Should I Stay or Should I Go? Transfers, Trade Gains, and Capital Accumulation in a Fiscal Union

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Abstract

I study the attainability of a fiscal union model for countries in the European Monetary Union (EMU). Recent research typically limits risk-sharing in a monetary union by introducing limited commitment frictions. Conversely, this paper contemplates productivity shocks for countries in the currency union to evaluate the viability of a fiscal union. The research proposes two models. Firstly, I propose a multi-country static model where countries share risk internationally through state-contingent transfers. In this setting, I compare welfare effects from transfers and trade gains for each country of the EMU. Therefore, I aim to establish whether the trade gains in the currency union compensate for the loss of transfers in the fiscal union, especially for high productivity and high-income countries. The second is a dynamic model where taxes and subsidies of the EU central government redistribute wealth across countries. The value-added of this dynamic model is the evaluation of the sustainability of the transfers in the fiscal union. Additionally, the government enforces capital accumulation to increase the future productivity of a low productive country and therefore overcomes moral hazard issues. Therefore, I expect to find that, in addition to the trade gains and the multiple social benefits, remaining in the union might represent an insurance mechanism against future shocks for all the members.

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1 Introduction

In the Optimum Currency Areas by Mundell (1961), McKinnon (1963), and Kenen (1969), countries sharing the same currency and participating in a monetary union should start to create the underpinnings for a fiscal union. In economic research, there is increasing interest in modeling a fiscal union for countries in a currency and monetary union, especially after the effects of the recent crisis in the European Monetary Union (EMU hereafter).

The relevant literature on this topic analyzes the fiscal union in a monetary union in a second-best problem with complete and incomplete financial market (Farhi and Werning, 2017); in a comparison under two different union regimes: Ferrari et al. (2020) study a fiscal union with independent monetary policy and a currency union with transfers; and in a subgame-perfect equilibrium framework (Auclert and Rognile, 2014). Similarly to this paper, the above studies focus on the implementation of a fiscal union for countries in a currency and monetary union. However, this paper first evaluates the system of transfers in comparison with the trade gains for each country in the EMU. Secondly, it enlarges the background of the risk-sharing topic with the evaluation of the sustainability of a fiscal union with the introduction of enforced capital accumulation.

The paper presents two models. The first will be structured as a multi-country static model with i countries where I focus on a single country called home (H). In the static framework, similarly to Farhi and Werning (2017), I study the case of incomplete markets for which agents have no opportunities to share risk internationally. Thus, the central government will arrange international fiscal transfers across countries to provide international risk-sharing using lump-sum transfers to redistribute wealth. With this approach, I model the fiscal union as a contract where the central government transfers ensure risk-sharing. These state-contingent transfers absorb the risk of the asymmetric component of the productivity shock, as specified later in the paper. In the static model, I compare the advantages deriving from trade gains in the currency union with the possible losses for the single country incurred with the transfers in the fiscal union. In this respect, I compute consumption and employment for each country in the EMU and evaluate which regime delivers higher welfare. This computation allows us to establish the feasibility of the fiscal union concerning the welfare achieved for each country. On the other hand, the dynamic setting extends the model to account for the time dimension, household savings, and the monetary policy in autarky and in the union to evaluate the sustainability of fiscal transfers. More importantly, it considers capital accumulation enforced through a system of taxes. Hence, the dynamic setting would determine the long-run trade-offs between trade gains and transfers for every single country. Additionally, it aims to establish the sustainability of transfers and a possible mechanism to overcome moral hazard issues through capital enforcement. Finally, I compare the size of transfers with the EMU budget. The sustainability argument would also carefully discuss the possible and reasonable magnitude of a European corporate tax that I impose as a financing instrument for the transfers.

Following Farhi and Werning (2017) and the Optimum Currency Area literature, the research considers the currency union an external constraint. Being in a currency union, like the EMU countries, does not allow countries to conduct independent monetary policy. Indeed, the central bank does not define policies specifically for individual countries. Therefore, given the differences among them and the limits of a central monetary policy, it seems reasonable that fiscal union and risk-sharing among countries become necessary.

In the static framework, the paper computes the trade gains for every single country of the EMU and compares those gains with the expected transfers of a fiscal union. To my knowledge, previous literature has

not compared and computed this analysis simultaneously for all the countries of the EMU. Additionally, the paper accounts for productivity and shows that a fiscal union is necessary for countries in a monetary union. I demonstrate that the fiscal union may resolve the imbalances created by asymmetric productivity shocks for the countries in the union. Furthermore, I account for the boost in trade gains experienced by countries entering the union. Therefore, I aim to compute the costs and benefits of the fiscal union for each country. Finally, I establish a fiscal union with a central monetary authority in a dynamic model where the central government enforces capital accumulation for low productivity countries. It also sets transfers levying taxes on corporate revenue while accounting for the optimal monetary policy of the union.
The rest of the paper examines the relevant literature in the second chapter and shows the static model in the third chapter. The fourth chapter encompasses the calibration and the first results of the static model. The fifth chapter introduces our idea for the dynamic model. The paper ends with conclusions.

2 Literature review and contribution to relevant research

The OCA literature poses that a currency union without a fiscal union may face severe difficulties in facing asymmetric shocks. Therefore, a fiscal union seems to be the natural step following the creation of a monetary union. Indeed, this argument relates to the sequencing theory that specifies how one type of economic cooperation begets the next step. Indeed, Balassa (1962), Gustavsson (1999), Estevadeordal and Suominen (2008), Baldwin (2012) all contribute to the "integration staircase" argument relative to the economic integration. This paper refers mainly to three recent and relevant papers. The following subsections explain the main claims of the three sources. Additionally, I present how and where this paper could contribute to these important studies and the previous literature in general.

Ferrari et al. (2020) consider two different models. Firstly, the authors build a model where two countries in a monetary union should establish a system of transfers to achieve better consumption smoothing to compensate for the loss of independent monetary authority. They prove that a currency union with transfers allows the countries to achieve an equivalent aggregate welfare level to the case of a fiscal union with independent monetary policy.
This paper, in contrast to Ferrari et al. (2020), does not consider the case of a fiscal union with independent monetary policy and does not allow countries to borrow from a risk neutral lender through state-contingent bond once outside the union. I argue that those hypotheses may not be a viable option. For example, an exit strategy for countries in the EMU would be extremely long and costly (see Brexit). Additionally, in our setting, it does not seem to be realistic that countries in the EMU will abandon the monetary union to form a fiscal union. Therefore, I rule out the case of fiscal union with independent monetary policy. Furthermore, I suppose that it may be difficult to find a risk neutral lender when exiting. In fact, in the crisis of 2008, the interest rates of the treasury bonds of Greece escalated as a reaction of the market to the possibility of Greece exiting the EMU.
In addition, Ferrari et al. (2020) show that when the participation constraint is binding the currency union with transfers could achieve the same welfare value achieved under fiscal union with independent monetary policies. This result seems consistent and in agreement with the basic idea of this paper that countries in a monetary union need a fiscal union to optimize welfare. Considering the possibility of a currency union break up, as shown by Cohen (1993), the dynamic model of the authors considers a contract with limited commitment (as shown by Kocherlakota (1996), Marcet and Marimon (2019)). Conversely, the dynamic

model of this paper considers a different setting than Ferrari et al. (2020). Indeed, while the authors consider an economy with two types of goods where they assume tradable goods shock in a currency union with transfers and a fiscal union with independent monetary policies, I study productivity shocks in the case of a fiscal union that accounts for the optimal monetary policy of the Central bank of the union. Additionally, the central government sets subsidies levying a corporate tax on the revenue of firms in our framework. The dynamic model would also consider capital accumulation and overcome the simplifying assumption of countries borrowing from a risk neutral lender through state-contingent bonds once outside the union. I aim to enrich the model by attempting to make borrowing contingent on the exit, in the sense of making it costly, especially for low productivity countries leaving the union, to find a source of financing outside the union.
Moreover, this paper agrees with these authors in concluding that a fiscal union helps to achieve welfare distribution and better consumption smoothing. On the other hand, they also state that independent monetary policies would optimally respond to consumption volatility. In fact, they find that a monetary union cannot perform better than a fiscal union with independent monetary policies. Thus, the optimal monetary policy at the union level should be accompanied by state-contingent transfers to achieve the same efficient allocation that an independent monetary policy would. In other words, they attain the same results as this paper (fiscal union effectiveness in a monetary union) but motivate it with respect to the level of welfare reached with a fiscal union with independent monetary policy. In this paper, I bypass this possibility and arrive at the same conclusions but contemplate the productivity of the countries. Adding productivity shocks could enhance a more efficient system of fiscal union in a monetary union because it accounts for outcome imbalances. Finally, Ferrari et al. (2020) also state that one flaw of the paper is that the study does not consider the asymmetry in the output level of the countries in the union, in contrast to one of the assumptions of this paper.

