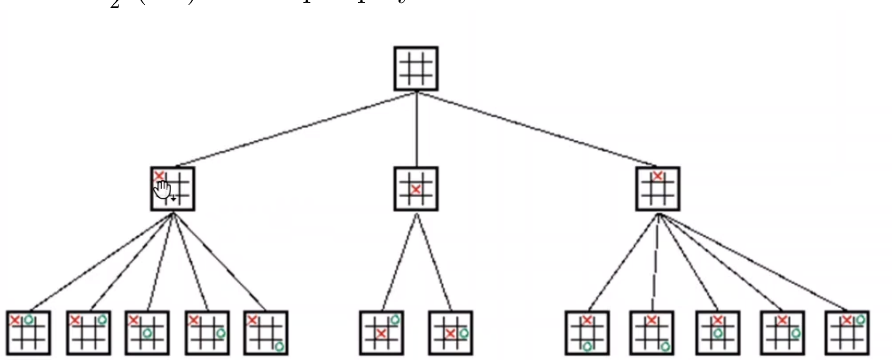
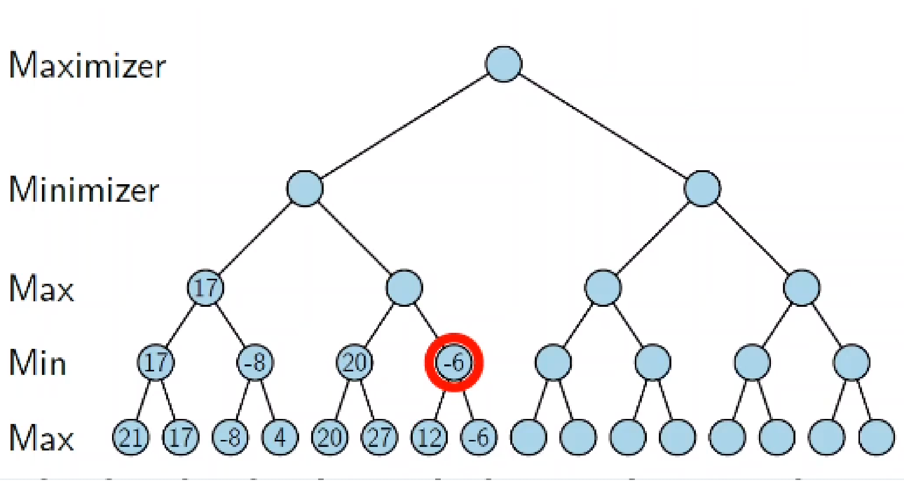
Classical Approach

- starting position as root
  - during game the current position
- states as nodes
- possible moves as edges to child nodes
  - states may be reached by multiple move sequences
  - performance improvement possible
    * no longer a tree
- tree depth is bounded
  - exponential overhead
  - combinatorial explosion
- half move
  - own move + opponents move = full move
  - must also consider the opponents answer move
  - k half moves
    * $\frac{k}{2}$ (+1) moves per player
- 
  - not shown possible moves are symmetrical
- leaves nodes are evaluated using heuristics
  - heuristic scores the game state
  - terminal states (win, lose) have extreme values
  - non-leave nodes evaluated by traversing the tree downwards
    * chooses maximum/minimum score of available moves
- maximizer and minimizer
  - one player wants to maximize the value of all child nodes
  - other player wants to minimize the value of all child nodes

