

R407 A SpecificHeat Characterization V1

March 24, 2020

```
[1]: #Import the necessary libraries
import math as math
import numpy as np
import pandas as pd
import pickle
import itertools
from CoolProp.CoolProp import PropsSI
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from sklearn import linear_model
from pandas import DataFrame
from mpl_toolkits.mplot3d import Axes3D
from pandas import ExcelWriter
from IPython.core.interactiveshell import InteractiveShell
import statsmodels.formula.api as sm
InteractiveShell.ast_node_interactivity = "all"
%matplotlib inline

[2]: # Iteration parameters
fluid = 'R407A.mix'
P_min_kPa = 100 #Minimum pressure in kPa we will be evaluating
P_max_kPa = 3000 #Maximum pressure in kPa we will be evaluating
T_min_K = -30+273 #Minimum temp
T_max_K = 60+273 #Max temp
n = 2900 #Number of samples we will collect
m = 180
P_kPa = np.linspace(P_min_kPa, P_max_kPa, n) #The array of the pressures we
↳will be evaluating
T_K = np.linspace(T_min_K, T_max_K, m) # This is the array of temperatures
↳divided into m spaces
Spec = [] # create a list
combo = list(itertools.product(P_kPa,T_K)) # assign to combo the combination of
↳pressures and arrays

[3]: # This step is only for extraction of data, if there is already a 'xxxx.p'
↳file, then it is not necessary to load data again
```

```

#Test values to make sure there was a coherent answer according to R407a
↳ tables, since those are mostly experimental tests
# the function C fro propsSi calculates the Mass specific constant pressure
↳ specific heat in [J/kg/K]
#print("Specific heat at 333K and 3000 kPa :", PropsSI('C', 'T', 273+60, 'P',
↳ 3000*1000, 'REFPROP::R407a.mix'), "J/kg/K") #Maxi
#print("Specific heat at 243K and 100 kPa :", PropsSI('C', 'T', 273-30, 'P',
↳ 100*1000, 'REFPROP::R407a.mix'), "J/kg/K") # Minimi

#for i in combo: # extract data from propsi, use only if library (coolprops) is
↳ loaded and functioning
# [p,t]= i
# try:
#     Spec.append(PropsSI('C','P', p*1000,'T',t, 'REFPROP::R407a.mix')) #
↳ use the function PropsSI to
# get the specific heat according to pressure defined and temperature
↳ defined, the pressure array has 2900 points
#according to 3000 kPa - 100 kPa Range - the temperature similarly has
↳ 90 points
# except:
#     Spec.append(0) # we don't know if there is values in all the spaces,
↳ or the extent of the ProposSI Function

# What happened above is to make sure or assess if there is data in the
↳ file that will adjust ot our workspace

```

```

[4]: #print(Spec) # This is Specific heat, I have used the variable Spec because it
↳ wass easy to start with and have a reference

```

```

[5]: # run this only when you want to extract data again from propsi, if there is
↳ already a SpecificHeat.p file; there is no need to run
#pickle.dump(Spec, open( "SpecificHeat.p", "wb" ) ) # save values from propsi
↳ in file so we don't have to run each time

```

```

[3]: X, Y = np.meshgrid(T_K,P_kPa)

```

```

[4]: Spec = pickle.load( open( "SpecificHeat.p", "rb" ) ) # load values from file

```

```

[4]: PropsSI('C','P',1162*1000,'T',264, 'REFPROP::R407a.mix')

```

```

[4]: 1358.0620567466892

```

```

[5]: X.shape
Y.shape

```

Y

```
[5]: (2900, 180)
```

```
[5]: (2900, 180)
```

```
[5]: array([[ 100.          ,  100.          ,  100.          , ...,
           100.          ,  100.          ,  100.          ],
          [ 101.00034495,  101.00034495,  101.00034495, ...,
           101.00034495,  101.00034495,  101.00034495],
          [ 102.00068989,  102.00068989,  102.00068989, ...,
           102.00068989,  102.00068989,  102.00068989],
          ...,
          [ 2997.99931011,  2997.99931011,  2997.99931011, ...,
           2997.99931011,  2997.99931011,  2997.99931011],
          [ 2998.99965505,  2998.99965505,  2998.99965505, ...,
           2998.99965505,  2998.99965505,  2998.99965505],
          [ 3000.          ,  3000.          ,  3000.          , ...,
           3000.          ,  3000.          ,  3000.          ]])
```

```
[6]: Z = np.split(np.array(Spec),2900)
      Z = np.array(Z)
      Z.shape
```

```
[6]: (2900, 180)
```

```
[7]: # plotting of raw data to visualize the behaviour and see Cp (propsSI) outputs
      from mpl_toolkits.mplot3d import Axes3D # import libraries for drawing in 3D
      import matplotlib.pyplot as plt
      from matplotlib import cm
      from matplotlib.ticker import LinearLocator, FormatStrFormatter
      # this is a graph of the data we are dealing with
      %matplotlib inline

      threedee = plt.figure().gca(projection='3d')
      threedee.scatter(X,Y, Z)
      threedee.set_xlabel('Pressure (kPa)')
      threedee.set_ylabel('Temp. (K)')
      threedee.set_zlabel('Cp(J/kg/K)')
      plt.show()
      #fig = plt.figure()
      #ax = fig.gca(projection='3d')
      #surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm, linewidth=0,
      →antialiased=False)# plot the plot will give two distinc surfaces with an
      →area in between
      #ax.set_xlabel('T (K)', fontsize = 14)
      #ax.set_ylabel('P(kPa)', fontsize = 14)
```

