- greedy [[Shortest Path Algorithms]] to find all destinations from one source
- For a start vertex $s$, compute shortest paths from $s$ to all $v \in V$ (tree structure + length).
- **Input:** A connected graph $G = (V, E, w)$ with non-negative edge weights $w(u, v)$ and a vertex $s \in V$.

  **Output:** The distances $d(s, v)$ in $G$ from $s$ to all vertices $v \in V$ and the tree with the according shortest paths.
  - Bellman Ford is an alternative for negative weights
- **Generic step**: Given a set $T$ of vertices where for all $v \in T$, $d(s, v)$ is already computed. Choose a vertex $u \in V \setminus T$ whose shortest path from $s$ "found so far" is minimal.

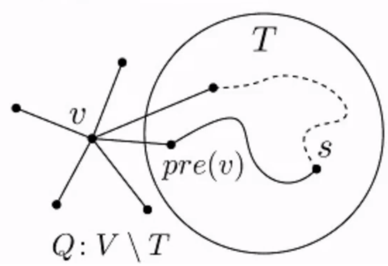
  Paths "found so far": paths that only go via vertices in $T$.

  For each vertex $v$, we maintain:

  $L(v)$: length of the shortest path from $s$ to $v$ "found so far".

  $pre(v)$: neighbor of $v$ in $T$ via which this shortest path goes.
  - similar to Prim's algorithm [[Minimum Spanning Tree]]
    * priority computation is different
  - $$L(v) = \begin{cases} d(s,v) & \text{if } v \in T \\ \infty & \text{if } v \text{ is not adjacent to } T \\ \text{shortest path from } s \text{ to } v \text{ via } T & \text{if } v \notin T,\ v \text{ adjacent to } T \end{cases}$$

    A **priority queue** $Q$ contains *all vertices that are not yet in* $T$, organized by their $L$-values (for example a min-heap; initially contains all vertices).

    
    
- pseudo code
  - **for** all $v \in V$ **do** $L(v) = \infty$ **od**<br>$L(s) = 0$; $pre(s) = nil$<br>$Q = V$ // build up $Q$<br>**while** $Q \neq 0$ **do**<br>  $u$ = MIN($Q$)<br>  remove $u$ from $Q$ // reorganize $Q$<br>  **for** all $v \in A(u)$ **do** // $A$: adjacency list of $G$<br>    **if** $L(v) > L(u) + w(u, v)$ **then**<br>      $L(v) = L(u) + w(u, v)$ // reorganize $Q$<br>      $pre(v) = u$

- **Runtime analysis** for graph with $n$ vertices and $m$ edges:
  - Min-heap with $n$ elements:
    - $\Theta(n)$ time for initialization $Q = V$.
    - $O(\log n)$ time for removal of the minimum.
    - $O(\log n)$ time per update of an $L$-value.
  - Processing vertex $u$ with $\deg(u)$ neighbors: removal of $u$ from $Q$ plus $O(\deg(u))$ updated $L$-values.

  $\Rightarrow$ Runtime in total for start vertex $s$:
  $\Theta(n) + \sum_{u \in V}(1 + \deg(u)) \cdot O(\log n)$
  $= \Theta(n) + \Theta(n + m) \cdot O(\log n) = O(m \log n)$,
  since the graph is connected.
- runtime may improve if Q is sorted
  - For dense graphs ($m = \Theta(n^2)$) the algorithm needs $\Theta(n^3 \log n)$ time to compute the distance matrix.

    If an unsorted list is used for the queue $Q$, a runtime of $O(\sum_{v \in V} v \in V(n + \deg(v) \cdot 1)) = O(n^2 + m) = O(n^2)$ for start vertex $s$ and $O(n^3)$ for the distance matrix is obtained (independent of $m$) $\Rightarrow$ good for dense graphs, bad for sparse graphs ($m = \Theta(n)$), works also for Prim.
- bad heuristics
  - only considers distance from start to current vertex
  - completely ignores distance from current vertex to goal
  - therefore slow
  - unlike [[A-Star Algorithm]]