

$$1. \int \int_R \frac{xy^2}{x^2+1} dA, R = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, -3 \leq y \leq 3\}$$

$$\begin{aligned} \blacksquare \int_{-3}^3 \frac{xy^2}{x^2+1} dy &= \\ \frac{xy^3}{3(x^2+1)} \Big|_{-3}^3 &= \\ 18 \cdot \frac{x}{x^2+1} & \\ \blacksquare 18 \cdot \int_0^1 \frac{x}{x^2+1} dx &= \\ u = x^2, du = 2x dx & \\ 9 \cdot \int_0^1 \frac{2x}{x^2+1} dx &= \\ 9 \cdot \int_0^1 \frac{1}{u+1} du &= \\ 9 \cdot \ln(u+1) du &= \\ 9 \cdot \ln(x^2+1) \Big|_0^1 &= \\ 9(\ln(2) - \ln(1)) & \end{aligned}$$

$$2. \int \int_R \frac{x}{1+xy} dA, R = [0, 1]x[0, 1]$$

$$\begin{aligned} \blacksquare \int_0^1 \frac{x}{1+xy} dy &= \\ u = xy, du = x dy & \\ \int \frac{x}{1+xy} dy &= \\ \int \frac{1}{1+u} du &= \\ \ln(1+xy) + C &\Rightarrow \\ \ln(1+xy) \Big|_0^1 &= \\ \ln(1+x) - \ln(1) & \\ \blacksquare \int_0^1 \ln(1+x) dx &= \\ (x+1) \ln(1+x) - x \Big|_0^1 &= \\ 2\ln(2) - 2 - \ln(1) + 1 & \end{aligned}$$