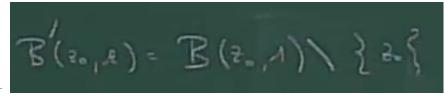
## Singularität

- $z_0 \in U$ ist Singularität, wenn  $f \in H(U/\{z_0\})$
- punktierte Kreisscheibe



- Singularitätsfälle
  - hebbare Singularität
  - Pol der Ordnung m
  - wesentliche Singularität
- Über<u>sicht</u>

```
U \in \mathbb{C} 2 \in \mathbb{C} f \in H(U \setminus \{a_n\}) \cap \mathbb{C}

1, f \in \mathbb{R} u_n \in
```

## Hebbare Singularität

- $\bullet\,$ sei f<br/> holomorph und beschränkt auf der punktierten Kreisscheibe<br/>  $B'(z_0,r)$
- f holomorph auf  $B(z_0, r)$

```
g(z) = \begin{cases} (z-z_{-1})^{2} f(z) & \text{fin } z+z_{+} \\ 0 & \text{fin } z=z_{-} \end{cases}
= \begin{cases} (z-z_{-1})^{2} f(z) & \text{fin } z+z_{+} \\ 0 & \text{fin } z=z_{-} \end{cases}
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• Beispiel

```
\frac{g_{2}p}{g_{2}}: f(z) = \frac{g_{2}p_{1}}{z} = 1 \Rightarrow f(z) \text{ is } \quad \text{on } z = 0 \text{ for larget, days large on } z = 0.
\frac{g_{2}p_{2}}{g_{2}p_{2}}: \frac{g_{2}p_{1}}{g_{2}p_{2}} = 1 \Rightarrow f(z) \text{ is } \quad \text{on } z = 0.
```

## Pol der Ordnung m

- $f \in H(U/\{z_0\})$ , wenn  $\exists m \in \mathbb{N} : (z-z_0)^m f(z)$ holomorph auf U
- $\bullet$  Ordnung des Pols = kleinste mögliche Wert von m
- Beispiel

```
\frac{1}{5^{2}(4)}, \frac{1}{5^{2}(4)} = 2 + RT \quad \text{$0 \in \mathbb{Z}$}
\frac{2}{5^{2}(4)} = \frac{2}{5^{2}(4)} = 2 + RT \quad \text{$0 \in \mathbb{Z}$}
\frac{1}{5^{2}(4)} = \frac{1}{2} = \frac{1}{2} + \frac{1}{2} +
```

## Wesentliche Singularität

•  $\forall r > 0$  gilt  $f(B'(z_0, r))$  dicht in  $\mathbb C$ 

• Beispiel



• Beweis

```
as presume \int (B'(e_{-,A})) \log t with d \cdot d \cdot t is \int F(e_{-,A}) = \int (B'(e_{-,A})) \log t with d \cdot d \cdot t is \int F(e_{-,A}) = \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t and \int (B'(e_{-,A})) \log t is \int (B'(e_{-,A})) \log t in \int (B'(e_{-,A}) \log t in \int (B'(e_{-,A})) \log t in \int (B'(e_{-,A}) \log t in \int (
```

- sei A⊆U, sodass A in U keinen Häufungspunkt hat
  - $f \in H(U/A)$ meromorph, wenn in A<br/> nur Pole hat

- harmonische Funktion
  - f(x+iy) = u(x,y) + iv(x,y) differenzierbar
    - \* wenn CR Gleichungen gelten
      - ♦ ==> beliebig oft



 $\bullet ==> u$  und v harmonisch

[[Komplexe Kurvenintegrale]]