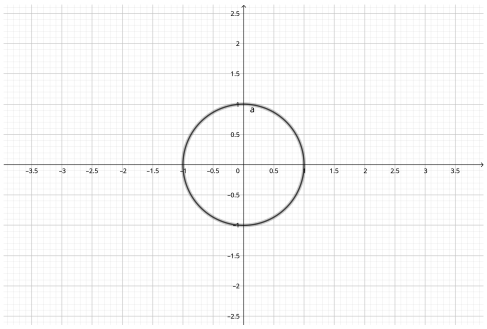
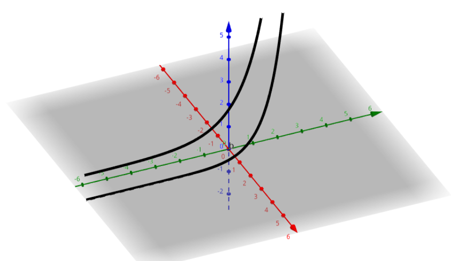
1. a) $\theta(t) = (\sin(t), \cos(t))$
   $\theta : \Re \to (x, y) \in \Re^2 : |x| \leq 1 \wedge |y| \leq 1$
   - $\sin(t)$ sabemos que es continua en todos los $\Re$
   - $\cos(t)$ sabemos que es continua en todos los $\Re$
   - $\sin(t) + \cos(t)$ por algebra de limites es contina en todos los $\Re$



   b) $\theta(t) = (\frac{\sin(t)}{t}, \ln\left(t^2 - t\right), t^2)$
   - $f(x) = \frac{\sin(t)}{t}$
     $f : \Re - 0 \to \Re$
     - $\sin(t)$ es continua
     - t es continua
     - $\frac{\sin(t)}{t}$ es continua en todos los puntos menos t = 0
   - $g(x) = \ln\left(t^2 - t\right)$
     - $t^2 - t$ es continua
     - $\ln\left(t^2 - t\right)$ es continua $\Leftrightarrow t^2 - t > 0$ $t^2 - t > 0 \Leftrightarrow t^2 > t \overset{por\ f\ t \neq 0}{\Leftrightarrow} |t| > \sqrt{t} \equiv 0 < t < 1$
     $g : \Re - (0, 1) \to \Re$
   - $h(x) = t^2$
     $h : \Re \to \Re$
     t^2 es continua en todo $\Re$

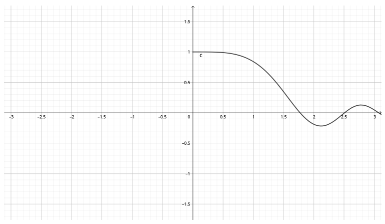
   $\theta(t) : \Re - (0, 1) \to \Re^3$



   c) $\theta(t) = (\theta_1(t), \theta_2(t))$
   - $\theta_1(t) = \sqrt{t}$
   - $\theta_2(t) = \begin{cases} \frac{\sin(t)}{t} & si\ t \neq 0 \\ 1\ si\ t = 0 \end{cases}$
   - $\theta_1(t) = \sqrt{t}$
     $\theta_1(t) : \Re_{\geq 0} \to \Re_{\geq 0}$
     $\sqrt{t}$ Es continua en todo su dominio
   - $\theta_2(t) = \begin{cases} \frac{\sin(t)}{t} & si\ t \neq 0 \\ 1\ si\ t = 0 \end{cases}$
     $\theta_2(t) : \Re \to [1, -1]$
     - $\frac{\sin(t)}{t}$ es continua en todos los puntos menos en $t = 0$
     - $\theta_2(t) es continua \Leftrightarrow$
       $\lim_{x \to 0} \theta_2(t) = \theta_2(0) \Leftrightarrow$
       $\lim_{x \to 0} \theta_2(t) = 1$
       $\lim_{x \to 0} \frac{\sin(t)}{t} \overset{L'H}{\equiv}$
       $\lim_{x \to 0} \frac{\cos(t)}{1} = 1$

