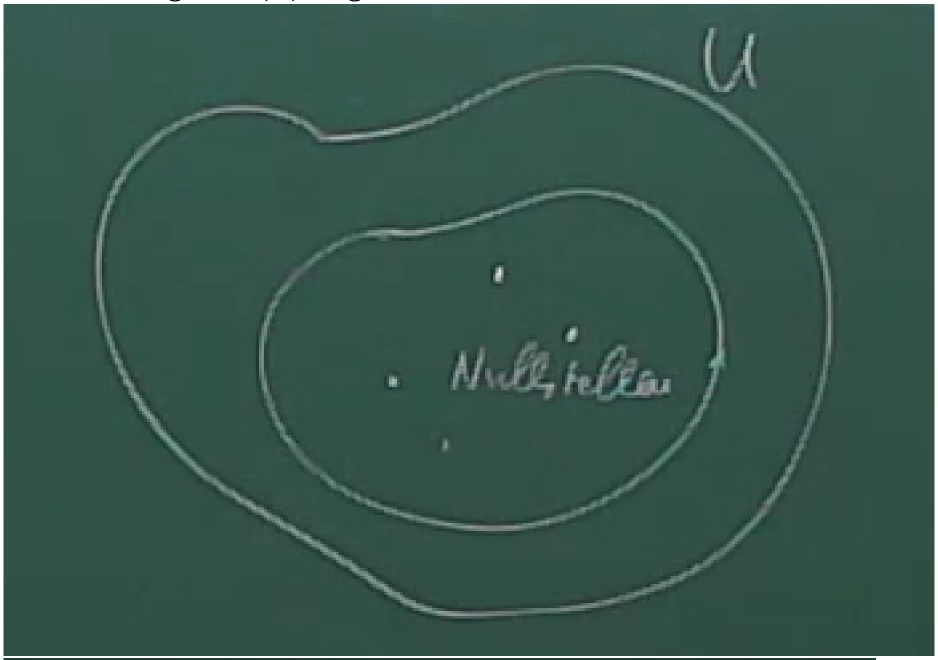
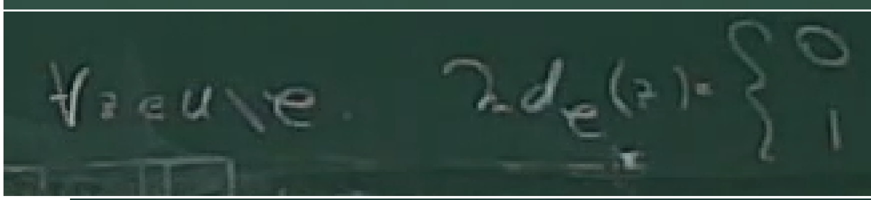
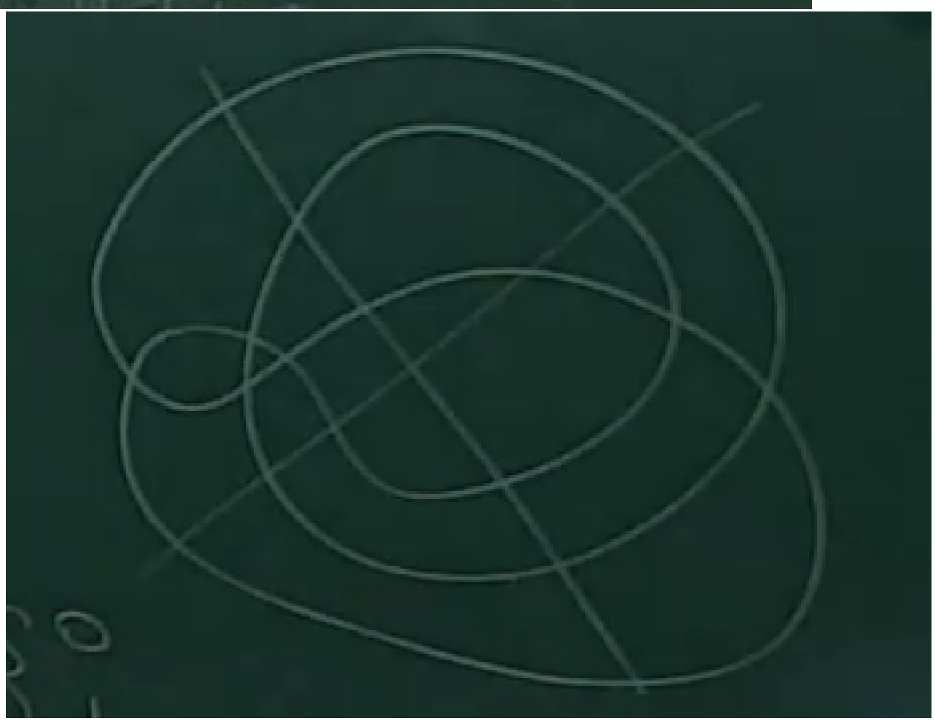
Zählen von Nullstellen