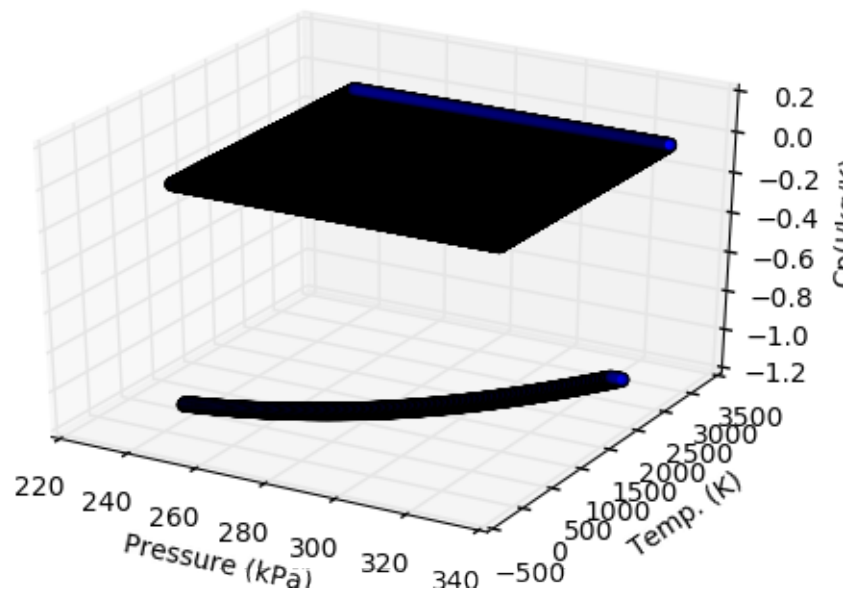
```
#ax.set_zlabel('C (kJ/kg/K)', fontsize = 14)
```

```
[7]: <mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7fa8af4bb518>
```

```
[7]: <matplotlib.text.Text at 0x7fa8af4f8860>
```

```
[7]: <matplotlib.text.Text at 0x7fa8af50c390>
```

```
[7]: <matplotlib.text.Text at 0x7fa8af512d68>
```



```
[8]: shf = pd.DataFrame.from_records( combo, columns= ['Pressure', 'Temperature'])  
shf = shf.assign(Cp = Spec)  
len (shf)
```

```
[8]: 522000
```

```
[9]: shf.head(3)
```

```
[9]:   Pressure  Temperature      Cp  
0    100.0    243.000000  770.671210  
1    100.0    243.502793  770.801452  
2    100.0    244.005587  770.950061
```

```
[10]: Mask = (shf>0)
```

```
[11]: shf[Mask]
```

```

[11]:      Pressure  Temperature      Cp
0      100.0    243.000000  770.671210
1      100.0    243.502793  770.801452
2      100.0    244.005587  770.950061
3      100.0    244.508380  771.116277
4      100.0    245.011173  771.299379
5      100.0    245.513966  771.498686
6      100.0    246.016760  771.713552
7      100.0    246.519553  771.943363
8      100.0    247.022346  772.187539
9      100.0    247.525140  772.445528
10     100.0    248.027933  772.716807
11     100.0    248.530726  773.000880
12     100.0    249.033520  773.297272
13     100.0    249.536313  773.605537
14     100.0    250.039106  773.925246
15     100.0    250.541899  774.255994
16     100.0    251.044693  774.597393
17     100.0    251.547486  774.949075
18     100.0    252.050279  775.310689
19     100.0    252.553073  775.681900
20     100.0    253.055866  776.062390
21     100.0    253.558659  776.451853
22     100.0    254.061453  776.849999
23     100.0    254.564246  777.256549
24     100.0    255.067039  777.671239
25     100.0    255.569832  778.093814
26     100.0    256.072626  778.524031
27     100.0    256.575419  778.961659
28     100.0    257.078212  779.406474
29     100.0    257.581006  779.858264
...
521970 3000.0    318.418994      NaN
521971 3000.0    318.921788      NaN
521972 3000.0    319.424581      NaN
521973 3000.0    319.927374      NaN
521974 3000.0    320.430168      NaN
521975 3000.0    320.932961      NaN
521976 3000.0    321.435754      NaN
521977 3000.0    321.938547      NaN
521978 3000.0    322.441341      NaN
521979 3000.0    322.944134      NaN
521980 3000.0    323.446927      NaN
521981 3000.0    323.949721      NaN
521982 3000.0    324.452514      NaN
521983 3000.0    324.955307      NaN
521984 3000.0    325.458101      NaN

```

521985	3000.0	325.960894	NaN
521986	3000.0	326.463687	NaN
521987	3000.0	326.966480	NaN
521988	3000.0	327.469274	NaN
521989	3000.0	327.972067	NaN
521990	3000.0	328.474860	NaN
521991	3000.0	328.977654	NaN
521992	3000.0	329.480447	NaN
521993	3000.0	329.983240	NaN
521994	3000.0	330.486034	NaN
521995	3000.0	330.988827	NaN
521996	3000.0	331.491620	NaN
521997	3000.0	331.994413	NaN
521998	3000.0	332.497207	NaN
521999	3000.0	333.000000	NaN