$\Rightarrow \theta_2(t)$ es continua en todo su dominio $\blacksquare$

$\theta : \Re_{\geq 0} \rightarrow \Re^2$



2. *a*)
   1) $\lim_{(x,y)\to(1,0)} x + y = 1$
      $\|(x-1, y)\| < \delta \Rightarrow |x + y - 1| < \epsilon$
      - $|x-1| \leq \|(x-1,y)\| < \delta$
      - $|y| \leq \|(x-1,y)\| < \delta$

      $|x+y-1| \leq |x-1| + |y| < 2\cdot\delta \Rightarrow$
      $\delta = \frac{\epsilon}{2}$
   2) $\lim_{(x,y)\to(-1,8)} xy = -8$
      $\|(x+1, y-8)\| < \delta \Rightarrow |xy+8| < \epsilon$
      - $|x+1| \leq \|(x+1,y-8)\| < \delta$
      - $|y-8| \leq \|(x+1,y-8)\| < \delta$

      $|xy+8| = |((x+1)-1)((y-8)+8)+8| =$
      $|(x+1)(y-8)+(x+1)(8)+(-(y-8))-8+8| =$
      $|(x+1)(y-8)+(x+1)(8)+(-(y-8))| \overset{DesigualdadTriangular}{\leq}$
      $|(x+1)(y-8)| + 8|x+1| + |y-8| <$
      $\delta\cdot\delta + 8\delta + \delta = \delta^2 + 9\delta \overset{\delta\leq 1}{\equiv}$
      $10\delta \Rightarrow \delta = \frac{\epsilon}{10}$

   b)
   1) $\epsilon = 1 \Rightarrow \delta = \frac{1}{10}$
   2) $\epsilon = \frac{1}{100} \Rightarrow \delta = \frac{1}{1000}$

3. $\lim_{(x,y)\to(2,3)} y\sin(x\cdot y - 6) = 0$

   $\|(x-2, y-3)\| < \delta \Rightarrow |y\cdot\sin(x\cdot y-6)| < \epsilon$
   - $|x-2| \leq \|(x-2,y-3)\| < \delta$
   - $|y-3| \leq \|(x-2,y-3)\| < \delta$

   $|y\cdot\sin(x\cdot y-6)| \equiv$
   $|((y-3)+3)\cdot\sin(((x-2)+2)\cdot((y-3)+3)-6)| \equiv$
   $|((y-3)+3)\cdot\sin((x-2)(y-3)+3(x-2)+2(y-3)+6-6)| \equiv$
   $|((y-3)+3)\cdot\sin((x-2)(y-3)+3(x-2)+2(y-3))| \equiv$
   $|(y-3)+3|\cdot|\sin((x-2)(y-3)+3(x-2)+2(y-3))|$
   $(x,y)\to(2,3) \Rightarrow (x-2)(y-3)+3(x-2)+2(y-3)\to 0$
   $\star : x \to 0 \Rightarrow \operatorname{sen}(x) \leq x$
   $|(y-3)+3|\cdot|\sin((x-2)(y-3)+3(x-2)+2(y-3))| \overset{\star}{\leq}$
   $|(y-3)+3|\cdot|(x-2)(y-3)+3(x-2)+2(y-3)| \overset{DesTrian}{\leq}$
   $(|y-3|+3)\cdot(|x-2|\,|y-3| + 3|x-2| + 2|y-3|) =$
   $(|y-3|+3)\cdot(|x-2|\,|y-3| + 3|x-2| + 2|y-3|) <$
   $(\delta+3)\cdot(\delta^2+5\delta) \overset{\delta\leq 1}{\leq}$
   $(\delta+3)\cdot(6\delta) =$

