Last lecture: Perceptron – Machine Learning

- Computational task classification: Assign one of a discrete number of labels to a (multidimensional) input.
  - A typical supervised machine learning task: Given a set of lablled training data $\{(x_1,y_1), (x_2,y_2) \ldots (x_n,y_n)\}$, discover a function f that approximtaes the true function that generates the labels y, such that $y=f_{TRUE}(x)$
- If two classes can be linearly separated, a perceptron can be iteratively parametrised, and the perceptron learning algorithm will converge towards a solution
  - Doesn‘t slowly converge but jumps around
  - Not necessarily the optimal one – inclusion of optimsiation criteria such as maximising distance to separation function is desirable
  - Convergence not guaranteed if classes are not linearly separable

Review: Which questions can different knowledge representation and reasoning paradigms answer?

Rules

- Represent: How are states/conditions related to each other?
- Questions: Which facts follow from the knowledge base? Is X true? Is the knowledge base consistent?

Object-oriented KR/Logic:

- Represent: How are different types of things (classes, concepts, frames) related to each other?
- Questions: Is the knowledge base consistent? What general (terminological) and assertional axioms (="truths") can be inferred? Is X true?

Graphs

- Represent: How are entities related to each other (weighted or labelled relationships)?
- Questions: How are X and Y related to each other? Which entities are important in the network? If X is important/affected, which other entities are important/affected? (spreading activation)?

Vector representation

- Represents: Complex entities as a collection of numbers (=vector), with informally defined semantics (=documentation of features in the vector)
- Questions: How similar are X and Y to each other – application in IR and RecSys? What category is X (classification)? Which groups can we identify in a larger dataset (clustering)?

Today: Shift away from entities towards variables = features = aspects of interest with respect to (interesting) complex entities or phenomena

Correlation

Introduction to Data Science and Artificial Intelligence

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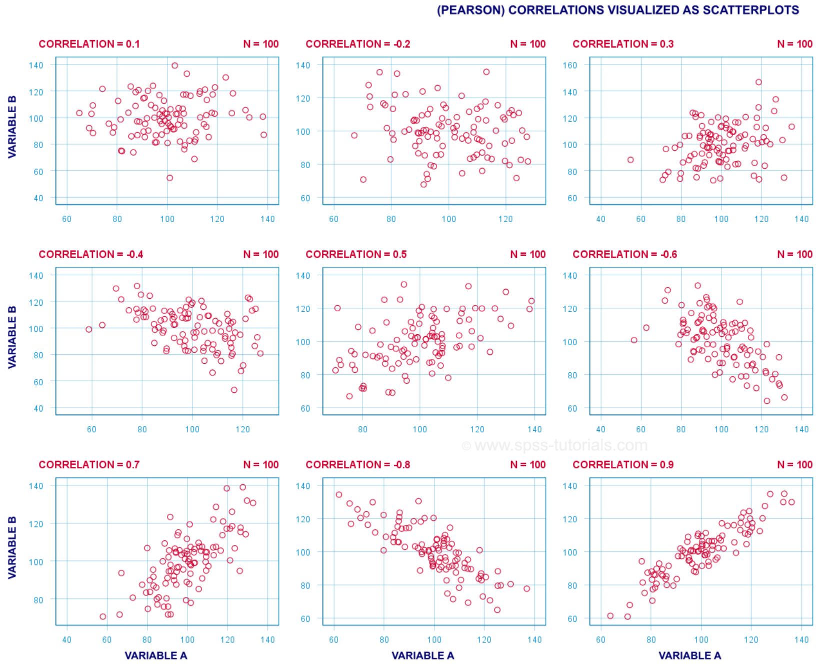
Correlation

To what extent are two variables related?

Pearson correlation (which we will use): To what extent are two quantitative variables related?

