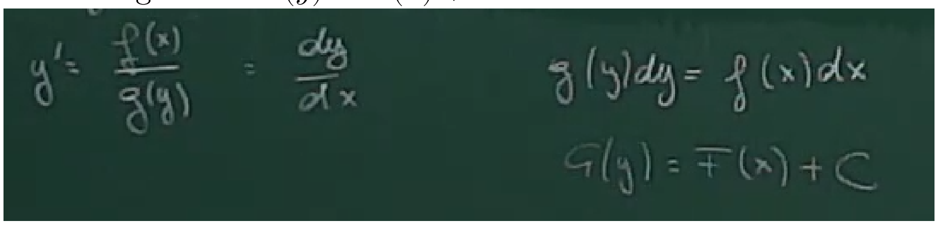
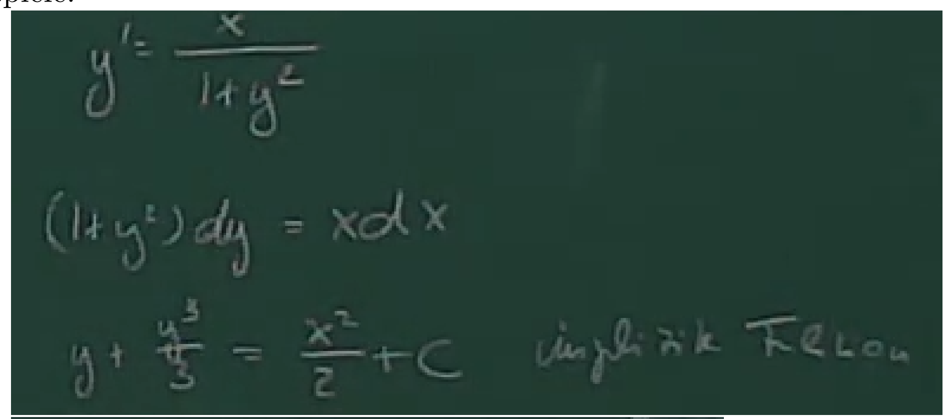
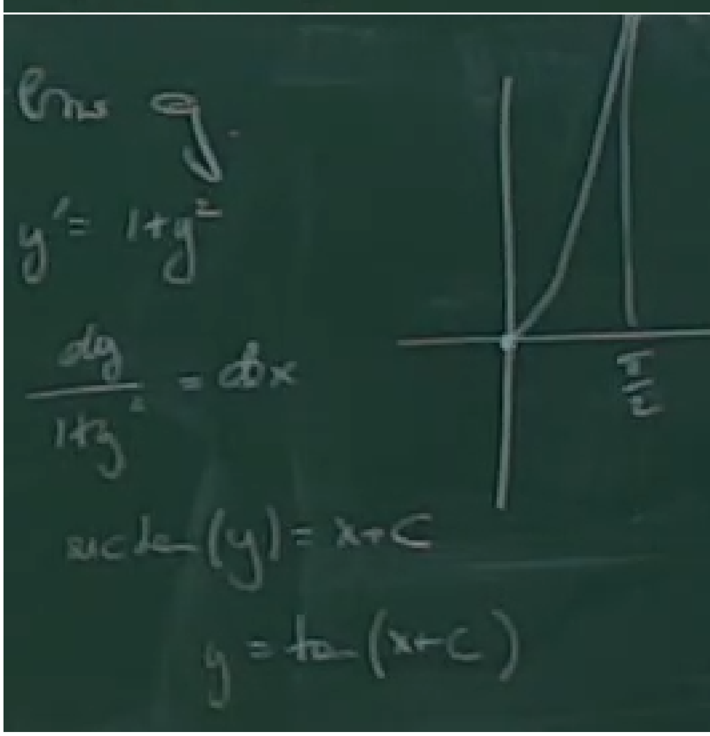
Getrennte Variablen

