Problem Set 4

Estimation of dynamic discrete choice models

ECON: 880

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Questions

The value function is given by:

$$V\left(i, c, p, \epsilon_t\right)=\max_{a \in\{0,1\}} U(a \mid i, c, p, \epsilon)+\beta \sum_{c^{\prime}, p^{\prime}} E_{\epsilon^{\prime}}\left[V\left(i^{\prime}, c^{\prime}, p^{\prime}, \epsilon^{\prime}\right)\right] \operatorname{Pr}\left(c^{\prime}, p^{\prime} \mid c, p, a\right)$$

$$\text{s.t. } i^{\prime}=\min\{\bar{i}, i+a-c\}$$

$$c^{\prime}=\begin{cases}0 & \text{With probability } 1/2\\ 1 & \text{With probability } 1/2\end{cases}$$

$$p^{\prime}=\begin{cases}p_s & \text{With probability } \pi(p)\\ p_r & \text{With probability } 1-\pi(p)\end{cases}$$

Question 1

Define $\bar{V}(s)=E_\epsilon[V(s,\epsilon)]$ (as a Fixed Point):

$$\begin{aligned}
\bar{V}(s)= &\; E_\epsilon[\max_a V(a; i, c, p, \epsilon)+\epsilon(a)]\\
= &\; \ln\Big(\sum_{a=0,1} e^{V(a,i,c,p,\epsilon)}\Big)+\gamma_{\text{euler}}\\
= &\ln\Big(e^{U(0)+\beta F(s'|s,a=0)\bar{V}(s)}+e^{U(1)+\beta F(s'|s,a=1)\bar{V}(s)}\Big)+\gamma_{\text{euler}}\\
= &\ln\Big(e^{\alpha c \mathbb{1}_{i>0}+\lambda \mathbb{1}_{c>0}+\beta F(s'|s,a=0)\bar{V}(s)}+e^{\alpha c-p+\beta F(s'|s,a=1)\bar{V}(s)}\Big)+\gamma_{\text{euler}}
\end{aligned}$$

The numerical solution for the value function is summarized by the following table:

Table 1

id	I	C	P	U(0)	U(1)	$\bar{V}(s)$
0	0	0	4	0	-4	61.1278
1	1	0	4	0	-4	65.0102
2	2	0	4	0	-4	68.4821
3	3	0	4	0	-4	71.6687
4	4	0	4	0	-4	74.6302
5	5	0	4	0	-4	77.3943
6	6	0	4	0	-4	79.9588
7	7	0	4	0	-4	82.2633
8	8	0	4	0	-4	84.0732
9	0	1	4	-4	-2	58.491
10	1	1	4	2	-2	63.1278
11	2	1	4	2	-2	67.0102
12	3	1	4	2	-2	70.4821
13	4	1	4	2	-2	73.6687
14	5	1	4	2	-2	76.6302
15	6	1	4	2	-2	79.3943
16	7	1	4	2	-2	81.9588
17	8	1	4	2	-2	84.2633
18	0	0	1	0	-1	63.2441
19	1	0	1	0	-1	66.8946
20	2	0	1	0	-1	70.2031
21	3	0	1	0	-1	73.2605
22	4	0	1	0	-1	76.1102
23	5	0	1	0	-1	78.7659
24	6	0	1	0	-1	81.2009
25	7	0	1	0	-1	83.2815
26	8	0	1	0	-1	84.2775
27	0	1	1	-4	1	61.0255
28	1	1	1	2	1	65.2441
29	2	1	1	2	1	68.8946
30	3	1	1	2	1	72.2031
31	4	1	1	2	1	75.2605
32	5	1	1	2	1	78.1102
33	6	1	1	2	1	80.7659
34	7	1	1	2	1	83.2009
35	8	1	1	2	1	85.2815

Question 2

Following the algorithm in the slides, we use a frequency estimator and compute the implied value function. We then compare it to that obtained in Question 1. We can see that at 4 decimal points, the differences are zero. If we consider more decimal points, we start seeing very negligible differences. (See Table 2).

