Idea: repeatedly plug in recurrence

Example: $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

$$= 2\left(2T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2\right) + n^2$$

$$= 4T\left(\frac{n}{4}\right) + \frac{n^2}{2} + n^2$$

$$= 4\left(2T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2\right) + \frac{n^2}{2} + n^2$$

$$= 8T\left(\frac{n}{8}\right) + \frac{n^2}{4} + \frac{n^2}{2} + n^2 = \ldots$$

$$\ldots = 2^k T\left(\frac{n}{2^k}\right) + \frac{n^2}{2^{k-1}} + \cdots + \frac{n^2}{2} + n^2$$

$$= 2^k T\left(\frac{n}{2^k}\right) + n^2 \sum_{i=0}^{k-1} \frac{1}{2^i}$$

Application +

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + n^2 \sum_{i=0}^{k-1} \frac{1}{2^i}$$

$$\frac{n}{2^k} = 1 \text{ implies } k = \log_2 n$$

$$\Rightarrow T(n) = 2^{\log_2(n)} T(1) + n^2 \sum_{i=0}^{\log_2(n)-1} \frac{1}{2^i}$$

$$= n \cdot \Theta(1) + n^2 \sum_{i=0}^{\log_2(n)-1} \frac{1}{2^i}$$

$$= n \cdot \Theta(1) + n^2 \cdot \Theta(1)$$

+

Recursion Tree

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