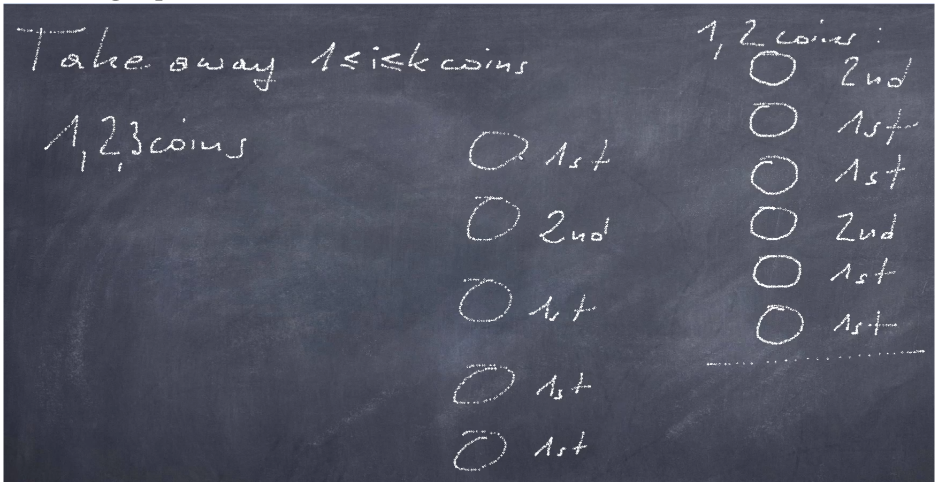
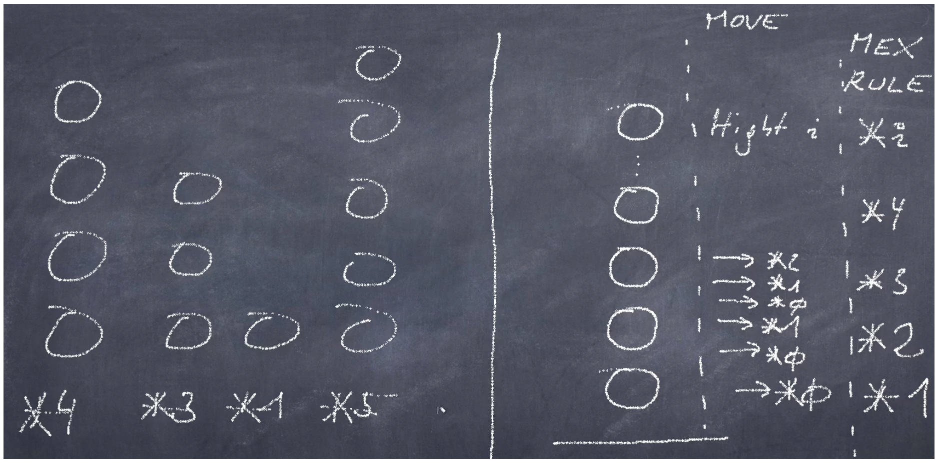
Sprague-Grundy-Theory

- First/Second-Player Win depends on number of coins
  - for a single pile
  - 

- Nimbers
  - Nimbers $*i, i \geq 0$, are a 'code' used for game-positions:
    - $*i, i \neq 0 \Rightarrow$ 1st player win (the player to move)
    - $*0 \qquad \Rightarrow$ 2nd player win (the one just moved)
  - A nimber code implies:
    - From a $*0$ situation no legal move leads to another $*0$ situation
      - Interpretation: If I made a winning move, my opponent can not
    - From any $*i, i \neq 0$, situation there is a legal move to a $*0$ situation
      - Interpretation: If my opponent gives me a (for them) non-optimal situation, I can make a winning move
  - nimber depends on height