- gesucht # Nullstellen innerhalb von C (Vielfachheit gezählt)
- U sternförmig, f   H(U), C geschlossen in U
- 

- 

    - 

Sei z_0 eine Nullstelle der Ordnung m von f. Dann gilt

$$f(z) = (z - z_0)^m g(z) \text{ mit } g(z_0) \neq 0.$$

Daraus ergibt sich

$$f'(z) = m(z - z_0)^{m-1} g(z) + (z - z_0)^m g'(z)$$

und weiter

$$\frac{f'(z)}{f(z)} = \frac{m}{z - z_0} + \frac{g'(z)}{g(z)},$$

also

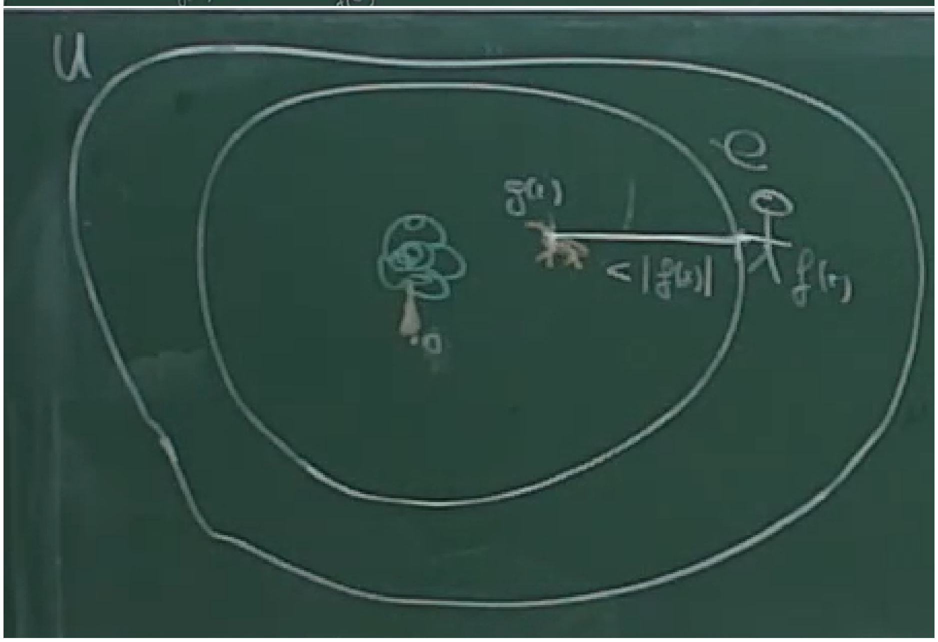
$$\operatorname*{Res}_{z=z_0} \frac{f'(z)}{f(z)} = m = \mathrm{ord}_{z_0}(f).$$

Es gilt daher

$$\frac{1}{2\pi i} \oint_C \frac{f'(z)}{f(z)}\, dz = \mathrm{Ind}_{f(C)}(0) = \sum_n \mathrm{ord}_{z_n}(f)\, \mathrm{Ind}_C(z_n).$$

-

- Satz vom logarithmischen Residuum
  - # Nullstelle von f innerhalb von $\mathcal{C}$ = $\frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{f'(z)}{f(z)}\,dz$ = # Umläufe von $f(\mathcal{C})$ um 0
- Satz von Rouchê
  - $|f(z) - g(z)| < |f(z)| \quad \forall z \in \mathcal{C}$

