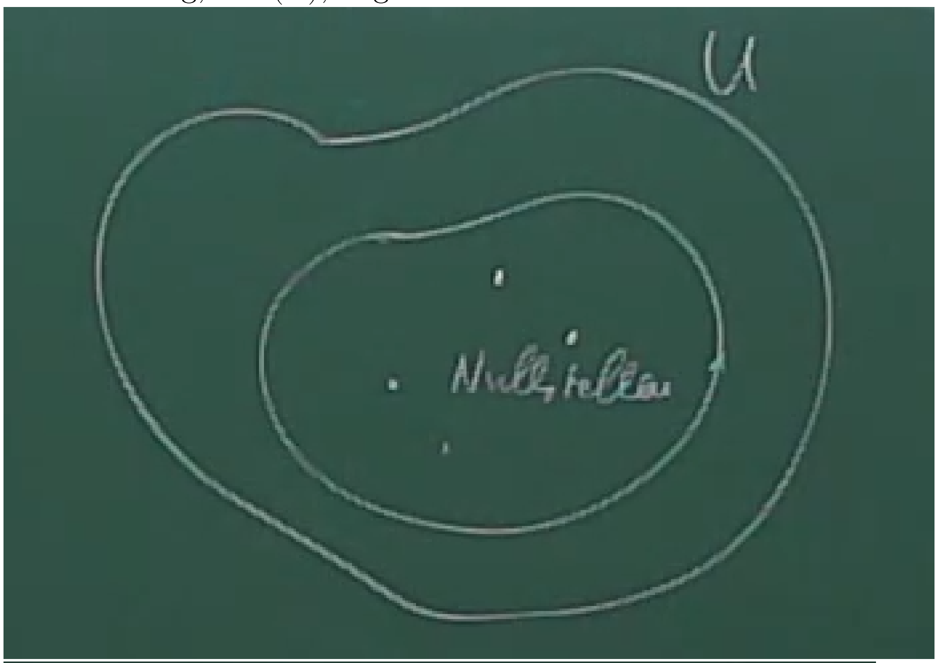
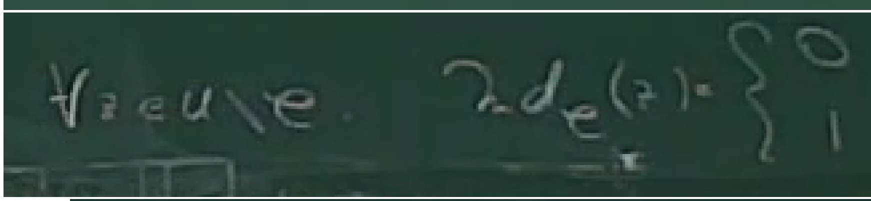
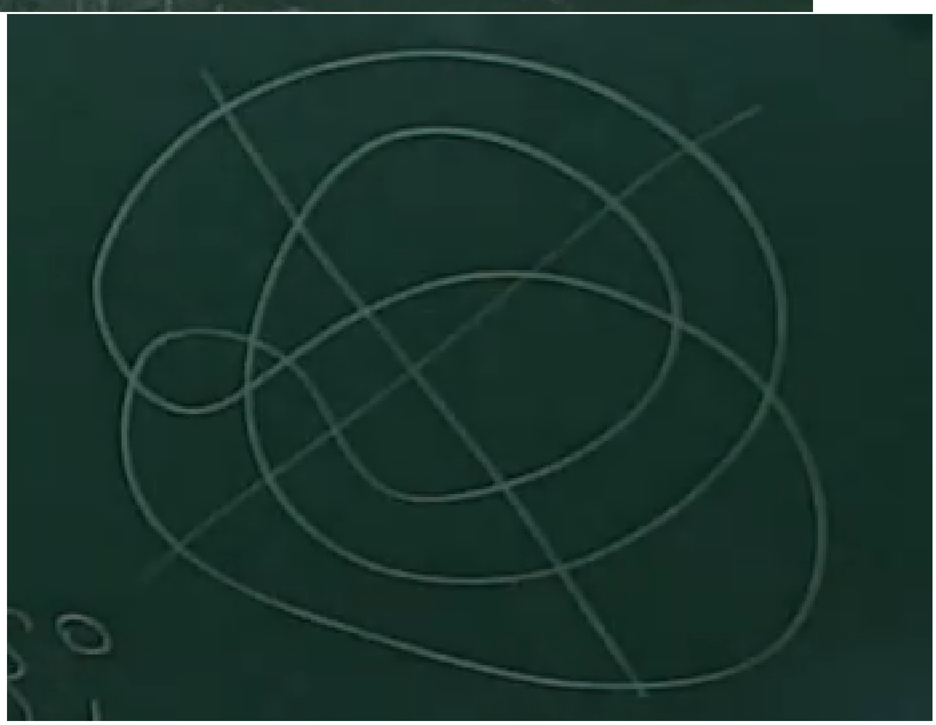
Zählen von Nullstellen

