

## Methods

With  $2 \times 2$  matrices:

$$\begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & g \\ f & h \end{pmatrix}$$

Traditional method:

$$r = ae + bf$$

$$s = ag + bh$$

$$t = ce + df$$

$$u = cg + dh$$

- In total 8 multiplications and 4 additions are needed.  
This method can be generalised for larger matrices:  
Let  $A$  and  $B$  be  $n \times n$  matrices with  $n = 2^k, k \in \mathbb{N}$  and  $C = A \cdot B$ . With the traditional method  $\Theta(n^3)$  multiplications are needed to calculate  $C$ .

Idea:

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{pmatrix}$$

with  $C_{ij} = A_{i1} \cdot B_{1j} + A_{i2} \cdot B_{2j}$

- If  $n \neq 2^k$ , it can be filled to the next higher dimension  $k$ .

With traditional method:

8  $n/2 \times n/2$  matrix multiplications

$$\Rightarrow T(n) = 8T(n/2) + \underbrace{\mathcal{O}(n^2)}$$

for matrix additions

$\Rightarrow T(n) = \mathcal{O}(n^3)$  multiplications  $\Rightarrow$  no improvement

With Strassen method:

7  $n/2 \times n/2$  matrix multiplications

$$\begin{aligned}\Rightarrow T(n) &= 7T(n/2) + \mathcal{O}(n^2) = \dots = \\ &= \mathcal{O}(n^{\lg(7)}) = \mathcal{O}(n^{2.81})\end{aligned}$$

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