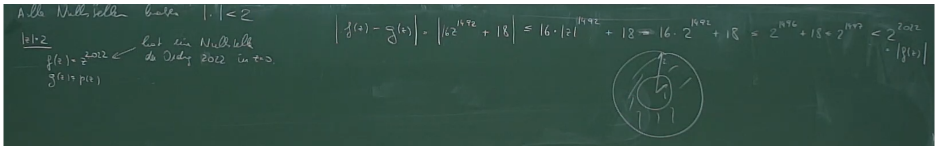
    $\mathrm{Ind}_{g(\mathcal{C})}(0) = \mathrm{Ind}_{f(\mathcal{C})}(0)$, wenn $\forall z \in \mathcal{C}: |f(z) - g(z)| < |f(z)|$
  - 

  - Beispiel
    * Bsp. $p(z) = z^{2022} + 16 z^{1492} + 18$

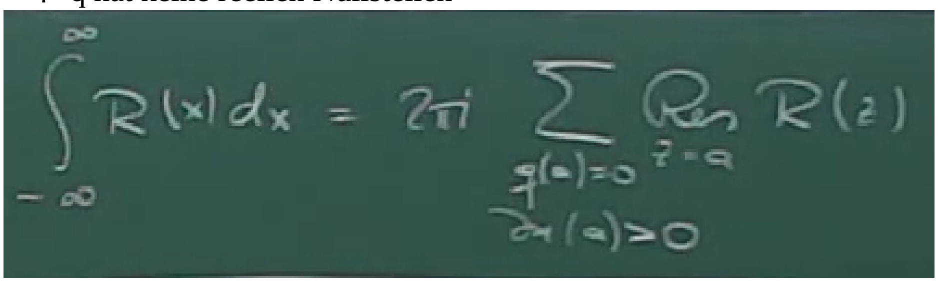
      Es gibt keine Nullstelle mit $|\cdot| < 1$ $\quad$ $|z| = 1$

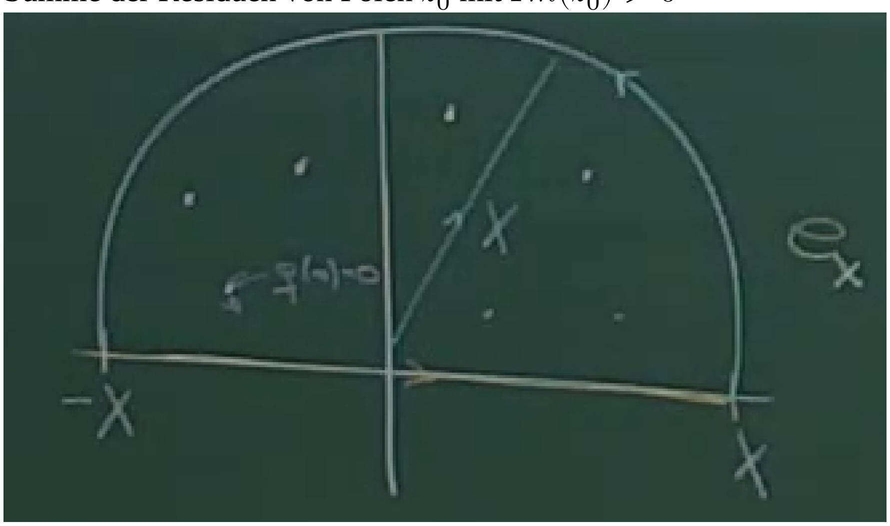
      $f(z) = 18$ $\quad$ $|f(z) - g(z)| = |z^{2022} + 16 z^{1492}| \leq |z|^{2022} + 16 |z|^{1492} = \underline{17} < 18 = |f(z)|$

