Problem set 1 - Tutorial

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1 Model 1: "A simple population growth".

This project will focus in trying to model the growth of a population of any certain animal. First of all, we are going to test a more basic model. For that purpose I will use the following function:

$$p_n = p_{n-1} + \alpha p_{n-1} \tag{1}$$

The variable p_n refers to the animal population for a given n moment. The α constant is the growth factor of the population.

1.1 An overview.

Here I test the model for an initial population of 2 (population at year 0) with an α factor of 0.1 throughout 100 years.

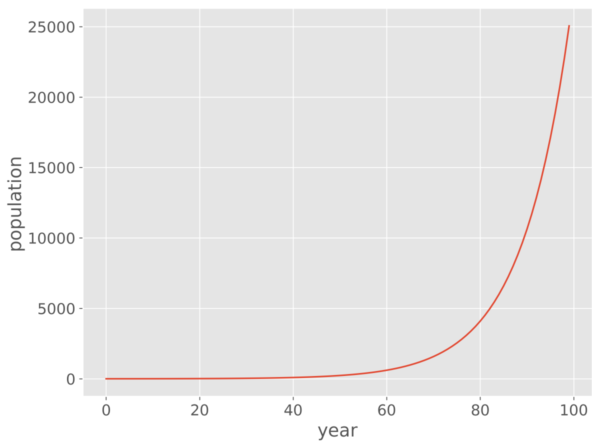

Figure 1: Line plot representing the growth of a population over 100 years. The x axis represents the $years$ and the y the number of individuals in the population.

The first thing that can be observed from Figure 1 the slow growth for the first 75 years and a rapid increase in the following 25 years. These results are predictable because as more individuals are in a given population more individuals are going to be reproducing, then, the population growth increases with the population too.

1.2 Different growth factors.

In this section I test different growth factors and their impact in the growth of this population. The α used for this section were: $[0.001, 0.1, 0.11, 0.12, 0.15]$.

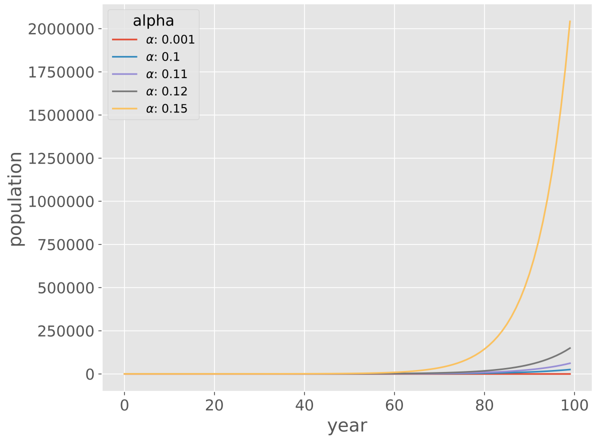

Figure 2: Line plot representing the growth of a population over 100 years. The x axis represents the $years$ and the y the number of individuals in the population. Each line represents a different population with a different α growth factor

It can be observed in Figure 2 how an increase from 0.1 to 0.15 produces a drastic increase, to the point that the growth of the populations with smaller α seem insignificant in comparison.

1.3 Different initial populations.

In this section, I tried how the value of the population at year 0 affected the growth of the population over the years. Five different p_0 were used: $[1, 2, 3, 5, 10]$.

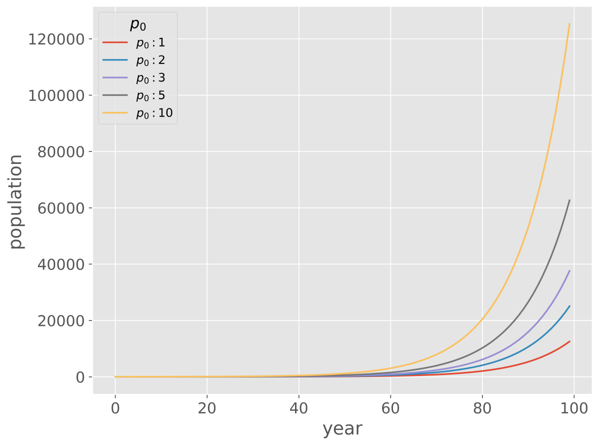

Figure 3: Line plot representing the growth of a population over 100 years. The x axis represents the years and the y axis the number of individuals in the population. Each line represents a different population with a different p_0 initial population

For this variable, it can be seen in Figure 3 that it produced an increase in the final population. However, this increment is smaller than the one produced by the variations in.

2 Model 2: adding a limiting factor.

2.1 Alpha limiting function.

Now it was included to the model a growth factor with a limiting factor, that simulates the resources that the population has. This is called as α and follows the function:

$$\alpha = 200 - p \tag{2}$$

It can be observed in Figure 4 how when the population reaches the 200 individuals the resources

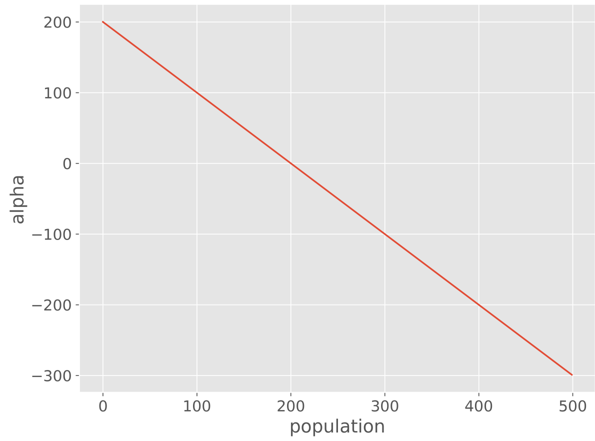

Figure 4: Line plot representing the relation between the number of individuals in a population (x axis) and the amount of resources (y axis).

are at 0 and afterwards they become negative.