- No statement about **causality**
  - … even if in some literature you read about “independent/explaining variable” and “dependent/outcome” variable!
  - Good practice: Experimental set-up that allows inferring causaility + theory-based explanation of why you would expect causality!



https://www.spss-tutorials.com/spss-correlation-analysis/

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Pearson correlation - formula

$$r_{XY} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}}$$

$\bar{X}, \bar{Y}$ = average of X,Y respectively

Correlation coefficient r describes the strength of the relationship between two variables

-1: Perfect descending linear relationship – if the value of x is higher of a given delta, the value of y will be respectively lower (fixed ratio)

0: Variables aren’t systematically related

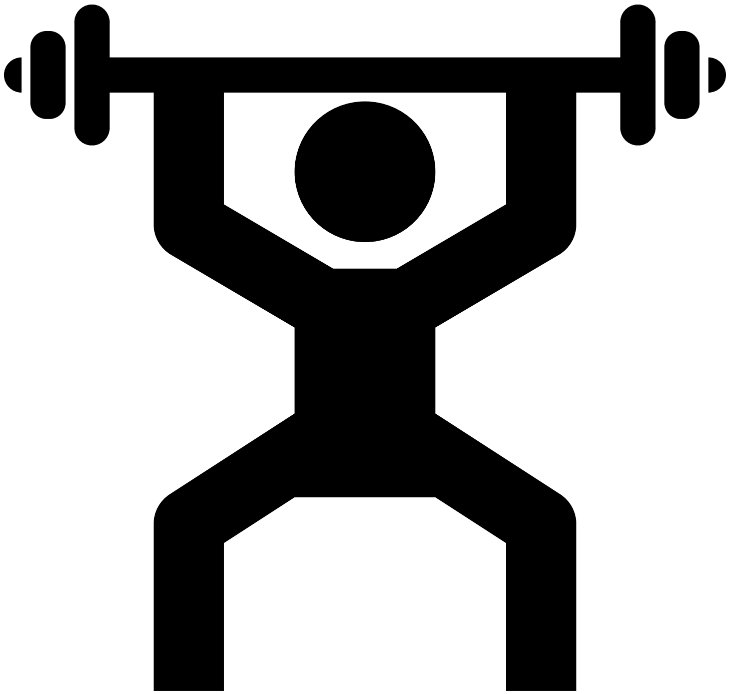
1: Perfect ascending linear relationship – if the value of x is higher of a given delta, the value of y will be respectively higher (fixed ratio)

Looks and sounds familiar? -> Pearson correlation == cosine similarity *IFF* each of two vectors can be understood as representing a (mean-adjusted) variable

Correlation Analysis

- Correlation coefficient *r* describes the strength of the linear relationship between two variables…
- … but in case of a large population, typically only a (hopefully good) sample is available.
  - How large is the likelihood that we are getting a correlation based on our sample even though there isn't really a correlation in the whole population? (null hypothesis)
    - We want this likelihood to be small! Typical threshold values for "small enough" are $p<0.05$, $p<0.01$, $p<0.005$

Exercise 18

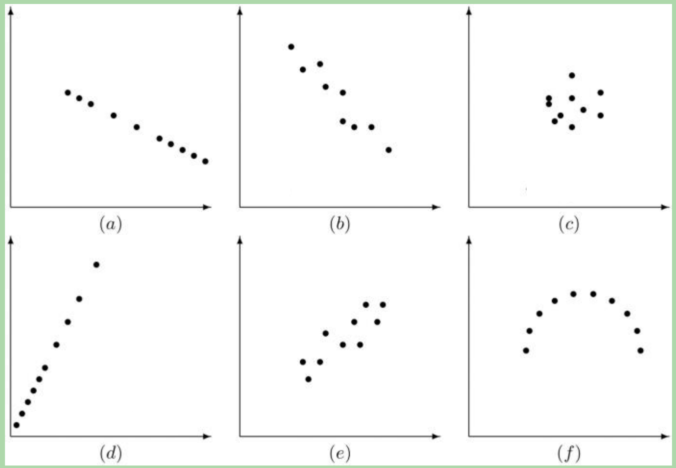


Correlation

Can you make a plausible guess as to the direction of the correlation of the following?

- The weight of an automobile and its fuel consumption?
- The age of an automobile and the distance driven?
- The weight of an automobile and the distance driven?
- The age and weight of a person?
- The number of pages of a book and the expected age of its readers?
- The number of pages of a book and the age of its author?

Is the correlation value in the scatterplots positive, negative, or rather around zero?