Farhi and Werning (2017) prove that the fiscal union would optimize the aggregate risk-sharing in an open economy where countries have the same currency. The paper shows that the benefits of a fiscal union for countries in a currency union positively correlate with the degree of incompleteness of financial markets, the asymmetry and persistence of the shocks, and the closeness of the economies of the members in the union. Thus, the study seems to analyze a wide spectrum of characteristics of a monetary union. The authors implement the fiscal union as a mechanism to achieve optimal aggregate risk-sharing through international transfers (an assumption maintained in this paper). The researchers demonstrate that optimal fiscal transfers result in a twofold improvement: they correct consumption smoothing of countries at the microeconomic level and enhance stability in the union at the macroeconomic level. In conclusion, I believe that Farhi and Werning (2017) provide a comprehensive study in this field because it ponders different aspects of a currency union applying a micro-founded New Keynesian model. Similarly to the authors, our model is micro-founded. I consider nominal rigidities and present a static and a dynamic model. The value added of this paper is that I define the net position of a country in terms of differences between transfers and trade gains. Additionally, I evaluate the convenience of each country to stay in the union. Finally, our dynamic model establishes the sustainability of transfers in the fiscal union with capital accumulation enforcement.

Auclert and Rognile (2014) demonstrate that a fiscal union leading to international risk-sharing is beneficial under any monetary regime. The study proves that monetary unions facing nominal rigidities may reach better risk-sharing under fiscal union. Thus, a fiscal union enhances less output dispersion at the aggregate level which implies an overall higher efficient allocation, equivalently to the first-best solution of our model. I share the view that fiscal union supports cooperation between countries to offset exogenous asymmetric shocks. The authors analyze a game-theory framework for the fiscal and monetary union attaining perfect risk-sharing. Countries in a monetary union beget risk-sharing because of the costs induced by the monetary union. In Auclert and Rognile (2014) benchmark model, the central bank is able to simultaneously stabilize the countries in the currency union attaining the risk-sharing miracle.

Additionally, Auclert and Rognile (2014) do not contemplate uncertainty in the production. Likewise, I propose a model with price rigidity frictions. In this scenario, the central government intervention would be beneficial in avoiding drifts towards an adverse economic structure that could lead to iniquitous conditions for the agents of a specific country. However, in Auclert and Rognile (2014) the first-best allocation for the tradable goods also yields the first-best in the nontradables because of the perfect risk-sharing (the risk-sharing miracle). Therefore, Auclert and Rognile (2014) can relax the assumption of monetary policy intervention on nontradables without losing the validity of the model.

3 The static model

Our static model uses the assumption of home bias as a core of all analyses of the Transfer Problem. In the same fashion of Avrai (2021) I add trade costs to extend and modify the model of Corsetti and Pesenti (2002). There is a single period and a continuum of countries indexed by $i \in [0,1]$. I focus on a single country called home indexed by $\mathrm{H} \in [0,1]$. I distinguish the foreign variables from the home ones using a *. While Ferrari et al. (2020), Farhi and Werning (2017) and Auclert and Rognile (2014) analyze a two-goods economy where the two types of countries receive endowments of tradable goods and produce nontradables, I model home bias in the consumption function. Productivity represents a source of uncertainty in our model. Goods are produced from labor and in each country, there is only one type of good. I compute the allocations for consumption and labor with nominal rigidities.

The paper studies cross-country risk-sharing with aggregate uncertainty and a continuum of countries. I aim to compare aggregate welfare in five different scenarios. In the first one, countries are outside the currency union and conduct independent monetary policy. They face trade, currency uncertainty, and exchange rate costs. I parametrize those costs in the same spirit of Alesina and Barro (2002), with a parameter identifying iceberg trade cost. In the second scenario, countries are in the currency union where the central bank fixes the exchange rate and maximizes the utility of all countries. There are no exchange rates, transaction, and trade costs. Thirdly, I refer to the social planner allocation as a benchmark model attaining the first-best. Additionally, I consider the constrained social planner problem, where the social planner can collect taxes from firms' revenues to set country specific transfers. Finally, I introduce the fiscal union for countries in the currency union. The central government aims to share risk internationally through state-contingent transfers.

Comparing the welfare attained in each scenario will allow us to make policy considerations. Indeed, from the comparison between fiscal union, constrained social planner, and social planner, I can study the wedges that arise in the first two scenarios with respect to the first best allocation. In this regard, I may be able to define a policy that aims at the first best allocation. For example, it may be the case that to aim for the first best and close the wedge in labor, the central government and the constrained planner should enforce labor mobility. This point would be close to one of the main findings of Mundell, (1961). Indeed, the author states that countries in a regime of fixed exchange rates should have free labor mobility. Finally, at the microeconomic level, I may evaluate whether it is convenient for high productivity and high-income countries to remain in the EMU and form a fiscal union of international transfers.

3.1 The Physical Structure of the Model

I assume that in autarchy there is Laissez-faire. In this scenario, there is no government intervention and the agents maximize their utility subject to their budget constraint.

3.1.1 Households' Problem

I focus on the home country (H) and consider all the others as the foreign country (F). Each country is populated by a mass one of identical individuals. For the single national good, there is a unit mass of brands $j \in [0,1]$ in the home country, and $f \in [0,1]$ in the foreign countries.

Preferences

The following additively-separable utility function represents preferences over consumption and labor supply for the representative household:

$$ln(C) - \kappa L \tag{1}$$

where L is working hours, κ is a coefficient for disutility of labor and C is the basket of consumption goods that consist of Home goods C_H and Foreign goods C_F consumption with elasticity of substitution of 1.

Consumption Bundles

I assume that the elasticity of substitution between home and foreign types is one. Therefore[1], I express the aggregate consumption as a combination of home and foreign consumption in the following Cobb-Douglas form:

$$C = C_H^{\gamma} C_F^{1-\gamma} \tag{2}$$

with home bias γ regulating the taste of households for H or F goods. In the same fashion, I define the aggregate foreign consumption $C^* = \int_0^1 {C_{iH}^*}^{1-\gamma} {C_i^*}^{\gamma} di$ where opposite to Corsetti and Pesenti (2002), I assume that also foreign countries have home bias and weight own goods with γ. I assume that a single home firm produces each brand j and sells them in all countries in a monopolistic market. Additionally, a single type of good is produced in each country. Therefore, I define C_H as an aggregator of different brands j with elasticity of substitution $\theta > 1$ (there is imperfect substitution among home brands):

$$C_H = \left[\int_0^1 C_H(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

where $j \in [0,1]$ is individual good variety. Similarly, the aggregator for the varieties produced in foreign countries (consumption index of imported goods) is $C_F = \int_0^1 C_i di$[2] where the index of consumption C_i for

[1]As in Corsetti and Pesenti (2002), under this assumption the consumption baskets of each individual can be written as a geometric average of the home and foreign consumption indexes. See the consumption aggregator (2).

[2]Recall that I assume that the elasticity of substitution between home and foreign types is one. The general formulation would be $C_F = \left[\int_0^1 C_i^{\frac{\delta-1}{\delta}} di \right]^{\frac{\delta}{\delta-1}}$ with δ elasticity between goods produced in different foreign countries.

goods imported from country i is an aggregator of different brands f, given by $C_i = \left[\int_0^1 C_i(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$ where θ is the elasticity between brands produced within a given country. Equivalently, I define the aggregator for the foreign countries:

$$C_H^* = \int_0^1 \left[\int_0^1 C_i^*(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} di, \quad C_F^* = \int_0^1 \int_0^1 C_i^* didi$$

where C_H^* is the foreign consumption of home brand j, while C_F^* is the foreign consumption of all the foreign goods, where the first integral aggregates the varieties produced abroad consumed in each country i (it would be the $C_{i,F}$ of each country - it includes the own good of the country i), while the second integral aggregates all those $C_{i,F}$ to have the total amount of foreign goods consumed abroad.