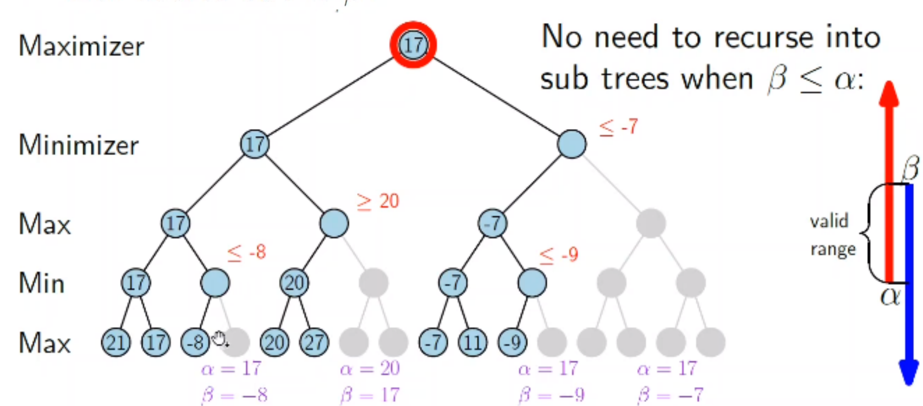


- Pruning

- prevents combinatorial explosion
- reduces states to consider
  - combines states with identical outcome
- more efficient and equal results as the classical approach
- stops traversing tree downwards upon reaching
  - a minimum value smaller than current maximum
  - a maximum value larger than current minimum

    α: Lower bound for maximizer: no need to consider states with scores below α.

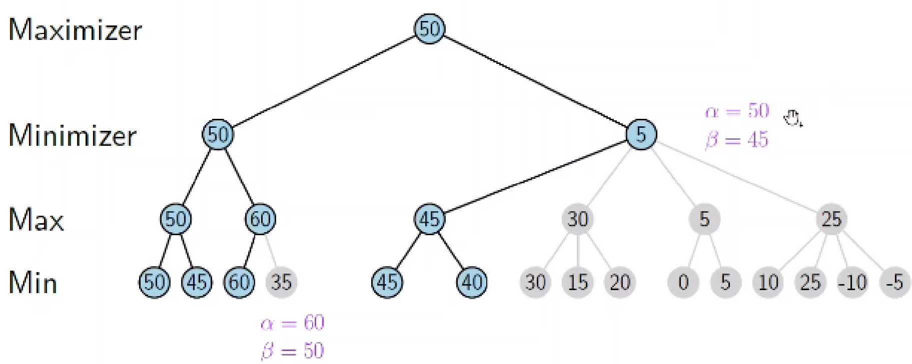
    β: Upper bound for minimizer: no need to consider states with scores above β.

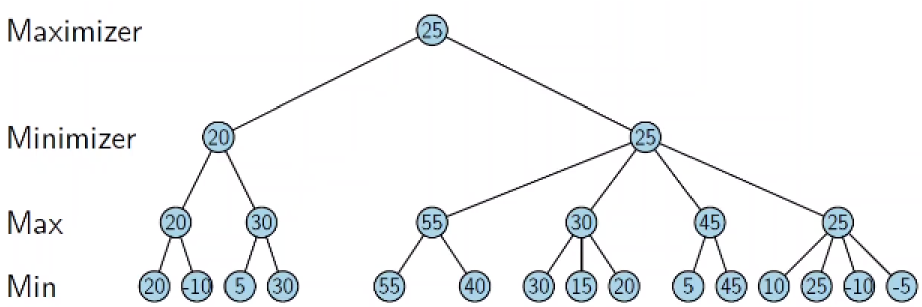
  - 

- pseudo code

```
evaluate (node, alpha, beta)
    if node is a leaf
       return (heuristic value of node)
    if node is a minimizing node
       for each child of node
          beta = min (beta, evaluate (child, alpha, beta))
          if beta <= alpha
             return (alpha)
       return (beta)
    if node is a maximizing node
       for each child of node
          alpha = max (alpha, evaluate (child, alpha, beta))
          if beta <= alpha
             return (beta)
       return (alpha)
```

– evaluate(root,−∞,∞);

- examples
  - 

  - 

    How much can we save with α-β pruning?
  - Sometimes nothing!
- pre-sorted pruning
  - efficiency depends on order of states
  - Consider potentially best states first (large values for maximizer, small (large negativ) for minimizer)