[522000 rows x 3 columns]

```
[12]: groupingtest = shf.loc[shf['Cp']>0]
      #len(shf.loc[shf['Cp']>0])
      groupingtest.head(2)
```

```
[12]:      Pressure  Temperature      Cp
0      100.0    243.000000  770.671210
1      100.0    243.502793  770.801452
```

```
[ ]: #writer = pd.ExcelWriter('pandas_simplecp.xlsx', engine='xlsxwriter')
      #shf.loc[shf['Cp']<0].to_excel(writer, sheet_name='Sheet1')
      #writer.save()
```

```
[13]: negatives = shf.loc[shf['Cp']<0]
      positives = shf.loc[shf['Cp']>0]
```

```
[14]: # Plot the extracted data from propsi
      print(shf.head());
      threedee = plt.figure().gca(projection='3d')
      threedee.scatter(positives['Temperature'],positives['Pressure'],
      ↪positives['Cp'])
      threedee.set_xlabel('Temp. (K)')
      threedee.set_ylabel('Pressure (kPa)')
      threedee.set_zlabel('Cp (J/kg/k)')
      plt.show()
```

	Pressure	Temperature	Cp
0	100.0	243.000000	770.671210
1	100.0	243.502793	770.801452
2	100.0	244.005587	770.950061

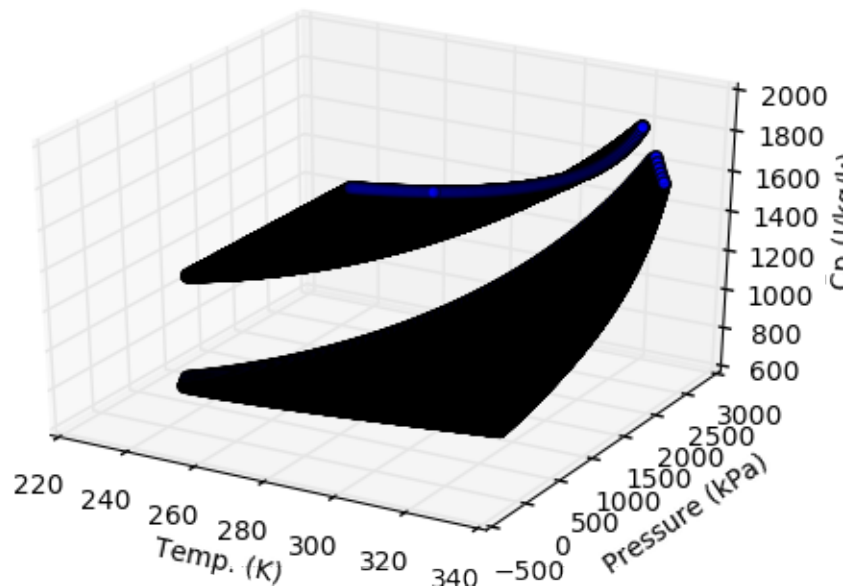
```
3      100.0    244.508380  771.116277
4      100.0    245.011173  771.299379
```

```
[14]: <mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7fa8a542aac8>
```

```
[14]: <matplotlib.text.Text at 0x7fa8a5bb22e8>
```

```
[14]: <matplotlib.text.Text at 0x7fa8a5bb7dd8>
```

```
[14]: <matplotlib.text.Text at 0x7fa8a5bc87f0>
```



```
[ ]: # Time to separate the characterization!! :)
```

Regression model for Mix (according to saturated characterization)

0.0.1 Characterization model for latent heat portion liquid portion

```
[15]: negatives.head(4) # negatives are the values selected to be outliers from the
    ↳ 'normal' Cp value
Latent_negatives = negatives[(negatives['Temperature'] > 148.074237046 + 25.
    ↳ 495200*negatives['Pressure']**0.24854063)&(negatives['Temperature'] < 151.12
    ↳ + 27.9058*negatives['Pressure']**0.237508)]
threedee = plt.figure().gca(projection='3d')
threedee.
    ↳ scatter(Latent_negatives['Temperature'], Latent_negatives['Pressure'], Latent_negatives['Cp'])
threedee.set_xlabel('Temperature (K)')
```

```

threedee.set_ylabel('Pressure (kPa)')
threedee.set_zlabel('Cp (J/kg/K)')
plt.show()

```

```

[15]:      Pressure  Temperature      Cp
9360  152.017937      243.0 -1.109744e+08
9540  153.018282      243.0 -1.109744e+08
9720  154.018627      243.0 -1.109744e+08
9900  155.018972      243.0 -1.109744e+08

```

```

[15]: <mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7fa8a41e6dd8>

```

```

[15]: <matplotlib.text.Text at 0x7fa8a5bdd2e8>

```

```

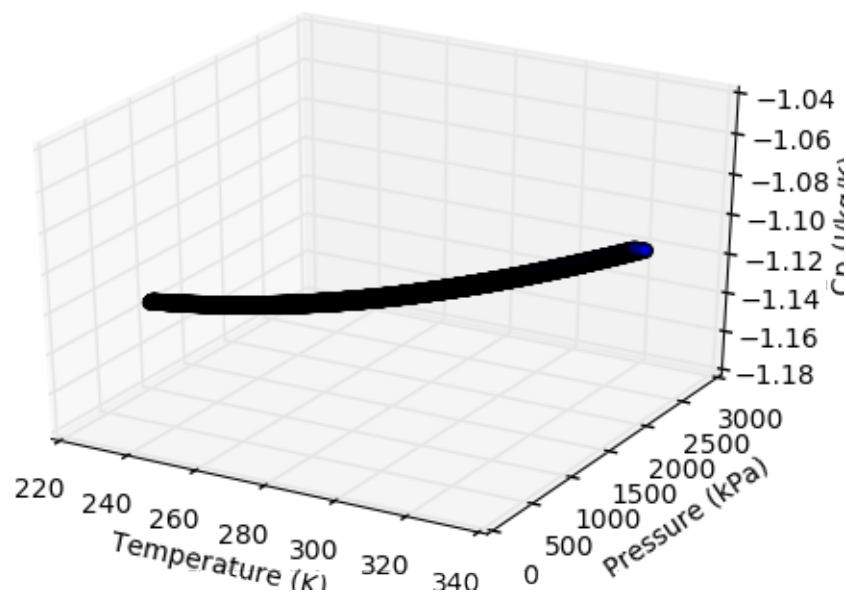
[15]: <matplotlib.text.Text at 0x7fa8a65ccba8>

