Motivation

- In case that σ^2 is unknown we can replace it with S_n^2 for large n
- We know that for large n: $\frac{\overline{X}_n-\mu}{S_n/\sqrt{n}} \approx N(0,1)$
- In practice, already for $n \geq 30$ we can compute Z-score confidence intervals
- But, what do we do for small $n < 30$?
- If $X_1 \sim N(\mu, \sigma^2)$ then $\frac{X_n-\mu}{S_n/\sqrt{n}} \sim t(n-1)$

  t distribution with $n-1$ degrees of freedom

  For normal population and small n we can use $t_{\alpha/2}$ instead of $z_{\alpha/2}$

  We obtain then $1-\alpha$ T-score confidence interval: $\overline{X}_n \pm t_{\alpha/2}\frac{S_n}{\sqrt{n}}$

Example

- Suppose $n = 4$ radar guns are set up along a stretch of road to catch people driving over the speed limit. Each radar gun is known to have a normal measurement error $N(0, \sigma^2)$ with σ^2 unknown. For a car passing at speed μ four readings are $(45.71, 47.41, 40.95, 50.65)$. Compute a random interval that covers the true unknown car speed μ with probability of 0.95.
- For our values, we get $\alpha = 0.05$, $S_n = 4.04$, $t_{\alpha/2} = 3.18$, $\overline{X}_n = 46.18$ and the 95% confidence interval is $(39.74, 52.62)$.