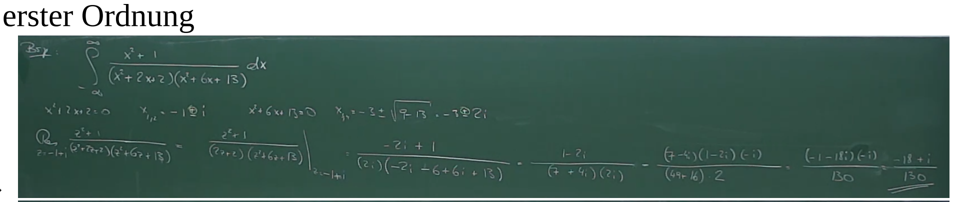
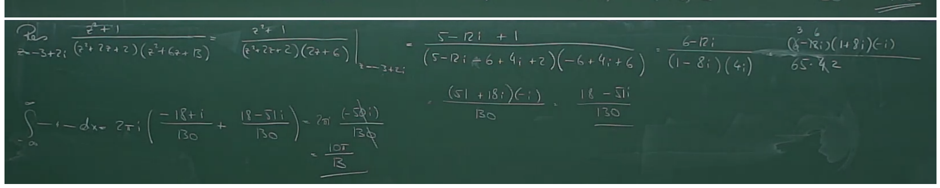
      $g(z) = p(z)$

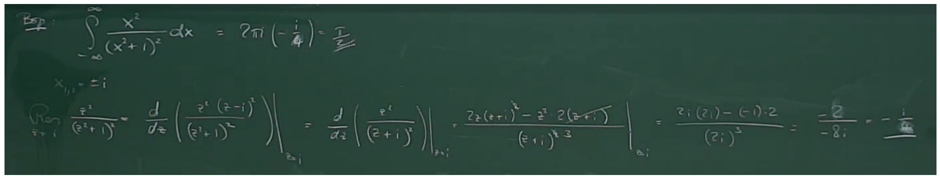
* 

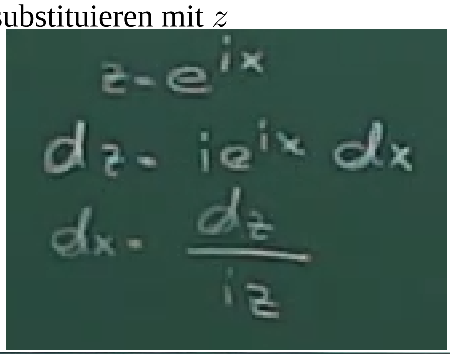
Bestimmung von Integralen

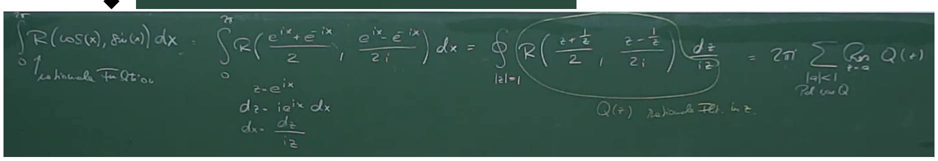
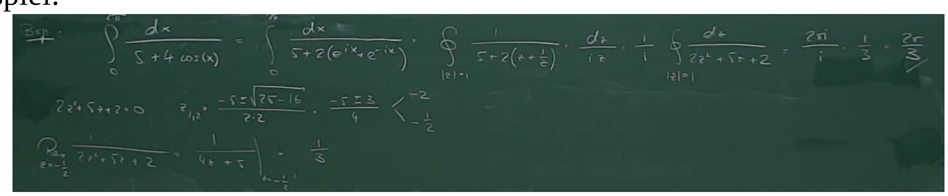
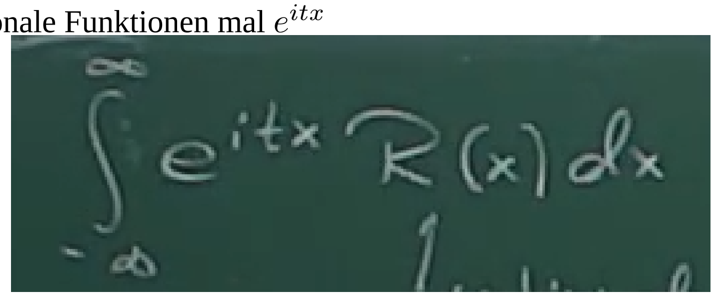
- rationale Funktionen
  - $R(x) = \frac{p(x)}{q(x)}$, für die gilt
    * p, q Polynome
    * $deg(p) <= deg(q) - 2$
    * q hat keine reellen Nullstellen
  - 