```

```

[15]: <matplotlib.text.Text at 0x7fa8a41b6470>

```



```

[ ]: # Due to the results, it is clear that propsSI for the latent portion of the
      ↳ specific heat data, is throwing a value that does not match with the
      ↳ physical phenomena.
      # The value for the portion should be instead in [J/mol] or [J/kg] since it is
      ↳ a rise described as the mass or molar latent heat at constant temperature
      # Therefore, this portion of the characterization will be performed in a
      ↳ separate crunchy book

```


0.1 Characterization model Specific Heat for liquid portion

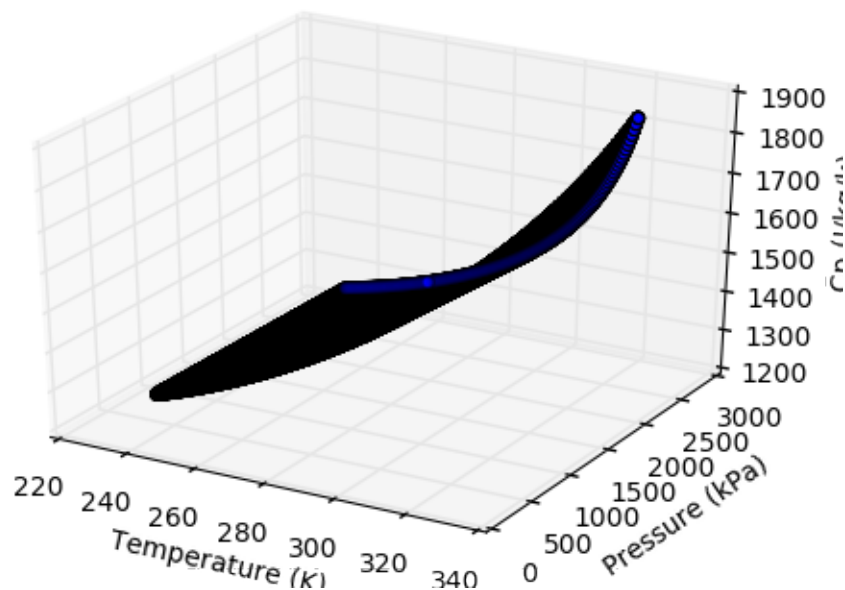
```
[16]: Liquid_positives = positives[(positives['Temperature'] < 148.074237046 + 25.  
    ↳495200534*positives['Pressure']**0.248540630499)]  
threedee = plt.figure().gca(projection='3d')  
threedee.  
    ↳scatter(Liquid_positives['Temperature'],Liquid_positives['Pressure'],Liquid_positives['Cp'])  
threedee.set_xlabel('Temperature (K)')  
threedee.set_ylabel('Pressure (kPa)')  
threedee.set_zlabel('Cp (J/kg/k)')  
plt.show()
```

```
[16]: <mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7fa8ab4450f0>
```

```
[16]: <matplotlib.text.Text at 0x7fa8a41c2780>
```

```
[16]: <matplotlib.text.Text at 0x7fa8ab3ab940>
```

```
[16]: <matplotlib.text.Text at 0x7fa8ab3ba358>
```



```
[17]: Liquid_positives = Liquid_positives.assign(Temp_sq =  
    ↳Liquid_positives['Temperature']**2) #1.56  
Liquid_positives = Liquid_positives.assign(Press_sq =  
    ↳Liquid_positives['Pressure']**2) #1.4  
Liquid_positives.head(3)
```

```
[17]:
```

	Pressure	Temperature	Cp	Temp_sq	Press_sq
17820	199.034150	243.0	1310.394142	59049.0	39614.592750
18000	200.034495	243.0	1310.390232	59049.0	40013.799051
18180	201.034840	243.0	1310.386321	59049.0	40415.006733

```
[39]: resultLiquid = sm.ols(formula="Cp ~ Temperature + Pressure + Temp_sq",
    ↪data=Liquid_positives).fit()
forecastLiquid = resultLiquid.
    ↪predict(Liquid_positives[['Temperature', 'Pressure', 'Temp_sq']])
ErrorLiquid = pd.DataFrame({'Error': forecastLiquid-Liquid_positives.Cp})

print(resultLiquid.params)
print(resultLiquid.summary())
ErrorLiquid.describe()
```

```
Intercept      5006.961681
Temperature     -30.457731
Pressure        -0.008074
Temp_sq         0.063061
dtype: float64
```

OLS Regression Results

```
=====
Dep. Variable:          Cp      R-squared:                0.991
Model:                  OLS      Adj. R-squared:            0.991
Method:                 Least Squares      F-statistic:          9.083e+06
Date:                  Wed, 13 Dec 2017      Prob (F-statistic):       0.00
Time:                  19:12:34      Log-Likelihood:          -9.0782e+05
No. Observations:      249026      AIC:                    1.816e+06
Df Residuals:          249022      BIC:                    1.816e+06
Df Model:               3
Covariance Type:       nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	5006.9617	3.377	1482.485	0.000	5000.342	5013.581
Temperature	-30.4577	0.024	-1249.461	0.000	-30.506	-30.410
Pressure	-0.0081	3.47e-05	-232.643	0.000	-0.008	-0.008
Temp_sq	0.0631	4.39e-05	1437.625	0.000	0.063	0.063

```
=====
Omnibus:                133221.287      Durbin-Watson:           0.155
Prob(Omnibus):           0.000      Jarque-Bera (JB):        2791171.881
Skew:                    2.127      Prob(JB):                0.00
Kurtosis:                18.840      Cond. No.                1.39e+07
=====
```

Warnings:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly
```

specified.