- gesucht # Nullstellen innerhalb von C (Vielfachheit gezählt)
- U sternförmig, f∈H(U), C geschlossen in U
- 

- 

  - 

Sei z_0 eine Nullstelle der Ordnung m von f. Dann gilt

$$f(z) = (z - z_0)^m g(z) \text{ mit } g(z_0) \neq 0.$$

Daraus ergibt sich

$$f'(z) = m(z - z_0)^{m-1} g(z) + (z - z_0)^m g'(z)$$

und weiter

$$\frac{f'(z)}{f(z)} = \frac{m}{z - z_0} + \frac{g'(z)}{g(z)},$$

also

$$\operatorname*{Res}_{z=z_0} \frac{f'(z)}{f(z)} = m = \operatorname{ord}_{z_0}(f).$$

Es gilt daher

$$\frac{1}{2\pi i} \oint_C \frac{f'(z)}{f(z)}\, dz = \operatorname{Ind}_{f(C)}(0) = \sum_n \operatorname{ord}_{z_n}(f) \operatorname{Ind}_C(z_n).$$

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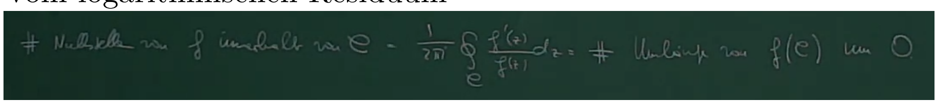
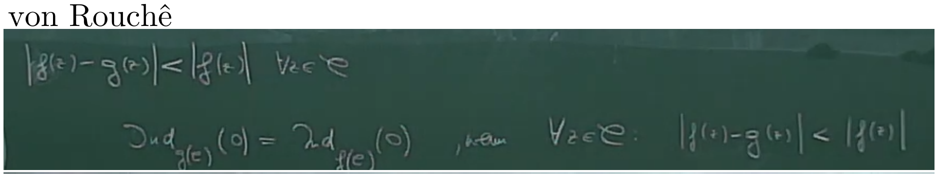
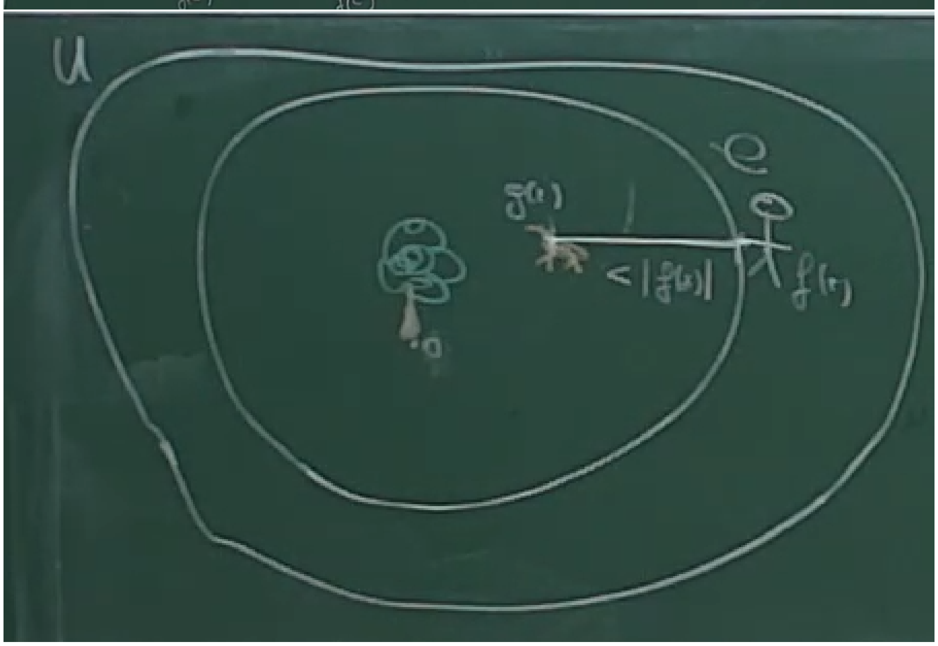
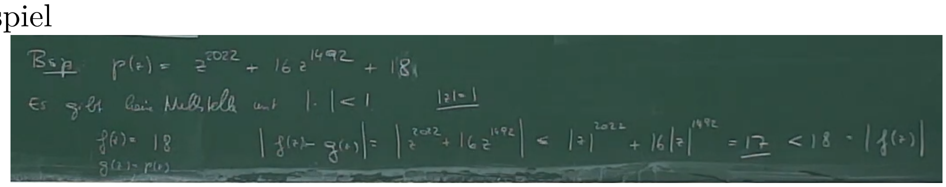
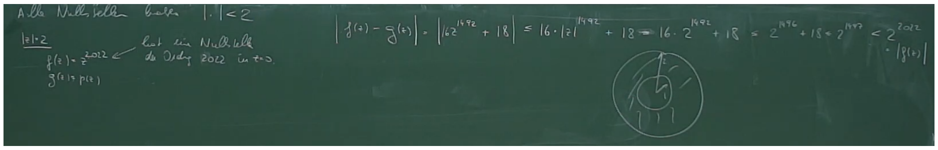
- Satz vom logarithmischen Residuum
  - 

