Motivation

For $n < 30$ and non-normal populations we use T-score confidence intervals with extreme caution

Alternatively, we can use non-parametric or computational methods

Our discussion so far concentrated on $\overline{X}_n$

But what happens if we want to estimate alternative parameters, e.g. median md

There are theoretical results for the sampling distribution of the sample median but they are much more involved than results for $\overline{X}_n$

- In many of these cases we can resort to **computer simulation**

Monte Carlo Method

Why is it ok to use simulation to estimate quantities of interest?

(Weak) Law of large numbers

$$\overline{X}_b = \frac{1}{b}\sum_{i=1}^{b} X_i \xrightarrow{P} E(X) \text{ as } b \to \infty.$$

For large enough b we can use $\overline{X}_b$ to approximate $E(X)$

- **Good thing**: in simulation we can make b as large as we like
- function approximation

If h is any function with finite mean:

$$\frac{1}{b}\sum_{i=1}^{b} h(X_i) \xrightarrow{P} E(h(X)) \text{ as } b \to \infty.$$

For example, to estimate the variance $Var(X)$:

$$\frac{1}{b}\sum_{i=1}^{b} (X_i - \overline{X}_b)^2 \xrightarrow{P} Var(X) \text{ as } b \to \infty.$$

[[Bootstrap Principle]]