[2] The condition number is large, 1.39×10^7 . This might indicate that there are strong multicollinearity or other numerical problems.

```
[39]:          Error
count  2.490260e+05
mean   -4.790373e-10
std     9.267901e+00
min    -1.045214e+02
25%    -5.547611e+00
50%    -1.018039e+00
75%     5.538027e+00
max     2.151780e+01
```

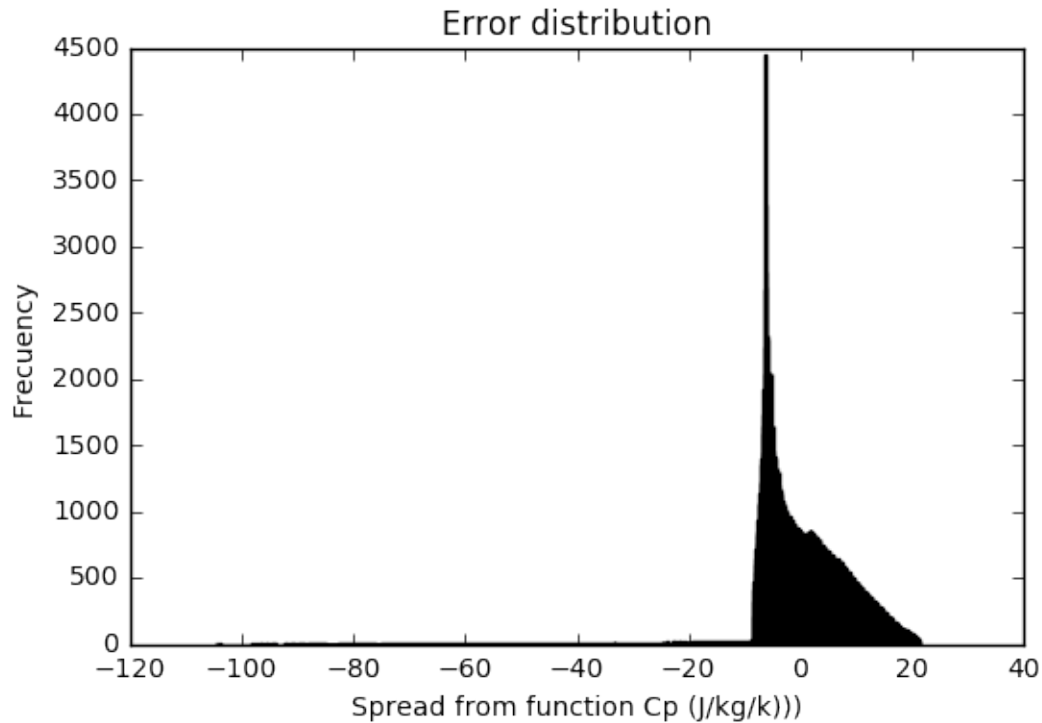
```
[40]: plt.hist(ErrorLiquid.Error, bins = 1500); # histogram of error distribution
plt.title('Error distribution')
plt.xlabel('Spread from function Cp (J/kg/K)')
plt.ylabel('Frecuency')
```

```
[40]: (array([ 1.,  1.,  0., ..., 29., 23., 14.]),
      array([-104.52140613, -104.43737999, -104.35335385, ..., 21.34974705,
              21.43377318, 21.51779932]),
      <a list of 1500 Patch objects>)
```

```
[40]: <matplotlib.text.Text at 0x7fa892b5efd0>
```

```
[40]: <matplotlib.text.Text at 0x7fa8930e70b8>
```

```
[40]: <matplotlib.text.Text at 0x7fa892e48eb8>
```



