sqrt{2} \end{cases} \Rightarrow$$

$\sin(k) = \frac{\sqrt{2}}{2} \Leftrightarrow k_1 = \frac{\pi}{4} \vee k_2 = \frac{3\pi}{4}$
$\cos(k_1) + 1 = \frac{\sqrt{2}}{2} + 1 \Rightarrow \cancel{k_1}$
$\cos(k_2) + 1 = -\frac{\sqrt{2}}{2} + 1 \checkmark$
$(\cos(k_2) + 1)^2 + \sin(k_2)^2 = (-\frac{\sqrt{2}}{2} + 1)^2 + 2 = \frac{1}{2} - \sqrt{2} + 1 + \frac{1}{2} = 2 - \sqrt{2} \checkmark$
$\Rightarrow r(k) = P \Rightarrow P \in C$
$(x, y, z) = \lambda \cdot r'(k) + P$

$$\begin{cases} r_1'(t) = -\sin(t) \\ r_2'(t) = \cos(t) \\ r_3'(t) = -2\sin(k) \end{cases} \Rightarrow$$

$$\begin{cases} r_1'(k) = -\frac{\sqrt{2}}{2} \\ r_2'(k) = -\frac{\sqrt{2}}{2} \\ r_3'(k) = -\sqrt{2} \end{cases} \Rightarrow$$

$(x, y, z) = \lambda \cdot (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\sqrt{2}) + (1 - \frac{\sqrt(2)}{2}, \frac{\sqrt{2}}{2}, 2 - \sqrt{2})$

2. a) $\lim_{(x,y)\to(0,0)} \frac{(x-1)^2 \sin(x^2) y}{x^2 + y^4}$

$f(x, y) = \frac{(x-1)^2 \sin(x^2) y}{x^2 + y^4}$
QvQ $\exists g(x, y) : lim_{(x,y)\to(0,0)} g(x, y) = 0 \wedge 0 \leq |f(x, y)| \leq g(x, y)$
$|\frac{(x-1)^2 \sin(x^2) y}{x^2 + y^4}| =$

$\frac{(x-1)^2 |\sin(x^2)||y|}{x^2 + y^4} \overset{|\sin(k)| \overset{k\to 0}{\leq} |k|}{\leq}$
$\frac{(x-1)^2 |x^2||y|}{x^2 + y^4} =$
$\frac{(x-1)^2 x^2 |y|}{x^2 + y^4} =$
$\frac{x^2}{x^2 + y^4} (x-1)^2 |y|$

$y^4 \leq 0 \Rightarrow x^2 + y^4 \leq x^2 \Rightarrow 1 \leq \frac{x^2}{x^2+y^4} : \star$

$\frac{x^2}{x^2+y^4}(x-1)^2|y| \overset{\star}{\leq}$

$(x-1)^2|y| \overset{(x,y)\to(0,0)}{\to 0} \Rightarrow$

$g(x,y) = (x-1)^2|y|$

$0 \leq |f(x,y)| \overset{(x,y)\to 0 \Rightarrow \to 0}{\leq (x-1)^2|y|}$

$\Rightarrow$ por sandwich $|f(x,y)| \to 0$ cuando $(x,y) \to 0$

$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{(x-1)^2 \sin(x^2) y}{x^2+y^4} = 0$

b) $\lim_{(x,y)\to(0,0)} \frac{e^{xy}-1}{x^2+y^2} = L_2$

- $y = 0 \Rightarrow \lim_{x\to 0} \frac{e^{x0}-1}{x^2} =$
  $\lim_{x\to 0} \frac{0}{x^2} = 0$
- $y = x \Rightarrow \lim_{x\to 0} \frac{e^{x^2}-1}{2x^2} \overset{"\frac{0}{0}\ por\ LH"}{=}$
  $\lim x \to 0 \frac{2\cancel{x}e^{x^2}}{4\cancel{x}} \overset{e^{x^2}\to 1}{=} \frac{1}{2}$

$\frac{1}{2} \neq 0 \Rightarrow \nexists L_2$

3. $f(x,y) = \begin{cases} \frac{x^2y^2 - \sin(x^4)}{x^2+\frac{1}{3}y^2} + 2 & si(x,y) \neq (0,0) \\ a & si(x,y) = (0,0) \end{cases} \Rightarrow$

$h(x,y) = \frac{x^2y^2 - \sin(x^4)}{x^2+\frac{1}{3}y^2}$

QvQ $\lim_{(x,y)\to(0,0)} h(x,y) = 0$

$\exists g(x,y) : \lim_{(x,y)\to(0,0)} g(x,y) = 0 \wedge 0 \leq |h(x,y)| \leq |g(x,y)|$

$|f(x,y)| = |\frac{x^2y^2 - \sin(x^4)}{x^2+\frac{1}{3}y^2}| =$

$\frac{|x^2y^2 - \sin(x^4)|}{x^2+\frac{1}{3}y^2} \leq$

$\frac{x^2y^2 + |\sin(x^4)|}{x^2+\frac{1}{3}y^2} \overset{|\sin(k)|\leq|k|}{\leq}$

$\frac{x^2y^2 + x^4}{x^2+\frac{1}{3}y^2} =$

$\frac{x^2}{x^2+\frac{1}{3}y^2} y^2 + x^4 \overset{\frac{x^2}{x^2+\frac{1}{3}y^2}\leq 1}{\leq}$

$y^2 + x^4 \overset{(x,y)\to 0}{\to} 0$

$\Rightarrow \lim_{(x,y)\to(0,0)} h(x,y) = 0$

$\Rightarrow f(x,y)$ es continua en todo $\Re^2 \Leftrightarrow a = 2$ ya que es un conciente de polinomios y trigonometricas continuas donde el denominador se anula en el $(x,y) = (0,0)$