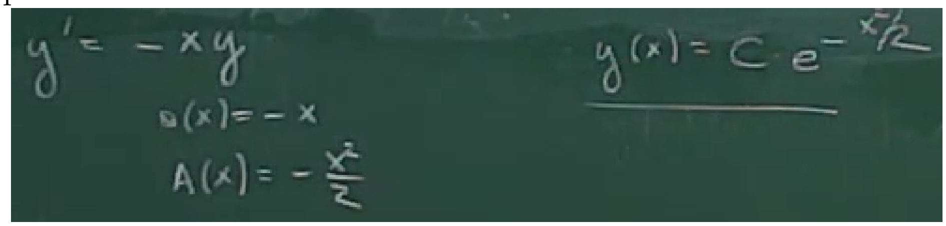
- x und y voneinander trennbar
- Vorgehensweise:
  - gegeben: $y' = \frac{f(x)}{g(y)} = \frac{dy}{dx}$
  - nach Umformung: $g(y)dy = f(x)dx$
  - nach Integration: $G(y) = F(x) + C$
  - 

- Beispiele:
  - 

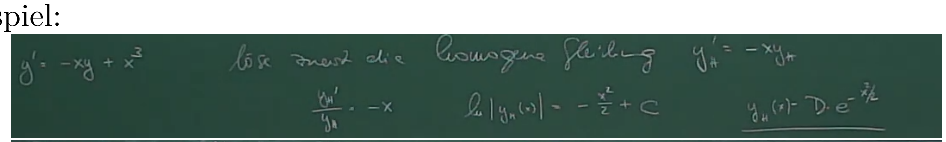
  - 

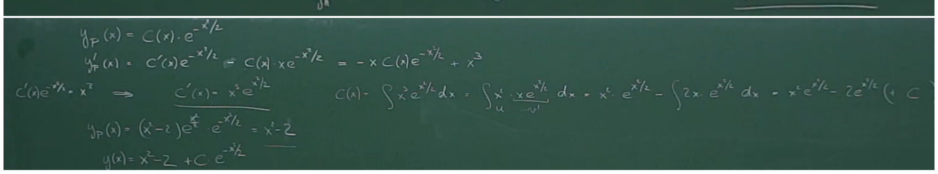
    - nicht möglich, da Lösung Definitionsbereich verlässt

Lineare DGL

- $y' = a(x)y{=}{=}{>}\frac{y'}{y} = a(x)$
- wenn $y(x_0) = 0$ ==> laut Satz von P-L $y(x) = 0$ für alle x
- $ln|y(x)| = \int a(x)dx$
  - $y(x) = C * e^{A(x)}$ ist allgemeine Lösung
- Beispiel:
  - 

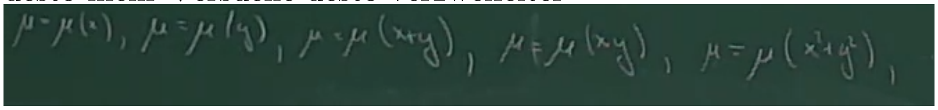
Inhomogene lineare DGL

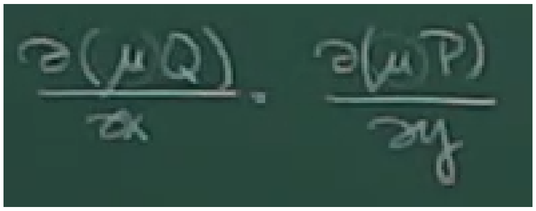
- $y' = a(x)y + b(x)$
  - $b(x)$ ==> Inhomogenität durch Störfunktion
- Vorgehensweise:
  - homogene GL lösen
    - $y'_H = a(x)y_H(x)$ ==> $y_H(x) = C * e^{A(x)}$
  - inhomogene GL lösen
    - $y_P(x) = C(x) * e^{A(x)}$
      - $C(x)$ gesuchte Funktion
      - Variation der Konstanten
    - $y_P = C'(x) * e^{A(x)} + C(x) * e^{A(x)} * a(x) = a(x) * C(x) * e^{A(x)} + b(x)$
      - ==> $C'(x) = b(x) * e^{-A(x)}$ ==> $C(x) = \int b(x) * e^{-A(x)}dx$
  - $y(x) = y_P(x) + D * e^{A(x)}$
    - allgemeine Lösung
    - homogene + partikuläre Lösung
- Beispiel:
  - 

