Cryptography 2: Encryption

Maria Eichlseder

Information Security – WT 2023/24

You Are Here

Crypto 1	Crypto 2	Crypto 3	Crypto 4
Symmetric Authentication	**Symmetric Encryption**	**Asymmetric Cryptography**	**Protocols and Applications**
Integrity	Confidentiality	Establishing communication	Theory meets Practice
▪ Hash functions ▪ MACs (Message Authentication)	▪ AEAD (Auth. Encryption) ▪ Symmetric primitives	▪ Key exchange ▪ Signatures ▪ Asymmetric primitives	▪ Protocols ▪ Applications

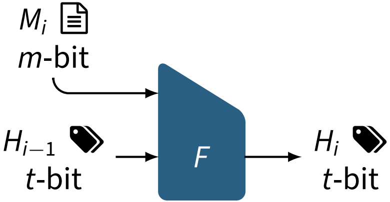
Recap of Last Week (1): Schemes for Message Authentication

Cryptographic schemes for message authentication compute a short, fixed-length Tag T from the Message M and (in some cases) a Key K.

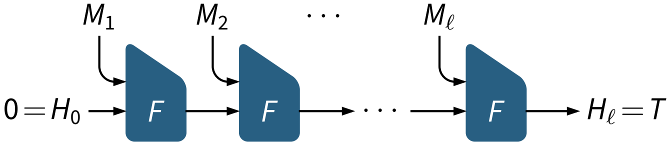
Hash Function $\mathcal{H}$	MAC $\mathcal{H}_{K_{AB}}$	Signature $\mathcal{S}_{K_A}$
Unkeyed	Symmetric Key K_{AB}	Asymmetric Key K_A
Anyone can compute T	A, B can compute T	A can compute T
Anyone can verify T	A, B can verify T	Anyone can verify T

Recap of Last Week (2): Merkle–Damgård Hashing

Primitive: Compression Function F (fixed-size inputs)



Mode: Merkle–Damgård (MD) Hash Function $\mathcal{H}(M) = T$ (variable-size inputs)



Outline

- Confidentiality
  - Goals and Applications
- Symmetric Primitives
  - Block Ciphers
  - The AES
- Encryption
  - Definition and Security
  - Constructions
- Authenticated Encryption
  - Definition
  - Constructions

Confidentiality

Introduction

Confidentiality

Confidentiality of Data

Prevent unauthorized entities from learning information (messages, data) that authorized parties are communicating or processing.

There are several related, but different concepts:

- Anonymity: The users' identity is unknown, they are not identifiable within a certain set of users
- Privacy: The users are able to seclude themselves, or information about themselves, and thereby express themselves selectively. This often refers to sensitive personal information.

