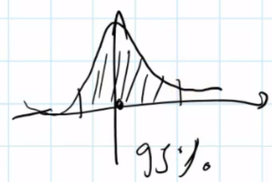
Example

Example 8 (Radar guns revisited)

- Suppose $n = 4$ radar guns are set up along a stretch of road to catch people driving over the speed limit. Each radar gun is known to have a normal measurement error $N(0, \sigma^2)$, $\sigma = 5km/h$. For a car passing at speed μ four readings are $(45.71, 47.41, 40.95, 50.65)$. Compute a random interval that covers the true unknown car speed μ with probability of 0.95.

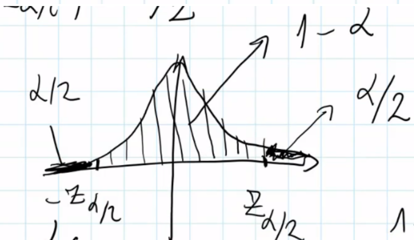
- $X_1 \sim N(\mu, \sigma^2)$ $\qquad \overline{X}_n \sim N(\mu, \frac{\sigma^2}{n})$

  $\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

  $z_{\alpha/2}$ the z-value s.t.

  $P(Z \geq z_{\alpha/2}) = \alpha/2$

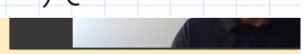
- 

  $-z_{\alpha/2}$ s.t.

  $P(Z \leq -z_{\alpha/2}) = \alpha/2$ $\qquad 1-\alpha = 0.95$

  $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$

- Continuing with $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$:

$$
\begin{aligned}
P(-z_{\alpha/2} < \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} < z_{\alpha/2}) &= 1 - \alpha \\
P(-z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \overline{X}_n - \mu < z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) &= 1 - \alpha \\
P(-\overline{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < -\mu < -\overline{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) &= 1 - \alpha \\
P(\overline{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \overline{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) &= 1 - \alpha
\end{aligned}
$$

  For our values, we get $\alpha = 0.05$, $z_{\alpha/2} = 1.96$, $\overline{X}_n = 46.18$ and the 95% confidence interval is $(41.28, 51.08)$.

General [[Konfidenzintervall]] Definition

- Let $X_1, X_2, \dots, X_n$ be a random sample of size n from density $f(\cdot)$. A $1-\alpha$ *confidence interval* for a parameter θ is an interval $C_n = (T_{n,1}, T_{n,2})$ with $T_{n,1} = t_1(X_1, X_2, \dots, X_n)$ and $T_{n,2} = t_2(X_1, X_2, \dots, X_n)$ such that:
$$P(\theta \in C_n) \geq 1-\alpha.$$
- C_n traps θ with probability $1-\alpha$. **Important:** C_n is random, θ is fixed.