System of Prices

I define the system of prices using the following price indices in the home country: home's Consumption Price Index (CPI) [3] $P = \frac{P_F^{1-\gamma} P_H^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$, home's Production Price Index (PPI) $P_H = [\int_0^1 p(j)^{1-\theta} dj]^{\frac{1}{1-\theta}}$ and the index for imported goods $P_F = [\int_0^1 P_i^{1-\gamma} di]^{\frac{1}{1-\gamma}}$ where $P_i = [\int_0^1 p_i(f)^{1-\theta} df]^{\frac{1}{1-\theta}}$ is the country i's PPI.
The exchange rate ξ_i defines the relation between the country i's currency and home currency. An increase in ξ_i means that with the foreign currency of country i it is possible to buy more units of the home currency, hence the home currency depreciates. In the same fashion of Gali and Monacelli (2005), the Law of One price assumption implies that a given variety has the same price in different countries when denominated in the same currency. This is the Producer Currency Pricing (PCP) assumption, for which the producers sell exported goods in their own currency. Therefore, I have that $p_i(f) = \xi_i p_i^*(f)$ where $p_i^*(f)$ is the country i's price of variety f expressed in its own currency. $P_i = \xi_i P_i^*$ where the country i's domestic PPI in terms of country i's own currency is $P_i^* = [\int_0^1 p_i^*(f)^{1-\theta} df]^{\frac{1}{1-\theta}}$. Furthermore, $P_F = \xi P^w$ where $P^w = [\int_0^1 P_i^{*1-\gamma} di]^{\frac{1}{1-\gamma}}$ is the world price index. Finally, the effective nominal exchange rate is $\xi = \frac{[\int_0^1 \xi_i^{1-\gamma} P_i^{*1-\gamma} di]^{\frac{1}{1-\gamma}}}{[\int_0^1 P_i^{*1-\gamma} di]^{\frac{1}{1-\gamma}}}$.
The effective terms of trade are:

$$S = \frac{P_F}{P_H} = \left(\int_0^1 S_i^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

where $S_i = P_i/P_H$ is the term of trade of home country versus country i. I use the term of trade to rewrite the home CPI as

$$P = P_H^{2\gamma-1}\left[\frac{S^{1-\gamma}}{\gamma_w}\right]$$

where $\gamma_w = \gamma^{\gamma}(1-\gamma)^{1-\gamma}$.

The maximization problem

The households receive wages from the labor supplied on a competitive market, profits from the firms they own, and interest rate from the financial market where they sell and buy home bonds B_H and foreign bonds B_i. Therefore, households maximize utility (1) subject to the consumption aggregator (2) and the nominal

[3] Avrai (2021) and Corsetti and Pesenti (2002) define the utility-based price index P of H as the consumption-based price index that can be obtained by minimizing expenditures to buy one unit of composite real consumption C.

budget constraint:

$$\int_0^1 p(j)C_H(j)dj + \int_0^1 \int_0^1 p_i(f)C_i(f)df di \leq WL + \int_0^1 \Pi(j)dj + T \tag{3}$$

where I can write the first two terms on the left-hand side in the following aggregate form: $P_H C_H + P_F C_F$, and n is the short-term nominal interest rate paid at the beginning of the period, T is the nominal lump-sum transfer, W is nominal wage and Π is nominal profits, all denominated in domestic currency. Solving the households' maximization problem, derived in the appendix section B.1.1, yields the following:

$$C_H = \frac{\gamma PC}{P_H}, \quad C_F = \frac{(1-\gamma)PC}{P_F} \tag{4}$$

where $P \equiv \frac{P_F^{1-\gamma} P_H^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$ is the utility-based price index in country h.

3.1.2 Firm's Problem

Before writing the firm's problem, I define the demand for goods as a function of the relative price and total consumption of home and foreign goods.

Demand for Goods

Following Corsetti and Pesenti (2002), the representative consumer in the home country has demand for brand j and brands f of foreign countries that are functions of relative prices and total home and foreign consumption:

$$C(j) = \left(\frac{p(j)}{P_H}\right)^{-\theta} C_H, \quad C_i(f) = \left(\frac{p_i(f)}{P_i}\right)^{-\theta} C_i \tag{5}$$

Additionally, using relative prices with respect to the overall price index CPI, I express consumption of goods as a function of total consumption:

$$C_H = \gamma \left(\frac{P_H}{P}\right)^{-1} C, \quad C_F = \int_0^1 (1-\gamma)\left(\frac{P_i}{P}\right)^{-1} C di \tag{6}$$

Production and Trade Costs

Firm in the home country produces brand j with labor input and a stochastic technology parameter a following the linear production function below:

$$Y(j) = aL(j) \tag{7}$$

The uncertainty in the model comes from the technological parameter, a for home country and a_i for foreign countries. It determines aggregate productivity for each single country.

Firm producing brand j faces demand from consumers in the home country and consumers in foreign countries. I extend the setup to include iceberg trade costs like in Alesina and Barro (2002). They posit that the transaction costs arising from the shipping of goods across country borders encompass transport expenses and trade barriers. Empirical literature, see McCallum (1995), Helliwell (2000), proves that political borders considerably affect the volume of trade. Therefore, a fraction ω of goods shipped in the foreign market is lost. Firms should ship $1+\omega$ units to the foreign country if one unit should arrive in a country abroad. These costs reflect different inefficiency of transporting goods from one country to another. The costs incurred are lost for the economy (e.g. currency conversion costs). It follows that participation in the currency union reduces these costs. Therefore, the total demand is:

$$\left(\frac{p(j)}{P_H}\right)^{-\theta} C_H + \int_0^1 (1+\omega_i)\left(\frac{p_i^*(f)}{P_{iH}^*}\right)^{-\theta} C_{iH}^* di \tag{8}$$

I assume that labor market is competitive and firms have the same marginal costs:

$$MC(j) = MC = \frac{W}{a}$$

Using the total demand (8) I express firm profits in the following fashion:

$$\Pi(j) = \left(p(j) - MC\right)\left(\frac{p(j)}{P_H}\right)^{-\theta} C_H +$$

$$\int_0^1 \left(\xi_i p_i^*(j) - (1+\omega_i)MC\right)\left(\frac{p_i^*(j)}{P_{iH}^*}\right)^{-\theta} C_{iH}^* di \tag{9}$$

The model replicates countries in a currency union using an open economy with nominal rigidities. Therefore, firms set prices $p(j)$ at the beginning of the period before the realization of the shock. They form expectations about productivity. Similarly, also the firms of the countries abroad set prices at the beginning of the period. Additionally, I assume that producers sell exported goods in their currency. In other words, I use the Producer Currency Pricing (PCP) model in the same fashion of Avrai (2021). This means that exchange rate variations do not impact the profits of firms, while they affect consumers buying foreign goods. Firms choose $\tilde{p}(j)$ as the price for exports, while foreign consumers face in their currency $\tilde{p}^*(j)$. Those prices are related through the exchange rate $\tilde{p_i}^*(j) = \tilde{p}(j)/\xi_i$. Additionally, I define ϕ as the level of monopolistic markup [4]:

$$\phi = \frac{\theta}{(\theta-1)}$$

Therefore, the firm optimally sets the domestic price equal to the marginal costs augmented by the equilibrium markup:

$$p(j) = P_H = \phi E[MC] \tag{10}$$

Furthermore, firms choose price for export goods $\tilde{p}(j)$ such that maximize profits (9) (see derivation in appendix section B.1.2).

$$\tilde{p}_i(j) = \phi(1+\omega_i)E[MC] \tag{11}$$

where ω_i is the factor that increases the prices of home brand j in the foreign country i, representing transportation cost. Symmetrically, I define the prices of the f brands from country i in the currency of H:

$$p_i(f) = \tilde{p}(f)\xi_i$$

[4] The markup price is a markup over marginal cost equal to $\frac{1}{1-\frac{1}{\theta}}$ where θ is the elasticity of demand.

and therefore I obtain in the same fashion the following prices for the foreign goods:

$$P_i^* = \phi^* E[MC], \quad P_i = \phi^*(1+\omega_i)\xi_i E[MC] \tag{12}$$

3.1.3 Market Clearing

Supply equal aggregate demand for each brand:

$$Y(j) = \left(\frac{p(j)}{P_H}\right)^{-\theta} C_H + \int_0^1 (1+\omega_i)\left(\frac{p_i^*(f)}{P_{iH}^*}\right)^{-\theta} C_{iH}^*$$

Labor markets clear:

$$L = \int_0^1 L(j)dj, \quad L_i^* = \int_0^1 L_i^*(f)df$$

3.2 The Country Outside the Union

After defining the general structure of the model, I derive the allocations for the different regimes I are considering. In the following sections, I consider the currency outside the trade, currency, and monetary union.

