Übersicht

- [[Anwendungen der Residuenrechnung]]
- f: $[a, \infty)$−>$\mathbb{R}$
- Laplace Transformation

$$\mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt$$

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  - $Lf(s)$ f existiert, wenn $\exists A, C \in \mathbb{R} \forall t \in \mathbb{R}^+ : |f(t)| \leq Ce^{At}$
- Konvergergenzabszisse $c_0(f)$
  - $\forall f \exists c_0(f) \in \mathbb{R} : \forall s \in \mathbb{C}$ mit $Re(s) > c_0(f)$ $Lf(s)$ existiert
  - $Lf(s)$ ist holomorphe Funktion für $Re(s) > c_0(f)$
- Doetsch Symbol
  - Beziehung zwischen f und Lf

$$\circ\!\!-\!\!\!-\!\!\!-\!\!\bullet$$

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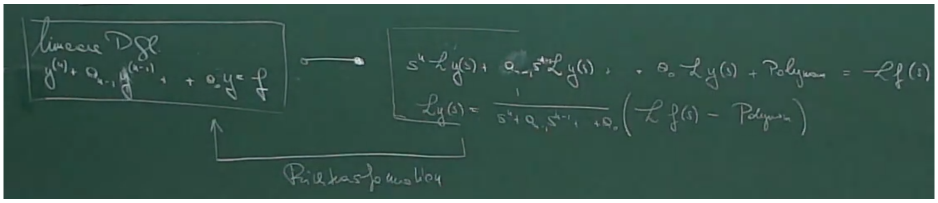
Rechenregeln für L

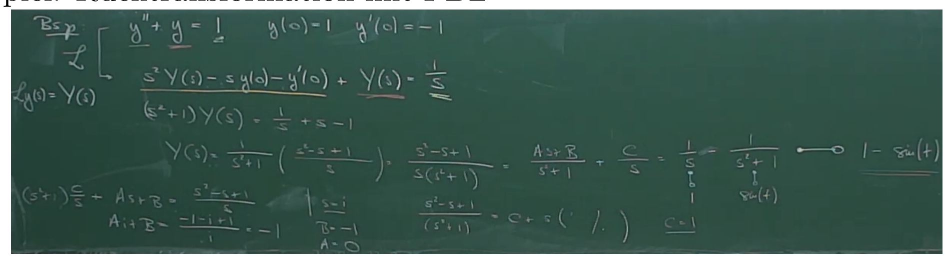
| | | |
|---|---|---|
| $F(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\mathcal{L}F(s)$ |
| $aF(t) + bG(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $a\mathcal{L}(F(s)) + b\mathcal{L}(G(s))$ |
| $F(t), G(t), H(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $f(s), g(s), h(s)$ |
| $1$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{1}{s}$ |
| $e^{\alpha t}$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{1}{s-\alpha}$ |
| $t^n$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{n!}{s^{n+1}}$ |
| $t^n e^{\alpha t}$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{n!}{(s-\alpha)^{n+1}}$ |
| $\sin \alpha t$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{\alpha}{s^2+\alpha^2}$ |
| $\cos \alpha t$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{s}{s^2+\alpha^2}$ |
| $e^{-\alpha t} \sin \beta t$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{\beta}{(s+\alpha)^2+\beta^2}$ |
| $e^{-\alpha t} \cos \beta t$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$ |
| $\alpha > -1, \quad t^\alpha$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$ |
| $\delta(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $1$ |
| $F^{(k)}(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $s^k f(s) - s^{k-1}F(0) - s^{k-2}F'(0) - \cdots - F^{(k-1)}(0)$ |
| $t^k F(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $(-1)^k f^{(k)}(s)$ |

| | | |
|---|---|---|
| $\lambda > 0, \quad \begin{cases} F(t-\lambda) & \text{für } t \geq \lambda \\ 0 & \text{sonst} \end{cases}$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $e^{-\lambda s} f(s)$ |
| $e^{-\alpha t}F(t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $f(s+\alpha)$ |
| $F(\rho t)$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{1}{\rho} f\left(\frac{s}{\rho}\right)$ |
| $F * G(t) = \int_0^t F(\tau)G(t-\tau)\,d\tau$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $f(s)g(s)$ |
| $\frac{F(t)}{t}$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\int_s^\infty f(v)\,d$ |
| $\int_0^t F(\tau)\,d\tau$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $\frac{f(s)}{s}$ |
| $\frac{1}{2\pi i} \int\limits_{c-i\infty}^{c+i\infty} f(s)e^{st}\,ds$ | $\circ\!\!-\!\!\!-\!\!\bullet$ | $f(s)$ |