Question 3

According to the Nested Fixed-Point MLE (NFMLE) explained in slide 22, the log-likelihood function is given by the following expression:

$$
\begin{aligned}
\mathbf{L}(\lambda) =& \sum_i a_i \ln(P(s_i)) + (1-a_i)\ln(1-P(s_i)) \\
& s.t \\
P(s_i) =& \Psi(s_i) \equiv (1+e^{-\tilde{v}(s_i)^{k-1}})^{-1}
\end{aligned}
$$

where $\Psi(s_i)$ follows the formula described in Algorithm 2 in the slides, as well as $\tilde{v}(s_i)^{k-1}$, which is defined as:

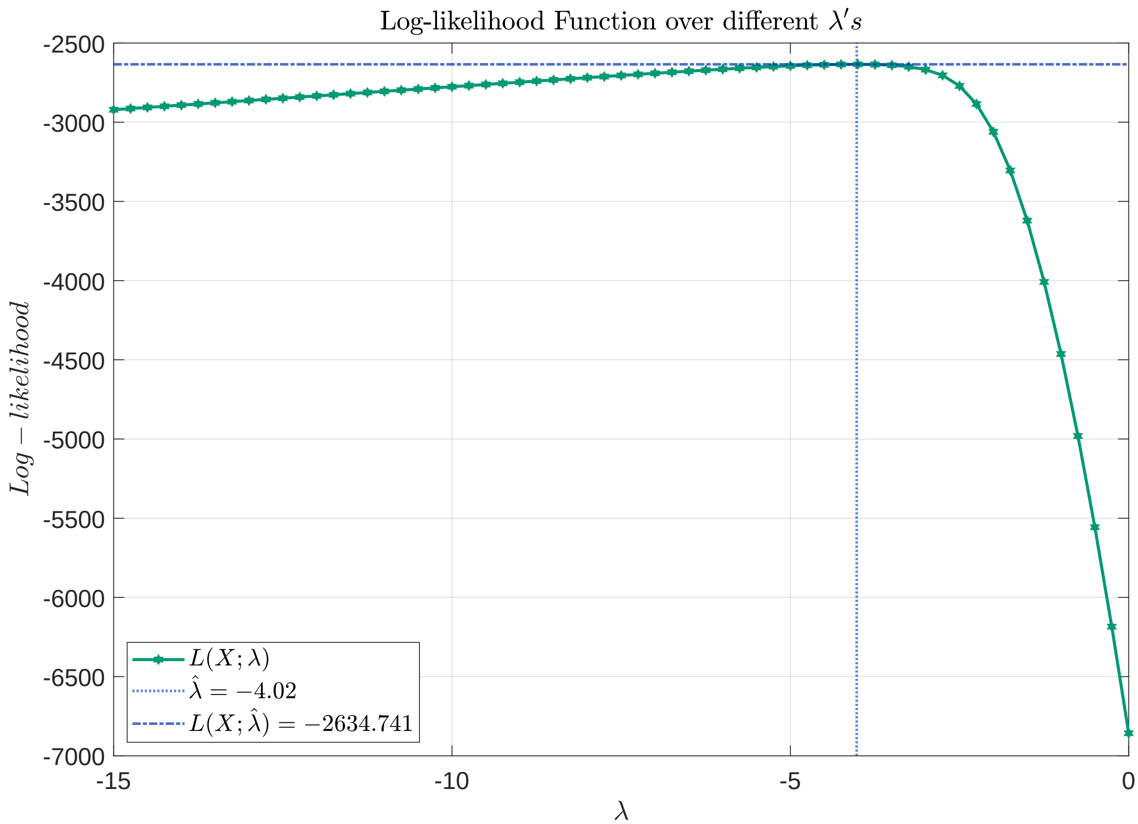
$$
\tilde{v}(s_i)^{k-1} = U(1) + \beta F\left(s_i' \mid s_i, a_i = 1\right)\bar{V}^{k-1} - \Big(U(0) + \beta F\left(s_i' \mid s_i, a_i = 0\right)\bar{V}^{k-1}\Big)
$$

