Counting Problem:

1. a) $\{m, e, e, t, i\}$ $\{m, e, e, t, n\}$ $\{m, e, e, t, g\}$ $\{m, e, e, i, n\}$
$\{m, e, e, i, g\}$ $\{m, e, e, n, g\}$ $\{e, e, t, i, n\}$ $\{e, e, t, i, g\}$
$\{e, e, t, n, g\}$ $\{e, e, i, n, g\}$
$\{m, e, t, i, n\}$ $\{m, e, t, i, g\}$ $\{m, e, t, n, g\}$ $\{m, e, i, n, g\}$
$\{e, t, i, n, g\}$
$\{m, t, i, n, g\}$.

$\therefore$ 16 unique subsets in total.

b). 2 e's : 10 sets. for each set : $\frac{5!}{2!} = 60$

$\therefore 10 \times 60 = 600$

0 e & 1 e's : 6 sets. for each: $5! = 120$

$\therefore 120 \times 6 = 720$

$720 + 600 = 1320$

2. a). $\binom{4}{2} \cdot \binom{13}{2} \cdot \binom{4}{2} \cdot \binom{11}{1} \cdot \binom{4}{1} = 123552$

suit for 1st pair

choosing first 2 cards for each of the 2 pairs

suit for 2nd pair

choosing the value for 5th card

suit for 5th card

b). same color within each pair:

$$\binom{13}{2} \cdot \underbrace{\binom{2}{1} \cdot \binom{2}{1}}_{\text{choosing color for 2 pairs}} \cdot \underbrace{\binom{11}{1} \cdot \binom{4}{1}}_{\text{the 5th card.}} = 13728$$

3、 $2 \times \binom{11}{8} \times \binom{8}{4} = 23100$

put best 2 players

select the rest 8 people for teams from all students

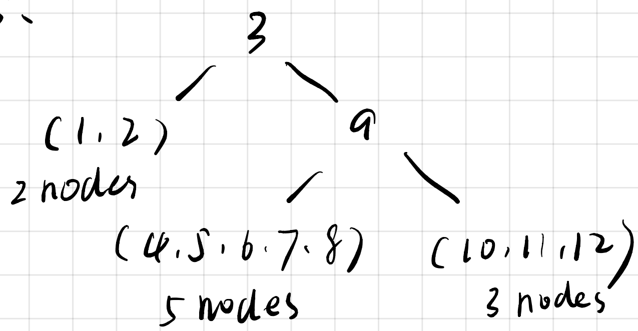
select 4 players for 1 team

4、 ① 1 song for couple at fight: $\binom{6-1+15}{6-1} = \binom{20}{5} = 15504$

② 0 song for couple at fight: $\binom{6-1+16}{6-1} = \binom{21}{5} = 20349$

$\therefore 15504 + 20349 = 35853$

5、



$2! \cdot \frac{\binom{10}{5}}{6} \cdot \frac{\binom{6}{3}}{4} = 420$

2 nodes　5 nodes　3 nodes

6、 ① 1 nurse break.

3 nurses + 10 friends

$1+1+8$　$2+3+5$

$1+2+7$　$2+4+4$

$1+3+6$　$3+3+4$

$1+4+5$

$2+2+6$

8 in total

② 0 nurse break

4 nurses + 10 friends

$1+1+1+7$　$1+2+3+4$

$1+1+2+6$　$1+3+3+3$

$1+1+3+5$　$2+2+2+4$

$1+1+4+4$　$2+2+3+3$

$1+2+2+5$

9 in total

$\therefore 8 + 9 = 17$

Probability:

1. $\dfrac{21P13}{21^{13}} = 0.008203$

2. $\underline{5} \times \underline{4} \times \underline{7} \times \underline{6} \times \underline{5} = 4200$

$P = \dfrac{4200}{10^5} = 0.042$

$\binom{10}{7} \times (0.042)^7 \times (1-0.042)^{10-7} = 2.43 \times 10^{-8}$

3. $P(B) = 1 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(A) = \binom{3}{2} \times (\frac{1}{2})^2 \times \frac{1}{2} + \binom{3}{3} \times (\frac{1}{2})^3 = \frac{1}{2}$

$P(A \cap B) = \dfrac{3}{6^3} = \dfrac{1}{72}$ $\quad P(A) \cdot P(B) = \frac{1}{36} \times \frac{1}{2} = \frac{1}{72} = P(A \cap B)$

$\therefore$ A and B are independent.

4. $P(\text{A gets a straight}) = \dfrac{\binom{10}{1} \cdot 4^{10}}{\binom{52}{5}} = \dfrac{128}{32487}$

$E(\text{A gets a straight}) = rt = 1$

$t = 1/r = 253.8 \approx 254$

5. $P(\text{superstar} \mid \text{won } 3) = \dfrac{P(\text{won } 3 \mid \text{superstar}) \cdot P(\text{superstar})}{P(\text{won } 3 \mid \text{superstar}) \cdot P(\text{superstar}) + P(\text{won } 3 \mid \text{no superstar}) \cdot P(\text{no superstar})}$

$= \dfrac{\binom{5}{3} \cdot (0.75)^3 \cdot (0.25)^2 \cdot 0.65}{\binom{5}{3} \cdot (0.75)^3 \cdot (0.25)^2 \cdot 0.65 + \binom{5}{3} \cdot (0.4)^3 \cdot (0.6)^2 \cdot 0.35}$

$= 0.68.$