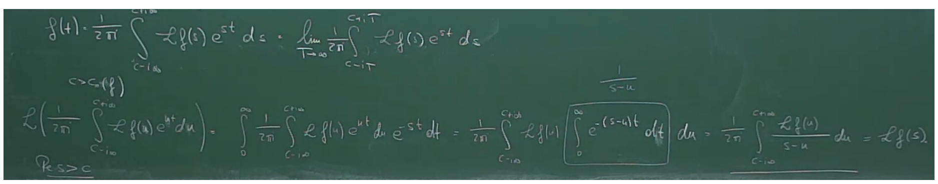
DGL lösen

- 

  - Laplace Transformation auf lineare DGL
  - dividieren statt DGL lösen

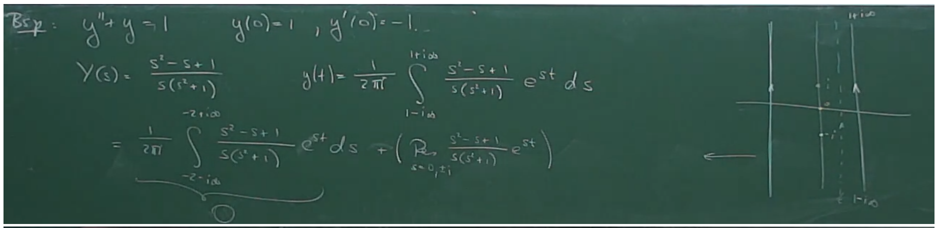
  - Rücktransformation
- Beispiel: Rücktransformation mit PBZ
  - 

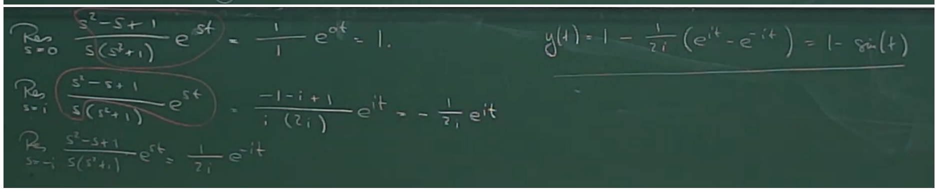
Laplace Umkehrformel

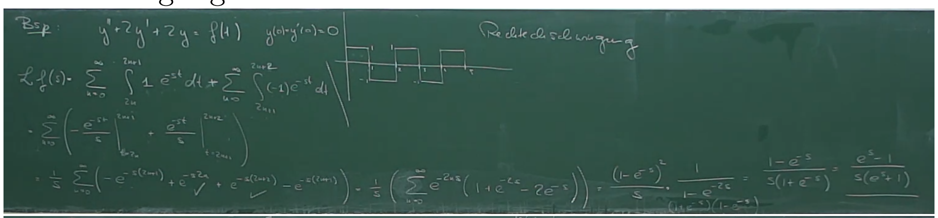
- 
- Rücktransformation mit CIF statt
  - PBZ
  - Bekanntsein der Rechenregeln für L
- Verschiebung in Gebiet mit Polstellen
  - ==> Residuensatz statt CIF
  - $$y(t) = \sum \mathrm{Res}\left(\frac{s^3+3s^2+2s+3}{(s^2+1)(s^2+4s+5)}e^{st}\right).$$
    - Verschieben des Kurvenintegrals im holomorphen Gebiet ändert nichts
    - Kurvenintegral = 0
    - Summe der Residuen bleibt übrig

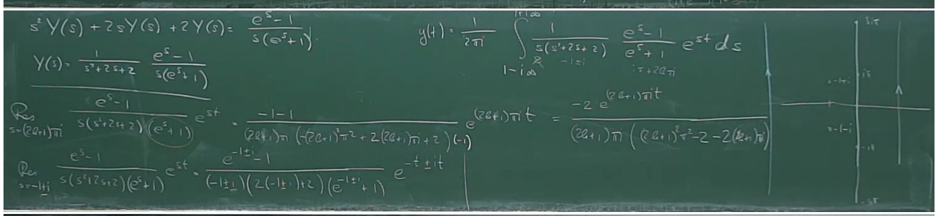
Beispiele mit Umkehrformel

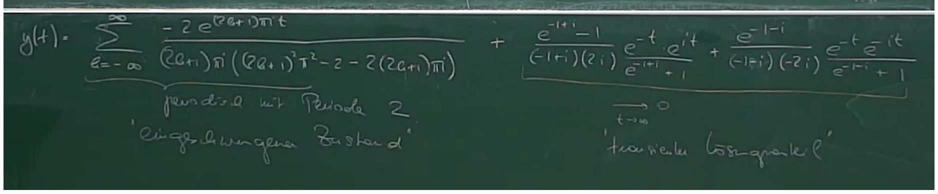
- gleiche Beispiel wie oben jedoch ohne PBZ