$6\delta^2 + 18\delta \overset{\delta \leq 1}{\leq}$

$24\delta = \epsilon \Rightarrow \delta = \text{mín}(1, \frac{\epsilon}{24})$

4. *a)* $\text{lím}_{(x,y)\to(7,2)} x^2 + y^2 - xy$
   $7^2 + 2^4 - (7 \cdot 2) = 39$

   b) $\text{lím}_{(x,y)\to(0,1)} xe^{xy}$
   $0 \cdot 1 = 0$

   c) $\text{lím}_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2}$
   $\frac{4-2}{4+3(1)} = \frac{2}{7}$

   d) $\text{lím}_{(x,y)\to(0,0)} = \frac{x-y}{x+y}$
   $\text{lím}_{x\to 0} f(x,0) = 1$
   $\text{lím}_{y\to 0} f(0,y) = -1$
   $\Rightarrow Por\ curvas \nexists L$

   e) $\text{lím}_{(x,y)\to(0,0)} = \frac{x^4-4y^2}{x^2+2y^2}$
   $\text{lím}_{x\to 0} f(x,0) = 0$
   $\text{lím}_{y\to 0} f(0,y) = -2$
   $\Rightarrow Por\ curvas \nexists L$

   f) $\text{lím}_{(x,y)\to(0,0)} = \frac{xy}{x^2+y^2}$
   $\text{lím}_{x\to 0} f(x,0) = 0$
   $\text{lím}_{y\to 0} f(0,y) = 0$
   $\text{lím}_{y\to 0} f(x,x) = \frac{1}{2}$
   $\Rightarrow Por\ curvas \nexists L$

   g) $\text{lím}_{(x,y)\to(1,0)} = \frac{xy-y}{(x-1)^2+y^2}$
   $\text{lím}_{x\to 1} f(x,0) = 0$
   $\text{lím}_{y\to 0} f(0,y) = 0$
   $\text{lím}_{x\to 1} f(x,x) = \frac{x(x-1)}{(x-1)^2+x^2} = 0$
   $\text{lím}_{x\to 1} f(x,(x-1)) = \frac{(x-1)^2}{2(x-1)^2} = \frac{1}{2}$
   $\Rightarrow Por\ curvas \nexists L$

   h) $\text{lím}_{(x,y)\to(0,0)} = \frac{xy}{\sqrt{x^2+y^2}}$
   $\text{lím}_{x\to 0} f(x,0) = 0$
   $\text{lím}_{y\to 0} f(0,y) = 0$
   Sospecho que $L = 0$, lo pruebo por definición
   $\|(x,y)\| < \delta \Rightarrow |\frac{xy}{\sqrt{x^2+y^2}}| < \epsilon$

   - $|x| \leq \|(x,y)\| < \delta$
   - $|y| \leq \|(x,y)\| < \delta$

   $|\frac{xy}{\sqrt{x^2+y^2}}| =$
   $\frac{|y||y|}{\sqrt{x^2+y^2}} =$
   $\frac{|y||y|}{\sqrt{x^2+y^2}} =$
   $\frac{|y||y|}{\|(x,y)\|} \leq$
   $\frac{\|(x,y)\|\|(x,y)\|}{\|(x,y)\|} =$
   $\|(x,y)\| \leq \delta \Rightarrow \delta = \epsilon$ ■

   i) $\text{lím}_{(x,y)\to(0,0)} = \frac{x^4-y^4}{x^2+y^2}$
   $\frac{x^4-y^4}{x^2+y^2} =$
   $\frac{(x^2-y^2)(x^2+y^2)}{x^2+y^2} =$
   $x^2 + y^2$
   $(x,y) \to (0,0) \Rightarrow x^2 + y^2 \to 0$ ■

j) $\lim_{(x,y)\to(0,0)} = \frac{y^2\sin^2(x)}{x^4+2y^4}$
$\lim_{x\to 0} f(x,0) = 0$
$\lim_{y\to 0} f(0,y) = 0$
$\lim_{x\to 0} f(x,x) = 0$
Segun wolpram no existe, consultar

k) $\lim_{(x,y)\to(0,3)} \frac{x^2(y-3)^2e^x}{x^2+(y-3)^2}$

- $\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{x^2(-3)^2e^x}{x^2+(-3)^2} = 0$
- $\lim_{y\to 3} f(0,y) = \lim_{y\to 3} \frac{0}{(y-3)^2} = 0$
- $\lim_{y\to 3} f(y-3,y) = \lim_{y\to 3} \frac{\cancel{(y-3)^2}(y-3)^2e^{y-3}}{2\cancel{(y-3)^2}} =$
  $\lim_{y\to 3} \frac{\overset{\to 0}{(y-3)^2}\overset{\to 1}{e^{y-3}}}{2} = 0$