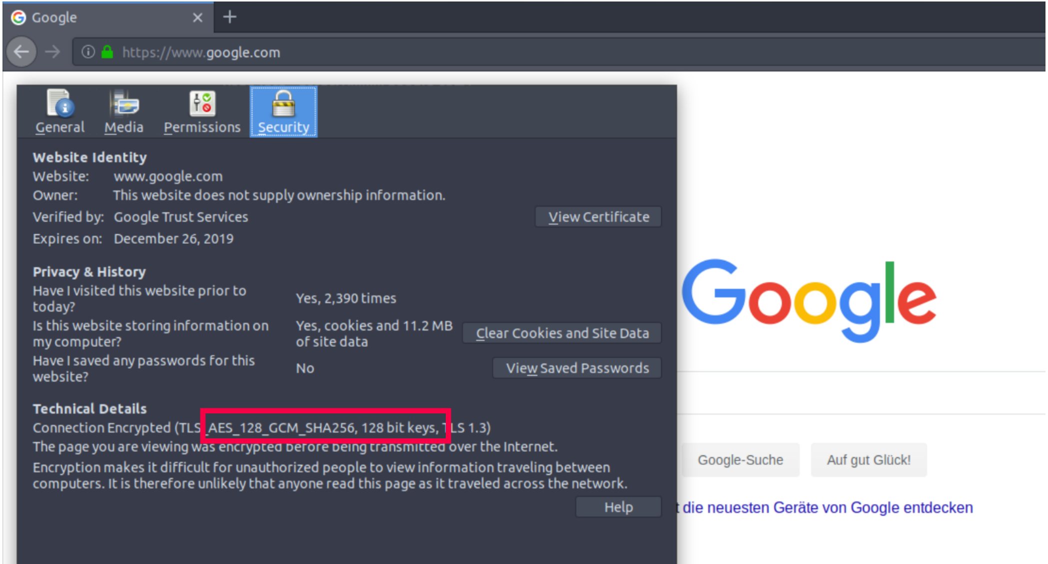
Cryptographic Schemes for Encryption

Encryption schemes transform a plaintext Message M of arbitrary length to a Ciphertext C of about the same length based on a Key K of fixed length.

Schemes may require additional inputs or produce an authentication Tag T.

Encryption $\mathcal{E}_{K_{AB}}$	Auth. Enc. $\mathcal{AE}_{K_{AB}}$	Key Encapsulation
Symmetric Key K_{AB}	Symmetric Key K_{AB}	Asymmetric Keys
Confidentiality only	Confid. + Authenticity	Confidentiality
A, B can encrypt	A, B can encrypt + auth	Anyone can encrypt
A, B can decrypt	A, B can decrypt + verify	A can decrypt