1 Result for liquid portion $Cp_{Liquid} = aT^2 + bT + cP + f$ [J/kg/K]

1.0.1 $R^2 = 0.991$

a is the parameter for Temp_sq : 0.063061

b is the parameter for Temperature : -30.457731

c is the parameter for Pressure : -0.008074

f is the Intercept: 5006.961681

2 Result for liquid portion $Cp_{Liquid} = bT + cP + f$

2.0.1 $R^2 = 0.916$

b is the parameter for Temperature : 4.558210

c is the parameter for Pressure : -0.002865

f is the Intercept: 165.200746

3 Result for liquid portion $C_{p\text{Liquid}} = bT + f$

3.0.1 $R^2 = 0.916$

b is the parameter for Temperature : 4.525411

f is the Intercept: 169.546153

3.1 Characterization model for Specific Heat for Vapor portion

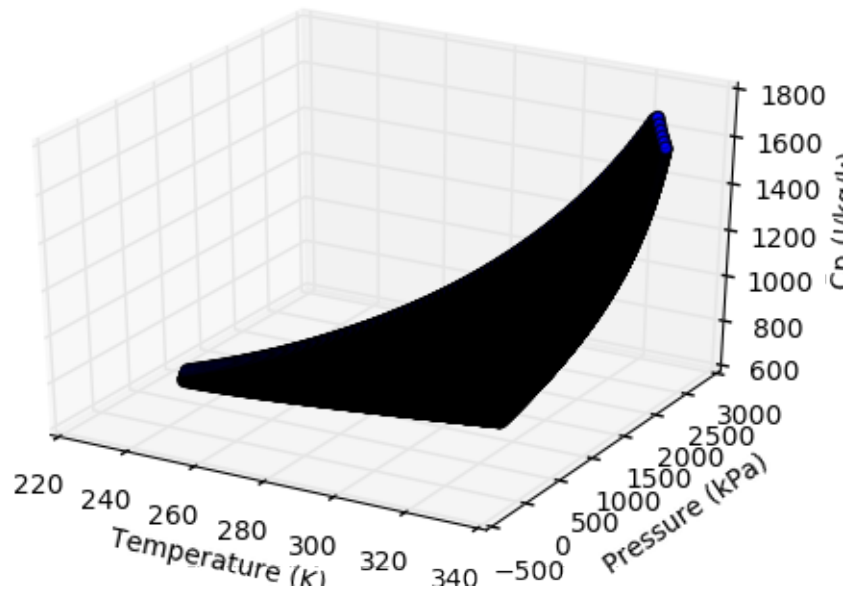
```
[24]: Vapor_positives = positives[(positives['Temperature'] > 151.12884541 + 27.  
    ↪ 9058175606*positives['Pressure']**0.23750882839)]  
threedee = plt.figure().gca(projection='3d')  
threedee.  
    ↪ scatter(Vapor_positives['Temperature'], Vapor_positives['Pressure'], Vapor_positives['Cp'])  
threedee.set_xlabel('Temperature (K)')  
threedee.set_ylabel('Pressure (kPa)')  
threedee.set_zlabel('Cp (J/kg/K)')  
plt.show()
```

```
[24]: <mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7fa8a9d74ac8>
```

```
[24]: <matplotlib.text.Text at 0x7fa8aa20a518>
```

```
[24]: <matplotlib.text.Text at 0x7fa8aa0aaef0>
```

```
[24]: <matplotlib.text.Text at 0x7fa8aa033358>
```



```
[25]: Vapor_positives.head(3)
```

```
[25]:   Pressure  Temperature      Cp
0    100.0    243.000000  770.671210
1    100.0    243.502793  770.801452
2    100.0    244.005587  770.950061
```

```
[34]: Vapor_positives = Vapor_positives.assign(Temp_sq =
↳ Vapor_positives['Temperature']**2) #1.56 as a suggestion, overall it is
↳ reasonable
Vapor_positives = Vapor_positives.assign(Press_sq =
↳ Vapor_positives['Pressure']**2) #1.4
```

```
[35]: resultVapor = sm.ols(formula="Cp ~ Temperature + Pressure ",
↳ data=Vapor_positives).fit()
forecastVapor = resultVapor.predict(Vapor_positives[['Pressure', 'Temperature']])
ErrorVapor = pd.DataFrame({'Error': forecastVapor-Vapor_positives.Cp})

print(resultVapor.params)
print(resultVapor.summary())
ErrorVapor.describe()
```

```
Intercept      887.750263
Temperature    -0.381473
Pressure        0.276162
dtype: float64
```

OLS Regression Results

```

=====
Dep. Variable:          Cp    R-squared:          0.937
Model:                  OLS   Adj. R-squared:       0.937
Method:                 Least Squares   F-statistic:         1.206e+06
Date:                   Wed, 13 Dec 2017   Prob (F-statistic):    0.00
Time:                   19:10:22   Log-Likelihood:       -8.3675e+05
No. Observations:      163429   AIC:                  1.673e+06
Df Residuals:          163426   BIC:                  1.674e+06
Df Model:               2
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	887.7503	1.788	496.464	0.000	884.246	891.255
Temperature	-0.3815	0.006	-62.475	0.000	-0.393	-0.370
Pressure	0.2762	0.000	1307.812	0.000	0.276	0.277

```

=====
Omnibus:                 41533.631   Durbin-Watson:          0.177
Prob(Omnibus):            0.000   Jarque-Bera (JB):       169829.334
Skew:                     1.209   Prob(JB):                0.00
Kurtosis:                 7.369   Cond. No.                1.88e+04
=====

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.88e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```

[35]:          Error
count  1.634290e+05
mean   4.501923e-13
std    4.048774e+01
min    -2.775680e+02
25%    -1.836774e+01
50%     3.330836e+00
75%     2.282027e+01
max     8.744900e+01

```

```

[33]: plt.hist(ErrorVapor.Error, bins = 150); # histogram of error distribution
plt.title('Error distribution')
plt.xlabel('Spread from function Cp (J/kg/k)'))')
plt.ylabel('Frecuency')

```

