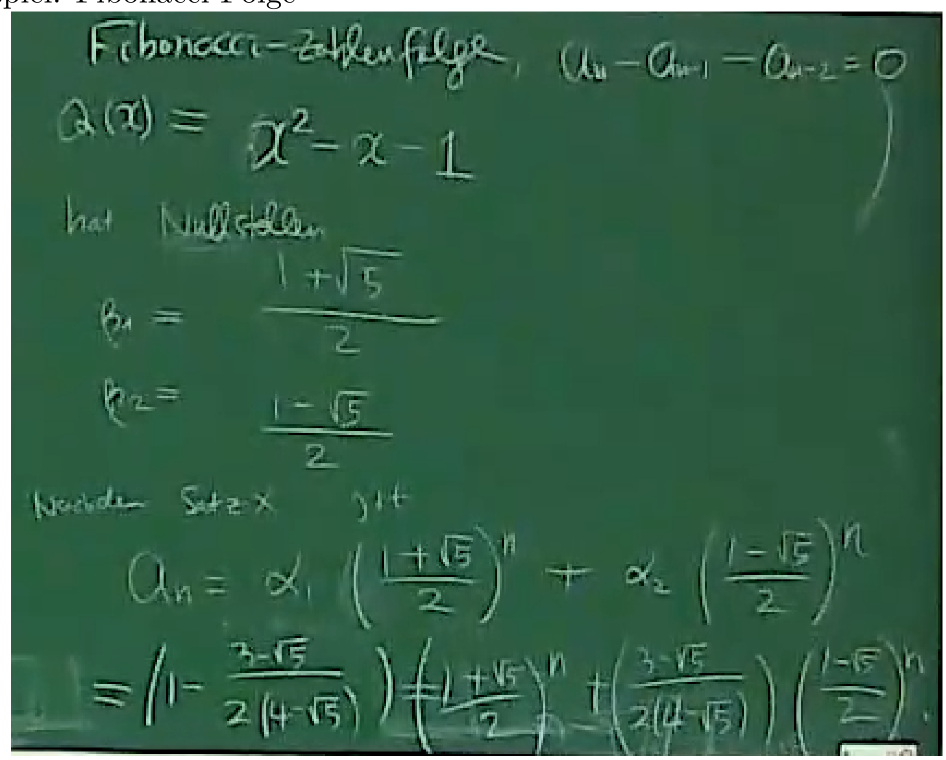
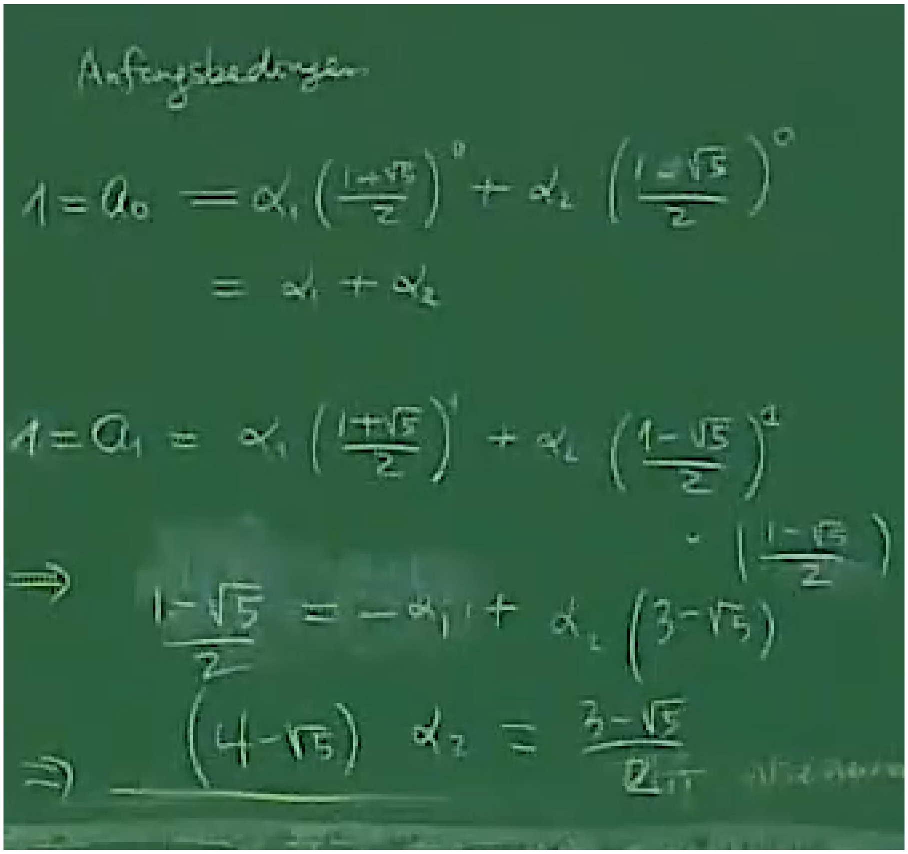
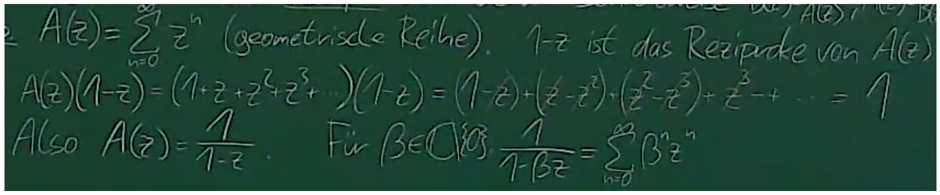
Lineare Rekursion