Examples (1): Secure Communication with HTTPS

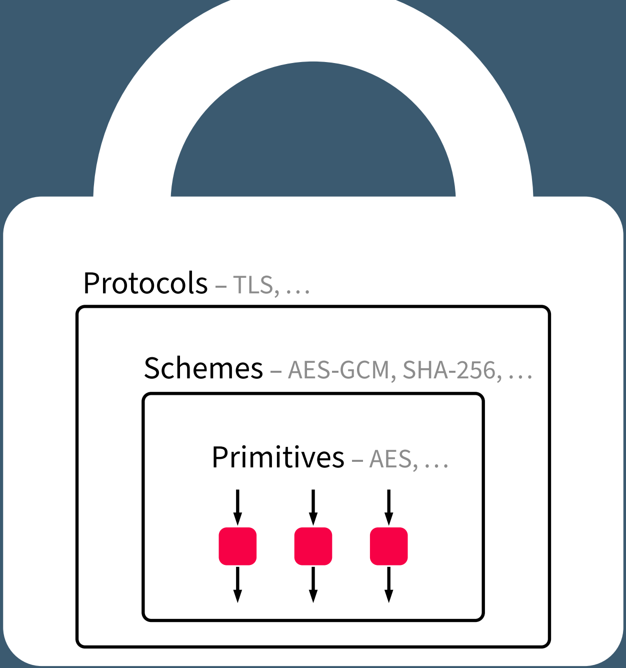


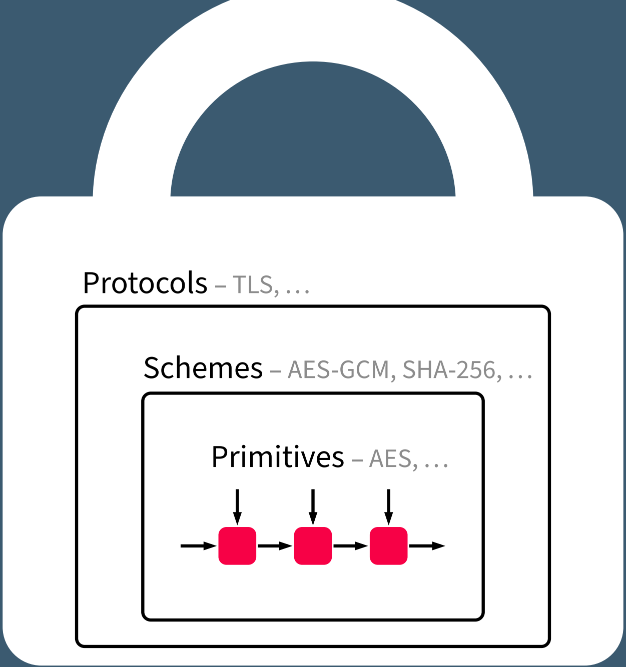
Example (2): Disk Encryption with LUKS

```
meichlseder@x1tblme ~ % sudo cryptsetup luksDump /dev/nvme0n1p3
[sudo] password for meichlseder:
LUKS header information
Version:        2
Epoch:          4
Metadata area:  16384 [bytes]
Keyslots area:  16744448 [bytes]
UUID:           087c56a9-a282-42f2-8361-869ec488e61e
Label:          (no label)
Subsystem:      (no subsystem)
Flags:          (no flags)

Data segments:
  0: crypt
        offset: 16777216 [bytes]
        length: (whole device)
        cipher: aes-xts-plain64
        sector: 512 [bytes]

Keyslots:
  0: luks2
        Key:        512 bits
        Priority:   normal
        Cipher:     aes-xts-plain64
        Cipher key: 512 bits
        PBKDF:      argon2i
        Time cost:  3
        Memory:     1048576
        Threads:    4
        Salt:       81 0d d7 18 01 e4 1d d9 6c 14 68 08 95 f5 f4 73
                    fc 8c 32 9a 4e 94 a0 aa 23 91 6b 2a 6d 66 51 13
        AF stripes: 4000
        AF hash:    sha256
```


Protocols – TLS, . . .
Schemes – AES-GCM, SHA-256, . . .
Primitives – AES, . . .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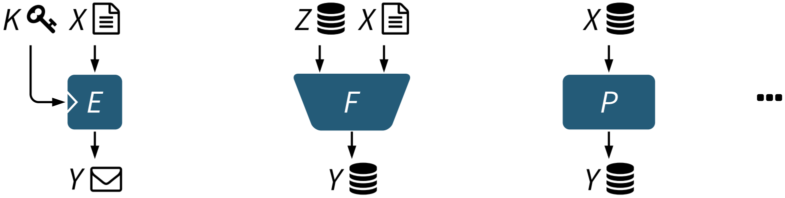
Symmetric Primitives

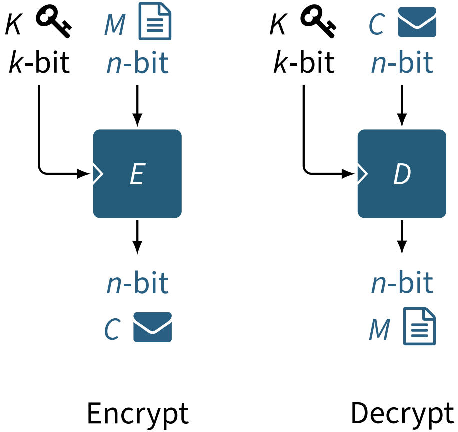
Secure Building Blocks

Symmetric Primitives



Block Cipher (BC) | Compression Function | Permutation

Block Ciphers – Key Space and Plaintext Space



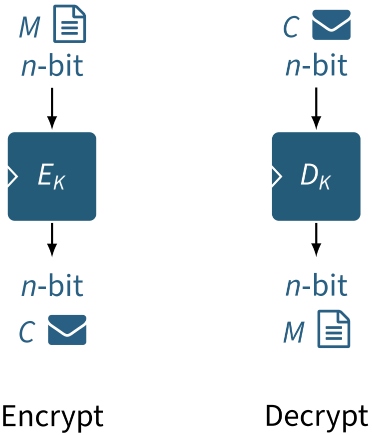
A block cipher is a family of permutations (bijective functions) E_K.

Each k-bit key K defines a permutation E_K that encrypts n-bit message blocks M to n-bit ciphertext blocks $C = E_K(M)$.

It also defines the inverse permutation $D_K = E_K^{-1}$ that maps C back to M.