```

[33]: (array([ 3.00000000e+00,  4.00000000e+00,  4.00000000e+00,
                4.00000000e+00,  5.00000000e+00,  6.00000000e+00,
                6.00000000e+00,  6.00000000e+00,  6.00000000e+00,

```

7.00000000e+00,	8.00000000e+00,	1.00000000e+01,
1.00000000e+01,	1.20000000e+01,	1.40000000e+01,
1.20000000e+01,	1.20000000e+01,	1.40000000e+01,
1.50000000e+01,	1.40000000e+01,	1.60000000e+01,
2.10000000e+01,	2.00000000e+01,	2.00000000e+01,
2.00000000e+01,	2.30000000e+01,	2.30000000e+01,
2.30000000e+01,	2.60000000e+01,	2.60000000e+01,
2.60000000e+01,	3.20000000e+01,	3.10000000e+01,
3.10000000e+01,	3.70000000e+01,	3.40000000e+01,
3.60000000e+01,	3.60000000e+01,	3.90000000e+01,
4.00000000e+01,	4.50000000e+01,	4.50000000e+01,
4.60000000e+01,	5.00000000e+01,	4.90000000e+01,
5.00000000e+01,	5.60000000e+01,	5.50000000e+01,
5.90000000e+01,	6.30000000e+01,	6.20000000e+01,
6.70000000e+01,	6.80000000e+01,	7.00000000e+01,
7.30000000e+01,	7.60000000e+01,	7.70000000e+01,
7.70000000e+01,	8.40000000e+01,	8.90000000e+01,
8.80000000e+01,	9.30000000e+01,	9.50000000e+01,
1.02000000e+02,	9.90000000e+01,	1.06000000e+02,
1.16000000e+02,	1.10000000e+02,	1.23000000e+02,
1.22000000e+02,	1.31000000e+02,	1.34000000e+02,
1.40000000e+02,	1.43000000e+02,	1.56000000e+02,
1.62000000e+02,	1.68000000e+02,	2.08000000e+02,
2.61000000e+02,	2.96000000e+02,	3.48000000e+02,
3.99000000e+02,	4.39000000e+02,	5.02000000e+02,
5.53000000e+02,	6.13000000e+02,	6.66000000e+02,
7.37000000e+02,	7.95000000e+02,	8.78000000e+02,
9.48000000e+02,	1.02900000e+03,	1.13500000e+03,
1.22300000e+03,	1.33800000e+03,	1.43500000e+03,
1.57000000e+03,	1.68500000e+03,	1.79900000e+03,
1.92700000e+03,	2.05500000e+03,	2.18200000e+03,
2.31700000e+03,	2.48400000e+03,	2.62400000e+03,
2.79200000e+03,	2.99700000e+03,	3.20100000e+03,
3.42100000e+03,	3.68800000e+03,	3.99800000e+03,
4.36800000e+03,	4.85600000e+03,	5.51500000e+03,
6.67600000e+03,	9.58600000e+03,	7.01800000e+03,
5.66400000e+03,	5.01300000e+03,	4.43500000e+03,
4.08400000e+03,	3.77500000e+03,	3.50600000e+03,
3.21200000e+03,	3.08600000e+03,	2.84000000e+03,
2.68900000e+03,	2.54500000e+03,	2.38600000e+03,
2.23000000e+03,	2.08100000e+03,	1.96000000e+03,
1.84200000e+03,	1.73800000e+03,	1.55900000e+03,
1.46500000e+03,	1.42300000e+03,	1.27300000e+03,
1.28000000e+03,	1.14400000e+03,	1.15100000e+03,
9.96000000e+02,	9.83000000e+02,	8.91000000e+02,
7.89000000e+02,	7.66000000e+02,	6.44000000e+02,
5.28000000e+02,	4.38000000e+02,	3.70000000e+02]]),


```

array([ -2.77568022e+02, -2.75134575e+02, -2.72701128e+02,
        -2.70267681e+02, -2.67834234e+02, -2.65400788e+02,
        -2.62967341e+02, -2.60533894e+02, -2.58100447e+02,
        -2.55667000e+02, -2.53233553e+02, -2.50800107e+02,
        -2.48366660e+02, -2.45933213e+02, -2.43499766e+02,
        -2.41066319e+02, -2.38632872e+02, -2.36199426e+02,
        -2.33765979e+02, -2.31332532e+02, -2.28899085e+02,
        -2.26465638e+02, -2.24032191e+02, -2.21598745e+02,
        -2.19165298e+02, -2.16731851e+02, -2.14298404e+02,
        -2.11864957e+02, -2.09431510e+02, -2.06998064e+02,
        -2.04564617e+02, -2.02131170e+02, -1.99697723e+02,
        -1.97264276e+02, -1.94830829e+02, -1.92397383e+02,
        -1.89963936e+02, -1.87530489e+02, -1.85097042e+02,
        -1.82663595e+02, -1.80230148e+02, -1.77796702e+02,
        -1.75363255e+02, -1.72929808e+02, -1.70496361e+02,
        -1.68062914e+02, -1.65629467e+02, -1.63196021e+02,
        -1.60762574e+02, -1.58329127e+02, -1.55895680e+02,
        -1.53462233e+02, -1.51028786e+02, -1.48595339e+02,
        -1.46161893e+02, -1.43728446e+02, -1.41294999e+02,
        -1.38861552e+02, -1.36428105e+02, -1.33994658e+02,
        -1.31561212e+02, -1.29127765e+02, -1.26694318e+02,
        -1.24260871e+02, -1.21827424e+02, -1.19393977e+02,
        -1.16960531e+02, -1.14527084e+02, -1.12093637e+02,
        -1.09660190e+02, -1.07226743e+02, -1.04793296e+02,
        -1.02359850e+02, -9.99264028e+01, -9.74929559e+01,
        -9.50595091e+01, -9.26260623e+01, -9.01926154e+01,
        -8.77591686e+01, -8.53257217e+01, -8.28922749e+01,
        -8.04588281e+01, -7.80253812e+01, -7.55919344e+01,
        -7.31584876e+01, -7.07250407e+01, -6.82915939e+01,
        -6.58581471e+01, -6.34247002e+01, -6.09912534e+01,
        -5.85578065e+01, -5.61243597e+01, -5.36909129e+01,
        -5.12574660e+01, -4.88240192e+01, -4.63905724e+01,
        -4.39571255e+01, -4.15236787e+01, -3.90902319e+01,
        -3.66567850e+01, -3.42233382e+01, -3.17898913e+01,
        -2.93564445e+01, -2.69229977e+01, -2.44895508e+01,
        -2.20561040e+01, -1.96226572e+01, -1.71892103e+01,
        -1.47557635e+01, -1.23223167e+01, -9.88886981e+00,
        -7.45542298e+00, -5.02197614e+00, -2.58852931e+00,
        -1.55082469e-01, 2.27836437e+00, 4.71181120e+00,
        7.14525804e+00, 9.57870488e+00, 1.20121517e+01,
        1.44455985e+01, 1.68790454e+01, 1.93124922e+01,
        2.17459391e+01, 2.41793859e+01, 2.66128327e+01,
        2.90462796e+01, 3.14797264e+01, 3.39131732e+01,
        3.63466201e+01, 3.87800669e+01, 4.12135138e+01,
        4.36469606e+01, 4.60804074e+01, 4.85138543e+01,
        5.09473011e+01, 5.33807479e+01, 5.58141948e+01,
        5.82476416e+01, 6.06810884e+01, 6.31145353e+01,

```

