Methods

With 2×2 matrices:

$$\begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & g \\ f & h \end{pmatrix}$$

Traditional method:

$$r = ae + bf$$
$$s = ag + bh$$
$$t = ce + df$$
$$u = cg + dh$$

- In total 8 multiplications and 4 additions are needed. This method can be generalised for larger matrices: Let A and B be $n \times n$ matrices with $n = 2^k$, $k \in \mathbb{N}$ and $C = A \cdot B$. With the traditional method $\Theta(n^3)$ multiplications are needed to calculate C.

  Idea:
  $$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{pmatrix}$$

  with $C_{ij} = A_{i1} \cdot B_{1j} + A_{i2} \cdot B_{2j}$

- If $n \neq 2^k$, it can be filled to the next higher dimension k.

With traditional method:

8 $n/2 \times n/2$ matrix multiplications

$\Rightarrow T(n) = 8T(n/2) + \underbrace{\mathcal{O}(n^2)}$

for matrix additions

$\Rightarrow T(n) = \mathcal{O}(n^3)$ multiplications $\Rightarrow$ no improvement

With Strassen method:

7 $n/2 \times n/2$ matrix multiplications

$$\Rightarrow T(n) = 7T(n/2) + \mathcal{O}(n^2) = \cdots =$$
$$= \mathcal{O}(n^{ld(7)}) = \mathcal{O}(n^{2.81})$$

•