Picture from: https://saylordotorg.github.io/text_introductory-statistics/s14-02-the-linear-correlation-coeffic.html

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Linear Regression

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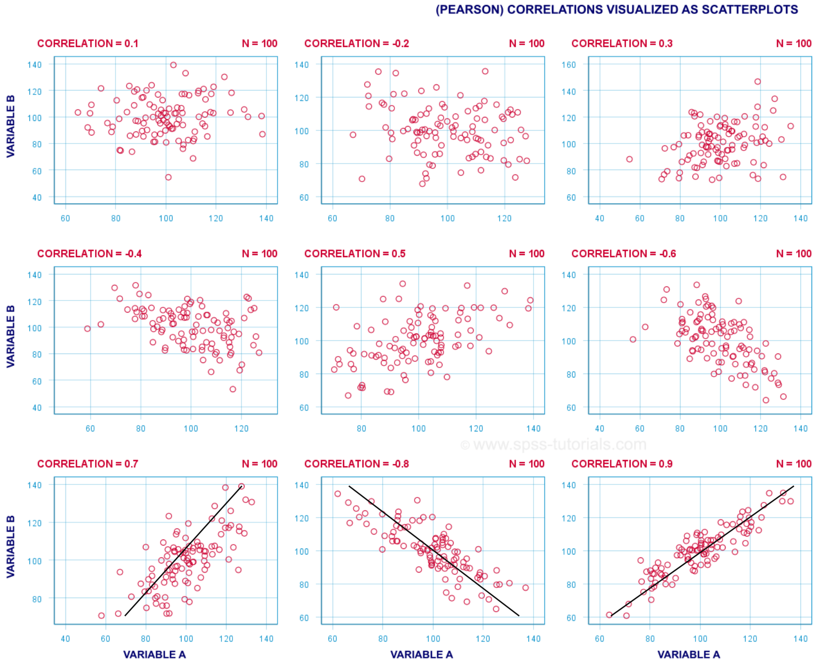
If we have two correlated variables – let‘s fit the line in! -> **linear regression**



https://www.spss-tutorials.com/spss-correlation-analysis/

Linear regression line via the least squares method

Goal: Identify the line that best fits the linearly correlated data

Regression line: $y = a + bx$

- Underlying assumption: We never know all data, the data that we know are just *training data*.
  - Typical ML approaches to data are taken, such as choosing a part of data as training data, keeping a part of data as test data for evaluating the quality of the model.
- Optimisation criterion: Minimal least squares error –minimal sum of distances (in whichever direction) of points to line.

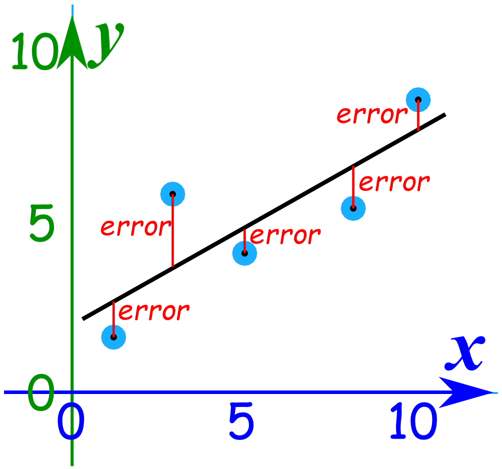
Linear regression line via the least squares method

Regression line: $y = a + bx$

Slope $b = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}$

OR $b = r\left(\frac{s_y}{s_x}\right)$ with *r* the correlation coefficient

Intercept $a = \frac{\sum y - b \sum x}{N}$



From: https://www.mathsisfun.com/data/least-squares-regression.html

- This method minimizes the sum of the squared errors.
- Error = difference between the estimated y-value (y=a+bx) for a given x value, and the real/measured y-value in the sample data for the same x value.