3.2.1 Closed form solutions for consumption and labor

Rearranging and combing the optimality conditions from the problems of households and of the firms, yield the following closed form solutions for consumption, derived in section B.1.3, and labor, derived in section B.1.4:

$$C = \frac{a\gamma_w \phi^{-\gamma}\phi^{*\gamma-1}}{\kappa\left[\int_0^1 (1+\omega_i)^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}}$$

$$L = \frac{\gamma}{\kappa\phi} + \frac{1-\gamma}{a}\int_0^1 \frac{(1+\omega_i)a_i}{\kappa\phi\left[\int_0^1 (1+\omega_i)^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}} di$$

The whole allocation for countries in autarky is in section B.1.5.

3.3 The Currency Union

In the currency union, the exchange rate is fixed and there are no transaction, transportation, or trade costs. Therefore, the union eliminates iceberg trade costs and the monetary policy erases exchange rate shocks. Indeed, countries in the union have the same currency and therefore there are no exchange rate costs in trade.

3.3.1 Closed form solutions for consumption and labor

$$C = \frac{a\gamma_w}{\kappa\phi^\gamma \phi_i^{*1-\gamma}}$$

$$L = \frac{\gamma}{\kappa\phi} + \frac{1-\gamma}{a\kappa\phi}\int_0^1 a_i di$$

The whole allocation for countries in a currency union is in section B.2.

3.4 The Social Planner's Problem

The social planner freely allocates consumption and labor maximizing the welfare of all agents subject to resource constraints. She does not face trade costs.

$$\max_{C_i,L_i} E\Bigg[ln(C_i) - \kappa L_i + \int_{-i} ln(C_{-i}) - \kappa L_{-i}di\Bigg]$$

$$s.t \quad Y = a_iL_i + \int_0^1 a_{-i}L_{-i}di = C_i + \int_{-i} C_{-i}di$$

$$C_i = C_{i,i}^{\gamma}C_{i,-i}^{1-\gamma}$$

The maximization problem of the social planner, solved in the appendix section B.3, yields the following allocations for labor and consumption:

$$C_H = \frac{\gamma}{\kappa}a \qquad C_F = \frac{1-\gamma}{\kappa}\int_0^1 a_i^* di \quad \rightarrow \quad C = \frac{\gamma_w a^{\gamma}}{\kappa}\Big(\int_0^1 a_i di\Big)^{1-\gamma}$$

$$C_{iH}^* = \frac{1-\gamma}{\kappa}a \qquad C_i^* = \frac{\gamma}{\kappa}\int_0^1 a_i^* di$$

$$L = \frac{1}{\kappa} \qquad L_i^* = \frac{1}{\kappa}$$

Therefore, it attains the first-best and the labor wedge $t = 1 + \frac{1}{a}\frac{U_L}{U_{C_H}}$ is zero.

3.5 The Constrained Social Planner

The general formulation of the social planner problem for the country i where the social planner is now levying a tax on corporate revenues to finance transfers between countries, is the following:

$$\max_{C_i,L_i} E\Bigg[ln(C_i) - \kappa L_i + \int_{-i} ln(C_{-i}) - \kappa L_{-i}di\Bigg]$$

$$s.t \quad Y = a_iL_i + \int_{-i} a_{-i}L_{-i}di = C_i + \int_{-i} C_{-i}di + T_i + \int_{-i} T_{-i}di$$

$$C_i = C_{i,i}^{\gamma}C_{i,-i}^{1-\gamma}$$

$$T = \int_0^1\int_0^1\int_0^1 \tau_i p_i(j)\left(\frac{p_i(j)}{P_i}\right)^{-\theta} C_i didjdi$$

$$T = T_i + \int_{-i} T_{-i}di = \tau\left(P_iC_i + P_i\int_{-i} C_{-i,i}di\right) + \tau\left(P_{-i}C_{i,-i} + P_{-i}\int_{-i} C_{-i,-i}di\right)$$

where T_i is planner transfers for country i. In the planner balanced budget, $(\frac{p_i(f)}{P_i})^{-\theta}C_i$ is demand for product j from all countries, home and foreign, $p_i(j)$ is the price for variety j (all prices are denominated in the common currency of the union). Additionally, the first integral expresses the tax on the revenues of the firm in country i producing j (it sums the tax revenue from the sales of product j in each country - integrate over i). The second integrates over all products in a single country (the tax revenues from all firms in a country - integrates over j). Finally, the third integral encompasses all the countries (tax revenues for all the products in all countries - integrates over i). The solution of the problem, derived in appendix section B.4, is:

$$C_H = \frac{2\gamma a}{\kappa(\phi\sqrt{4\phi+1}-\phi+2)} \qquad C_F = \frac{2(1-\gamma)\tilde{a}_i}{\kappa(\phi\sqrt{4\phi+1}-\phi+2)} \qquad \rightarrow \qquad C = \frac{\gamma_w a^\gamma}{\kappa(1+\tau\phi)}\Big(\int_0^1 a_i di\Big)^{1-\gamma}$$

$$C^*_{iH} = \frac{2(1-\gamma)a}{\kappa(\phi\sqrt{4\phi+1}-\phi+2)} \qquad C^*_i = \frac{2\gamma\tilde{a}_i}{\kappa(\phi\sqrt{4\phi+1}-\phi+2)}$$

$$\tau = \frac{\sqrt{4\phi+1}-1}{2}$$

$$T_H = \frac{\tilde{a}_i + a}{a}\left[\frac{\tau\phi-1+\tau}{1-\tau}(C_H + C^*_{iH}) + \frac{\tau\phi+1-\tau}{1-\tau}(C_F + C^*_i)\right]$$

$$L = \frac{1}{a}(C_H + C^*_{iH} + T_H)$$

3.6 The Fiscal Union

In the fiscal union, there is a central government using fiscal transfers to face imbalances created by productivity shocks. I assume that the European government collects resources from firms' revenues setting a tax τ equal for all firms of the EMU countries. This tax would encompass the idea of a corporate tax at the union level that distorts monopolistic markups. Therefore, it would represent a consolidated taxation system. I define $\phi_\tau = \frac{\theta}{(\theta-1)(1-\tau)}$ as the level of monopolistic markup corrected by distortionary taxation (for $\phi_\tau = 1$ taxes completely eliminate monopolistic distortions). I adopt the dual approach to have the government choose the taxes directly. The government chooses the allocation indirectly where the taxes affect the allocations by changing the equilibrium prices. The government problem of choosing a tax function yields a constrained Pareto optimal solution. The central government maximizes the utility of all agents subject to the market economy equilibrium conditions, the resource constraints and the government budget is balanced:

$$\max_\tau E\left[\int_0^1 ln(C_i) - \kappa L_i di\right]$$

$$C_i = a_i \phi_\tau^{-\gamma}\phi_{\tau,-i}^{\gamma-1}, \qquad L_i = \frac{\gamma}{\kappa\phi_\tau} + \frac{1-\gamma}{a_i\kappa\phi_\tau}\int_{-i} a_{-i} di$$

$$P_i C_i \leq W_i L_i + \Pi_i + T_i$$

$$C_i = C_{i,i}^\gamma C_{i,-i}^{1-\gamma}$$

$$Y^U = \int_0^1 a_i L_i = \int_0^1 (C_i + T_i) di$$

$$T^U = \int_0^1\int_0^1\int_0^1 \tau_i p_i(j)\left(\frac{p_i(j)}{P_i}\right)^{-\theta} C_i didjdi$$

I think that transfers will be sent to countries experiencing low productivity shocks. It should be the case that, given the home bias, the transfers increase the demand for domestic goods. Therefore, they boost demand to help alleviate the imbalances created by the productivity shock. The solution to the problem is derived in the appendix section B.5

4 The Static Model Calibration

The calibration of the model uses data from the countries in the EMU. The literature helps to calibrate different parameters for the European Union and I will consider a range of trade cost parameters to highlight different levels of trade gains enhanced with the union. I assume that countries are symmetric in their parameters.

