$$\begin{split} P_2(x,y) &= 1 - x^2 - y^2 + R_2(x,y) \\ \lim_{(x,y)\to(0,0)} \frac{f(x,y) + x^2 + y^2 - 1}{x^2 + y^2} &\equiv \\ \lim_{(x,y)\to(0,0)} \frac{P_2(x,y) + x^2 + y^2 - 1}{x^2 + y^2} &\equiv \\ \lim_{(x,y)\to(0,0)} \frac{P_2(x,y) + x^2 + y^2 - 1}{x^2 + y^2} &\equiv \\ \lim_{(x,y)\to(0,0)} \frac{R_2(x,y)}{x^2 + y^2} &\equiv \\ \lim_{(x,y)\to(0,0)} \frac{R_2(x,y)}{\|(x,y)\|^2} &= 0 \end{split}$$