Übersicht

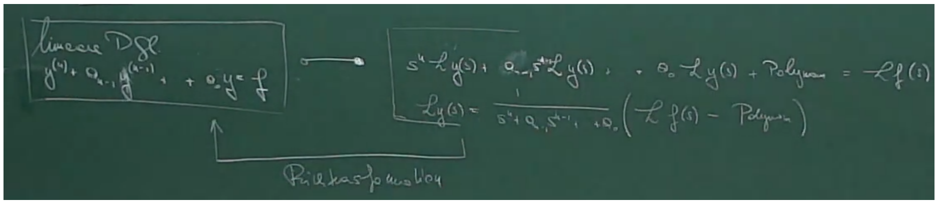
- [[Anwendungen der Residuenrechnung]]
- f: $[a, \infty)$–>
- Laplace Transformation
  - $$\mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt$$
  - $Lf(s)$ f existiert, wenn $\exists A, C \in \mathbb{R} \forall t \in \mathbb{R}^+ : |f(t)| \leq Ce^{At}$
- Konvergergenzabszisse $c_0(f)$
  - $\forall f \exists c_0(f) \in \mathbb{R} : \forall s \in \mathbb{C}$ mit $Re(s) > c_0(f)$ $Lf(s)$ existiert
  - $Lf(s)$ ist holomorphe Funktion für $Re(s) > c_0(f)$
- Doetsch Symbol
  - Beziehung zwischen f und Lf
  - $\circ\!\!-\!\!\!-\!\!\bullet$.

Rechenregeln für L

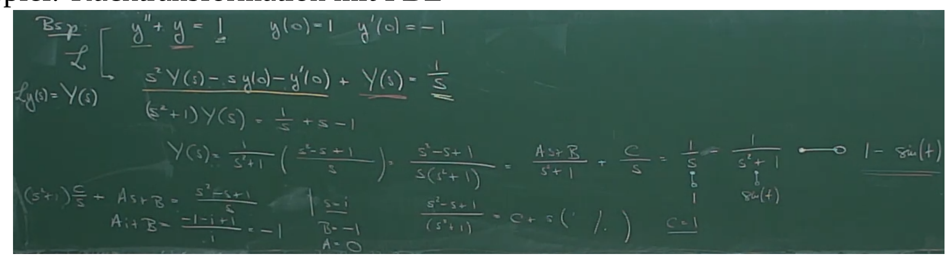
$F(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\mathcal{L}F(s)$
$aF(t) + bG(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$a\mathcal{L}(F(s)) + b\mathcal{L}(G(s))$
$F(t), G(t), H(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$f(s), g(s), h(s)$
1	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{1}{s}$
$e^{\alpha t}$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{1}{s-\alpha}$
t^n	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{n!}{s^{n+1}}$
$t^n e^{\alpha t}$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{n!}{(s-\alpha)^{n+1}}$
$\sin \alpha t$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{\alpha}{s^2+\alpha^2}$
$\cos \alpha t$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{s}{s^2+\alpha^2}$
$e^{-\alpha t} \sin \beta t$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
$e^{-\alpha t} \cos \beta t$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$
$\alpha > -1, \quad t^\alpha$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$
$\delta(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	1
$F^{(k)}(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$s^k f(s) - s^{k-1}F(0) - s^{k-2}F'(0) - \cdots - F^{(k-1)}(0)$
$t^k F(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$(-1)^k f^{(k)}(s)$

$\lambda > 0, \quad \begin{cases} F(t-\lambda) & \text{für } t \geq \lambda \\ 0 & \text{sonst} \end{cases}$	$\circ\!\!-\!\!\!-\!\!\bullet$	$e^{-\lambda s} f(s)$
$e^{-\alpha t} F(t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$f(s+\alpha)$
$F(\rho t)$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{1}{\rho} f\left(\frac{s}{\rho}\right)$
$F * G(t) = \int_0^t F(\tau)G(t-\tau)\,d\tau$	$\circ\!\!-\!\!\!-\!\!\bullet$	$f(s)g(s)$
$\frac{F(t)}{t}$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\int_s^\infty f(v)\,dv$
$\int_0^t F(\tau)\,d\tau$	$\circ\!\!-\!\!\!-\!\!\bullet$	$\frac{f(s)}{s}$
$\frac{1}{2\pi i} \int\limits_{c-i\infty}^{c+i\infty} f(s)e^{st}\,ds$	$\circ\!\!-\!\!\!-\!\!\bullet$	$f(s)$

