

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$P = (1, 2, f(1, 2))$$

$$\Phi : z = x + 2y - 1$$

$$f(1, 2) = z = 1 + 4 - 1 = 4$$

$$P = (1, 2, 4)$$

$$F(s, t) = f(3s + t^2, 2s^2 + 2t)$$

$$F(0, 1) = f(1, 2)$$

$$P_2 = (0, 1, 4)$$

$$\nabla F(s, t) = (F_s(s, t), F_t(s, t))$$

$$\begin{aligned} \blacksquare \quad F_s(s, t) &= f_x(x(s, t), y(s, t)) \cdot x_s(s, t) + f_y(x(s, t), y(s, t)) \cdot y_s(s, t) = \\ &= f_x(x(s, t), y(s, t)) \cdot 3 + f_y(x(s, t), y(s, t)) \cdot 4s \\ \Rightarrow F_s(0, 1) &= f_x(1, 2) \cdot 3 + f_y(1, 2) \cdot 0 \\ \Rightarrow F_s(0, 1) &= 1 \cdot 3 + 2 \cdot 0 = 3 \end{aligned}$$

$$\begin{aligned} \blacksquare \quad F_t(s, t) &= f_x(x(s, t), y(s, t)) \cdot x_t(s, t) + f_y(x(s, t), y(s, t)) \cdot y_t(s, t) = \\ &= f_x(x(s, t), y(s, t)) \cdot 2t + f_y(x(s, t), y(s, t)) \cdot 2 \\ \Rightarrow F_t(0, 1) &= f_x(1, 2) \cdot 2 + f_y(1, 2) \cdot 2 \\ \Rightarrow F_t(0, 1) &= 1 \cdot 2 + 2 \cdot 2 = 6 \end{aligned}$$

$$z = \nabla F(0, 1) \cdot (s, t - 1) + F(0, 1) =$$

$$\nabla(3, 6) \cdot (s, t - 1) + 4 =$$

$$3s + 6(t - 1) + 4$$

$$\Rightarrow z = 3s + 6t - 2$$