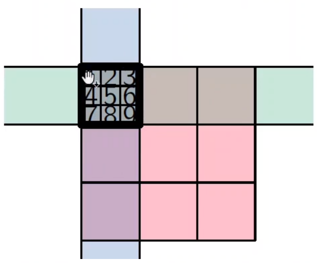
Observations

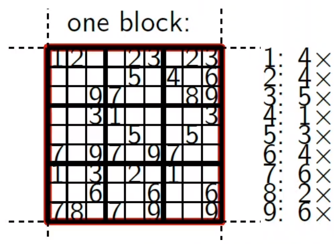
- groups
  - 9 horizontal rows
  - 9 vertical rows
  - 9 blocks
- each cell is part of 3 groups

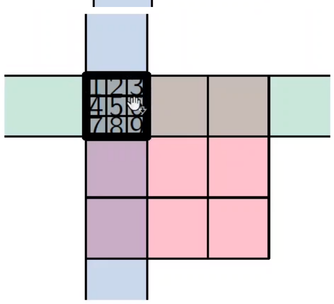
  For each square sq (81×):
  - still possible numbers for sq (initially 1,...,9)

  For each group G (27×):
  - For each number $i = 1, \ldots, 9$ a counter $c(G, i)$ of how many squares in G could still be i (initially 9)

  Each square points to its 3 groups, each group to its 9 squares

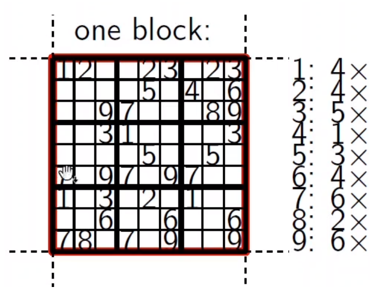
- 

- 

  Observation 1: If a square sq has only one possible number left, then we can fix it. Plus update of all groups of sq and the squares of these groups. And the groups of these squares.

  - a cell has only one possible number =>
    - remove this number from other cells
- **Observation 2:** If a counter $c(G, i)$ is one, then we can fix this number in a square sq in G. Plus update of all groups of sq and the squares of these groups.

  - a number is in only one cell possible =>
    - remove all other possible entries for this cell
    - decrement counters for these entries

Pseudocode

- PROCEDURE FIX-SQUARE (sq, nr)

  Set square sq to nr
  For all three groups G which contain sq
      reduce counter $c(G, nr)$
      For all squares sq' of G, $sq' \neq sq$ DO
          remove nr from sq'
          FOR each group $G' \neq G$ which contains sq' DO
              reduce counter $c(G', nr)$
  For all sq', not fixed and only one possible number nr' DO
      fix-square(sq', nr')
  For all $c(G, nr)$ which have been reduced to 1 DO
      search for square sq' of G which still allows nr
      fix-square(sq', nr)

  Call FIX-SQUARE() for all given numbers of a sudoku
  If all 81 squares get fixed, the Sudoku is solved

  For all simple (Level 1) Sudokus FIX-SQUARE() will provide the solution

- Add check if a counter $c(G, nr)$ gets zero or if a square has no valid number left $\Rightarrow$ return('No valid solution')

- more sophisticated backtracking version
  - able to solve level 2 sudokus
  - guesses a number for a square
    - Choose a square sq with a small number of still valid numbers.
    - For all valid numbers nr, fix the square sq with this number by calling FIX-SQUARE(sq, nr)
    - If the answer is "No valid solution" choose next valid number
    - Otherwise we have either a solution, or we need to backtrack recursively for another square

    *
  - level 3 sudokus would need 2 backtracking steps

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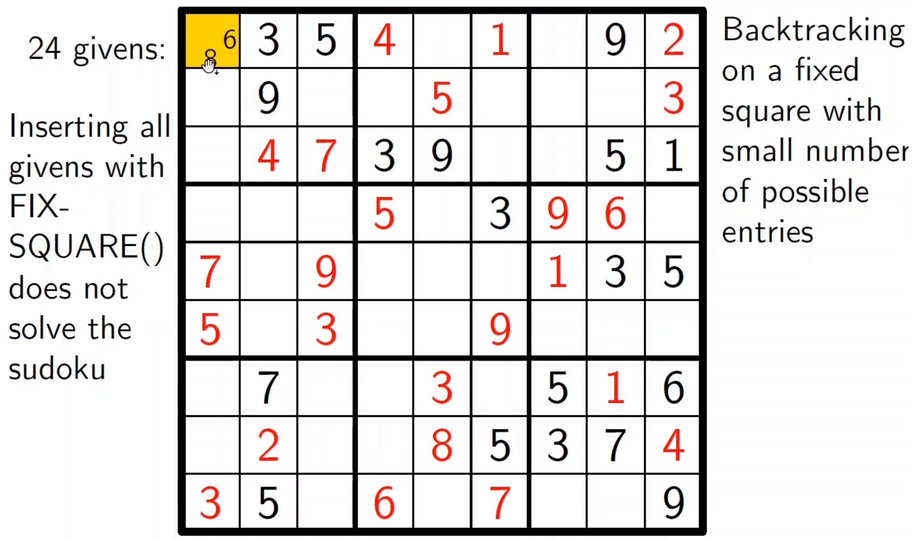

  - ∗ 8 would lead to a contradiction => try 6
  - ∗ 6 solves the sudoku

- constant time and space complexity
  - – assuming a regular 9x9 sudoku
  - – otherwise NP complete?