Problem Set 1

Zachary Orlando, Nicolas Moreno Arias & Cutberto Frias Sarraf

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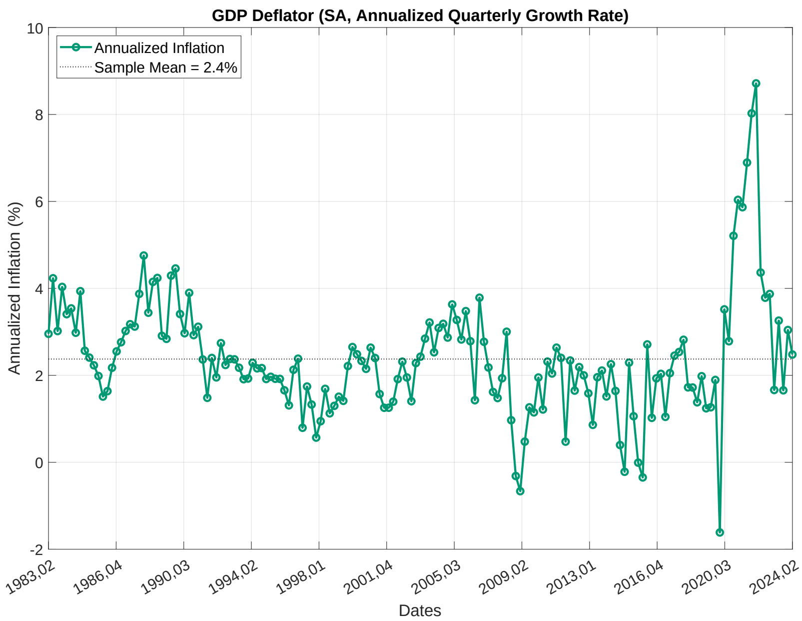
Use US inflation data to compute various quantities discussed in class.

1. Download the US GDP deflator, 1983:1 to present. In the FRED database, this is series **GDPDEF**. Call this series P_t.
2. Compute annualized inflation, 1983:2-present as $x_t = 400 \times \ln\left(\frac{P_t}{P_{t-1}}\right)$.
3. Just in case you do the assignment correctly but accidentally download the wrong data: report the values of the deflator, $\ln(P_t)$, and inflation x_t for the last two quarters. **Inflation for 2024Q1:** 3.0418 and **Inflation for 2024Q2:** 2.4784.



Questions

(10) Fit an AR(2) to x_t, 1984:1-2019:4. (Note the end date, which is different from the up-to-date end date used in class. So your estimates will be slightly different.) Report the coefficients, $\hat{\sigma}^2$ (the estimate of the variance of e_t), and the implied unconditional mean $\hat{\mu}$.

The second order autoregressive model AR(2) is expressed as:

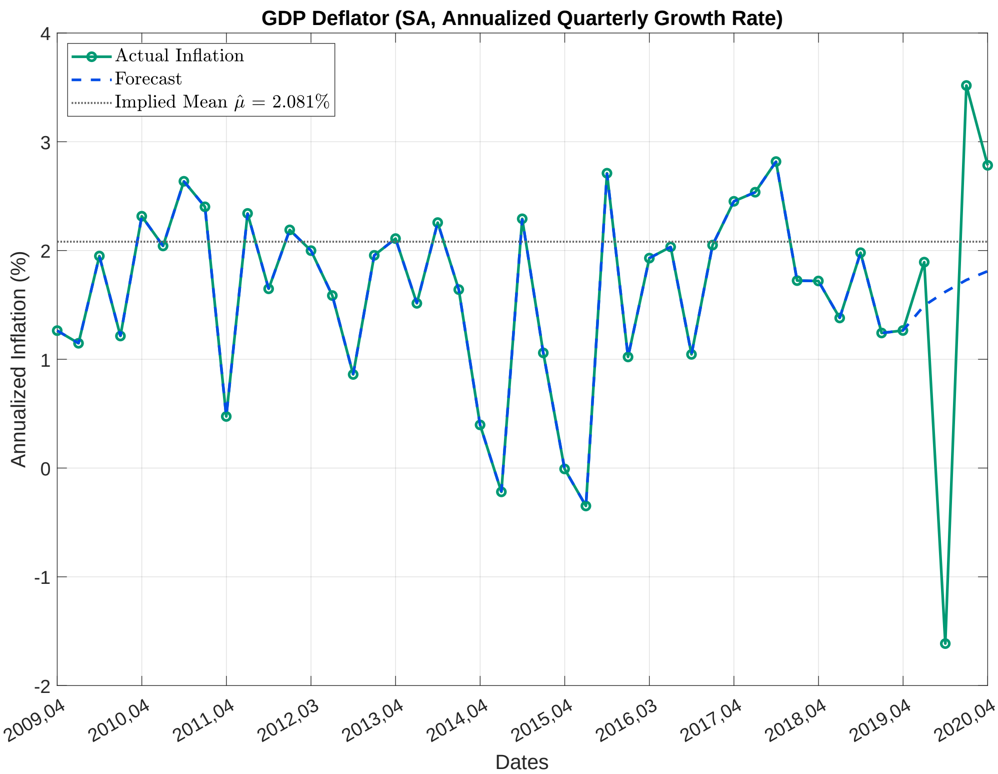
$$x_t = m + \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + e_t$$

