Methods

Calculate $x^n, x \in \mathbb{R}, n \in \mathbb{N}$ efficiently.
Example x^{23}:
trivial: 22 multiplications
better:

$$x \cdot x \to x^2$$
$$x^2 \cdot x^2 \to x^4$$
$$x^4 \cdot x^4 \to x^8$$
$$x^8 \cdot x^8 \to x^{16}$$
$$x^{23} = x^{16} \cdot x^4 \cdot x^2 \cdot x^1$$

7 multiplications

- Idea: Calculate $x^2, x^4, x^8, \ldots, x^{2^k}, k = \lfloor ld(n) \rfloor$ by repeated squaring.

  Multiply the terms x^{2^i} for which the binary representation $(b_k, b_{k-1}, \ldots, b_1, b_0)$ of n has a 1 in the i-th position, i.e., $b_i = 1$.

- In total at most $2 \cdot \lfloor ld(n) \rfloor = \mathcal{O}(log(n))$ multiplications.

  Another example: x^{62}

  $x^2, x^4, x^8, x^{16}, x^{32}$
  $x^{62} = x^2 \cdot x^4 \cdot x^8 \cdot x^{16} \cdot x^{32}$
  $\Rightarrow$ 9 multiplications

  But there exists an even better option:

  $x^{62} = x^{20} \cdot x^{20} \cdot x^{20} \cdot x^2$
  $x^{20} = x^{16} \cdot x^4$
  x^2, x^4, x^8, x^{16}
  $\Rightarrow$ 8 multiplications

  Finding the optimal solution for general n is an open research problem.

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