

Example

Example 8 (Radar guns revisited)

Suppose $n = 4$ radar guns are set up along a stretch of road to catch people driving over the speed limit. Each radar gun is known to have a normal measurement error $N(0, \sigma^2)$, $\sigma = 5 \text{ km/h}$. For a car passing at speed μ four readings are (45.71, 47.41, 40.95, 50.65). Compute a random interval that covers the true unknown car speed μ with probability of 0.95.

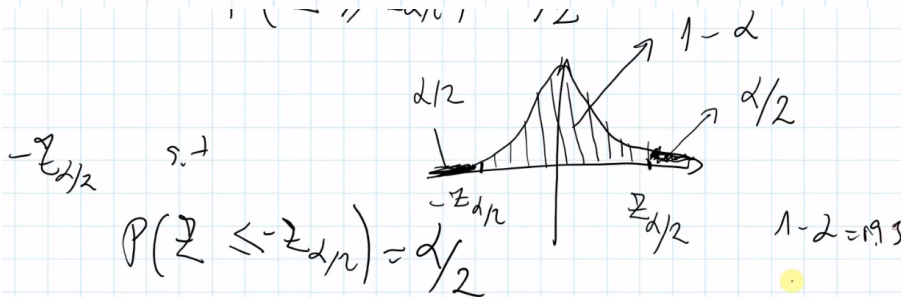
$$X_i \sim N(\mu, \sigma^2) \quad \bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$



$z_{\alpha/2}$ the z -value s.t.

$$P(Z \geq z_{\alpha/2}) = \alpha/2$$



$$P(Z \leq -z_{\alpha/2}) = \alpha/2$$

$$P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$$

Continuing with $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$:

$$P(-z_{\alpha/2} < \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} < z_{\alpha/2}) = 1 - \alpha$$

$$P(-z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \bar{X}_n - \mu < z_{\alpha/2} \frac{\sigma}{\sqrt{n}}) = 1 - \alpha$$

$$P(-\bar{X}_n - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < -\mu < -\bar{X}_n + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}) = 1 - \alpha$$

$$P(\bar{X}_n - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X}_n + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}) = 1 - \alpha$$

For our values, we get $\alpha = 0.05$, $z_{\alpha/2} = 1.96$, $\bar{X}_n = 46.18$ and the 95% confidence interval is (41.28, 51.08).

General [[Konfidenzintervall]] Definition

Let X_1, X_2, \dots, X_n be a random sample of size n from density $f(\cdot)$. A $1 - \alpha$ *confidence interval* for a parameter θ is an interval $C_n = (T_{n,1}, T_{n,2})$ with $T_{n,1} = t_1(X_1, X_2, \dots, X_n)$ and $T_{n,2} = t_2(X_1, X_2, \dots, X_n)$ such that:



$$P(\theta \in C_n) \geq 1 - \alpha.$$

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- C_n traps θ with probability $1 - \alpha$. **Important:** C_n is random, θ is fixed.