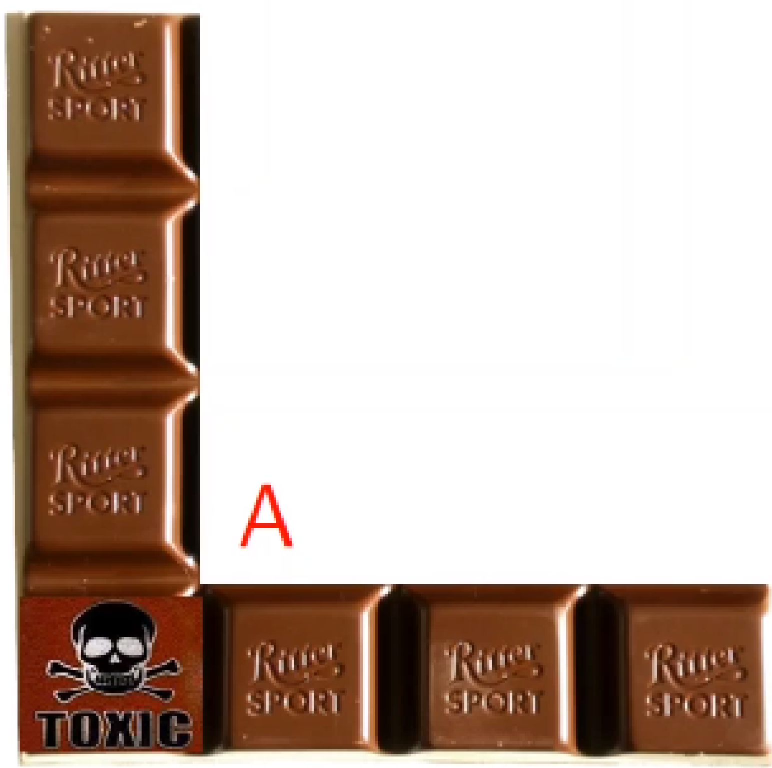
Chocolate Game - Chomp

- First-Player Win
- quadratic board
  - 

  - 

    - optimal strategy
        - take piece top right of the toxic piece
        - creates two independent fields
        - Tweedledum-Tweedledee-Principle
            - first player copies moves of second player
- rectangle board (of arbitrary size)
    - draws are not possible
        - must be a first or second player win
    - assuming A does not have a winning strategy
        - A can just take the top right piece
        - B makes a winning move
        - A could have just started with the move B just made
            - strategy stealing
        - contradiction
            - A must have a winning strategy for every possible game board size
            - First-Player Win
            - A's winning strategy exists but is unknown
                - for general board sizes

Tic Tak Toe

- [[2 Player Combinatorial Game]]
- no winner if played optimally
- [[Min-Max Decision Tree]]
    - Storing a board:
      2 bit per square:
      $2 \times 9 = 18$ bit, thus $2^{18} = 262144$ possible boards.
      3 possibilities per square:
      $3^9 = 19683$ possible boards with $\lceil \log_2 3^9 \rceil = 15$ bit.

n half-moves	game-tree	different boards
0	1	1
1	9	3
2	72	12
3	504	38
4	3024	108
5	15120	174
6	60480	228
7	181440	174
8	362880	89
9	362880	23
sum	986410	850

- 986410 = game-tree complexity
- $262144 = 2^{18}$
- $19683 = 3^9$
- 850 different boards = state space complexity

   * only 765 states when stopping after winning

Nine Men's Morris - Mühle

- http://ninemensmorris.ist.tugraz.at:8080/
- 3 phases
  - placing stones
  - moving stones
    * allowed along the lines
  - moving stones
    * jumping allowed
- 3 stones along a line
  - choose opponent's stone to remove
- draw if played optimally
- operations to combine equivalent game states