Question 4

Using the steps for NFPMLE, we obtained $\hat{\lambda} = -4.02$ and the log-likelihood obtained was $\mathbf{L}(\hat{\lambda}) = -2634.741$. The graph 1 illustrates the optimization by plotting the log-likelihood over different parameter values and on top of it the blue lines give us the optimal parameter and the corresponding log-likelihood value.

Table 2

id	I	C	P	$\hat{P}_0$	$\hat{P}_1$	$\bar{V}(s)$	$\hat{\bar{V}}(s)$	$\Delta\bar{V}(s)$
0	0	0	4	0.4754	0.5246	61.1278	61.1278	0
1	1	0	4	0.6085	0.3915	65.0102	65.0102	0
2	2	0	4	0.6828	0.3172	68.4821	68.4821	0
3	3	0	4	0.7204	0.2796	71.6687	71.6687	0
4	4	0	4	0.7725	0.2275	74.6302	74.6302	0
5	5	0	4	0.8278	0.1722	77.3943	77.3943	0
6	6	0	4	0.8241	0.1759	79.9588	79.9588	0
7	7	0	4	0.8182	0.1818	82.2633	82.2633	0
8	8	0	4	0.999	0.001	84.0732	84.0732	0
9	0	1	4	0.1146	0.8854	58.491	58.491	0
10	1	1	4	0.4686	0.5314	63.1278	63.1278	0
11	2	1	4	0.6111	0.3889	67.0102	67.0102	0
12	3	1	4	0.6731	0.3269	70.4821	70.4821	0
13	4	1	4	0.7807	0.2193	73.6687	73.6687	0
14	5	1	4	0.7429	0.2571	76.6302	76.6302	0
15	6	1	4	0.8358	0.1642	79.3943	79.3943	0
16	7	1	4	0.8718	0.1282	81.9588	81.9588	0
17	8	1	4	0.7273	0.2727	84.2633	84.2633	0
18	0	0	1	0.001	0.999	63.2441	63.2441	0
19	1	0	1	0.0672	0.9328	66.8946	66.8946	0
20	2	0	1	0.0826	0.9174	70.2031	70.2031	0
21	3	0	1	0.0795	0.9205	73.2605	73.2605	0
22	4	0	1	0.1538	0.8462	76.1102	76.1102	0
23	5	0	1	0.2174	0.7826	78.7659	78.7659	0
24	6	0	1	0.3333	0.6667	81.2009	81.2009	0
25	7	0	1	0.2857	0.7143	83.2815	83.2815	0
26	8	0	1	0.999	0.001	84.2775	84.2775	0
27	0	1	1	0.001	0.999	61.0255	61.0255	0
28	1	1	1	0.0455	0.9545	65.2441	65.2441	0
29	2	1	1	0.0522	0.9478	68.8946	68.8946	0
30	3	1	1	0.1	0.9	72.2031	72.2031	0
31	4	1	1	0.0968	0.9032	75.2605	75.2605	0
32	5	1	1	0.1765	0.8235	78.1102	78.1102	0
33	6	1	1	0.2857	0.7143	80.7659	80.7659	0
34	7	1	1	0.1667	0.8333	83.2009	83.2009	0
35	8	1	1	0.3333	0.6667	85.2815	85.2815	0



Log-likelihood Function over different $\lambda's$
-2500
-3000
-3500
-4000
-4500
-5000
-5500
-6000
-6500
-7000
$Log - likelihood$
-15
-10
-5
0
λ
$L(X;\lambda)$
$\hat{\lambda} = -4.02$
$L(X;\hat{\lambda}) = -2634.741$

Figure 1