MATH 2418: Linear Algebra

Assignment# 2

Due :01/30, Tuesday before 11:59pm   Term :`Spring 2024`

[Last Name]	[First Name]	[Net ID]	[Lab Section]

Recommended Problems:(Do not turn in)
Sec 1.2: 1, 2, 5, 6, 7, 8, 12, 13, 14, 25, 28, 29, 31. Sec 1.3: 1, 2, 3, 4, 5, 6, 8, 9, 10, 11,14, 16, 18, 20, 21, 23.

1. Let $\mathbf{u} = (3, -2, 1)$ and $\mathbf{v} = (-1, 1, 1)$ be two vectors in $\mathbb{R}^3$.

   (a) Calculate the dot product $\mathbf{u} \cdot \mathbf{v}$. What does it say about the angle between $\mathbf{u}$ and $\mathbf{v}$?

   (b) Compute the lengths $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ of the vectors.

   (c) Compute $\cos\theta$, where θ $(0 \leq \theta \leq \pi)$ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (d) Find the unit vector $\hat{\mathbf{u}}$ in the direction of $\mathbf{u}$.

   (e) Find two vectors parallel to $\mathbf{v}$ with length 3.

2. Determine all real values q so that the angle between the vectors $\mathbf{u} = (q-3, 1+3q)$ and $\mathbf{v} = (1+3q, q-3)$ is $\frac{\pi}{3}$.

3. Given a 3×4 matrix $A = \begin{bmatrix} 7 & 0 & -2 & 2 \\ 1 & 5 & 2 & 0 \\ 0 & 2 & -1 & -4 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 2 \end{bmatrix}$, calculate $A\mathbf{b}$

(a) as a linear combination of columns of A

(b) with the entries as dot products of rows of A with vector $\mathbf{b}$.

4. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$,

(a) write the linear system corresponding to the matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) solve the linear system.

(c) Write the vector $\mathbf{b}$ as a linear combination of the columns of the matrix A.

5. (a) Are the columns of the matrix $A = \begin{bmatrix} 3 & -2 & -1 \\ 2 & -3 & -4 \\ -1 & -2 & -5 \end{bmatrix}$ linarly dependent or independent? Justify your answer. Is the column space of the matrix A the single point $(\mathbf{0}, \mathbf{0}, \mathbf{0})$ in $\mathbf{R}^3$, a line in $\mathbf{R}^3$ through the zero verctor, a plane in $\mathbf{R}^3$ through the zero vector or the whole space $\mathbf{R}^3$?

(b) Are the columns of the matrix $B = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 3 & -4 \\ -1 & -2 & -1 \end{bmatrix}$ linearly dependent or independent? Justify your answer. Is the column space of the matrix B the single point $(\mathbf{0}, \mathbf{0}, \mathbf{0})$ in $\mathbf{R}^3$, a line in $\mathbf{R}^3$ through the zero verctor, a plane in $\mathbf{R}^3$ through the zero vector or the whole space $\mathbf{R}^3$?

6. (a) Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be three linearly independent vectors, and α, β, γ be any three nonzero real numbers. Prove that the vectors $\alpha\mathbf{u}$, $\beta\mathbf{v}$ and $\gamma\mathbf{w}$ are also linearly independent.

   (b) Is the following set of vectors linearly independent or dependent? If it is linearly dependent, find a linear dependence relation. For each vector in the set, find whether it lies in the set spanned by the other vectors.

   (i) $\left\{ \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}.$

   (ii) $\left\{ \mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}.$

   (c) **T or F and justify your answer**. If $\mathbf{x}$, $\mathbf{y} \in \mathbf{R}^3$ and $\mathbf{x}$ is not a multiple of $\mathbf{y}$, then the set $\{\mathbf{x}, \mathbf{y}\}$ is independent.