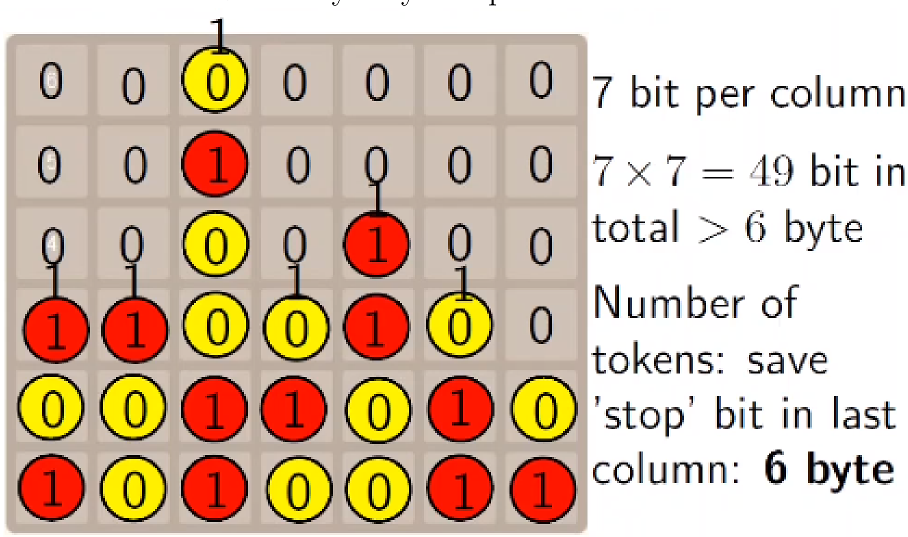
  Pólya-Redfield Enumeration Theorem: 16 Operations:
  R_0: ID: $r_0 = \binom{24}{2} \times 22 = 6072$
  R_1 Rotation $90°$ (R_3 Rotation $270°$): $r_1 = r_3 = 0$
  R_2 Rotation $180°$: $r_2 = 0$
  $R_4 \ldots R_7$ Reflections: $r_4 = \ldots = r_7 = 6 \times (9 + \binom{5}{2}) = 114$
  R_8: In-Out Inversion: $r_8 = 8 \times (8 + \binom{7}{2}) = 232$
  $R_9 \ldots R_{15}$: In-Out-Inversion plus $R_1 \ldots R_7$
  $r_9 = r_{10} = r_{11} = 0$
  $r_{12} = \ldots = r_{15} = 2 \times 11 = 22$
  Number of orbits$=\frac{6072+4\times114+232+4\times22}{16} = \frac{6848}{16} = 428$

  $24 * 23 * 22 =$ 12144 games

Connect 4

- http://connect4.ist.tugraz.at:8080/

- First-Player Win
- states (7x6 board)
  - 0 to 42 fields which have a
    - yellow token
    - red token
    - no token
  - For each column from above: write 0 for each empty field, then a 1 befor the first non-empty field. Starting from there write 0 for a yellow token, and 1 for a red token.
  - 7 bit per column
    - $7 * 7 = 49$ bit require 6 byte + 1 bit

    - first 1 acts as separator
      - marks the first token
      - afterwards only the color is stored
    - last separator is not needed
      - number of half moves = total number of tokens
      - count tokens in first 6 columns
      - tokens in last column = total number of tokens - tokens in first 6 columns
    - only store empty fields and colors without separator
      - saves 1 bit $\Rightarrow$ exactly 6 byte required
    - 

- move generator
  - up to 7 successors
  - add a token to a non-full column
- identify final states
  - draw
    - 42 tokens placed and no win

    - lose
        * check if previous player has won
    - win
        * check 11 4-tuples which include just placed token
        * fields above just placed token not considered
- hybrid approach
    - store first 23 half moves in DB
    - compute remaining decision tree online
    - maximum remaining search depth $42 - 23 = 19$
        * with ~5 possible moves on average

half-moves	different boards	half-moves	different boards	half-moves	different boards
0	1	8	91295	16	177841160
1	4	9	269531	17	363798195
2	25	10	809464	18	767435580
3	121	11	2148087	19	1448894267
4	568	12	5832236	20	2818993420
5	2144	13	14105207	21	4907390200
6	8231	14	35045629	22	8788132016
7	27109	15	77785047	23	14066554884
				sum	33475164421

    - 33475164421 states with 6 byte each: 200 GB + 34 GB