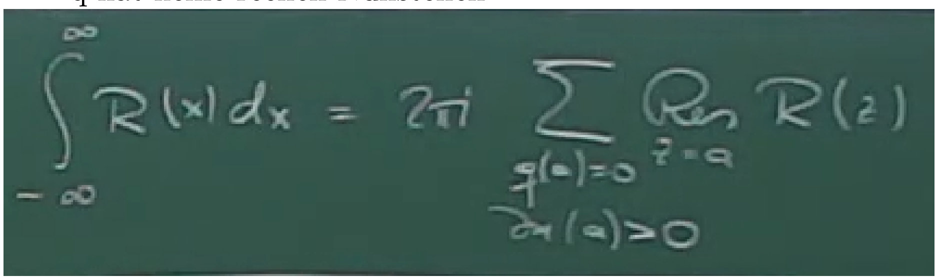
- Satz von Rouchê
  - 

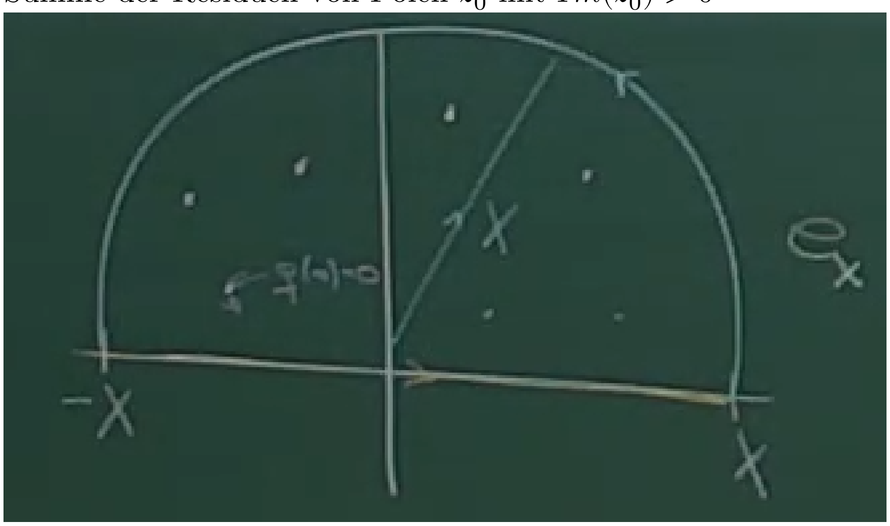
  - 

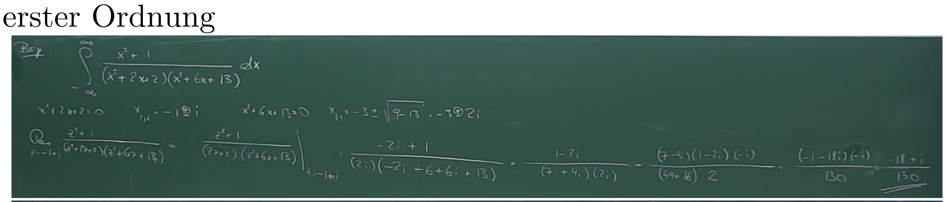
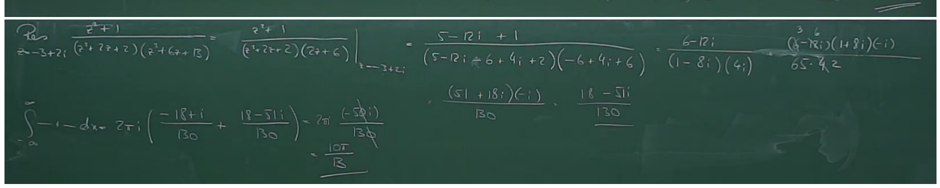
  - Beispiel
    * 

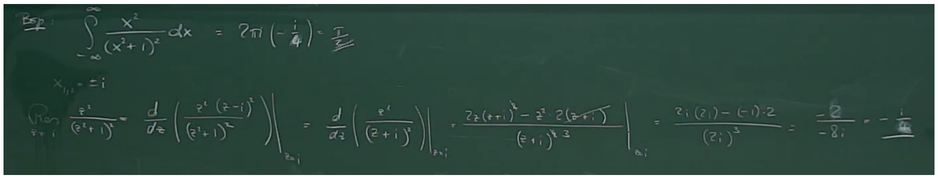
- * 

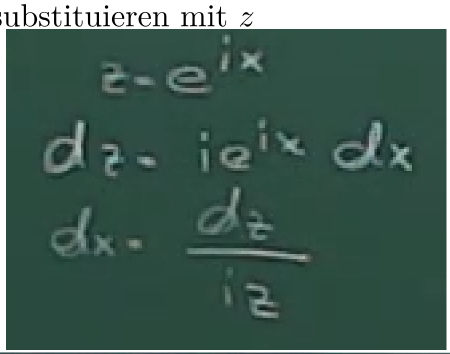