```

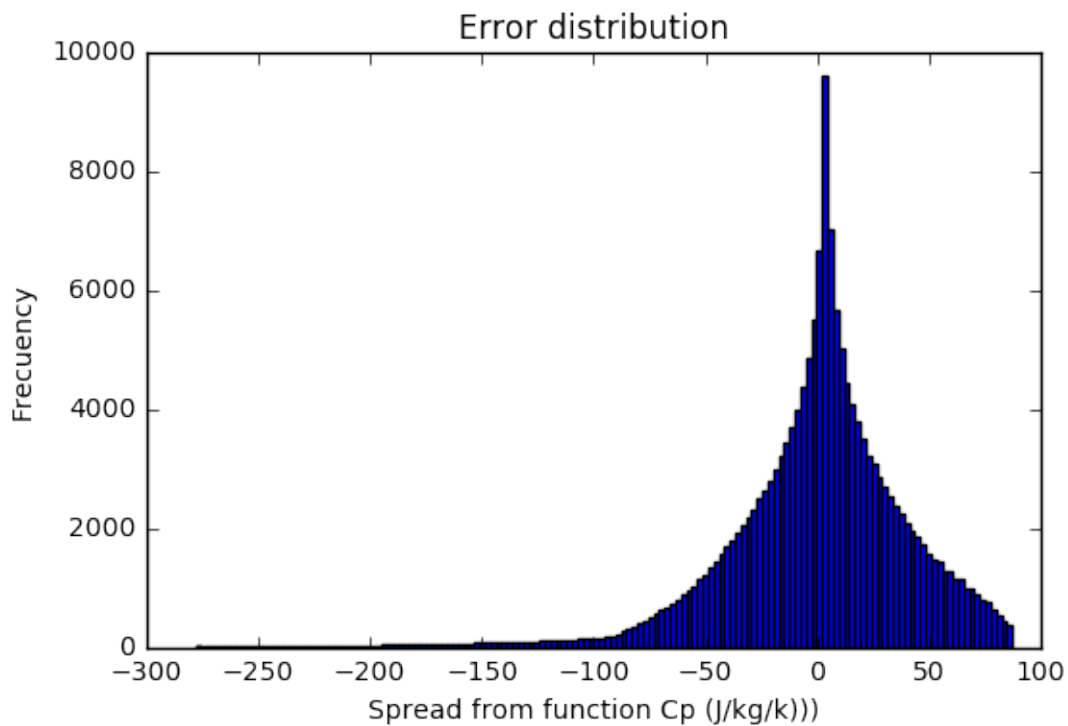
        6.55479821e+01, 6.79814290e+01, 7.04148758e+01,
        7.28483226e+01, 7.52817695e+01, 7.77152163e+01,
        8.01486631e+01, 8.25821100e+01, 8.50155568e+01,
        8.74490036e+01]),
    <a list of 150 Patch objects>)

```

```
[33]: <matplotlib.text.Text at 0x7fa8a4d44be0>
```

```
[33]: <matplotlib.text.Text at 0x7fa8a9c37630>
```

```
[33]: <matplotlib.text.Text at 0x7fa892c097f0>
```



4 Result for Vapor portion $Cp_{\text{Vapor}} = aT^2 + bT + cP + f$

4.1 [J/kg/K]

4.1.1 $R^2 = 0.939$

a is the parameter for Temp_sq : -0.017369

b is the parameter for Temperature : 9.963247

c is the parameter for Pressure : 0.278669

f is the Intercept: -645.880275

5 Result for Vapor portion $C_p^{\text{Vapor}} = bT + cP + f$

5.0.1 $R^2 = 0.937$

b is the parameter for Temperature : -0.381473

c is the parameter for Pressure : 0.276162

f is the Intercept: 887.750263

```
[ ]: #Spec.append(PropsSI('C','P', p*1000,'T',t, 'REFPROP::R407a.mix')) # test
      #in case there is a need to export data to excel
      #writer = pd.ExcelWriter('pandas_simplifiedensity.xlsx', engine='xlsxwriter')
      #Liquid_df.to_excel(writer, sheet_name='Sheet1')
      #writer.save()
```