- 2^n possible inputs/outputs M
- 2^k possible keys (mappings) K

Block Ciphers – Key Space and Plaintext Space



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- 2^n possible inputs/outputs M
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Block Ciphers – Security

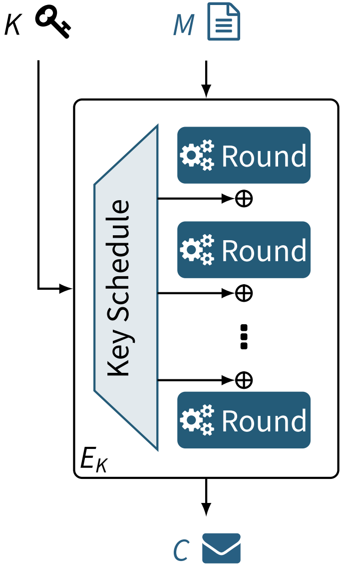
Pseudorandomness

An attacker must be unable to learn *M* from *C* (or vice-versa).

Key Recovery Security

An attacker must be unable to recover *K*, even if they can obtain ciphertexts *C* for any messages *M* of their choice (or vice-versa).

Anatomy of a Block Cipher – The Key-Alternating Construction



Two fundamental ideas:

1. Repeat simple circuit r times: the "round function"
   - Make it easy to implement
2. Make the round circuit public but XOR input with round key
   - Avoid key-dependent circuitry

The AES Competition (1997–2000)

- AES – Advanced Encryption Standard
- Goals: A block cipher to replace DES
  - The previous Data Encryption Standard (DES) was co-designed by NSA
  - Its security level was no longer adequate (small key, cryptanalysis)
- Organized by NIST (US Institute of Standards and Technology)
- Announced 1997, 15 submissions from 50 cryptographers
- Winner: `Rijndael/AES`, designed by Joan Daemen and Vincent Rijmen
  - Now used *everywhere* for secure encryption

AES – State and Operations

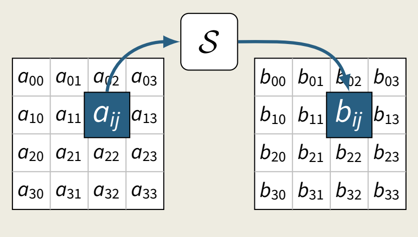
- Block size $n = 128$ bits
- Key size $k \in \{128, 192, 256\}$ bits $\rightarrow$ ciphers AES-128, AES-192, AES-256
- The 16-byte input block $M = s_{00} \| s_{10} \| s_{20} \| s_{30} \| s_{01} \| \ldots \| s_{33}$ is written as a 4×4 matrix of bytes, the $\{16, 24, 32\}$-byte key K as a $4 \times \{4, 6, 8\}$ matrix:

$$M = \begin{array}{|c|c|c|c|} \hline s_{00} & s_{01} & s_{02} & s_{03} \\ \hline s_{10} & s_{11} & s_{12} & s_{13} \\ \hline s_{20} & s_{21} & s_{22} & s_{23} \\ \hline s_{30} & s_{31} & s_{32} & s_{33} \\ \hline \end{array}, \qquad K = \begin{array}{|c|c|c|c|c|c|c|c|} \hline k_{00} & k_{01} & k_{02} & k_{03} & k_{04} & k_{05} & k_{06} & k_{07} \\ \hline k_{10} & k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} \\ \hline k_{20} & k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} \\ \hline k_{30} & k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} \\ \hline \end{array}$$

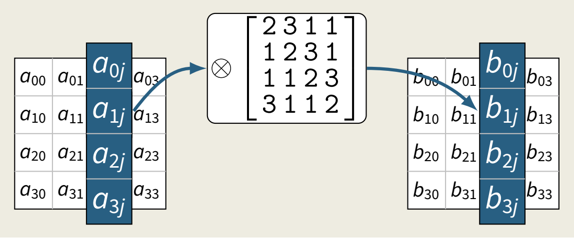
- The state is initialized to M and updated in 10 rounds (for AES-128) or 12 rounds (AES-192) or 14 rounds (AES-256).