Sospecho que $L=0$ pruebo por definición
$\|(x,y-3)\| < \delta \Rightarrow |\frac{x^2(y-3)^2e^x}{x^2+(y-3)^2}| < \epsilon$

- $|x| \leq \|(x,y-3)\| < \delta$
- $|y-3| \leq \|(x,y-3)\| < \delta$

$|\frac{x^2(y-3)^2e^x}{x^2+(y-3)^2}| =$
$\frac{|x^2(y-3)^2e^x|}{x^2+(y-3)^2} =$
$\frac{x^2(y-3)^2|e^x|}{x^2+(y-3)^2} \overset{x\to 0 \Rightarrow e^x\to 1}{\equiv}$
$\frac{x^2(y-3)^2}{x^2+(y-3)^2} \leq$
$\frac{\cancel{\|(x,y-3)\|^2}\|(x,y-3)\|^2}{\cancel{\|(x,y-3)\|^2}} \overset{\delta<1}{\leq}$
$\delta = \epsilon \Rightarrow$
$\delta = \min(1,\epsilon)$ ■

5.
- $x = r\cdot\cos(\theta)$
- $y = r\cdot\sin(\theta)$

$\cos^2(\theta)\sin^2(\theta)$

a) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} \equiv$
$\lim_{(r,\theta)\to(0,0)} \frac{\cancel{r^3}(\sin^3(\theta)\cos^3(\theta))}{\cancel{r^2}}$
$\lim_{(r,\theta)\to(0,0)} \sin^3(\theta)\cos^3(\theta) = 0$

b) $\lim_{(x,y)\to(0,0)} (x^2+y^2)\cdot\ln\left(x^2+y^2\right) \equiv$
$\lim_{(r,\theta)\to(0,0)} r^2(\overset{1}{\cancel{(\cos^2(\theta)\sin^2(\theta))}})\cdot\ln\left(r^2(\overset{1}{\cancel{(\cos^2(\theta)\sin^2(\theta))}})\right) \equiv$
$\lim_{(r,\theta)\to(0,0)} r^2\cdot\ln\left(r^2\right) \equiv$
$\lim_{(r,\theta)\to(0,0)} r^2\cdot 2\cdot\ln(r) = 0$

c) $\lim_{(x,y)\to(0,0)} \frac{e^{-(x^2+y^2)}-1}{x^2+y^2} \equiv$
$\lim_{(r,\theta)\to(0,0)} \frac{e^{r^2}-1}{r^2} = 1$

d) $\lim_{(x,y)\to(0,0)} \frac{\sin\left(x^2+y^2\right)}{x^2+y^2} \equiv$
$\lim_{(r,\theta)\to(0,0)} \frac{\sin\left(r^2\right)}{r^2} = 1$

6. *a*) $f(x,y) = \frac{x^4y^4}{(x^2+y^4)^3}$

   $\lim_{x\to 0} f(x,mx) \equiv$

   $\lim_{x\to 0} \frac{x^8m^4}{(x^2+(mx)^4)^3} =$

   $\lim_{x\to 0} \frac{x^8m^4}{(x^4+2x^2(mx)^4+(mx)^8)((x^2+(mx)^4)}$

   $\lim_{x\to 0} \frac{x^8m^4}{x^6+3x^8m^4+3x^{10}m^8+m^{12}x^{12}} = 0$

   b) $f(x,y) = \frac{x^2}{x^2+y^2-x}$

   $\lim_{x\to 0} f(x,mx) \equiv$

   $\lim_{x\to 0} \frac{x^2}{x^2+mx^2-x} =$

   $\lim_{x\to 0} \frac{x^{\cancel{2}}}{\cancel{x}(x(1+m)-1)} = 0$

   $\lim_{x\to 0} f(x,x) =$

   $\lim_{x\to 0} \frac{x^2}{x^2\cancel{+x}\cancel{-x}} = 1$

   Si $m = 0$ da 1, sino 0 $\Rightarrow \nexists L$

7.

8.

9.

10.

11.

12.