

Definition

- equation (inequality) describing function in terms of its value for smaller inputs

Example: factorial function

$$f(n) = n \cdot f(n-1) \text{ for } n > 1, f(1) = 1$$

Example: fibonacci numbers

– $f(n) = f(n-1) + f(n-2) \text{ for } n > 2, f(1) = f(2) = 1$

- relevant for [[Laufzeitanalyse]]
 - runtime and consumption analysis

Asymptotic Bounds

- [[O-Notation]] and [[Θ-Notation]]
- Limit theorem

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f(n) = \mathcal{O}(g(n)) \\ 0 < c < \infty & f(n) = \Theta(g(n)), \\ & \mathcal{O}(g(n)), \Omega(g(n)) \\ \infty & f(n) = \Omega(g(n)) \end{cases}$$

$$\liminf_{n \rightarrow \infty} \frac{f(n)}{g(n)} > 0 \quad f(n) = \Omega(g(n))$$

$$\limsup_{n \rightarrow \infty} \frac{f(n)}{g(n)} < \infty (\text{and } \geq 0) \quad f(n) = \mathcal{O}(g(n))$$

– if both are true: $f(n) = \Theta(g(n))$

- computation

- Addition:

$$\Theta(f(n)) + \Theta(g(n)) = \Theta(\max\{f(n), g(n)\})$$

$$\sum_i \Theta(f_i(n)) = \Theta\left(\sum_i f_i(n)\right)$$

Attention: not iteratively!

- Multiplication:

$$c \cdot \Theta(f(n)) = \Theta(f(n)) \text{ for constant } c > 0$$

$$\Theta(f(n)) \cdot \Theta(g(n)) = \Theta(f(n) \cdot g(n))$$

- Attention: equations always “from left to right”:

– $f(n) = \Theta(g(n))$ vs. $\Theta(g(n)) = f(n)$

Methods to solve recurrence relations

- [[Iterative Method]]
- [[Master Theorem]]
- [[Substitution Method]]