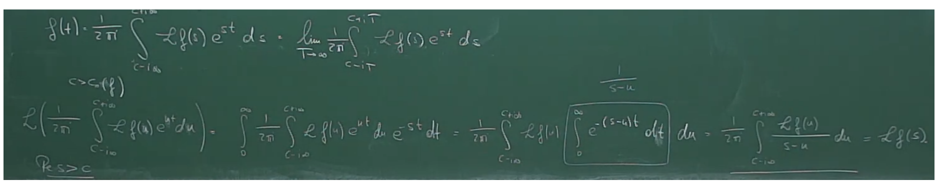
DGL lösen

- 

  - Laplace Transformation auf lineare DGL
  - dividieren statt DGL lösen

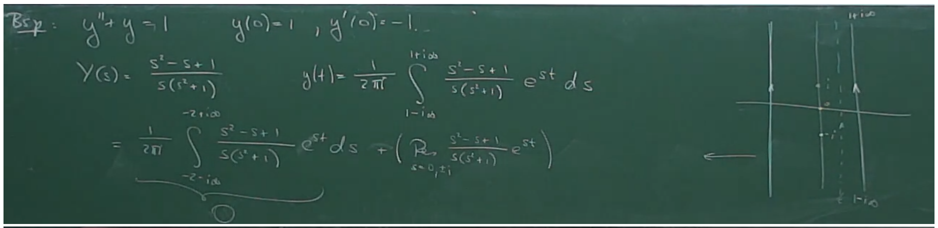
  - Rücktransformation
- Beispiel: Rücktransformation mit PBZ
  - 

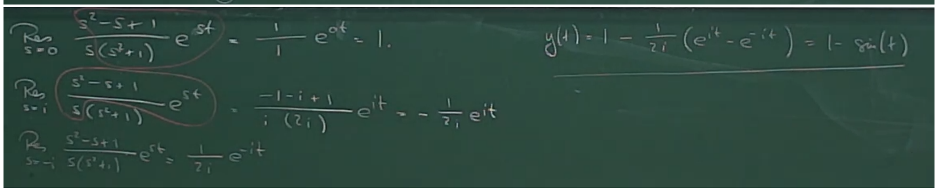
Laplace Umkehrformel

- 
- Rücktransformation mit CIF statt
  - PBZ
  - Bekanntsein der Rechenregeln für L
- Verschiebung in Gebiet mit Polstellen
  - ==> Residuensatz statt CIF
  - $$y(t) = \sum \mathrm{Res}\left(\frac{s^3+3s^2+2s+3}{(s^2+1)(s^2+4s+5)}e^{st}\right).$$
    - Verschieben des Kurvenintegrals im holomorphen Gebiet ändert nichts
    - Kurvenintegral = 0
    - Summe der Residuen bleibt übrig

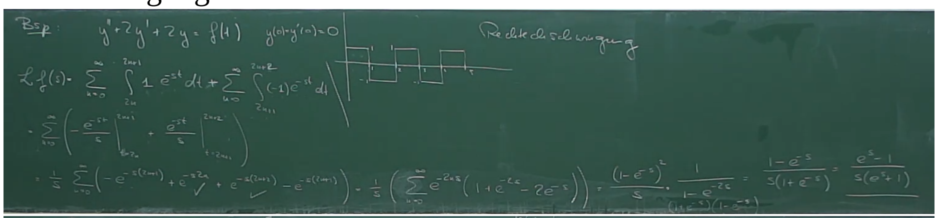
Beispiele mit Umkehrformel

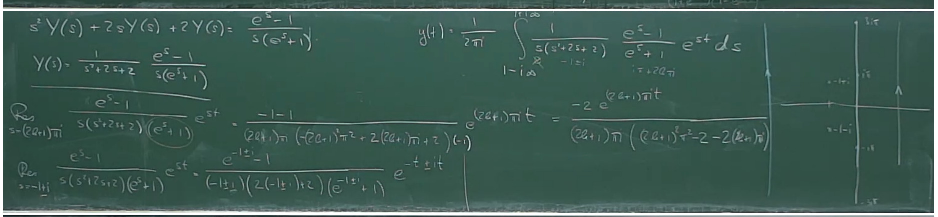
- gleiche Beispiel wie oben jedoch ohne PBZ