Galì (2015) obtains a markup of 20% setting to 6 the elasticity of substitution between goods in the same country. The home bias parameter γ reflects the trade openness of the country which I measure as the ratio of imports over GDP. Furthermore, I have that consumption utility is logarithmic because I implicitly assumed that the intertemporal consumption elasticity is equal to 1. In the same fashion of Corsetti and Pesenti (2002), labor linearity implies an infinite Frisch elasticity of labor supply. Therefore, households satisfy labor demand and I have that households spend one-third of their life working setting $\kappa = 8/3$. The consumption aggregator (2) is Cobb-Douglas because I assumed that the elasticity of substitution between home and foreign goods is 1[5]. Following Head and Mayer (2014), I have a trade elasticity of 1. With lower values of the trade elasticity, in the case of lower trade cost, there would be limited effect on the trade volume.

A central assumption of the model is that the parameter ω embodies the trade gains, that come specifically from cost reduction. Therefore, the benefits delivered by the currency union, in the form of trade gains, are symmetric for all countries. To embrace the wide range of trade that the literature estimates, I use different specifications of gains. Indeed, in the same fashion of Avrai (2021) I adopt small, medium, and large trade gains. The literature on trade gains deriving from the European Union does not unanimously agree with the estimates. Rose's (2000) estimate is lower than the 5% of Baldwin et al. (2008), while Micco et al. (2003) shows that the common currency increases bilateral trade in the range of 4 to 16%. The value assigned to ω reflects three different scenarios. In the case of large gains, $\omega = 0.1$, I have that countries experience an increase of 10% in bilateral trades. With medium and small gains, respectively 0.066 and 0.005, I have that trade costs reduction amount to 6.5% and 5%.

Table 1 - Static calibration

	Value	Description	Target
γ	0.75	Home bias for each country	Trade openness Italy 2015
$\theta(\theta^*)$	6	Elasticity of substitution of Home(F*) varieties	Gali (2008)
ω	0.1 (0.066) (0.05)	Transportation costs	Large/Medium/Small gains
κ	8/3	Disutility of labor	HH life working time

Italian imports relative to GDP in 2015 were 26.7% according to Eurostat. For the exchange rate I use the one fixed when entering the European Exchange Rate Mechanism (ERM).

Therefore, I compute and compare consumption and labor for all countries of the EMU in the case of autarky, currency union, social planner, constrained social planner, and fiscal union.

[5] Avrai (2021) shows that the assumption of elasticity of substitution between home and foreign goods equal to 1 and the log consumption implies that the current account is always balanced.

5 The Dynamic Model

In this model, I initially assume that there is a home country and two foreign countries. This specification simplifies the computation of the model that I enrich with the financial sector and capital. The dynamic model includes time dimension and productivity shock distribution. I allow the agent to save by buying domestic and foreign bonds. Additionally, I introduce government borrowing, especially in light of the sovereign debt crisis of 2011 that involved different countries of the Union. Ferrari et al. (2020) consider the case in which countries can borrow from a risk neutral lender through state-contingent bond once outside the union. I aim to enrich the model trying to make borrowing contingent on the exit. In other words, I would model borrowing to be costly for low productivity countries leaving the union. Finally, I aim to overcome the moral hazard problem concerning the system of transfers from high productivity countries to low productivity ones, establishing a system of transfers to enforce capital accumulation.

6 Conclusion

This paper considers two settings accounting for productivity shocks, nominal rigidities, optimal monetary policies, and capital accumulation to evaluate the feasibility of a fiscal union model in a monetary union. Firstly, I consider a static model for the monetary and fiscal union that aims to reduce distortions for countries in a currency union. The fiscal union provides a mechanism of transfer to eliminate or reduce the idiosyncratic risk on productivity. I should estimate the degree of depletion of trade gains when establishing transfers for each single country of the EMU. It follows that I should be able to define for which countries it would be convenient to establish the fiscal union and for which ones it would be desirable to leave the union or remain in the status quo. In the dynamic model, I aim to solve moral hazard issues related to the system of transfers from high productive countries to low productive countries, enforcing capital accumulation. Consequently, I might outline the sustainability of transfers with respect to the European Union budget and the corporate tax level levied on finance transfers.

Overall, I expect to prove that the fiscal union is necessary to reach an efficient allocation. After the calibration, I estimate welfare under different regimes and compare the results. Additionally, I compute welfare for every country under different regimes. This comparison allows us to evaluate the overall welfare state experienced by countries inside and outside the fiscal union. I expect to find that the gains outweigh the costs of the transfers, especially for smaller countries like the Netherlands and Austria. This estimation may be relevant in response to the difficulties that the European central government faces nowadays in the decision process to promulgate measures to mitigate the effects of the Covid-19 crisis (see Recovery fund). Additionally, it may boost the unification process for fiscal cooperation.

To sum up, in a currency union with productivity shock, and nominal rigidities I expect to find that a fiscal union providing transfers is sustainable.

To conclude, in this paper I aim to evaluate the sustainability of fiscal transfers. In this regard, I focus the estimation at the country level and establish the convenience of a fiscal union for every member of the EMU. Finally, I would conclude that, in addition to trade gains and multiple social benefits, staying in the union may represent a convenient insurance mechanism against future shocks for all the members.

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A Appendix: Notation

A.1 Consumption

Aggregate home consumption:

$$C = C_H^{\gamma} C_F^{1-\gamma}$$

Aggregate foreign consumption:

$$C^* = \int_0^1 C_{iH}^{*\,1-\gamma} C_i^{*\gamma} di$$

Home aggregator of different brands j:

$$C_H = \left[\int_0^1 C_H(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

Home aggregator for the varieties produced in foreign countries (consumption index of imported goods):

$$C_F = \int_0^1 C_i di$$

where the index of consumption C_i for goods imported from country i is aggregator of different brands f:

$$C_i = \left[\int_0^1 C_i(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$$

Foreign countries aggregator of home good (exported good aggregator: C_H^* is the foreign consumption of home brand j):

$$C_H^* = \int_0^1 \left[\int_0^1 C_i^*(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} di$$

Foreign countries aggregator of foreign goods:

$$C_F^* = \int_0^1 \int_0^1 C_i^* didi$$

is the foreign consumption of all the foreign goods, where the first integral aggregates the varieties produced abroad consumed in each country i (it would be the $C_{i,F}$ of each country - it includes the own good of the country i), while the second integral aggregates all those $C_{i,F}$ to have the total amount of foreign goods consumed abroad.