Bestimmung von Integralen

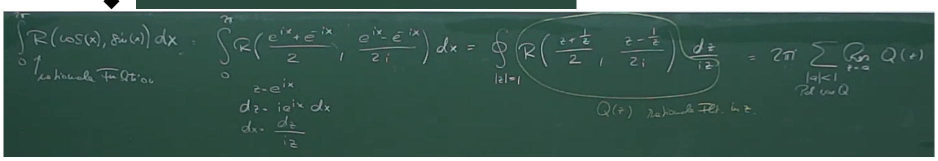
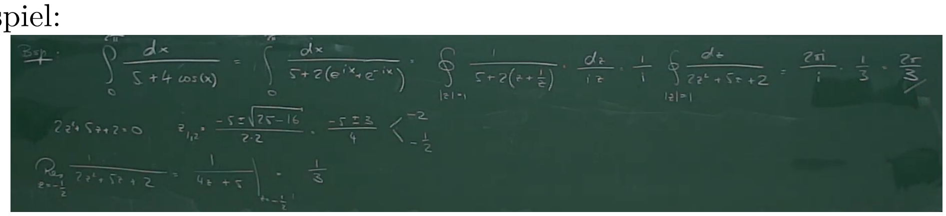
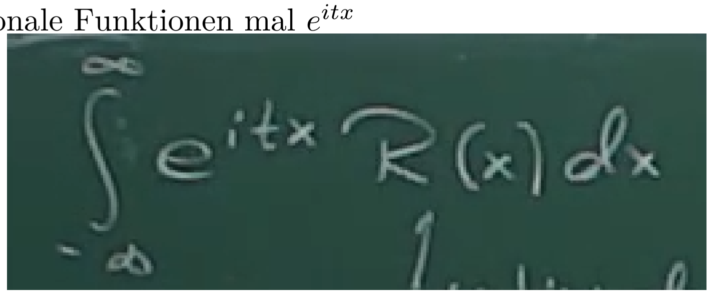
- rationale Funktionen
  - $R(x) = \frac{p(x)}{q(x)}$, für die gilt
    * p, q Polynome
    * $deg(p) <= deg(q) - 2$
    * q hat keine reellen Nullstellen
  - 