2.2 The model.

In this model α is the limiting function and a new growth factor β is added to the model. Then the model follows the map:

$$p_n = p_{n-1} + \beta p_{n-1}(200 - p_{n-1}) \tag{3}$$

This model was applied to a population with an initial number of individuals of 2 and with a β of 0.001 over 100 years.

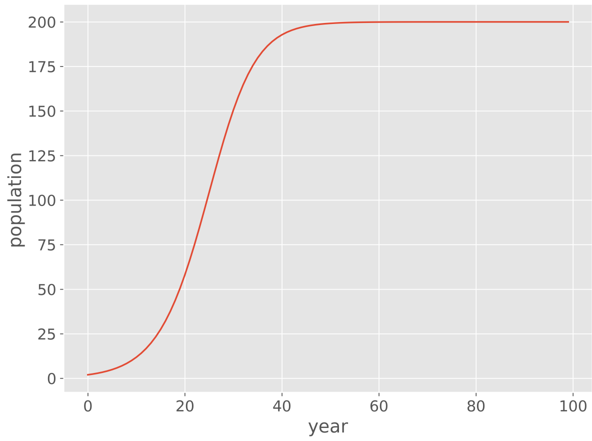

Figure 5: The growth of the population following this model over 100 years. The *years* are represented in x and the individuals in the population in y

It can be observed in Figure 5 that the population has a rapid increase, however when reaches the 200 individuals stops its growth permanently. This is due to the α limiting factor, that when reaches 0 ends the growth of the population.

2.2.1 Different β growth factors.

Here different beta factors were tested in order to observed its effect in Model 2. Five different β were used: $[0.0001, 0.001, 0.005, 0.01]$.

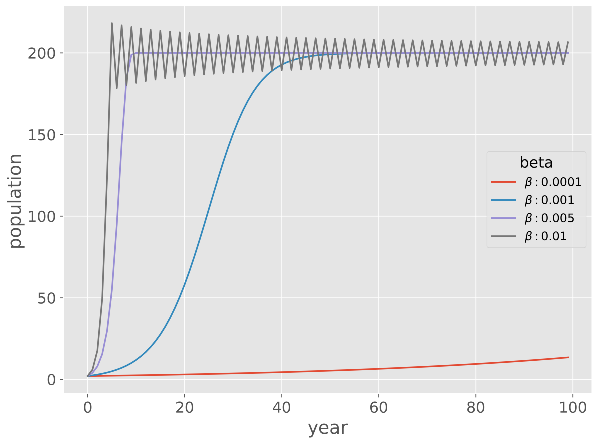

Figure 6: Line plot representing the growth of a population over 100 years. The x axis represents the *years* and the y the number of individuals in the population. Each line represents a different population with a different β growth factor

In Figure 6 it can be observed that with a smaller β the population has a really slow growth, meanwhile, with a bigger β the population reaches a point of oscillation. This is due that with that factor the population never reaches the exact number of 200, therefore, the alpha factor never becomes 0 and stop the growth of the population.

2.2.2 Different initial populations.

Here the effect of the initial population was tested. The p_0 used were: $[1, 2, 3, 5, 10]$.

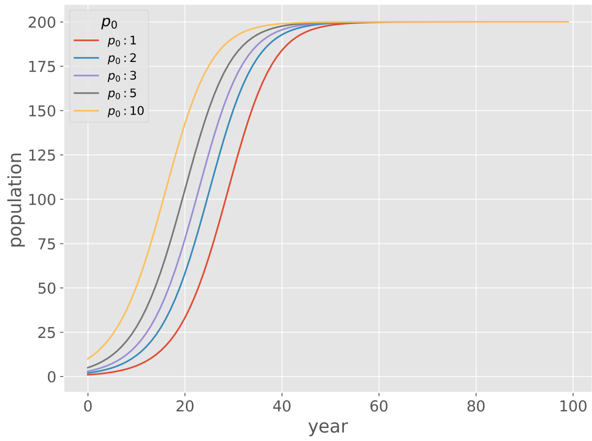

Figure 7: Line plot representing the growth of a population over 100 years. The x axis represents the years and the y axis the number of individuals in the population. Each line represents a different population with a different p_0 initial population

The Figure 7 shows how the variation in p_0 only moves the function to the left proportionally to the initial population.

3 Conclusions.

Model 1 enable to produce a fast and infinitely growing population, while Model 2 simulates a population that only can increase until it reaches a point. The variation of the growth factor seems to have stronger effects over the Model 1, however, in Model 2 produces strange behaviors that should need to be solved.