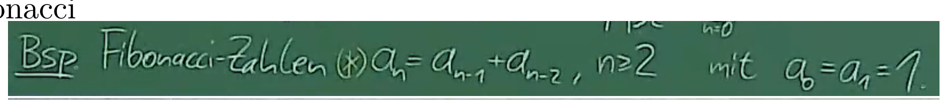
- a_n mit n∈ℕ
  - $a_n = c_1 * a_{n-1} + c_2 * a_{n-2} + ... + c_l * a_{n-l}$
    - ==>$a_n - c_1 * a_{n-1} - c_2 * a_{n-2} - ... - c_l * a_{n-l} = 0$
  - c_i Konstante
  - Anfangsbedingung/startwert fix gegeben
- Charakteristische Polynom
  - $Q(x) = x_l - c_1 * x_{l-1} - c_2 * x_{l-2} - ... - c_l$
  - hat Q(x) l verschiedene Nullstellen ${}_1, {}_2, ..., {}_l$
    - $a_n = \alpha_1 * {}^n_1 + \alpha_2 * {}^n_2 + ... + \alpha_l * {}^n_l$
    - Konstanten $\alpha_1, \alpha_2, ..., \alpha_l$, sodass Anfangsbedingungen erfüllt
- Beispiel: Fibonacci-Folge
  - 

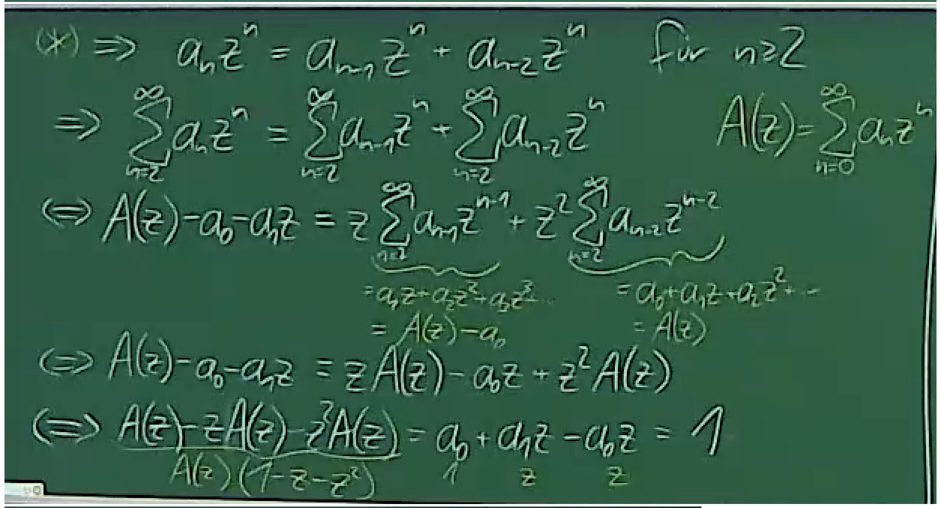
- 

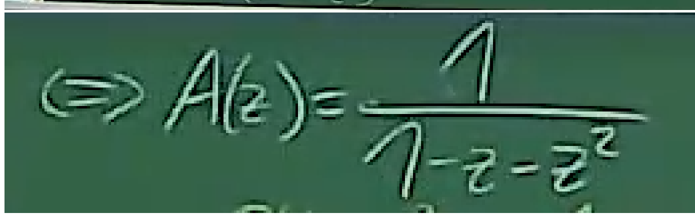
Erzeugende Funktionen

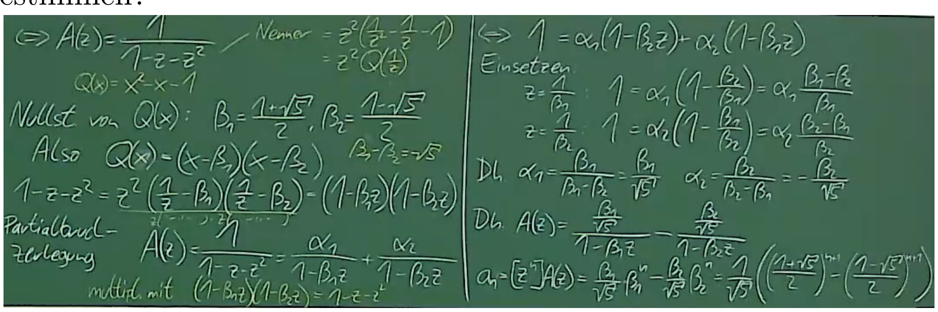
- formale Potenzreihe
  - $A(z) := a_0 + a_1 * z + a_2 * z^2 + ... = \sum_{n=0}^{\infty} a_n * z^n$
  - z formale Variable
- A(z) heißt erzeugende Funktion/Potenzreihe der Folge a_n
- $[z^n]A(z) := a_n$
  - Reskalierung
    - $[z^n]A(z) = {}^n[z^n]A(z)$
  - Rechenregeln
    - $A(z) + B(z) = \sum_{n\geq 0}(a_n + b_n)z^n$
    - $A(z) * B(z) = \sum_{n\geq 0}^{\infty}(\sum_{k=0}^{n} a_k * b_{n-k})z^n$
    - $\frac{\partial}{\partial z}(\sum_{n\geq 0} a_n * z^n) = \sum_{n\geq 1} n * a_n * z^{n-1}$
- $B(z)$ ist Reziprokes von $A(z)$, wenn $A(z) * B(z) = B(z) * A(z) = 1$
  - $A(z) = \frac{1}{B(z)}$
  - Beispiele:
    - geometrische Reihe

        - 

    * Fibonacci
        - 

        - 

        - 

- a_n bestimmen?
    - 

[[Kombinatorik]]