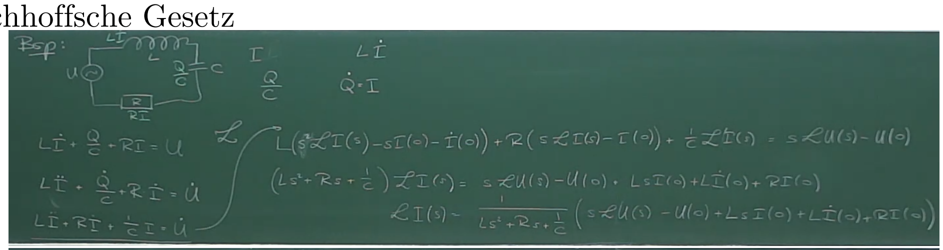
- 

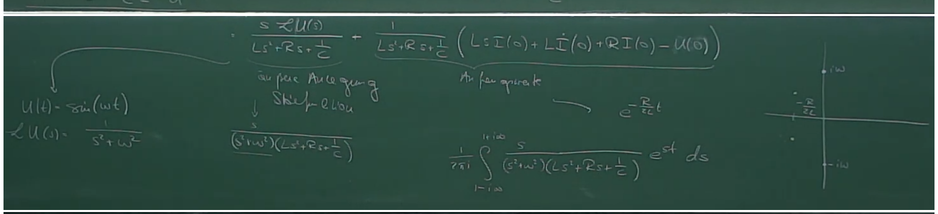
- 

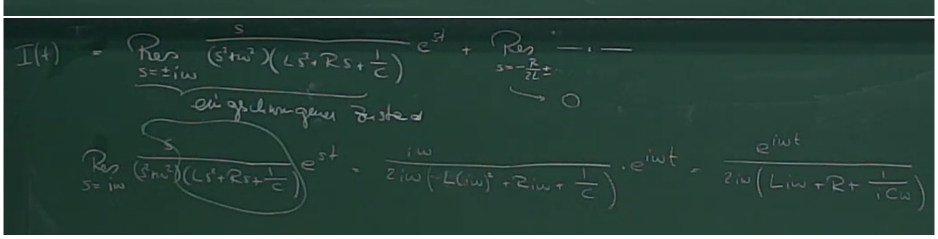

- Rechteckschwingung mit unendlich Polen
  - 

  - 

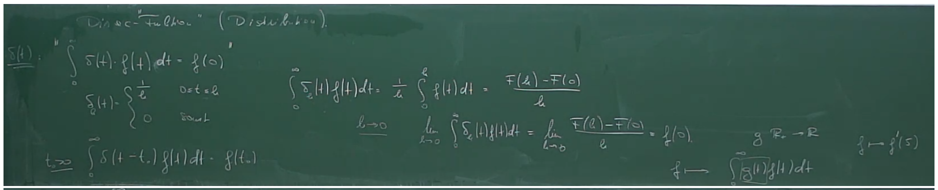
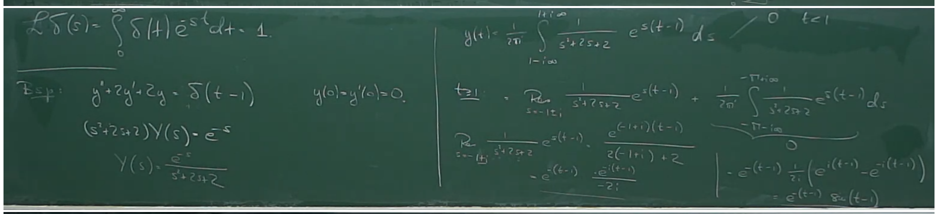
  - 


- Kirchhoffsche Gesetz
  - 

  - 

  - 

Exkurs: Distributionen

- 
- 
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