Table 1: Estimation Results

AR(2)	Intercept $\hat{m}$	$\hat{\varphi}_1$	$\hat{\varphi}_2$
	0.596	0.532	0.182
	(0.160)	(0.083)	(0.083)
$\hat{\sigma}^2$	0.528		
$\hat{\mu}$	2.081		
Observations	142		

Note: Standard errors are in parentheses. We use information from 1984.1 to 2019.4, since we use 2 lagged values the number of observations is 142.

(40) Forecast inflation for the next four quarters, 2020:1-2020:4. Compute the forecast error for all four quarters. How accurate is this forecast relative to the 2023-based forecast presented in class? (You will have to think about how to gauge forecast accuracy.) An informal discussion is okay.



They are similar in the fact that both forecasting exercises predict mean reversion, missing the series crossing their unconditional mean in both periods. However, they differ in that the magnitude of the forecast errors for 2020 were qualitatively larger than the forecast errors for 2023 in the post-Covid period. This relates to the significant larger swings in inflation around its mean during 2020.

(10) Compute and report the weights $j = 0, \dots, 4$ in the moving average representation of x_t.

By stationary we have $\mathbb{E}(x_t) = \mu$ for all t. Using expectations in our AR(2) equation we obtain $\mu = m + \varphi_1\mu + \varphi_2\mu$ that implies that $m = \mu(1 - \varphi_1 - \varphi_2)$. Then we can express the second order autoregressive model AR(2) as follows:

$$\begin{aligned}(x_t - \mu) - \varphi_1(x_{t-1} - \mu) - \varphi_2(x_{t-2} - \mu) &= e_t \\ \implies (x_t - \mu) &= \frac{1}{1 - \varphi_1 L - \varphi_2 L^2} e_t\end{aligned}$$

Further, since x_t is stationary, we can use the Wold representation:

$$\begin{aligned}(x_t - \mu) &= \sum_{j=0}^{\infty} \Psi_j e_{t-j} \\ \implies (x_t - \mu) &= (\Psi_0 + \Psi_1 L + \Psi_2 L^2 + \Psi_3 L^3 + ...)e_t\end{aligned}$$

for some sequence $\{\Psi_j\}$.

$$1 = (1 - \varphi_1 L - \varphi_2 L^2)(\Psi_0 + \Psi_1 L + \Psi_2 L^2 + \Psi_3 L^3 + ...)$$

expanding the product we obtain a relationship between the sequence Ψ_j and φ_1 and φ_2. The estimation of weights for $j = 0, ..., 4$ are:

$$\begin{aligned}&\Psi_0 = 1 \\ &\Psi_1 = \varphi_1 = 0.5319 \\ &\Psi_2 = \varphi_1\Psi_1 + \varphi_2\Psi_0 = 0.4648 \\ &\Psi_3 = \varphi_1\Psi_2 + \varphi_2\Psi_1 = 0.3440 \\ &\Psi_4 = \varphi_1\Psi_3 + \varphi_2\Psi_2 = 0.2676\end{aligned}$$

(40) Plot and report the numerical values of the following impulse response functions, for horizons $j = 0, \dots, 4$:

(a) A 1% positive shock.

