

$$1. \int_0^3 \int_0^x \int_{x-y}^{x+y} y dz dy dx$$

$$\begin{aligned} & \int_{x-y}^{x+y} y dz = \\ & yz \Big|_{x-y}^{x+y} = \\ & (x+y)y - (x-y)y = xy + y^2 - xy + y^2 = \\ & 2y^2 \\ & \int_0^x 2y^2 dy = \\ & \frac{2y^3}{3} \Big|_0^x = \\ & \frac{2x^3}{3} \\ & \int_0^3 \frac{2x^3}{3} dx = \\ & \frac{2x^4}{12} \Big|_0^3 = \\ & \frac{2 \cdot 81}{12} \end{aligned}$$

$$2. \int \int_E e^{\frac{z}{y}} dv, E = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq y \leq 1, y \leq x \leq 1, 0 \leq z \leq xy\} = \\ \int_0^1 \left( \int_y^1 \left( \int_0^{xy} e^{\frac{z}{y}} dz \right) dx \right) dy$$

$$\begin{aligned} & \int_0^{xy} e^{\frac{z}{y}} dz = \\ & ye^{\frac{z}{y}} \Big|_0^{xy} = \\ & ye^x - y \\ & \int_y^1 ye^x - y dx = \\ & ye^x - yx \Big|_y^1 = \\ & ye^y - y^2 + ye + y \\ & \int_0^1 ye^y - y^2 + ye + y dy = \\ & -\frac{y^2}{2} + \frac{y^3}{3} + \frac{ey^2}{2} - e^y y + e^y \Big|_0^1 = \\ & -\frac{7}{6} + \frac{e}{2} \end{aligned}$$

$$3. \int \int_E xy dV, E, y = x^2, x = y^2, z = 0, z = x + y \\ D = \{(x, y, z) \in \mathbb{R}^3, 0 \leq x \leq 1 \wedge x^2 \leq y \leq \sqrt{x} \wedge 0 \leq z \leq x + y\} \\ \int_0^1 \left( \int_{x^2}^{\sqrt{x}} \left( \int_0^{x+y} xy dz \right) dy \right) dx$$

$$\begin{aligned} & \int_0^{x+y} xy dz = \\ & xyz \Big|_0^{x+y} = \\ & xy(x+y) = yx^2 + xy^2 \\ & \int_{x^2}^{\sqrt{x}} yx^2 + xy^2 dy = \\ & \frac{y^2 x^2}{2} + \frac{xy^3}{3} \Big|_{x^2}^{\sqrt{x}} = \\ & \frac{x^3}{2} + \frac{x\sqrt{x}^3}{3} - \frac{x^6}{2} - \frac{x^7}{3} = \\ & \frac{x^3}{2} + \frac{x^{\frac{5}{2}}}{3} - \frac{x^6}{2} - \frac{x^7}{3} = \\ & \int_0^1 \frac{x^3}{2} + \frac{x^{\frac{5}{2}}}{3} - \frac{x^6}{2} - \frac{x^7}{3} dx = \\ & \frac{x^4}{8} + \frac{x^{\frac{7}{2}}}{\frac{21}{2}} - \frac{x^7}{14} - \frac{x^8}{24} \Big|_0^1 = \\ & \frac{1}{8} + \frac{1}{42} - \frac{1}{14} - \frac{1}{24} = \frac{1}{28} \end{aligned}$$

$$4. \int \int_E x dV, E : x = 4y^2 + 4z^2 \wedge x = 4 \\ D = \{(x, y, z) \in \mathbb{R}^3, -1 \leq y \leq 1, 4y^2 \leq x \leq 4 \wedge -4y^2 \leq z \leq 4y^2\} \\ \int_{-1}^1 \left( \int_{4y^2}^4 \left( \int_{-4y^2}^{4y^2} x dz \right) dx \right) dy$$

$$\begin{aligned} \blacksquare \int_{-4y^2}^{4y^2} x dz &= \\ xz \Big|_{-4y^2}^{4y^2} &= \\ 4xy^2 + 4xy^2 &= \\ 8xy^2 & \\ \blacksquare \int_{4y^2}^4 8xy^2 dx &= \\ 4x^2y^2 \Big|_{4y^2}^4 &= \\ 64y^2 - 16y^6 & \\ \blacksquare \int_{-1}^1 64y^2 - 16y^6 dy &= \\ 64 - 16 - 64 + 16 & \end{aligned}$$