* 

NIM Rules

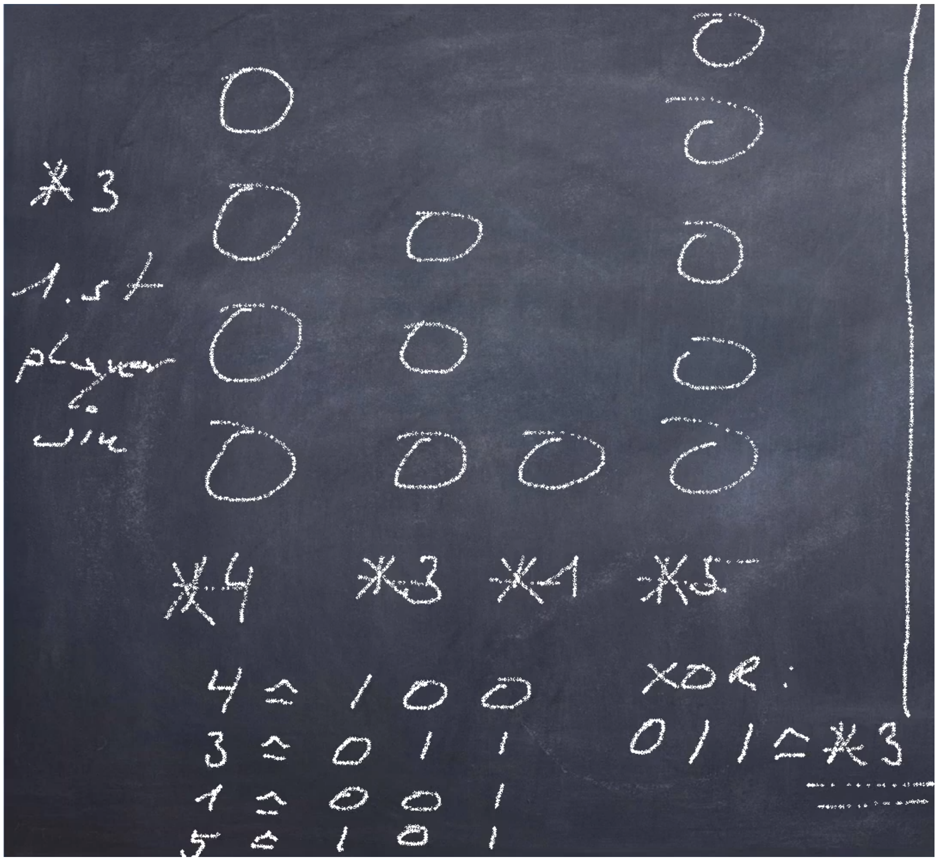
MEX-rule (Minimal Excluded):

- The nimber of a position P is the smallest value which is NOT a nimber of any position which is reachable by a valid move from P.

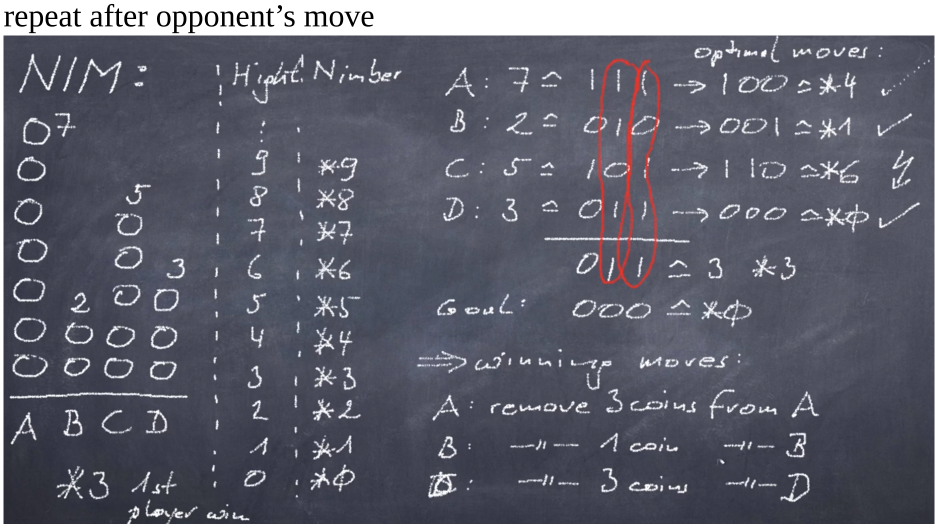
- ◦ The MEX-rule guarantees a good code!
  ◦ From a $*0$ situation no legal move leads to another $*0$ position
  ◦ From any $*i, i \neq 0$, situation there is a legal move to a $*0$ situation.

