**Overview**

- all these games (and many more) can be reduced to NIM
- NIM rules
  -
  - **Games:**
    - NIM
    - Prime-game
    - Poker NIM
    - Northcott's Game
    - Kayles
    - Dawson's Kayles
    - Monochromatic Triangle
    - Triangulation Coloring Game
  -

**Prime Game**

- $n$ integers $f_1, \ldots, f_n > 1$
- Valid move:
  - Choose a non-prime integer $f_i > 1$
  - Split $f_i$ into (one or more) prime factors $p_1, \ldots, p_k > 1, k \geq 1$ and a rest $f' > 1$
  - Replace $f_i$ with $p_1, \ldots, p_k$ and $f'$
- **Normal play**: the last one to make a valid move wins
-
- each integer with k prime factors corresponds to a coin pile with k coins
- pile height = number of coins = number of prime factors

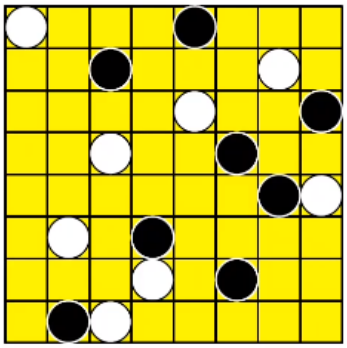
**Poker NIM**

- Startposition:
  - Same as for NIM
- Possible moves:
  - Similar to NIM, but instead of removing coins you may also put an arbitrary number of coins from your pool (built by previously taken coins) on a heap.
- **Normal play**: the last one to make a valid move wins

- exactly the same as NIM with finite number of coins in the pool
- placing coins from the pool just postpones the game end
  - opponent just removes the coins you just placed

**Northcott's Game**

- $n \times m$ chess board
  one black, one white coin per row in different columns
- Valid move:
  - Choose a row
  - move the coin of your color left or right arbitrarily many steps, but a least one
  - don't jump over your opponent's coin



- **Normal play**: the last one to make a valid move wins

- asymmetric version of Poker NIM
- each row corresponds to a pile
  - number of spaces between the coins = pile height
  - spaces behind a coin (inaccessible for the opponent) = number of coins in the pool

**Kayles**

- Setting:
  - as for NIM
- Possible moves:
  - Chose an arbitrary, non-empty stack
  - Remove one or two coins from this stack
  - Optional: split the remaining stack into two non-empty, smaller stacks
  - Bowling: Row of $n$ pins.
    In a move hit one or two neighbored pins.
- 
- Dawson's Kayles
  - always hit two neighbored pins
  - remove single pins

**Monochromatic Triangle**

- $n$ points in the plane, in general position
- Valid move:
  - Draw a straight line segment connecting two points, not crossing any other line
- The game ends when an empty triangle occurs
- 
- same as Dawson's Kayles

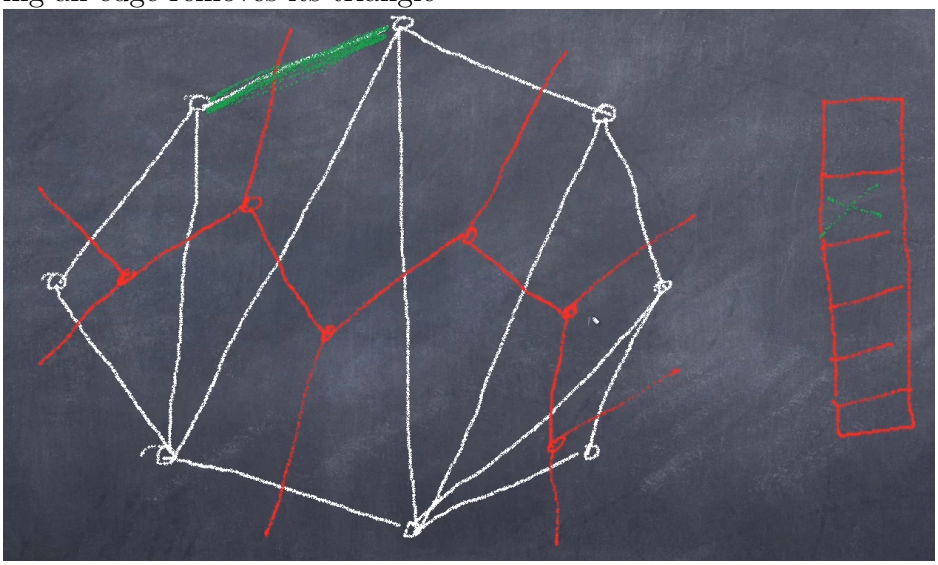
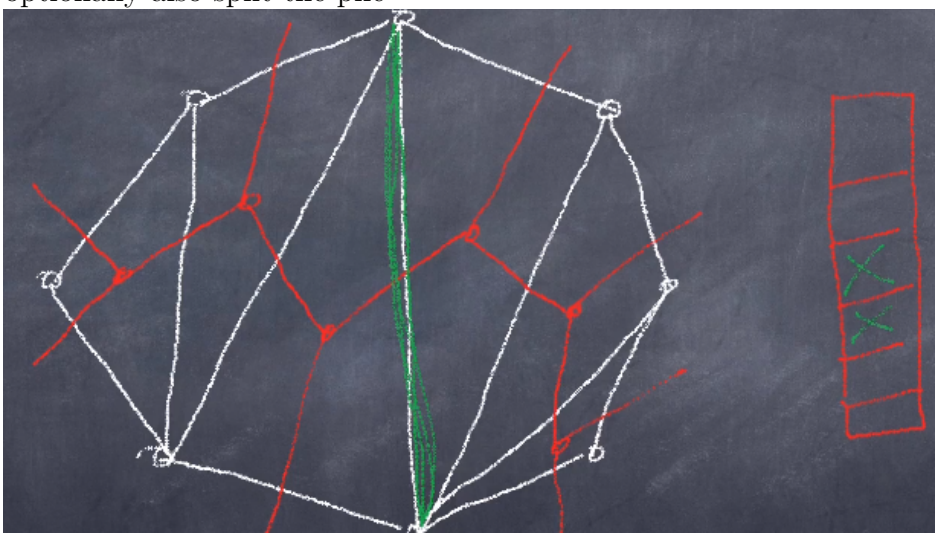
**Triangulation Coloring Game**

Triangulation on $n$ points in the plane, all edges are black (white on the blackboard)

Valid moves:

- Select a black edge, color it green

The game ends when the first green empty triangle occurs.

- 
- dual structure
  - voronoi diagram
  - coloring an edge removes its triangle
    - 
  - may optionally also split the pile
    - 
- reduces to Kayles