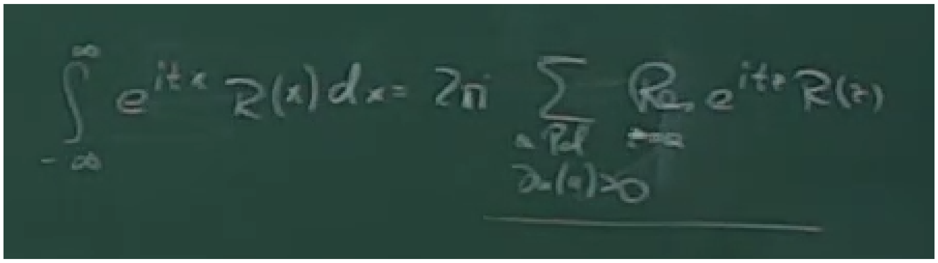
    * Summe der Residuen von Polen z_0 mit $Im(z_0) > 0$
    * 

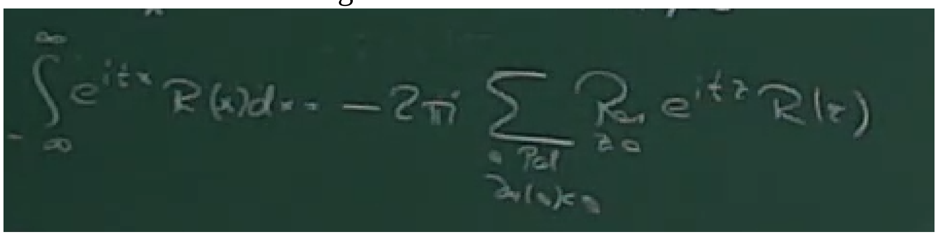
  - Beispiel:
    * Pol erster Ordnung
      - 
      - 
    * Pol höherer Ordnung

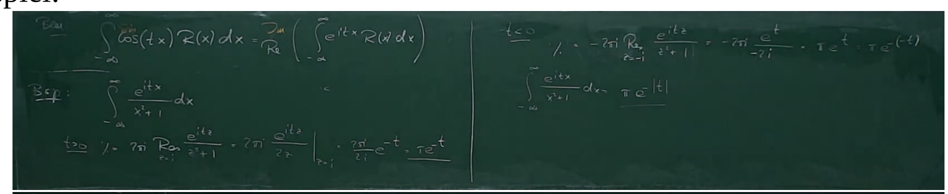
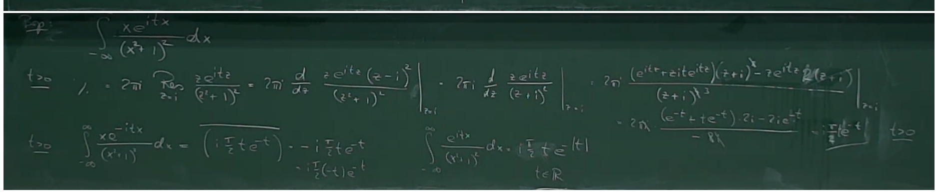
        - 

- rationale Funktionen in Winkelfunktionen
  - $R(cos(x), sin(x))$
    - Winkelfunktionen durch komplexe Äquivalent ersetzen
      - $cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$
      - $sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$
    - e^{ix} substituieren mit z
      - 

  - 
  - Beispiel:
    - 
- rationale Funktionen mal e^{itx}
  - 
  - $R(x)$ mit $deg(p) < deg(q) - 2$
  - $t = 0$
    - $e^{itx} = e^0 = 1$ ==> rationale Funktion ohne e^{itx}
  - $t > 0$
    - obere Halbkreis wird integriert

* 

- $t < 0$
  * untere Halbkreis wird integriert
  * 

- R reell ==> Konjugation von Integral für $t > 0$ = Integral für $t < 0$
- Beispiel:
  * 
  * 

[[Laplace Transformation]]

[[Residuensatz]]