Mathematical Model

Objective Function

Minimize total cost:

$$\text{Minimize} \quad \sum_{i,j} c_{ij} x_{ij} + \sum_{i,j,k} c_{ijk} y_{ijk} + \sum_{k} h_p^k \sum_{i,j} y_{ijk} + \sum_{i,k,l} c_{lk} z_{ikl} + \sum_{l} h_w^l \sum_{i,k} z_{ikl} + \sum_{i,l,m} c_{km} w_{ilm}$$

Constraints

Port Capacity $\sum_{i,j} y_{ijk} \leq \text{Cap}_p^k \quad \forall k$

Warehouse Capacity $\sum_{i,k} z_{ikl} \leq \text{Cap}_w^l \quad \forall l$

Supplier Balance $\sum_{k} y_{ijk} = x_{ij} \quad \forall i, j$

Port Balance $\sum_{j} y_{ijk} = \sum_{l} z_{ikl} \quad \forall i, k$

Warehouse Balance $\sum_{k} z_{ikl} = \sum_{m} w_{ilm} \quad \forall i, l$

Nutritional Requirement $\sum_{i,l} w_{ilm} \cdot \text{nutrient}_{i,n} \geq \text{requirement}_{m,n} \quad \forall m, n$

Variable Definitions

The model uses the following variables and parameters:

Decision Variables

- x_{ij}: Quantity of product i shipped from supplier i to demand node j
- y_{ijk}: Quantity of product i shipped from supplier i to port k for demand node j
- z_{ikl}: Quantity of product i transported from port k to warehouse l
- w_{ilm}: Quantity of product i delivered from warehouse l to market m

Cost Parameters

- c_{ij}: Cost per unit of shipping product i from supplier i to demand node j
- c_{ijk}: Cost per unit of transporting product i from supplier i to port k for demand node j
- c_{lk}: Cost per unit of transporting product i from port k to warehouse l
- c_{km}: Cost per unit of transporting product i from warehouse l to market m

Handling Costs

- h_p^k: Handling cost per unit at port k
- h_w^l: Handling cost per unit at warehouse l

Capacity Parameters

- Cap_p^k: Capacity of port k
- Cap_w^l: Capacity of warehouse l

Nutritional Parameters

- $\mathrm{nutrient}_{i,n}$: Amount of nutrient n in product i
- $\mathrm{requirement}_{m,n}$: Minimum required amount of nutrient n at market m