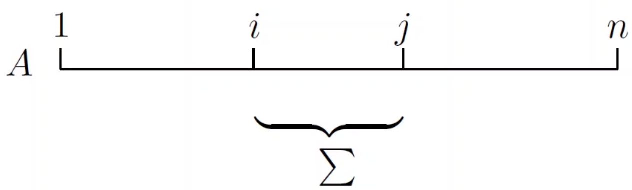
Methods

Example 1: Maximum Subarray Sum

Given: Array A[1,. . . ,n] of integers (also negative values)

Goal: continuous subarray A[i,. . . ,j] with maximum sum

$$\begin{array}{|c|c|c|c|c|}\hline 4 & -5 & 4 & 2 & -3 \\ \hline\end{array} \Rightarrow \sum = 6$$



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- Method 1:
  Check all subsets of A and check for connectedness.

  $n \ldots$ sum of a subset
  $2^n \ldots$ number of subsets
  $\mathcal{O}(n2^n)$ runtime

  A computer with 10^6 Operations per second needs for
  $n = 1000$ numbers $\approx 10^{304}$ operations, i.e., $\approx 10^{290}$ years.

- Same computer: for $n = 10^6$ numbers $\approx 10^{300000}$ years.

Method 2:
Only checking connected sequences:

$$\max_{1\leq i\leq j\leq n}\left\{\sum_{k=i}^{j} A[k], 0\right\}$$

i, j, k run for at most n steps.
Three nested loops $\Rightarrow \mathcal{O}(n^3)$

For $n = 1000 \Rightarrow 10^9$ steps ~16 min.

For $n = 10^6 \Rightarrow\approx 32000$ years.

- Method 3 (pseudo code):
  Computing the sum 'online' with j.

$max := 0;\ from := 0;\ to := 0;$
for $i := 1$ to n do
    $sum := 0;$
    for $j := i$ to n do
        $sum := sum + A[j];$
        if $sum > max$ then $max := sum;\ from = i;\ to = j$
        fi
    od
od
$output($"$A[$"$, from,$" $-$ "$, to,$"$]$maximum sum = "$, max)$

- – almost equivalent to method 2
  – does not recalculate existing sum
  – instead adds next element to previous

i, j go through at most n values $\Rightarrow \mathcal{O}(n^2)$ steps

For $n = 1000 \Rightarrow 10^6$ steps ~1 sec

For $n = 10^6 \Rightarrow\approx 11,5$ days.

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Method 4:

Run through the input once with a 'scanline' and only consider the part of the input currently covered by the scanline.
Idea: Calculate for every index k the maximum sequence T_k ending at k.
From this get a k with a global maximum sequence.

Observe:
$$T_k \geq 0,$$
$$T_k = \max\{T_{k-1} + A[k], 0\}$$

- ## Method 4 (pseudo code):

$max := 0;\ from := 0;\ to := 0;\ f := 1;\ T := 0$
for $k := 1$ to n do
  $T := T + A[k]$
  if $T < 0$ then $T := 0;\ f = k + 1$
  fi
  if $T > max$ then $max := T;\ from := f;\ to := k$
  fi
od
$output("A[", from, "-", to, "]\text{maximum sum} = ", max)$

$n = 1000 \Rightarrow \approx 1/1000$ second; for $n = 10^6 \Rightarrow \approx 1$ second.
Compare: 10^{300000} years (Method 1) or 32000 years (M. 2).

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