AES Round Function – Overview

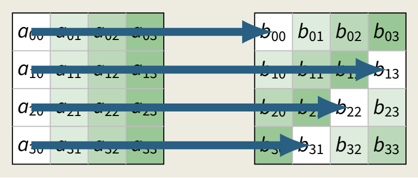
1 SubBytes (SB)



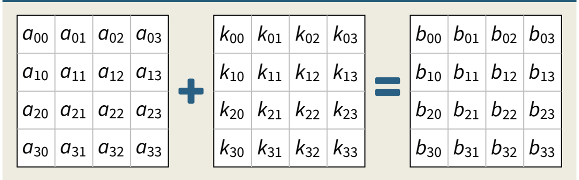
3 MixColumns (MC)



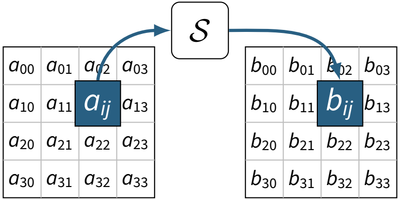
2 ShiftRows (SR)



4 AddRoundKey (AK)

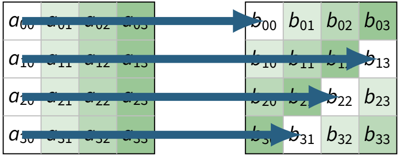


AES Round Function – 1 SubBytes (SB)



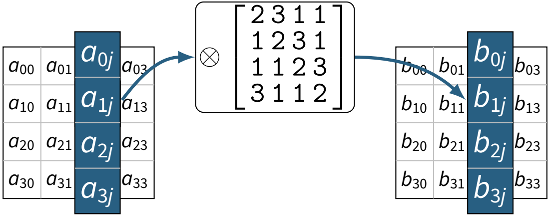
- S-box layer: $b_{ij} = \mathcal{S}[a_{ij}]$
- Each of the 16 state bytes a_{ij} is substituted using an 8-bit lookup table
  $\mathcal{S}[\texttt{0x00}] = \texttt{0x63}, \quad \mathcal{S}[\texttt{0x01}] = \texttt{0x7C}, \quad \mathcal{S}[\texttt{0x02}] = \texttt{0x77}, \quad \ldots, \quad \mathcal{S}[\texttt{0xFF}] = \texttt{0x16}$
- The S-box $\mathcal{S}$ has strong cryptanalytic properties to defend against attacks

AES Round Function – 2 ShiftRows (SR)



- Part of the linear layer: $b_{i,j} = a_{i,j+i\,\%\,4}$
- Each row i of the state is rotated to the left by i bytes
- The values of one column are shifted to four different columns

AES Round Function – 3 MixColumns (MC)



- Part of the linear layer: $(b_{0j}, b_{1j}, b_{2j}, b_{3j}) = M \cdot (a_{0j}, a_{1j}, a_{2j}, a_{3j})$
- Each column of the state is updated using a multiplication with a matrix M
(this multiplication is over a “finite field”, not normal integer multiplication!)
- If one byte at the input changes, all output bytes in the column will change
- This step is omitted in the last round

AES Round Function – 4 `AddRoundKey` (AK)

Play with it here

$$\begin{array}{|c|c|c|c|}\hline a_{00} & a_{01} & a_{02} & a_{03} \\ \hline a_{10} & a_{11} & a_{12} & a_{13} \\ \hline a_{20} & a_{21} & a_{22} & a_{23} \\ \hline a_{30} & a_{31} & a_{32} & a_{33} \\ \hline \end{array} + \begin{array}{|c|c|c|c|}\hline k_{00} & k_{01} & k_{02} & k_{03} \\ \hline k_{10} & k_{11} & k_{12} & k_{13} \\ \hline k_{20} & k_{21} & k_{22} & k_{23} \\ \hline k_{30} & k_{31} & k_{32} & k_{33} \\ \hline \end{array} = \begin{array}{|c|c|c|c|}\hline b_{00} & b_{01} & b_{02} & b_{03} \\ \hline b_{10} & b_{11} & b_{12} & b_{13} \\ \hline b_{20} & b_{21} & b_{22} & b_{23} \\ \hline b_{30} & b_{31} & b_{32} & b_{33} \\ \hline \end{array}$$