(b) A -2% negative shock.

(c) One standard deviation shock.

Note: This question asks for impulse responses, not forecasts. For a given AR(2) process, the forecast for the next 4 periods will vary as data in the current and previous quarter vary. An impulse response function does not vary with the data.

The following impulse response functions reflect a shock when inflation is equal to the unconditional mean 2.081. A standard deviation $\hat{\sigma}$ is 0.7265.

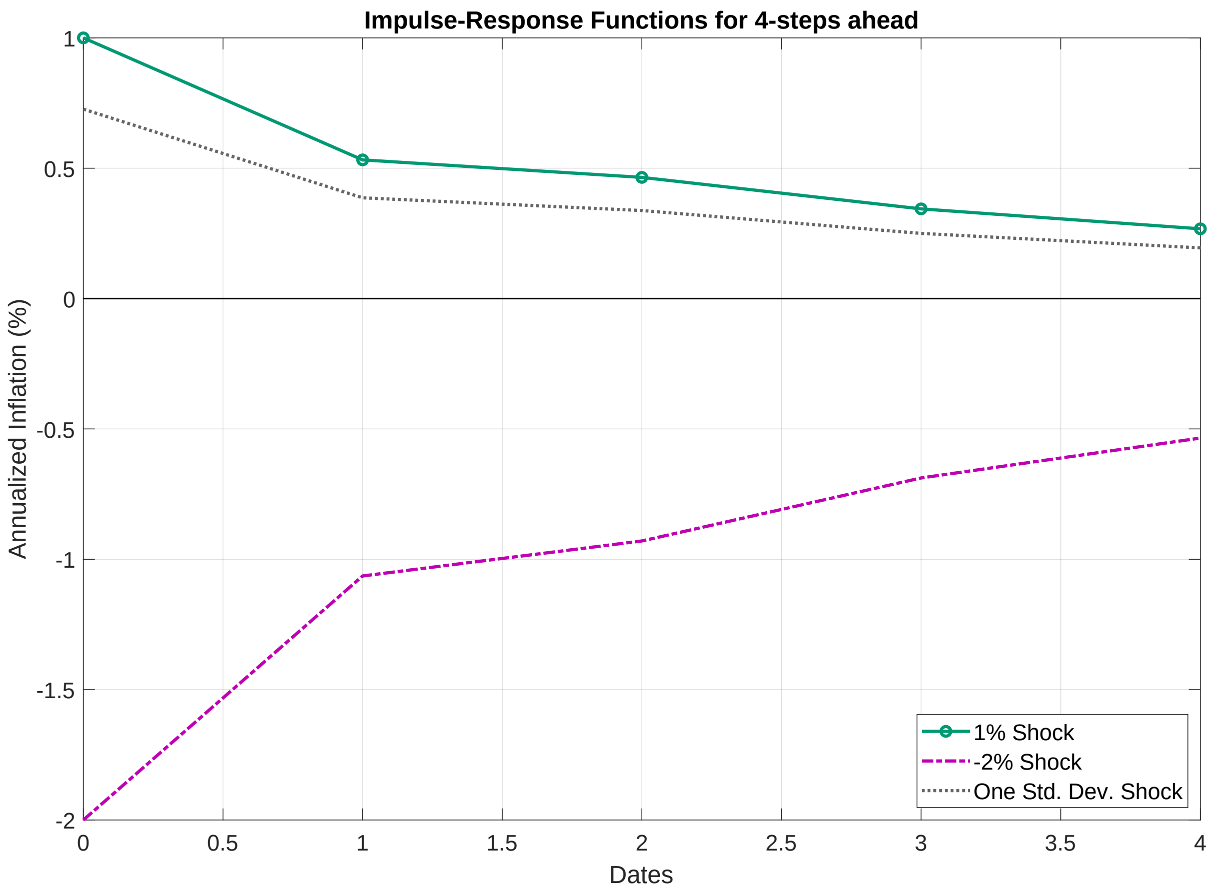


Table 2: Impulse Response Function Output

Shock	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1.00	3.0813	2.6132	2.5461	2.4253	2.3488
-2.00	0.0813	1.0173	1.1516	1.3932	1.5462
σ	2.8078	2.4678	2.4190	2.3312	2.2756