A.2 Prices

System of prices for the home country
Consumption Price Index (CPI):

$$P = \frac{P_F^{1-\gamma} P_H^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

Production Price Index (PPI):

$$P_H = \left[\int_0^1 p(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

Index for imported goods

$$P_F = \left[\int_0^1 P_i^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

where $P_i = [\int_0^1 p_i(f)^{1-\theta} df]^{\frac{1}{1-\theta}}$ is the country i's PPI.
Producer Currency Pricing (PCP):

$$p_i(f) = \xi_i p_i^*(f)$$

where $p_i^*(f)$ is country i's price of f in its own currency. In aggregate terms:

$$P_i = \xi_i P_i^*$$

where $P_i^* = [\int_0^1 p_i^*(f)^{1-\theta} df]^{\frac{1}{1-\theta}}$ is country i's domestic PPI in country i's own currency.
I alternatively define the index for imported goods P_F as:

$$P_F = \xi P^w$$

where

$$P^w = \left[\int_0^1 P_i^{*1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

is the world price index. Finally, the effective nominal exchange rate is

$$\xi = \frac{[\int_0^1 \xi_i^{1-\gamma} P_i^{*1-\gamma} di]^{\frac{1}{1-\gamma}}}{[\int_0^1 P_i^{*1-\gamma} di]^{\frac{1}{1-\gamma}}}$$

The effective terms of trade are:

$$S = \frac{P_F}{P_H} = \left(\int_0^1 S_i^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

where $S_i = P_i / P_H$ is the term of trade of home country versus country i.
Using the term of trade to rewrite the home CPI:

$$P = P_H^{2\gamma-1} \left[\frac{S^{1-\gamma}}{\gamma_w} \right]$$

where $\gamma_w = \gamma^\gamma (1-\gamma)^{1-\gamma}$.

B Appendix: Derivations for the Static Model

In this section I derive five different allocations: for the currency outside the union, currency union, social planner, constrained social planner, and fiscal union

B.1 Country Outside the Union

In the case of autarchy, I have Laissez faire with no government and central bank intervention.

B.1.1 The Households' problem

Households maximize their own utility (1) subject to (2) and (3). The maximization problem has the following Lagrangian:

$$L = E\Bigg[ln(C) - \kappa L + \lambda\Bigg(WL + T + \int_0^1 \Pi(j)dj - P_H C_H - P_F C_F\Bigg)\Bigg]$$

Recalling that $C = C_H^{\gamma} C_F^{1-\gamma}$, I derive the following FOC:

$$[C_H] : \frac{\gamma}{C_H} = \lambda P_H$$

$$[C_F] : \frac{1-\gamma}{C_F} = \lambda P_F$$

$$[L] : \kappa = \lambda W$$

Taking a geometric average of the FOC for consumption using weights γ and $1-\gamma$ yields:

$$\gamma^{\gamma}(1-\gamma)^{1-\gamma} = \lambda (P_H C_H)^{\gamma}(P_F C_F)^{1-\gamma}$$

which gives $\lambda = \frac{1}{PC}$, where $P \equiv \frac{P_F^{1-\gamma} P_H^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$ is the utility-based price index in country h. Hence, home and foreign consumption are corresponding fraction of overall consumption:

$$PC = \frac{1}{\gamma} P_H C_H = \frac{1}{1-\gamma} P_F C_F \quad \rightarrow \begin{cases} C_H = \frac{\gamma PC}{P_H} \\ C_F = \frac{(1-\gamma)PC}{P_F} \end{cases}$$

Similarly, for country i: $\lambda^* = \frac{1}{P^* C^*}$, where $P^* \equiv \frac{P_i^{*1-\gamma} P_{iH}^{*\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$ and

$$P^* C^* = \frac{1}{\gamma} P_{i,F}^* C_{i,F}^* = \frac{1}{1-\gamma} P_{i,H}^* C_{i,H}^*$$

B.1.2 The firms' problem

I consider the version of the model where prices are higher than marginal costs. Therefore, if firms do not meet the participation constraints $P_H \geq MC$ and $P_{iH}^* \geq \frac{1+\omega_i}{\xi_i} MC$, they will not sell goods.
Here, I show how firms choose optimal price for exported goods $\tilde{p}_i(j)$ in each country i. They maximize the following profit function:

$$\max_{p(j),\tilde{p}_i(j)} E\Bigg[\Bigg(p(j) - MC\Bigg)\Bigg(\frac{p(j)}{P_H}\Bigg)^{-\theta} C_H + \int_0^1 \Bigg(\tilde{p}_i(j) - (1+\omega_i)MC\Bigg)\Bigg(\frac{\tilde{p}_i(j)}{\tilde{P}_{iH}}\Bigg)^{-\theta} C_{iH}^* di\Bigg]$$

differentiating the above with respect to $\tilde{p}_i(j)$ yields:

$$E\Bigg[(1-\theta)\Bigg(\frac{\tilde{p}_i(j)}{\tilde{P}_H}\Bigg)^{-\theta} C_{iH}^* + \theta(1+\omega_i)MC\Bigg(\frac{\tilde{p}_i(j)}{\tilde{P}_{iH}}\Bigg)^{-\theta}\frac{C_{iH}^*}{\tilde{p}_i(j)}\Bigg] = 0 \rightarrow$$

$$\tilde{p}_i(j) = \frac{\theta}{(\theta-1)}(1+\omega_i)E[MC] = \phi(1+\omega_i)E[MC]$$

$$p_i^*(j) = P_i^* = \phi^*(1+\omega_i)\frac{E[MC]}{\xi_i}$$

The price of foreign varieties in the home country are:

$$p_i(f) = \tilde{p}_i(f)\xi_i$$

and for foreign goods

$$P_i^* = \phi_i^* E[MC_i^*], \qquad P_i = \phi_i^*(1+\omega_i)\xi_i E[MC_i^*] = \phi_i^*(1+\omega_i)E[MC_i]$$

and therefore I express the index for imported goods in the following way:

$$P_F = \left[\int_0^1 P_i^{1-\gamma} di\right]^{\frac{1}{1-\gamma}} = \left[\int_0^1 \left(\phi^*(1+\omega_i)E[MC_i]\right)^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}$$

B.1.3 Consumption

From the first two FOCs in consumption I obtain:

$$\frac{\gamma}{P_H C_H} = \frac{1-\gamma}{P_F C_F} \qquad \rightarrow S = \frac{1-\gamma}{\gamma}\frac{C_H}{C_F}$$

therefore I explicitly relate the terms of trade with the ratio of home consumption of home good over the home consumption of foreign goods (imports).
To obtain the closed form solution for consumption, I use the FOCs derived in the households problem. From the first FOC in C_H I have that $\lambda = \frac{\gamma}{P_H C_H}$. Plugging it into the FOC for labor yields:

$$C_H = \frac{\gamma}{\kappa}\frac{W}{P_H}$$

recalling that $W = aMC$ and that from firm's problem $P_H = \phi MC$ I express wage as:

$$W = a\frac{P_H}{\phi} \qquad \rightarrow \qquad C_H = \frac{\gamma}{\kappa}\frac{a}{\phi}$$

From the second FOC in C_F I have that $\lambda = \frac{1-\gamma}{P_F C_F}$. Plugging it into the FOC for labor yields:

$$C_F = \frac{1-\gamma}{\kappa}\frac{W}{P_F}$$

recalling that $W = aMC$ and that from firm's problem $P_F = \left[\int_0^1 (\phi^*(1+\omega_i)MC)^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}$, I express wage as:

$$W = a\frac{P_F}{\phi^*\left[\int_0^1 (1+\omega_i)^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}} \qquad \rightarrow \qquad C_F = \frac{(1-\gamma)a}{\kappa\phi^*\left[\int_0^1 (1+\omega_i)^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}}$$

Therefore, imposing $\gamma_w = \gamma^\gamma 1-\gamma^{1-\gamma}$ given that $C = C_H^\gamma C_F^{1-\gamma}$ I obtain:

$$C = \frac{a\gamma_w \phi^{-\gamma}\phi^{*\gamma-1}}{\kappa\Big[\int_0^1 (1+\omega_i)^{1-\gamma} di\Big]^{\frac{1}{1-\gamma}}}$$

B.1.4 Labor

The firm choice of labor correspond to the global demand it faces:

$$L(j) = \frac{1}{a}\left[\left(\frac{p(j)}{P_H}\right)^{-\theta} C_H + \int_0^1 (1+\omega_i)\left(\frac{p_i^*(f)}{P_{iH}^*}\right)^{-\theta} C_{iH}^* di\right]$$

In a symmetric equilibrium $p(j) = P_H$ ($p_i^*(f) = P_{iH}^*$). Utilizing the expression obtained for home consumption and noting that the consumption C_{iH}^* of foreign country of good from H is symmetric to C_F, therefore:

$$C_{iH}^* = \frac{(1-\gamma)a_i}{\kappa\phi\Big[\int_0^1 (1+\omega)^{1-\gamma} di\Big]^{\frac{1}{1-\gamma}}}$$

