MATH 2418: Linear Algebra

Assignment# 3

Due :02/06, Tuesday before 11:59pm   Term :Spring 2024

[Last Name]	[First Name]	[Net ID]	[Lab Section]

Recommended Problems:(Do not turn in)
Sec 1.4: 1,2,3,4,5, 6, 7, 8, 11, 12, 13, 17, 19, 20. **Sec 2.1:** 1, 2, 5, 6, 7, 8, 9, 10, 12, 16, 17, 18, 19, 26.

1. Let $B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ and $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

   (a) For any 3×4 matrix A, what is the column 2 of AB? Write down all components in the column 2 of AB, and explain your answer.

   (b) For any 4×5 matrix C, what is the row 3 of BC? Write down all components in the row 3 of BC. Explain your answer.

   (c) Is it possible to find a 4×4 matrix D such that $DB = I_4$? Explain your answer.

   (d) Is it possible to find a 4×4 matrix F such that $BF = I_4$? Explain your answer.

2. (a) Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 6 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 4 \\ 5 & 3 & 7 \end{bmatrix}$. Without calculating the complete matrices AB and BA, compute (if possible) the following:

(i) The entry $(AB)_{22}$ of AB (ii) The entry $(BA)_{22}$ of BA (iii) Column 2 of AB (iv) Row 3 of BA.

(b) For matrices $A = \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}$, and $D = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & 0 \end{bmatrix}$,

compute the following products if they exist.
(i) AB and BA (ii) BC and CB (iii) CD and DC (iv) verify $B(DC) = (BD)C$.

3. Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 1 \\ 3 & 6 & 0 & 0 \end{bmatrix}$

    (a) Factor the matrix A into CR.

    (b) Find the column space of the matrix A.

    (c) Write each row of the matrix A as combinations of the rows of the matrix R.

    (d) Find the rank r of the matrix A.

    (e) Write A as sum of r rank one matrices.

4. Given the linear system $\begin{cases} 2x + y = 12 \\ -3x + y = 2 \end{cases}$

   (a) Write the corresponding matrix equation $A\mathbf{x} = \mathbf{b}$.

   (b) Solve the system.

   (c) Write $\mathbf{b}$ as a linear combination of columns of A.

   (d) Draw the row picture and the column picture.

5. Given the linear system

$$\begin{cases} \alpha x - 3y = 6 \\ 4x + 6y = -12 \end{cases}$$

(a) For what value(s) of α does the elimination fail (i) temporarily (ii) permanently?

(b) Solve the system after fixing the temporary failure of the elimination.

(c) Solve the system in case of permanent break down.

6. Let k be a real number. Consider the following linear system:

$$\begin{cases} x + \ y - \qquad\quad z = 2 \\ x + 2y + \qquad\quad z = 3 \\ x + \ y + (k^2 - 5)z = k \end{cases} \tag{1}$$

(a) Find all possible values of k such that system (1)

 (i) has a unique solution;

 (ii) has no solutions;

 (iii) has infinitely many solutions.

(b) For the values of k for which solutions exist, find those solutions.

7. (a) Consider the 3×3 matrix $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & -1 \\ 1 & 1 & 4 \end{bmatrix}$

    (i) Perform elimination to convert B into upper triangular matrix U.

    (ii) List the elimination matrices E_{21}, E_{31}, and E_{32} corresponding to row operations in part (a).

    (iii) Compute the matrix M that takes B directly to U ie such that $MB = U$.

  (b) Consider the linear system $\begin{cases} x + 2y + \ z = 1 \\ 2x + 5y - \ z = 4 \\ x + \ y + 4z = 2 \end{cases}$

    (i) Write the corresponding augmented matrix.

    (ii) Solve the system by reducing the augmented matrix into upper triangular form.