  - 

Exakte DGL

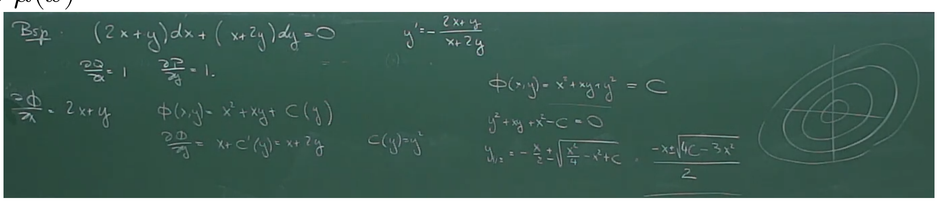
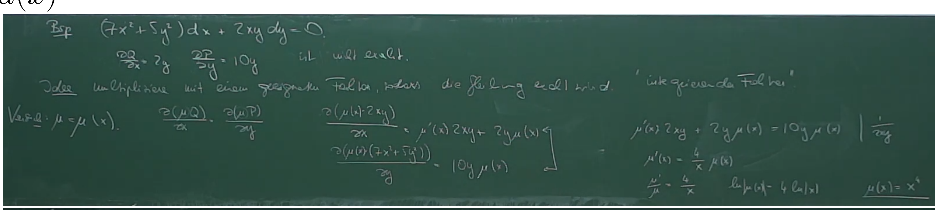
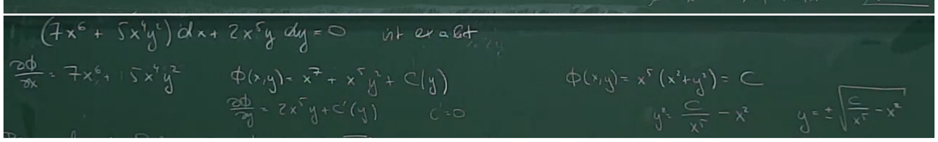
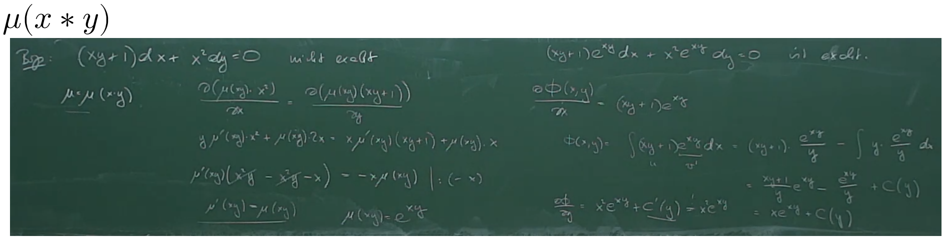
- $P(x,y)dx + Q(x,y)dy = 0 <{=}{=}{>} y' = \frac{dy}{dx} = -\frac{P(x,y)}{Q(x,y}$
  - Stammfunktion existiert, wenn $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

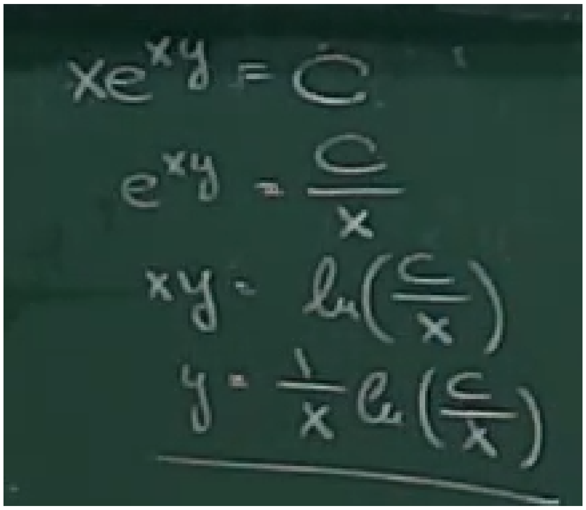
- wenn nicht exakt ==> Multiplikation mit integrierenden Faktor μ(x)
    - alternative Formen $\mu(x)$ in Reihenfolge zum Versuchen
    - desto mehr Versuche desto verzweifelter
    - 