- XOR-rule:
  ◦ The nimber of a set of positions is the XOR-sum of the nimbers of the positions.

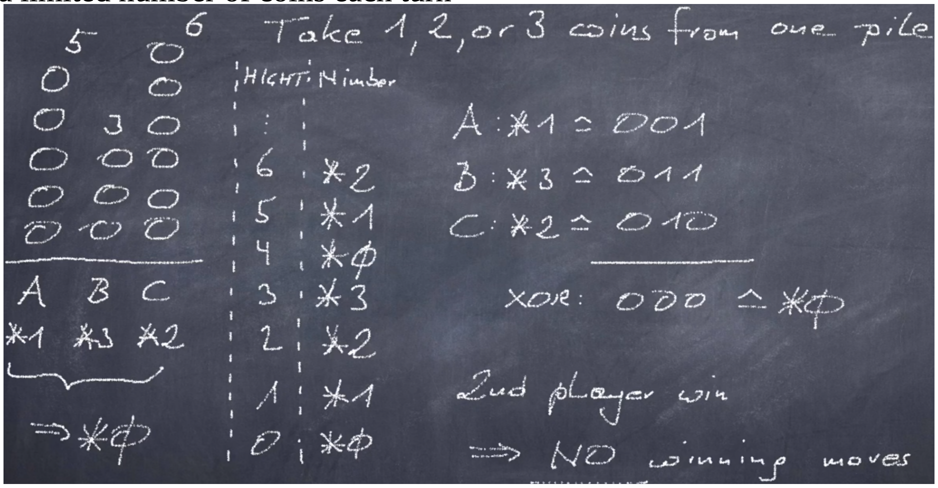
- ◦ Simplifies computation of nimbers for several piles.
  ◦ That is, using the MEX-rule AND the XOR-rules makes computations much more efficient!
  ◦ For correctness the MEX-rule is sufficient.