- Key-alternating construction: $b_{ij} = a_{ij} \oplus k_{ij}^{(r)}$
- XOR the round key $k^{(r)}$ of round r to the state
- The round keys $k_{ij}^{(r)}$ are derived from the key K using the key schedule
  (details omitted – the key schedule uses similar operations to the round function)
- An additional `AddRoundKey` step happens before the first round

Symmetric Primitives – Conclusion

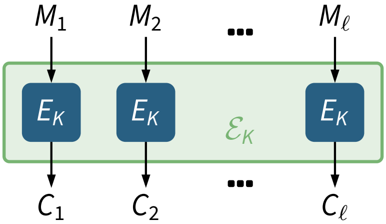
- Primitives are the foundation of security in symmetric cryptography
- Their security cannot be “proven”, but only “analyzed”

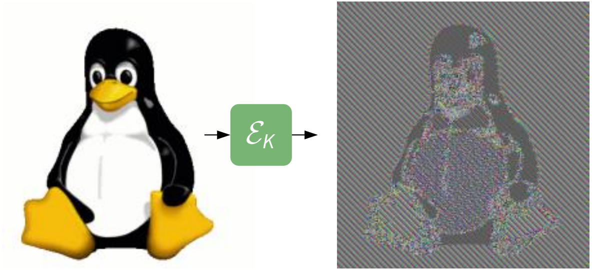
- Symmetric primitives in TLS 1.3:
  - $\texttt{AES}$-$\{128, 256\}$ block cipher
  - $\texttt{ChaCha20}$ stream cipher
  - $\texttt{SHA}$-$\{256, 512\}$ compression function
- All of these are expected to provide long-term security (also in a post-quantum world)

Encryption

Protecting Confidentiality

How NOT To Do It – The Electronic CodeBook mode (ECB)





Split M into blocks $M_1, M_2, \ldots, M_\ell$ and encrypt each block with block cipher E_K.

This simple mode has 2 major problems:

- Patterns: Two identical blocks M_i, M_j get encrypted to the same C_i, C_j
- Context: Two identical messages M, M' get encrypted to the same C, C'

Encryption Schemes – Definition

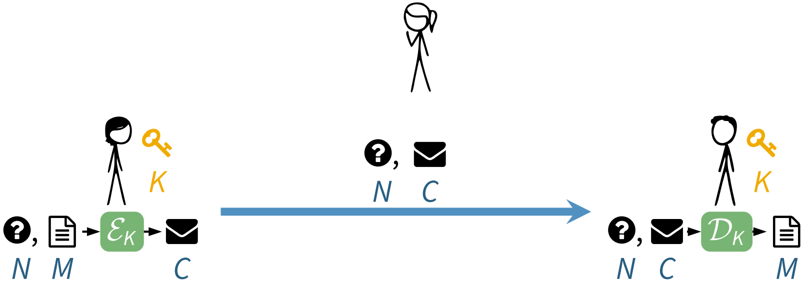
An encryption scheme is a keyed function $\mathcal{E}_K$ that maps a k-bit key K, n-bit nonce N, and a message M of arbitrary length to a ciphertext C, together with its inverse decryption function $\mathcal{D}_K$, to protect the confidentiality of M:

$$\begin{aligned} \mathcal{E}_K &: \mathbb{F}_2^k \times \mathbb{F}_2^n \times \mathbb{F}_2^* \to \mathbb{F}_2^*, \qquad & \mathcal{E}_K(N, M) &= C \\ \mathcal{D}_K &: \mathbb{F}_2^k \times \mathbb{F}_2^n \times \mathbb{F}_2^* \to \mathbb{F}_2^*, \qquad & \mathcal{D}_K(N, C) &= M \end{aligned}$$

The **nonce** (**n**umber used only **once**) makes sure that an adversary can't tell if two encrypted messages are the same! It is sometimes also called "IV" = Init. Vector.