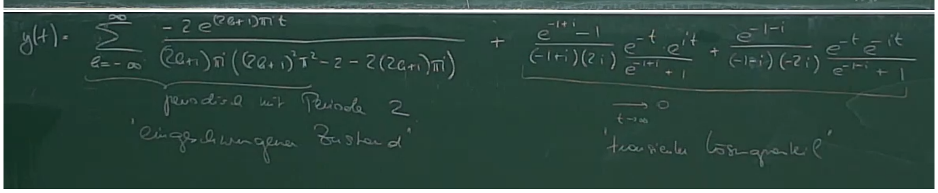
- 

- 

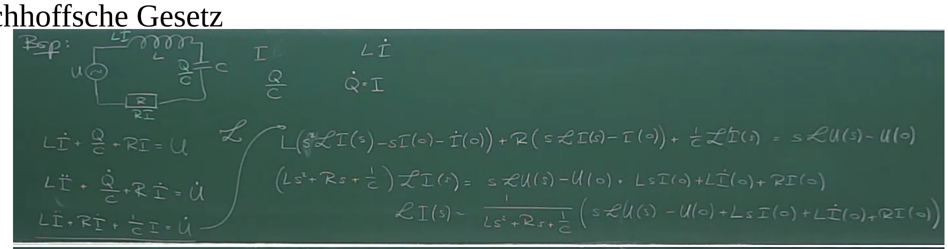
• Rechteckschwingung mit unendlich Polen

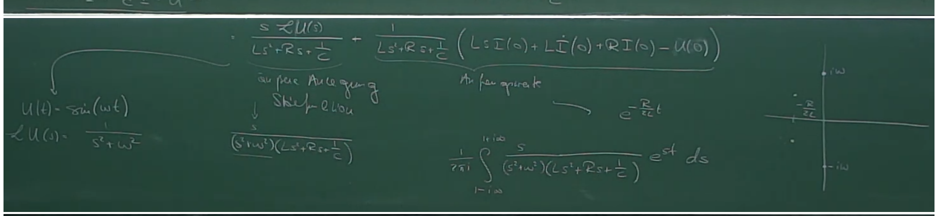
- 

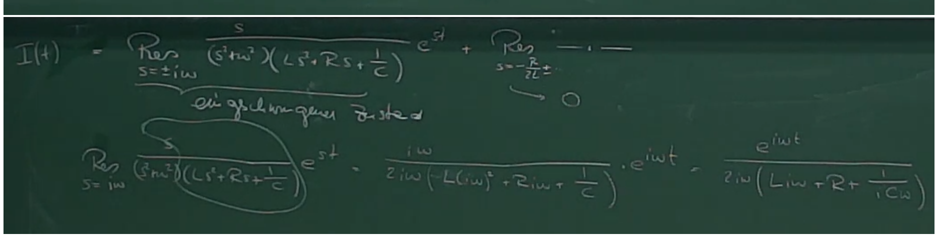
- 

- 

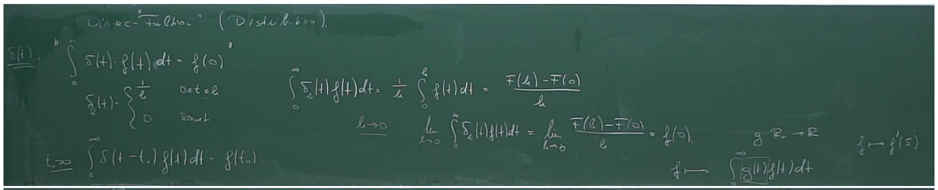
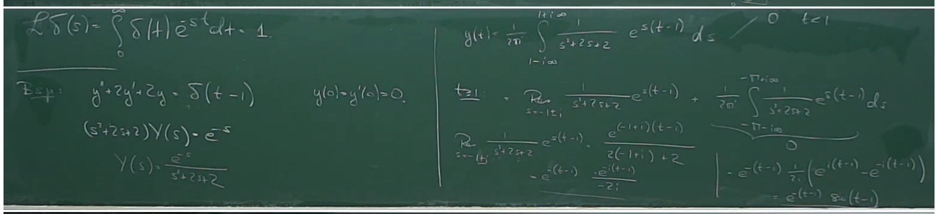
• Kirchhoffsche Gesetz

- 

- 

- 

Exkurs: Distributionen

- 
- 
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