Idea: repeatedly plug in recurrence

Example:
$$\begin{aligned}
T(n) &= 2T\left(\tfrac{n}{2}\right) + n^2 \\
&= 2\left(2T\left(\tfrac{n}{4}\right) + \left(\tfrac{n}{2}\right)^2\right) + n^2 \\
&= 4T\left(\tfrac{n}{4}\right) + \tfrac{n^2}{2} + n^2 \\
&= 4\left(2T\left(\tfrac{n}{8}\right) + \left(\tfrac{n}{4}\right)^2\right) + \tfrac{n^2}{2} + n^2 \\
&= 8T\left(\tfrac{n}{8}\right) + \tfrac{n^2}{4} + \tfrac{n^2}{2} + n^2 = \dots \\
\dots &= 2^k T\left(\tfrac{n}{2^k}\right) + \underline{\tfrac{n^2}{2^{k-1}}} + \dots + \tfrac{n^2}{2} + \underline{n^2} \\
&= 2^k T\left(\tfrac{n}{2^k}\right) + n^2 \sum_{i=0}^{k-1} \tfrac{1}{2^i}
\end{aligned}$$

Application + +

$$T(n) = 2^k T\left(\tfrac{n}{2^k}\right) + n^2 \sum_{i=0}^{k-1} \tfrac{1}{2^i}$$

$$\tfrac{n}{2^k} = 1 \text{ implies } k = \log_2 n$$

$$\begin{aligned}
\Rightarrow T(n) &= 2^{\log_2(n)} T(1) + n^2 \sum_{i=0}^{\log_2(n)-1} \tfrac{1}{2^i} \\
&= n \cdot \Theta(1) + n^2 \sum_{i=0}^{\log_2(n)-1} \tfrac{1}{2^i} \\
&= n \cdot \Theta(1) + n^2 \cdot \Theta(1)
\end{aligned}$$

Recursion Tree

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