I can rewrite the labor expression as:

$$L = \frac{1}{a}\left(\frac{\gamma}{\kappa}\frac{a}{\phi} + \int_0^1 (1+\omega_i)\frac{(1-\gamma)a_i}{\kappa\phi\Big[\int_0^1 (1+\omega_i)^{1-\gamma} di\Big]^{\frac{1}{1-\gamma}}} di\right)$$

which simplifies to:

$$L = \frac{\gamma}{\kappa\phi} + \frac{1-\gamma}{a}\int_0^1 \frac{(1+\omega_i)a_i}{\kappa\phi\Big[\int_0^1 (1+\omega_i)^{1-\gamma} di\Big]^{\frac{1}{1-\gamma}}} di$$

Hence, the trade and exchange rate costs influence firms choice of labor. Similarly, for the foreign countries:

$$L_i^* = \frac{\gamma}{\kappa\phi^*} + \frac{1-\gamma}{a_i}\int_0^1 \frac{(1+\omega_i)a_i}{\kappa\phi^*\Big[\int_0^1 (1+\omega_i)^{1-\gamma} di\Big]^{\frac{1}{1-\gamma}}} di$$

Taking derivative of L with respect to ω_i yields:

$$\frac{dL}{d\omega_i} = \frac{1-\gamma}{a\kappa\phi}\frac{1+\omega_i}{\frac{\omega_i^2}{2}+\omega_i}\int_0^1 a_i di > 0$$

therefore, labor increases with trade costs. Indeed, it seems reasonable to think that production increases if I want to have same quantity produced when countries do not face iceberg trade costs.

B.1.5 Closed form solutions for the Country Outside the Union

$$C = \frac{a\gamma_w \phi^{-\gamma}\phi^{*\gamma-1}}{\kappa\Big[\int_0^1 (1+\omega_i)^{1-\gamma} di\Big]^{\frac{1}{1-\gamma}}}$$

$$C_i^* = \frac{a_i \gamma_w \phi^{*-\gamma} \phi^{\gamma-1}}{\kappa \Big[\int_0^1 (1+\omega_i^*)^{1-\gamma} di \Big]^{\frac{1}{1-\gamma}}}$$

$$L = \frac{\gamma}{\kappa\phi} + \frac{1-\gamma}{a} \int_0^1 \frac{(1+\omega_i)a_i}{\kappa\phi \Big[\int_0^1 (1+\omega_i)^{1-\gamma} \Big]^{\frac{1}{1-\gamma}}} di$$

$$L_i^* = \frac{\gamma}{\kappa\phi^*} + \frac{1-\gamma}{a_i} \int_0^1 \frac{(1+\omega_i)a_i}{\kappa\phi^* \Big[\int_0^1 (1+\omega_i)^{1-\gamma} di \Big]^{\frac{1}{1-\gamma}}} di$$

$$P_H = \phi E[MC]$$

$$P_F = \phi^*(1+\omega_i)\xi_i E[MC^*]$$

$$P_{iH}^* = \phi(1+\omega_i)\frac{E[MC]}{\xi_i}$$

$$P_i^* = \phi^* E[MC^*]$$

$$\xi_i = \frac{P_H}{P_i^*}$$

B.2 Currency Union

The market economy derivations for households and firms are similar to the currency outside the union case. Indeed, the main differences are that under the regime of currency union there are no iceberg trade costs and exchange rates are fixed.

Rearranging FOC in C_H and L and using the fact that $W = aMC$ and $P_H = \phi MC$ gives wages $W = a\frac{P_H}{\phi}$, I obtain:

$$C_H = \frac{\gamma}{\kappa} \frac{a}{\phi}$$

Similarly for C_F where $W = aMC$ and $P_F = \phi MC$ gives wages $W = a\frac{P_F}{\phi}$ and therefore:

$$C_F = \frac{(1-\gamma)a}{\kappa\phi_i^*}$$

Given that $C = C_H^{\gamma} C_F^{1-\gamma}$:

$$C = \frac{\gamma_w a}{\kappa\phi\gamma\phi_i^{*1-\gamma}}$$

B.2.1 Closed form solutions in the Currency Union

$$C = \frac{a\gamma_w}{\kappa\phi^{\gamma}\phi_i^{*1-\gamma}}$$

$$C_i = \frac{a_i\gamma_w}{\kappa\phi^{*\gamma}\phi_i^{*1-\gamma}}$$

$$L = \frac{\gamma}{\kappa\phi} + \frac{1-\gamma}{a\kappa\phi} \int_0^1 a_i di$$

$$L_i^* = \frac{\gamma}{\kappa\phi^*} + \frac{1-\gamma}{a_i\kappa\phi^*}\int_0^1 a_i di$$

$$P_H = \phi E[MC]$$

$$P_F = \phi^* E[MC^*]$$

$$P_{iH}^* = \phi E[MC]$$

$$P_i^* = \phi^* E[MC^*]$$

B.3 Social Planner

The social planner allocates consumption and labor for each country subject to the resources constraints. Therefore, the Social Planner Problem is:

$$\max_{C,L,C_i^*,L_i^*} E\Bigg[ln(C) - \kappa L + \int_0^1 ln(C_i) - \kappa L_i di\Bigg]$$

$$s.t \quad Y = aL = C_H + \int_0^1 C_{iH}^* di$$

$$Y^* = a^*L^* = C_F + \int_0^1 C_i^* di$$

$$C = C_H^{\gamma} C_F^{1-\gamma}$$

$$C_i^* = C_{iH}^{*1-\gamma} C_i^{*\gamma}$$

which gives the following Lagrangian:

$$\max_{C_H,C_F,C_{iH}^*,C_i^*,L,L_i^*} L = \gamma ln(C_H) + (1-\gamma)ln(C_F) - \kappa L + \int_0^1 \Big[(1-\gamma)ln(C_{iH}^*) + \gamma ln(C_i^*) - \kappa L_i^*\Big] di +$$

$$\lambda_1\Big(aL - C_H - \int_0^1 C_{iH}^* di\Big) + \lambda_2\Big(-C_F + \int_0^1 a_i^* L_i^* - C_i^* di\Big)$$

The FOCs are:

$$[C_H] : \frac{\gamma}{C_H} = \lambda_1 \qquad [C_F] : \frac{1-\gamma}{C_F} = \lambda_2$$

$$[C_{iH}^*] : \frac{1-\gamma}{C_{iH}^*} = \lambda_1 \qquad [C_i^*] : \frac{\gamma}{C_i^*} = \lambda_2$$

$$[L] : \kappa = \lambda_1 a \qquad [L_i^*] : \kappa^* = \lambda_2 \int_0^1 a_i^* di$$

Rearranging the above yields the allocation of the social planner:

$$C_H = \frac{\gamma}{\kappa} a \qquad C_F = \frac{1-\gamma}{\kappa}\int_0^1 a_i^* di$$

$$C_{iH}^* = \frac{1-\gamma}{\kappa} a \qquad C_i^* = \frac{\gamma}{\kappa}\int_0^1 a_i^* di$$

$$L = \frac{1}{\kappa} \qquad L_i^* = \frac{1}{\kappa}$$

B.4 Constrained Social Planner Problem

The central government maximizes weighted utility of all agents. I rewrite the problem as:

$$\max_{C,L,C_i^*,L_i^*} E\Bigg[ln(C) - \kappa L + \int_0^1 ln(C_i) - \kappa L_i di\Bigg]$$

$$s.t \quad Y = aL + \int_0^1 a_i L_i di = C_H + \int_0^1 C_{iH}^* di + T_H + C_F + \int_0^1 (C_i^* + T_i) di$$

$$C = C_H^{\gamma} C_F^{1-\gamma}, \qquad C_i^* = C_{iH}^{*1-\gamma} C_i^{*\gamma}$$

$$T_H + \int_0^1 T_i = \tau\Bigg(P_H C_H + P_H \int_0^1 C_{iH}^*\Bigg) + \tau\Bigg(P_F C_F + P_F \int_0^1 C_i^* di\Bigg)$$

$$P_H = \phi_\tau MC, \qquad P_F = \phi_\tau MC^*, \qquad \phi_\tau = \phi_\tau = \frac{\theta}{(\theta-1)(1-\tau)}$$