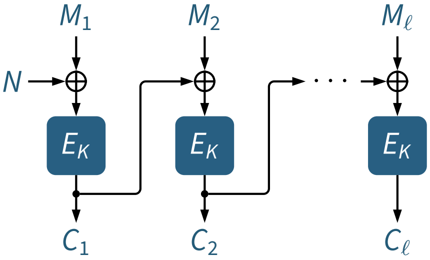
In practice, N can be randomly generated or (in some cases only!) a counter.

Encryption Schemes – Application



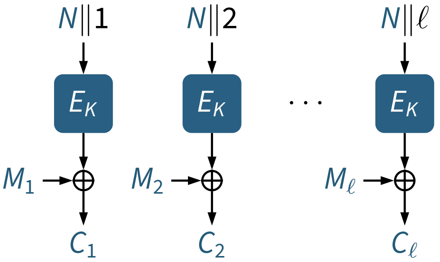
1. Alice computes $C = \mathcal{E}_K(N, M)$
2. Alice transmits N and C to Bob (over an insecure channel controlled by Eve)
3. Bob computes $M = \mathcal{D}_K(N, C)$

Cipher Block Chaining mode (CBC)



- **Goal**: C_i should depend on the "context", i.e., on blocks $M_1, \ldots, M_i$ and the nonce N.
- **Idea**: XOR $\oplus$ previous ciphertext block C_{i-1} (= chaining value) to msg block M_i, then encrypt with E_K
- **Idea**: Start with random (!) nonce N to hide repeated messages
- Must be combined with a suitable padding scheme for the message M.

CounTeR mode (CTR)



- **Goal**: C_i should depend on the "context", i.e., on block M_i, position i, and the nonce N.
- **Idea**: Create a streaming mode that produces a keystream depending on K, N and XOR it to M
- Nonce N can be random (unpredictable) or a counter (predictable), as long as it never repeats for the same K
- No padding needed, $\texttt{len}(C) = \texttt{len}(M)$

Encryption in Practice

- `CBC` and `CTR` provide only confidentiality, no authenticity
- There are VERY FEW applications that need pure (unauthenticated) encryption or where authenticated encryption doesn't fit.

  Example: some file system encryption schemes (no space for tags)
- Usually you instead want Authenticated Encryption!

Authenticated Encryption

Protecting Confidentiality and Authenticity

Authenticated Encryption – Goals

If your data is worth encrypting,
you almost certainly don’t want it modified!

- Confidentiality

  as provided by encryption modes $\mathcal{E}_K$

- Authenticity, integrity

  as provided by message authentication codes $\mathcal{H}_K$

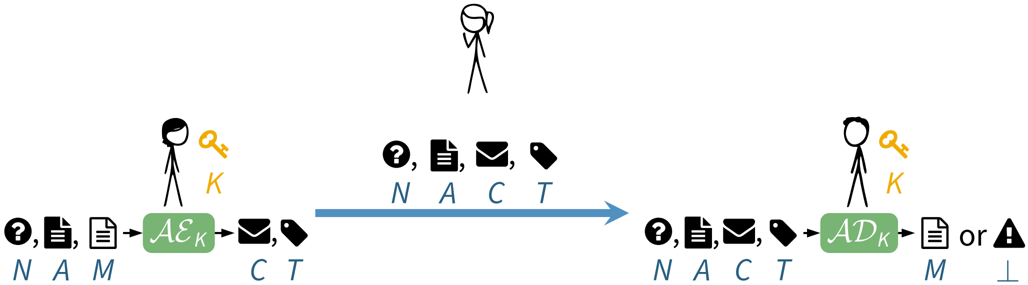
AEAD – Authenticated Encryption (with Associated Data)