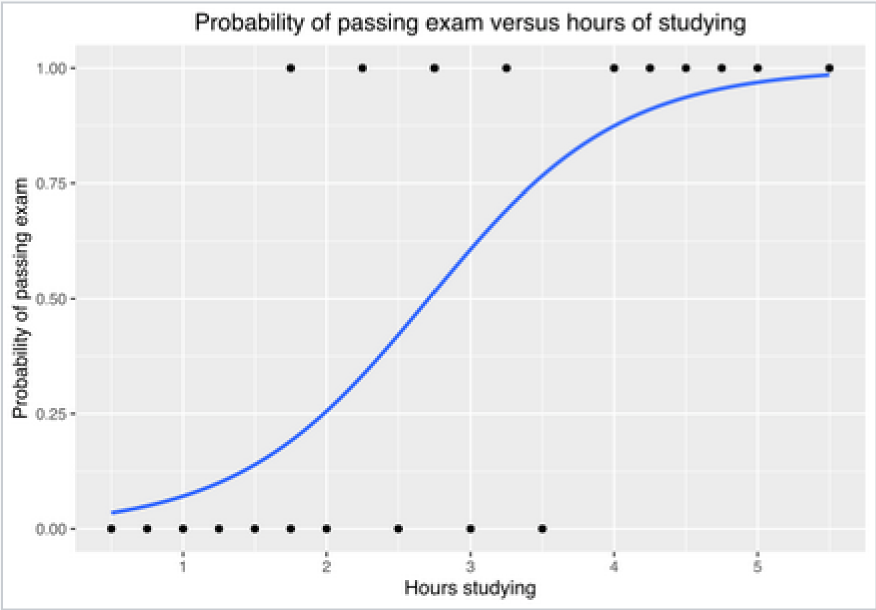
Other types of regression

Non-linear regression – curve fitting: Fitting a non-linear function to data

Logistic regression: Fitting a logarithmic function to continuous independent data and dichotomous outcome data

- Looks familiar?

➢ Logistic regression is a method for classification into two classes



Example graph of a logistic regression curve fitted to data. The curve shows the probability of passing an exam (binary dependent variable) versus hours studying (scalar independent variable). See § Example for worked details.

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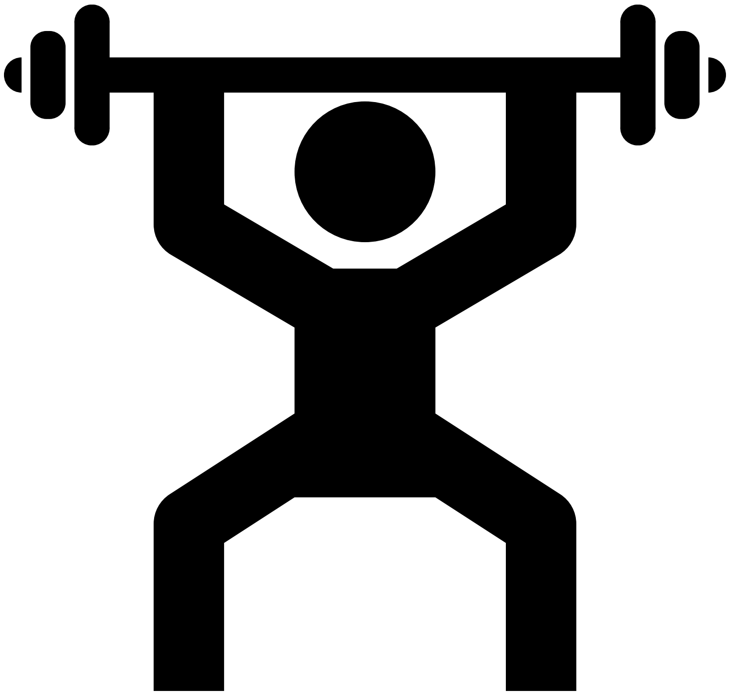
Exercise 19



Linear Regression

For the following sample data, compute

- The correlation value
- The best fitting regression line via least squares regression

As an exception – do use a calculator, Excel/spreadsheets, R, … to do the calculation!

X	**Y**
6	45
12	47
13	39
17	58
22	68
25	76
27	75
29	74
30	78
32	81

Discussion

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Correlation, linear regression, and machine learning

Prediction:

Estimating the value y_i (y is the dependent, the outcome variable), given x_i (x is the independent, the explaining variable)? – Use the regression line

- What example use cases can you think of for prediction?

Feature engineering:

Sometimes features are desirable that are not highly correlated pairwise

Learning Goals

- Understand correlation
- Be able to compute correlation between two numerical variables
- Understand the computational task of (linear) regression analysis
- Be able to carry out least-squares linear regression
- Understand the computational task of prediction, and regression analysis as a method for prediction.