Random Sample

- Let the random variables $X_1, X_2, \dots, X_n$ have a joint density $f_{X_1,X_2,\dots,X_n}(x_1, x_2, \dots, x_n)$ that factors as follows:

  $$f_{X_1,X_2,\dots,X_n}(x_1, x_2, \dots, x_n) = f(x_1)f(x_2) \cdot \dots \cdot f(x_n),$$

  where $f(\cdot)$ is the common density of each X_i. We then define $X_1, X_2, \dots, X_n$ to be a *random sample* of size n from a population with density $f(\cdot)$. Thus, a random sample is a sequence of independent, identically distributed (*i.i.d.*) random variables.

- Remark 1 (Sampling with/without replacement)

  When sampling from a finite population our definition requires to always sample with replacement as otherwise the drawings are not independent.

- We define X_i as 1 (ith person watched the movie) or as 0 (ith person did not watch the movie). If we sample people so that the variables $X_1, X_2, \dots, X_n$ are independent and have the same density (all people have the same probability of watching the movie) then the sample is random.

Sampled Population

- Let $X_1, X_2, \dots, X_n$ be a random sample from a population with density $f(\cdot)$; then this population is called *sampled population*.

- Remark 2 (Distinction between the sampled and the target population)

  - With random samples we can only make valid probability statements about sampled population
  - Statements about target population are not valid
  - Unless the target population is also the sampled population

- Example 3 (Rise of Skywalker)

  All people living in Austria form the *target population*. We draw a sample from Graz. Thus, Graz residents form the *sampled population*.