An Authenticated Encryption scheme is a keyed function $\mathcal{AE}_K$ that maps a k-bit key K, n-bit nonce N, associated data A, and a message M of arbitrary length to a ciphertext C with attached tag T. Its inverse verified decryption function $\mathcal{AD}_K$ returns either the message M or, on invalid ciphertexts, an error $\perp$. AEAD protects

- the confidentiality and authenticity of message M.
- the authenticity of associated data A (e.g., metadata).

$$
\begin{aligned}
\mathcal{AE}_K &: \mathbb{F}_2^k \times \mathbb{F}_2^n \times \mathbb{F}_2^* \times \mathbb{F}_2^* && \to \mathbb{F}_2^* \times \mathbb{F}_2^t, && \mathcal{AE}_K(N, A, M) = C, T \\
\mathcal{AD}_K &: \mathbb{F}_2^k \times \mathbb{F}_2^n \times \mathbb{F}_2^* \times \mathbb{F}_2^* \times \mathbb{F}_2^t && \to \mathbb{F}_2^* \cup \{\perp\}, && \mathcal{AD}_K(N, A, C, T) = M
\end{aligned}
$$

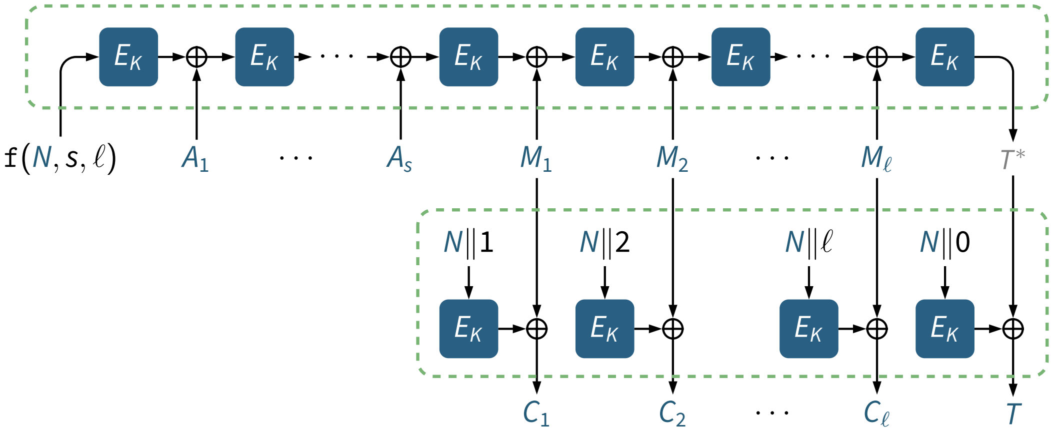
AEAD – Authenticated Encryption (with Associated Data)



Important:

- $\mathcal{AE}_K$: Nonce N must never repeat for the same K; a counter is usually ok
- $\mathcal{AD}_K$: (Parts of) Message M must never be released before verifying T

Example: CCM Mode – CTR encryption with CBC-MAC authentication



$A_1, \ldots, A_s$ and $M_1, \ldots, M_\ell$ are the blocks of the padded A and M.
$\mathtt{f}(N, s, \ell)$ encodes various parameters in one block (details here).

Popular Authenticated Ciphers

In TLS 1.3:

- `AES-GCM` (the TLS default), with `AES-`$\{128, 256\}$
- `AES-CCM`
- `ChaCha20-Poly1305` (not based on `AES`, uses `ChaCha20` stream cipher)

New NIST standard:

- `Ascon`

Conclusion

Conclusion

- Symmetric schemes protect data confidentiality and/or authenticity
- Their security builds on secure primitives by using a secure mode
- Confidentiality can be protected with
  - Encryption (⚠ no authenticity)
  - Authenticated Encryption
  - Asymmetric encryption, key encapsulation (next lecture)