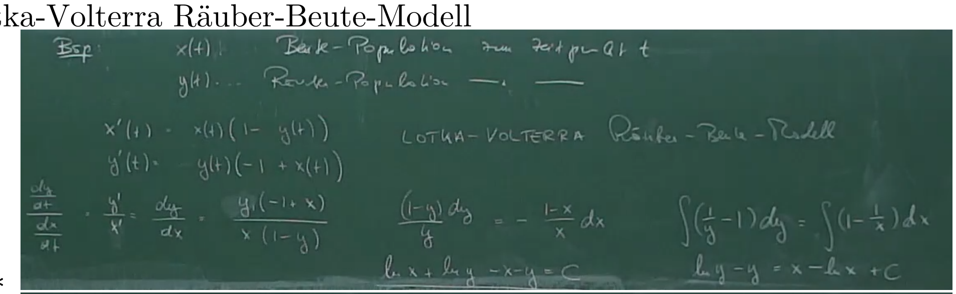
- $\mu(x)$ bestimmen
    - 

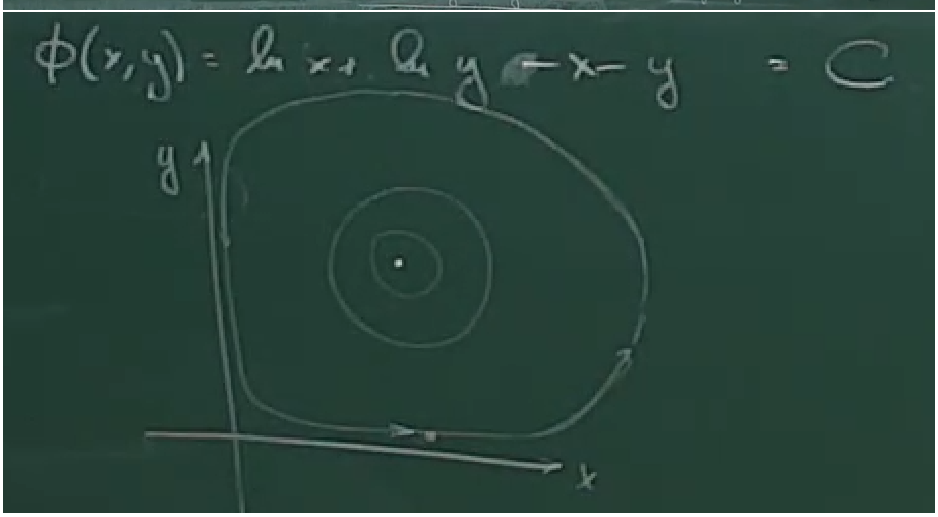
    - DGL nach $\mu(x)$ lösen
- siehe 2. Beispiel

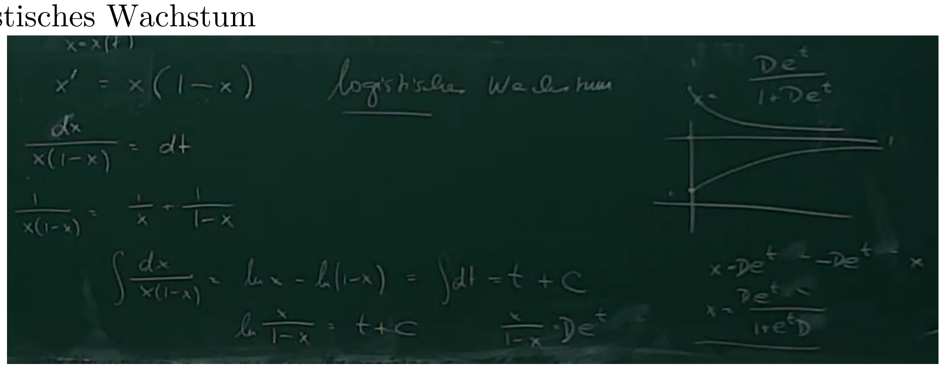
– Stammfunktion: $d\phi = 0 ==> \phi(x, y) = const$
- Lösungen von $P(x, y)dx + Q(x, y)dy = 0$ sind Niveaulinien von ϕ

- Beispiele:
  – ohne $\mu(x)$
    - 
  – mit $\mu(x)$
    - 
    - 
  – mit $\mu(x * y)$
    - 

    * 

  - Lotka-Volterra Räuber-Beute-Modell
    * 

    * 

  - logistisches Wachstum
    * 

[[Differentialgleichungen]]