Methods

Calculate x^n, $x \in \mathbb{R}$, $n \in \mathbb{N}$ efficiently.
Example x^{23}:
trivial: 22 multiplications
better:

$$x \cdot x \rightarrow x^2$$

$$x^2 \cdot x^2 \rightarrow x^4$$

$$x^4 \cdot x^4 \rightarrow x^8$$

$$x^8 \cdot x^8 \rightarrow x^{16}$$

$$x^{23} = x^{16} \cdot x^4 \cdot x^2 \cdot x^1$$

- 7 multiplications

Idea: Calculate $x^2, x^4, x^8, \ldots, x^{2^k}$, $k = \lfloor ld(n) \rfloor$ by repeated squaring.

Multiply the terms x^{2^i} for which the binary representation $(b_k, b_{k-1}, \ldots, b_1, b_0)$ of n has a 1 in the i-th position, i.e., $b_i = 1$.

- In total at most $2 \cdot \lfloor ld(n) \rfloor = \mathcal{O}(log(n))$ multiplications.

Another example: x^{62}

$x^2, x^4, x^8, x^{16}, x^{32}$
$x^{62} = x^2 \cdot x^4 \cdot x^8 \cdot x^{16} \cdot x^{32}$
$\Rightarrow$ 9 multiplications

But there exists an even better option:

$x^{62} = x^{20} \cdot x^{20} \cdot x^{20} \cdot x^2$
$x^{20} = x^{16} \cdot x^4$
x^2, x^4, x^8, x^{16}
$\Rightarrow$ 8 multiplications

Finding the optimal solution for general n is an open research problem.