Overview

- [[Graph Terminology]]
- [[Graph Algorithms]]
- [[Graphentheorie]]

Storing Graph

- **Size** of a graph $G = (V, E)$:
  - Number of vertices: $n = |V|$
  - Number of edges: $m = |E|,\ 0 \leq m \leq n^2$

  $\Rightarrow$ In total: size $\Theta(n + m)$

  $\Rightarrow$ **Analysis** needs two parameters !
- - **dense** graphs: $m \approx n^2$, for example complete graphs
  - **sparse** graphs: $m \ll n^2$, for ex. trees $(m = n - 1)$ or hypercubes ($m = \frac{d}{2} \cdot n = O(n \log n)$, where d is the dimension of the hypercube)
- **Adjacency-matrix**: Matrix $A[1 \ldots n, 1 \ldots n]$ with

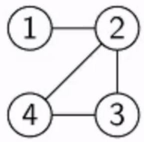
  $$A[i, j] = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{else} \end{cases}$$

  - Memory: $\Theta(n^2)$
  - convenient for dense graphs
  - test for existence of an edge in $\Theta(1)$ time

  Adjacency-List: Array $F[1 \ldots n]$ with pointers,
  $F[i]$ points to a linear list with all vertices that are incident to the i^{th} vertex

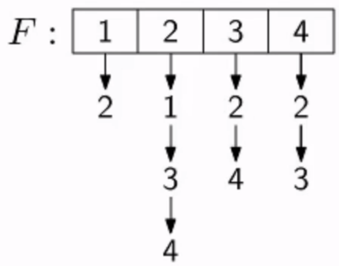
  - Memory: $\Theta(n + m)$
  - test for existence of an edge in $\Omega(1), O(n)$ time

A small example:



$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

↑ symmetric if graph is undirected



← 1 Entry for every node,
$\Theta(n + m)$ memory for all edges.
The k neighbours of a node can be obtained in $\Theta(k)$ time.