Similarly to the solution method used before, I write the Lagrangian as the following:

$$\max_{C_H,C_F,C_{iH}^*,C_i^*,L,L_i^*} L = \gamma ln(C_H) + (1-\gamma) ln(C_F) - \kappa L + \int_0^1 \Big[(1-\gamma) ln(C_{iH}^*) + \gamma ln(C_i^*) - \kappa L_i^*\Big] di +$$

$$\lambda\Big(aL - C_H - \int_0^1 C_{iH}^* di - \tau P_H C_H - \tau P_H C_{iH}^* + \int_0^1 (a_i^* L_i^* - C_i^* - \tau P_F C_i^* di) - C_F - \tau P_F C_F\Big)$$

The FOCs are:

$$[C_H] : \frac{\gamma}{C_H} = \lambda(1+\tau P_H) \qquad [C_F] : \frac{1-\gamma}{C_F} = \lambda(1+\tau P_F) \qquad [L] : \kappa = \lambda a$$

$$[C_{iH}^*] : \frac{1-\gamma}{C_{iH}^*} = \lambda(1+\tau P_H) \qquad [C_i^*] : \frac{\gamma}{C_i^*} = \lambda(1+\tau P_F) \qquad [L_i^*] : \kappa^* = \lambda \int_0^1 a_i^* di$$

Plugging the two λ from labor FOCs into the FOCs for consumption yields:

$$C_H = \frac{\gamma a}{\kappa(1+\tau P_H)} \qquad C_F = \frac{(1-\gamma)\int_0^1 a_i^* di}{\kappa^*(1+\tau P_F)}$$

$$C_{iH}^* = \frac{(1-\gamma)a}{\kappa(1+\tau P_H)} \qquad C_i^* = \frac{\gamma \int_0^1 a_i^* di}{\kappa^*(1+\tau P_F)}$$

To simplify notation I impose $\int_0^1 a_i^* di = \tilde{a}_i$. The planner wants to smooth consumption for all agents and complete markets equalizing consumption across countries. Therefore, imposing $C = C_i \rightarrow C_H^{\gamma} C_F^{1-\gamma} = C_i^{*\gamma} C_{iH}^{*1-\gamma}$:

$$\left(\frac{\gamma a}{\kappa(1+\tau P_H)}\right)^{\gamma} \left(\frac{(1-\gamma)\tilde{a}_i}{\kappa^*(1+\tau P_F)}\right)^{1-\gamma} = \left(\frac{\gamma \tilde{a}_i}{\kappa^*(1+\tau P_F)}\right)^{\gamma} \left(\frac{(1-\gamma)a}{\kappa(1+\tau P_H)}\right)^{1-\gamma}$$

considering that $\kappa = \kappa^*$ yields

$$\tau = \frac{a - \tilde{a}_i}{\tilde{a}_i P_H - a P_F}$$

I use the relation for prices, where the social planner knows the realized productivity shock. Therefore, $P_H = \phi_\tau E[MC]$ and $P_F = \phi_\tau E[MC^*]$ where I have that marginal costs are the same across countries $\frac{W}{a} = \frac{W_i}{a_i}$ and therefore I normalize $MC = 1$ without loss of generality. It yields:

$$\tau = -\phi_\tau = -\frac{\theta}{(\theta - 1)(1 - \tau)} \quad \rightarrow \quad \tau = \frac{1 + \sqrt{4\phi + 1}}{2}, \qquad \tau = \frac{1 - \sqrt{4\phi + 1}}{2}$$

For $\theta = 6$ I have $\tau = 1.47$ and $\tau = -0.47$. Considering that the distortionary taxation that completely eliminates monopolistic markup $\phi_\tau = 1$ when I have $\theta = 6$ to have monopolistic markup of 20% is indeed $\tau = -\frac{1}{5} = -0.2$, I have that $\tau = -0.47$ is the tax that I will use. Therefore, to make the argument consistent, I express the absolute value and say that the taxation on corporate revenues is:

$$\tau = \frac{\sqrt{4\phi + 1} - 1}{2}$$

whereas, for $\theta = 6 \rightarrow \phi = 6/5 \rightarrow \tau = 0.47$ I have a 47% tax on corporate revenues. Substituting for τ yields the allocation for consumption:

$$C_H = \frac{2\gamma a}{\kappa(\phi\sqrt{4\phi + 1} - \phi + 2)} \qquad C_F = \frac{2(1 - \gamma)\tilde{a}_i}{\kappa(\phi\sqrt{4\phi + 1} - \phi + 2)}$$

$$C^*_{iH} = \frac{2(1 - \gamma)a}{\kappa(\phi\sqrt{4\phi + 1} - \phi + 2)} \qquad C^*_i = \frac{2\gamma\tilde{a}_i}{\kappa(\phi\sqrt{4\phi + 1} - \phi + 2)}$$

The constrained social planner also equalize employment across countries. Therefore, I first find the formulation for transfers and then the employment level. I will maintain the transfers and labor formulation in function of consumption to avoid carrying cumbersome equations. To determine the transfers for home country I equalize employment of home country with the aggregate employment of foreign countries:

$$L = \frac{1}{a}\left(C_H + \int_0^1 C^*_{iH} di + T_H\right) = \frac{1}{\tilde{a}_i}\left(C_F + \int_0^1 (C^*_i + T_i) di\right) = L^U$$

and then use the planner resource constraint:

$$T_H + \int_0^1 T_i di = \tau\left(P_H C_H + P_H \int_0^1 C^*_{iH} di\right) + \tau\left(P_F C_F + P_F \int_0^1 C^*_i di\right)$$

Therefore I have a system of two equations in two unknowns: the transfer for home country T_H and the homogeneous aggregation of foreign countries transfers $\int_0^1 T_i di$. From the second equation I express the aggregate transfers

$$\int_0^1 T_i di = \frac{\tau\phi}{1 - \tau}(C_H + C^*_{iH} + C_F + C^*_i) - T_H$$

and then I plug it into the first equation

$$C_H + C^*_{iH} + T_H = \frac{a}{\tilde{a}_i}\left(C_F + C^*_i + \frac{\tau\phi}{1 - \tau}(C_H + C^*_{iH} + C_F + C^*_i) - T_H\right) \rightarrow$$

$$T_H = \frac{\tilde{a}_i + a}{a}\left[\frac{\tau\phi - 1 + \tau}{1-\tau}(C_H + C_{iH}^*) + \frac{\tau\phi + 1 - \tau}{1-\tau}(C_F + C_i^*)\right]$$

which allow us to obtain the employment level for home country:

$$L = \frac{1}{a}(C_H + C_{iH}^* + T_H)$$

It seems that the planner enforces labor mobility to efficiently redistribute imbalances created by productivity shocks.

B.5 Fiscal Union

I rewrite the problem for the European Ramsey Planner in the following fashion:

$$\max_{\tau} E\left[ln(C) - \kappa L + \int_0^1 ln(C_i) - \kappa L_i di\right]$$

$$P_H = \phi_\tau E[MC], \qquad P_F = \phi_\tau^* E[MC^*]$$

$$C = a\phi_\tau^{-\gamma}\phi_\tau^{*\gamma-1}, \qquad C = C_H^{\gamma}C_F^{1-\gamma}$$

$$L = \frac{\gamma}{\kappa\phi_\tau} + \frac{1-\gamma}{a\kappa\phi_\tau}\int_0^1 a_i di$$

$$P_H C_H + P_F C_F \leq WL + \Pi + T_H$$

$$s.t \quad Y = aL + \int_0^1 a_i L_i di = C_H + \int_0^1 C_{iH}^* di + T_H + C_F + \int_0^1 (C_i^* + T_i) di$$

$$T_H + \int_0^1 T_i = \tau(P_H C_H + P_H C_{iH}^*) + \tau\left(P_F C_F + P_F \int_0^1 C_i^* di\right)$$