Overview

- all these games (and many more) can be reduced to NIM
- NIM rules
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    - **Games:**
      - NIM
      - Prime-game
      - Poker NIM
      - Northcott's Game
      - Kayles
      - Dawson's Kayles
      - Monochromatic Triangle
      - Triangulation Coloring Game

Prime Game

- n integers $f_1, \ldots, f_n > 1$
- Valid move:
  - Choose a non-prime integer $f_i > 1$
  - Split f_i into (one or more) prime factors $p_1, \ldots, p_k > 1, k \geq 1$ and a rest $f' > 1$
  - Replace f_i with $p_1, \ldots, p_k$ and f'
- **Normal play**: the last one to make a valid move wins

- each integer with k prime factors corresponds to a coin pile with k coins
- pile height = number of coins = number of prime factors

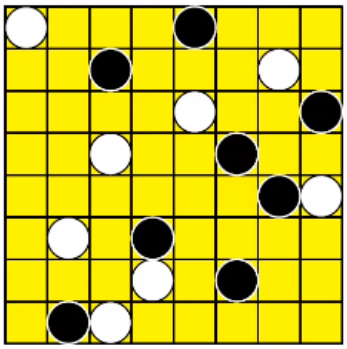
Poker NIM

- Startposition:
  - Same as for NIM
- Possible moves:
  - Similar to NIM, but instead of removing coins you may also put an arbitrary number of coins from your pool (built by previously taken coins) on a heap.
- **Normal play**: the last one to make a valid move wins

- exactly the same as NIM with finite number of coins in the pool
- placing coins from the pool just postpones the game end
  - opponent just removes the coins you just placed

Northcott's Game

- $n \times m$ chess board
  one black, one white coin per row in different columns
- Valid move:
  - Choose a row
  - move the coin of your color left or right arbitrarily many steps, but a least one
  - don't jump over your opponent's coin



- **Normal play**: the last one to make a valid move wins

- asymmetric version of Poker NIM
- each row corresponds to a pile
  - number of spaces between the coins = pile height
  - spaces behind a coin (inaccessible for the opponent) = number of coins in the pool

Kayles

- Setting:
  - as for NIM
- Possible moves:
  - Chose an arbitrary, non-empty stack
  - Remove one or two coins from this stack
  - Optional: split the remaining stack into two non-empty, smaller stacks
  - Bowling: Row of n pins.
    In a move hit one or two neighbored pins.

- Dawson's Kayles
  - always hit two neighbored pins
  - remove single pins

Monochromatic Triangle

- n points in the plane, in general position
- Valid move:
  - Draw a straight line segment connecting two points, not crossing any other line
- The game ends when an empty triangle occurs

- same as Dawson's Kayles

Triangulation Coloring Game

Triangulation on n points in the plane, all edges are black (white on the blackboard)

Valid moves:

- Select a black edge, color it green

The game ends when the first green empty triangle occurs.

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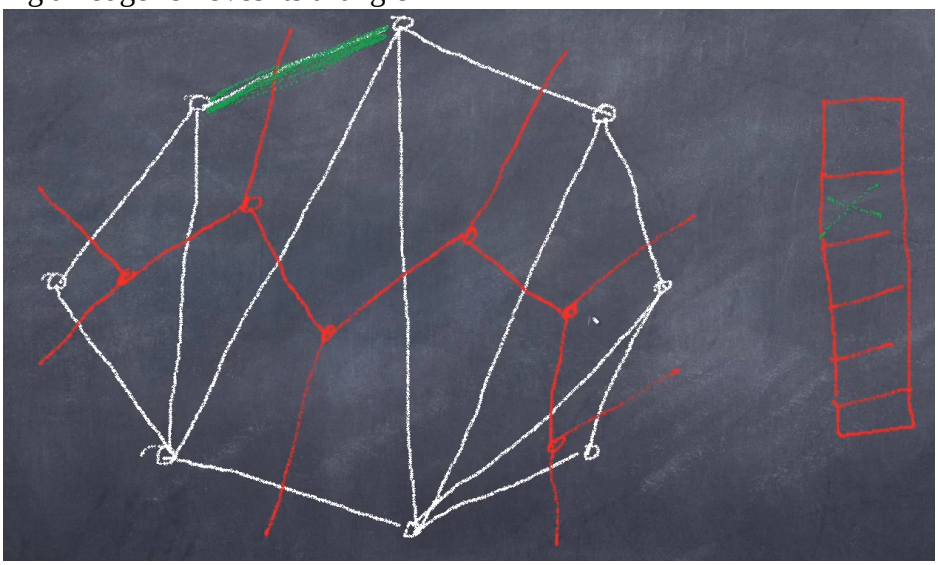
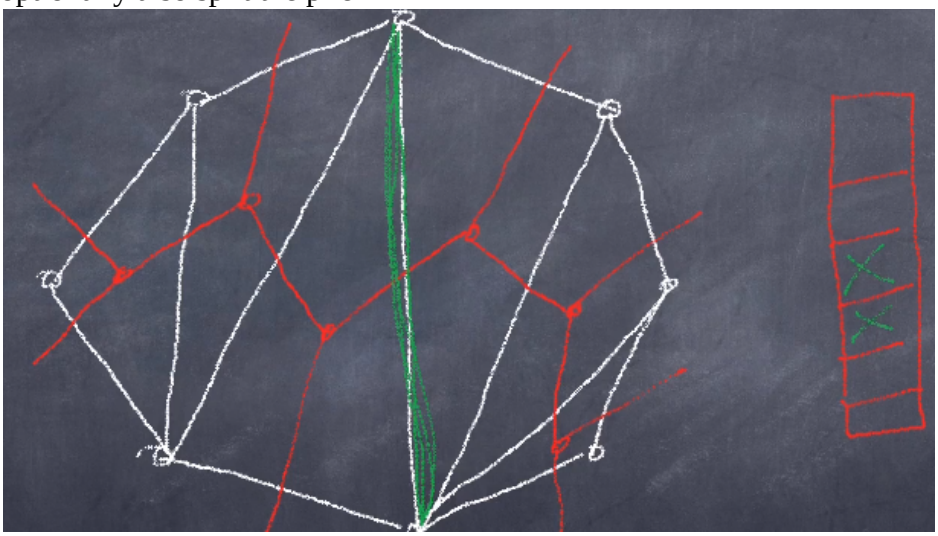
- dual structure
  - voronoi diagram
  - coloring an edge removes its triangle
    - 
  - may optionally also split the pile
    - 
- reduces to Kayles