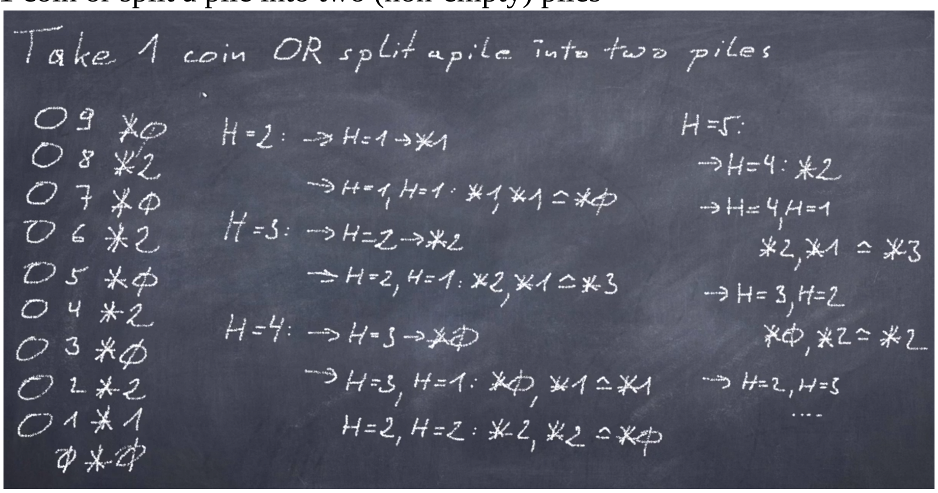
- 

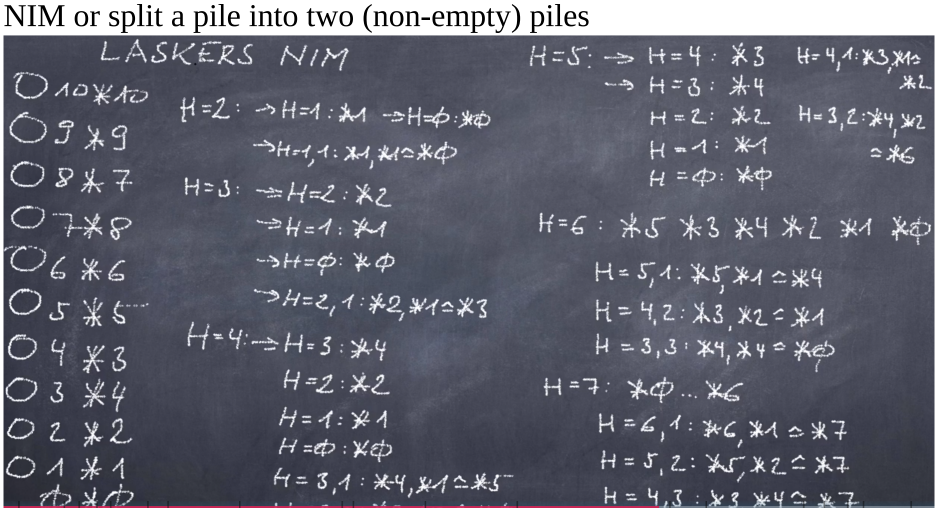
Optimal Strategy

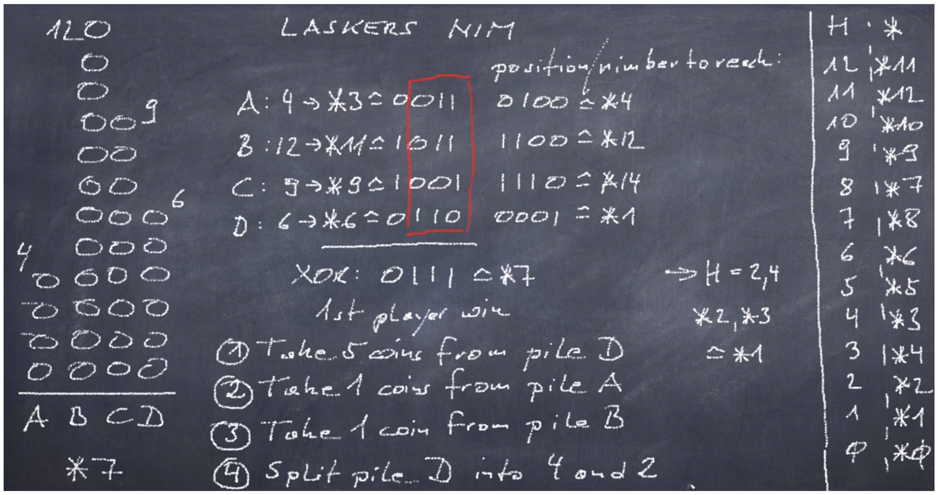
- compute each pile’s nimber
- XOR to find out if a winning move exists
  - zero ⇒ no winning move
- compute for each pile if there is a winning move for this pile
- execute a optimal move
- repeat after opponent’s move
- 

Variations of [[NIM-type Games]]

- take a limited number of coins each turn
  - 

- take 1 coin or split a pile into two (non-empty) piles
  - 

- Laskers NIM
  - NIM or split a pile into two (non-empty) piles
  - 

- 

- Kayles and Dawson's Kayles
  - no repeating nimber patterns

[[Algorithms and Games]]