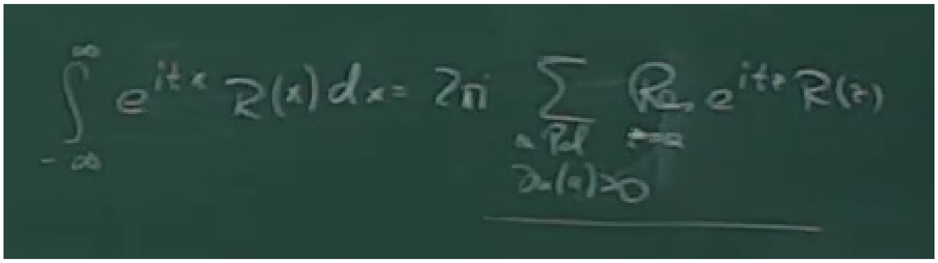
    * Summe der Residuen von Polen z_0 mit $Im(z_0) > 0$
    * 

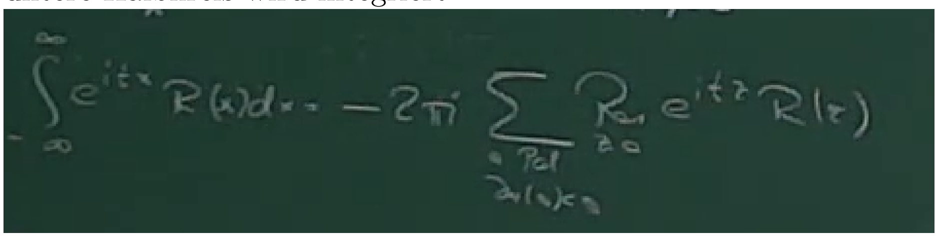
  - Beispiel:
    * Pol erster Ordnung
      - 
      - 
    * Pol höherer Ordnung

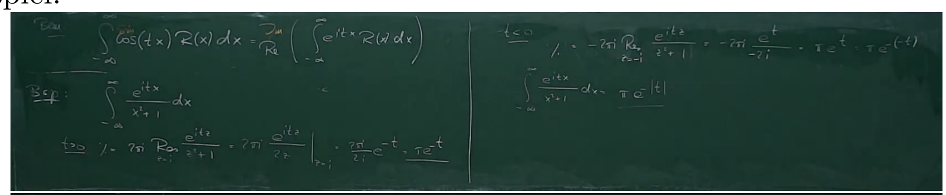
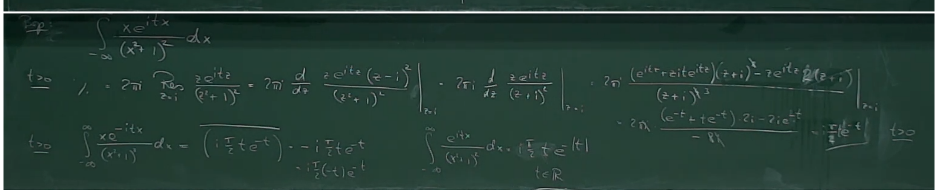
            - 

- rationale Funktionen in Winkelfunktionen
    - $R(cos(x), sin(x))$
        - Winkelfunktionen durch komplexe Äquivalent ersetzen
            - $cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$
            - $sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$
        - e^{ix} substituieren mit z
            - 

    - 
    - Beispiel:
        - 
- rationale Funktionen mal e^{itx}
    - 
    - $R(x)$ mit $deg(p) < deg(q) - 2$
    - $t = 0$
        - $e^{itx} = e^0 = 1$ ==> rationale Funktion ohne e^{itx}
    - $t > 0$
        - obere Halbkreis wird integriert

    - 

- $t < 0$
    - untere Halbkreis wird integriert
    - 

- R reell ==> Konjugation von Integral für $t > 0$ = Integral für $t < 0$
- Beispiel:
    - 
    - 

[[Laplace Transformation]]

[[Residuensatz]]