$g : \mathbb{R}^2 \to \mathbb{R} \land g \in C^2$
$p(x, y) = x^2 + 3xy + y^2$ polinomio de taylor de orden 2 en $(0, 0)$
$f : \mathbb{R}^2 \to \mathbb{R}$
$f(x, y) = \sin^2(x - y) + 2g(x, y) \Rightarrow$

Analizo las derivadas de p

- $p_x(x, y) = 2x + 3y$
- $p_y(x, y) = 3x + 2y$
- $p_{xx}(x, y) = 2$
- $p_{xy}(x, y) = 3$
- $p_{yy}(x, y) = 2$

Como p es el polinomio de taylor de g en $(0, 0)$ en $(0, 0)$ sus derivadas coinciden

- $g(0, 0) = p(0, 0) = 0$
- $g_x(0, 0) = p_x(0, 0) = 0$
- $g_y(0, 0) = p_y(0, 0) = 0$
- $g_{xx}(0, 0) = p_{xx}(0, 0) = 2$
- $g_{yy}(0, 0) = p_{yy}(0, 0) = 3$
- Como $g, p \in C^2 : g_{xy}(0, 0) = g_{yx}(0, 0) = p_{xy}(0, 0) = p_{yx}(0, 0) = 2$

1. **Analizo f y sus derivadas**

   Propongo $r(x, y) = \sin^2(x - y) \in C^2$ al ser trigonometrica

   - $r_x(x, y) = 2\sin(x - y)\cos(x - y)$
   - $r_y(x, y) = -2\sin(x - y)\cos(x - y)$
   - $r_{xx}(x, y) = 2(\cos(x - y)\cos(x - y) + \sin(x - y)(-\sin(x - y))) =$
     $2(\cos^2(x - y) - \sin^2(x - y))$
   - $r_{xy}(x, y) = r_{yx}(x, y) = 2\cos^2(x, y)(-1) - 2\sin^2(x - y)(-1) =$
     $2\sin^2(x - y) - 2\cos^2(x - y) =$
     $2(\sin^2(x - y) - \cos^2(x - y))$
   - $r_{yy}(x, y) = -2\cos^2(x - y)(-1) + 2\sin^2(x - y)(-1) =$
     $2\cos^2(x - y) - 2\sin^2(x - y) =$
     $2(\cos^2(x - y) - \sin^2(x - y))$

   $f(x, y) = r(x, y) + 2g(x, y)$

   - $f(0, 0) = r(0, 0) + 2g(0, 0) = 0 + 2(0) = 0$
   - $f_x(0, 0) = r_x(0, 0) + 2g_x(0, 0) = 0 + 2(0) = 0$
   - $f_y(0, 0) = r_y(0, 0) + 2g_y(0, 0) = 0 + 2(0) = 0$
   - $f_{xx}(0, 0) = r_{xx}(0, 0) + 2g_y(0, 0) = 2 + 2(2) = 6$
   - $f_{xy}(0, 0) = f_{yx}(0, 0) = r_{xy}(0, 0) + 2g_y(0, 0) = -2 + 2(2) = 2$
   - $f_{yy}(0, 0) = r_{yy}(0, 0) + 2g_y(0, 0) = 2 + 2(3) = 8$

Desarrollo el polinomio de taylor de orden 2 en $(0,0)$

$t(x,y) = f(0,0) + f_x(0,0)x + f_y(0,0)y + \frac{f_{xx}(0,0)x^2}{2} + \frac{f_{yy}(0,0)y^2}{2} + f_{xy}(0,0)xy =$

$t(x,y) = 0 + 0x + 0y + \frac{\cancelto{3}{6}\, x^2}{\cancelto{1}{2}} + \frac{\cancelto{4}{8}\, y^2}{\cancelto{1}{2}} + 2xy =$

$t(x,y) = 3x^2 + 4x^2 + 2xy$

2. $\boldsymbol{\nabla} f(0,0) = (0,0) \Rightarrow$ es un punto critico

Analizo por el criterio del Hessiano

$det(H_f(0,0)) = f_{xx} \cdot f_{yy} - f_{xy} \cdot f_{yx} = 6 \cdot 8 - 2 \cdot 2 = 44$

$det(H_f(0,0)) > 0 \wedge f_{xx} > 0 \overset{por\ el\ criterio\ del\ Hessiano}{\Rightarrow} (0,0)$ es un mínimo local de f