MATH 2418: Linear Algebra

Assignment# 1

Due :01/23, Tuesday, 11:59pm Term :Spring 2024

[Last Name]	[First Name]	[Net ID]	[Lab Section]

Recommended Problems:(Do not turn in) Sec 1.1: 1, 2, 3,4,5, 8, 9, 10, 11, 12, 13, 14, 15,17, 18,19,25, 26, 27.

1. Let $\mathbf{u}+\mathbf{v}=(-2,3)$ and $\mathbf{u}-\mathbf{v}=(4,1)$.
   (a) Compute $\mathbf{u}$ and $\mathbf{v}$.
   (b) Draw the vectors $\mathbf{u},\ \mathbf{v},\ -\mathbf{u},\ -\mathbf{v},\ \mathbf{u}+\mathbf{v},\ (\mathbf{u}-\mathbf{v}),\ (-\mathbf{u}+\mathbf{v}),\ (-\mathbf{u}-\mathbf{v})$ in the single $xy-$plane.

2. (a) Determine all real values of s such that the set of all linear combination of $\mathbf{u} = (s, 3)$ and $\mathbf{v} = (2, 7)$ is all of $\mathbb{R}^2$. Justify your answer.

   (b) Determine all real values of p and q such that the set of all linear combinations of $\mathbf{u} = (-1, 3, p)$ and $\mathbf{v} = (q, 2, 1)$ is a plane in $\mathbb{R}^3$. Justify your answer.

3. Let $\mathbf{u} = (3, -1)$ and $\mathbf{v} = (1, 4)$ be two given vectors in $\mathbb{R}^2$. Let $\mathbf{b} = (r, s)$ be any vector in $\mathbb{R}^2$. Suppose that $\mathbf{b}$ can be written as a linear combination of $\mathbf{u}$ and $\mathbf{v}$ as $c\mathbf{u} + d\mathbf{v} = \mathbf{b}$.

   (a) Write two equations in c and d corresponding to the vector equation $c\mathbf{u} + d\mathbf{v} = \mathbf{b}$.

   (b) Solve the equations in part (a) for c and d.

   (c) Express $\mathbf{b}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$ (if possible).

4. Determine whether the set of all linear combinations of the following set of vector in $\mathbb{R}^3$ is **a line or a plane or all of** $\mathbb{R}^3$. Justify, your answer.

   (a) $\{(0, 0, -4), (0, 2, 1), (1, 1, 0)\}$

   (b) $\{(1, 2, 0), (1, 1, 1), (3, 4, 2)\}$

   (c) $\{(2, -5, 3), (6, -15, 9), (-10, 25, -15)\}$