a) dimensionslos:

$$r' = \frac{r}{a}, \quad s' = \frac{s}{a}, \quad z' = \frac{z}{a}$$

$$\frac{dr'}{ds'} = \frac{dr}{ds} = \cos\psi$$

$$\frac{dz'}{ds'} = \frac{dz}{ds} = \sin\psi$$

$$\frac{d\psi}{ds'} = \frac{d\psi}{ds} \cdot a = \frac{p_L a}{\gamma} - \frac{\rho g z' a^2}{\gamma} - \frac{\sin\psi}{r'}$$

b)

$$\lim_{s\to 0} \frac{d\psi}{ds} = \lim_{s\to 0} \left(\frac{p_L}{\gamma} - \underbrace{\frac{\rho g z}{\gamma}}_{=0} - \frac{\sin\psi}{r} \right)$$

$$= \frac{p_L}{\gamma} - \lim_{s\to 0} \left(\frac{\sin\psi}{r} \right) \overset{\text{"}\frac{0}{0}\text{"}}{=} \frac{p_L}{\gamma} - \lim_{s\to 0} \left(\frac{\psi \cos\psi}{\cos\psi} \right)$$

$$= \frac{p_L}{\gamma} - \lim_{s\to 0} \left(\frac{p_L}{\gamma} - \underbrace{\frac{\rho g z}{\gamma}}_{=0} - \frac{\sin\psi}{r} \right)$$

$$= \lim_{s\to 0} \frac{\sin\psi}{r} = \lim_{s\to 0} \left(\frac{p_L}{\gamma} - \frac{\rho g z}{\gamma} - \frac{\sin\psi}{r} \right)$$

$$\Rightarrow \lim_{s\to 0} \frac{\sin\psi}{r} = \frac{1}{2} \left(\frac{p_L - \rho g z}{\gamma} \right)$$