$f(x,y)$ es diferencibale en el $(0,0) \Leftrightarrow$

$\exists L : \ lim_{(x,y)\to(0,0)} \frac{f(x,y) - f(0,0) - \nabla f(0,0)\cdot(x,y)}{\|(x,y)\|} = L \wedge L = 0$

$\nabla f(x,y) = (f_x(x,y), f_y(x,y))$

- $f_x(0,0)$ $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$

  $f_x(0,0) = \lim_{h\to 0} \frac{\frac{-\sin(h^4)}{h^2}+2-2}{h} =$

  $f_x(0,0) = \lim_{h\to 0} \frac{-\sin(h^4)}{h^2} \cdot \frac{1}{h} =$

  $f_x(0,0) = \lim_{h\to 0} \frac{-\sin(h^4)}{h^3} \overset{"\cdot\frac{h}{h}"}{=}$

  $f_x(0,0) = \lim_{h\to 0} \frac{-h\sin(h^4)}{h^4} \overset{"\cdot\frac{h}{h}"}{=}$

$f_x(0,0) = \text{lím}_{h\to 0} \frac{-h\sin\left(h^4\right)}{h^4} \stackrel{"lim_{k\to 0}\frac{\sin(k)}{k}=1"}{=}$
$f_x(0,0) = \text{lím}_{h\to 0} 1\cdot(-h) = 0$

- $f_y(0,0)$
  $f_y(0,0) = \text{lím}_{h\to 0} \frac{f(0,h)-f(0,0)}{h} =$
  $\text{lím}_{h\to 0} \frac{\frac{0}{\frac{1}{3}h^2}+2-2}{h} =$
  $\text{lím}_{h\to 0} \frac{0}{h} = 0$

$\Rightarrow \nabla f(0,0) = (0,0)$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\nabla f(0,0)\cdot(x,y)}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-(0,0)\cdot(x,y)}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}+2-2}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}\cdot\frac{1}{\|(x,y)\|} =$

$\Rightarrow lim_{(x,y)\to(0,0)} \frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}\cdot\frac{1}{\|(x,y)\|} =$

$|\frac{x^2y^2-\sin\left(x^4\right)}{x^2+\frac{1}{3}y^2}\cdot\frac{1}{\|(x,y)\|}| =$

$\frac{|x^2y^2-\sin\left(x^4\right)|}{x^2+\frac{1}{3}y^2}\cdot\frac{1}{\|(x,y)\|} \stackrel{"des.\ triang"}{\leq}$

$\frac{x^2y^2+|\sin\left(x^4\right)|}{x^2+\frac{1}{3}y^2}\cdot\frac{1}{\|(x,y)\|} \stackrel{|\sin(k)|\leq|k|}{\leq}$

$\frac{x^2(y^2+x^2)}{x^2+\frac{1}{3}y^2}\cdot\frac{1}{\|(x,y)\|} =$

$\frac{x^2}{x^2+\frac{1}{3}y^2}\cdot(y^2+x^2)\cdot\frac{1}{\|(x,y)\|} \stackrel{\frac{x^2}{x^2+\frac{1}{3}y^2}\leq 1}{\leq}$

$\frac{y^2+x^2}{\|(x,y)\|} \stackrel{\frac{x^2}{x^2+\frac{1}{3}y^2}\leq 1}{\leq}$

$\frac{y^2+x^2}{\|(x,y)\|} =$

$\frac{\|(x,y)\|^{\cancel{2}}}{\cancel{\|(x,y)\|}} =$

$\|(x,y)\| \wedge \|(x,y)\| \stackrel{(x,y)\to(0,0)}{\to} 0$

$\Rightarrow$ entonces f es diferencibale en (0,0)

4. $f:\Re^2\to\Re$
   $P = (1,2,f(1,2))$
   $\Phi: z = x+2y-1$
   $f(1,2) = z = 1+4-1 = 4$
   $P = (1,2,4)$
   $F(s,t) = f(3s+t^2, 2s^2+2t)$
   $F(0,1) = f(1,2)$
   $P_2 = (0,1,4)$

   $\nabla F(s,t) = (F_s(s,t), F_t(s,t))$

- $F_s(s,t) = f_x(x(s,t),y(s,t)) \cdot x_s(s,t) + f_y(x(s,t),y(s,t)) \cdot y_s(s,t) =$
  $f_x(x(s,t),y(s,t)) \cdot 3 + f_y(x(s,t),y(s,t)) \cdot 4s$
  $\Rightarrow F_s(0,1) = f_x(1,2) \cdot 3 + f_y(1,2) \cdot 0$
  $\Rightarrow F_s(0,1) = 1 \cdot 3 + 2 \cdot 0 = 3$
- $F_t(s,t) = f_x(x(s,t),y(s,t)) \cdot x_t(s,t) + f_y(x(s,t),y(s,t)) \cdot y_t(s,t) =$
  $f_x(x(s,t),y(s,t)) \cdot 2t + f_y(x(s,t),y(s,t)) \cdot 2$
  $\Rightarrow F_t(0,1) = f_x(1,2) \cdot 2 + f_y(1,2) \cdot 2$
  $\Rightarrow F_t(0,1) = 1 \cdot 2 + 2 \cdot 2 = 6$

$z = \boldsymbol{\nabla} F(0,1) \cdot (s, t-1) + F(0,1) =$

$\boldsymbol{\nabla}(3,6) \cdot (s, t-1) + 4 =$

$3s + 6(t-1) + 4$

$\Rightarrow z = 3s + 6t - 2$