Problem set 4
NETWORKS

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1 Problem 1: *Neuron with Autapse*

Neurons communicate can communicate with different neurons though chemical synapses but also can stimulate themselves. A synapses that occurs with the the same neuron is often refer to as *autapse* neurons. In order to model this kind of neurons' firing rate we will assume that is given by the differential equation:

$$\dot{x}(t) = -x(t) + f\big(wx(t) + I\big) \tag{1}$$

For this problem the activation function that will be used is:

$$f(s) = 50(1 + \tanh(s)) \tag{2}$$

The values used for the constants of this problem are:

- $w = 0.04$
- $I = -2$
- $dt = 0.1$
- $T = 25$

1.a

First, the activation function (Equation 2) was plotted.

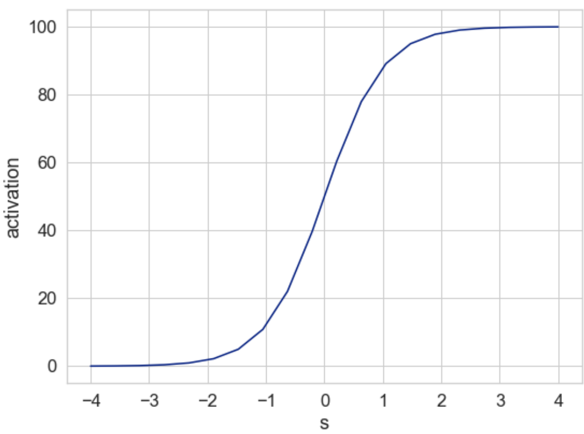

Figure 1: Activation function given by equation 2 plotted against s.

It can be observed that the activation function is set at 0 and at 100 ranging only between this values.

1.b

Afterwards, the derivatives $\dot{x}$of the differential equation (1) was plotted as a function of the neuron's firing rate x.

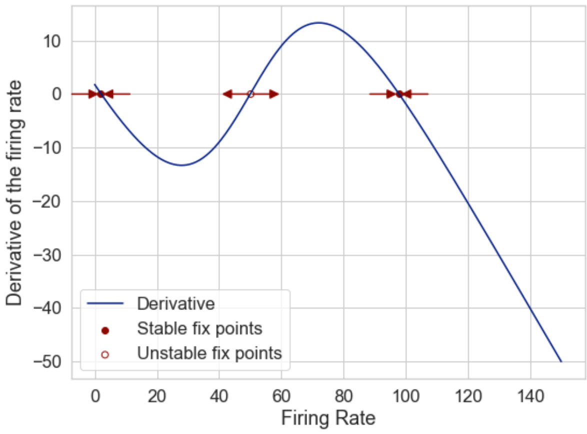

Figure 2: Derivatives of $\dot{x}$ against firing rate x. The filled red dots indicate the stable fix points, and empty red circles indicate the unstable fix point.

It can be seen in that the plot crosses three times the x axis (zero-crossings). Each of this crossings indicates what is known as a 'fix point'. There are two different kind of fix points stables and unstables. Stable fix points are attractors of the dynamics and can be observed when the derivative function is decreasing and crosses the x axis. On the other hand, unstable fix point are observed when the derivative function reaches zero and is increasing, these fix points produced the opposite effect of stable ones and repel.

1.c

In order to simulate the system the differential equation had to be solved using Euler's method:

$$\begin{aligned}
\frac{\Delta x(t)}{\Delta t} &= -x(t) + f\big(wx(t) + I\big) \\
\frac{x(t+\Delta t) - (t)}{\Delta t} &= -x(t) + f\big(wx(t) + I\big) \\
x(t+\Delta t) - x(t) &= -x(t)\Delta t + f\big(wx(t) + I\big)\Delta t \\
x(t+\Delta t) &= x(t) - x(t)\Delta t + f\big(wx(t) + I\big)\Delta t
\end{aligned}$$

Being the final equation:

$$x(t+\Delta t) = x(t)(1-\Delta t) + f\big(wx(t) + I\big)\Delta t \tag{3}$$

Using this equation the system was simulated three times using three different $x(0)$ values 49, 50 and 51.

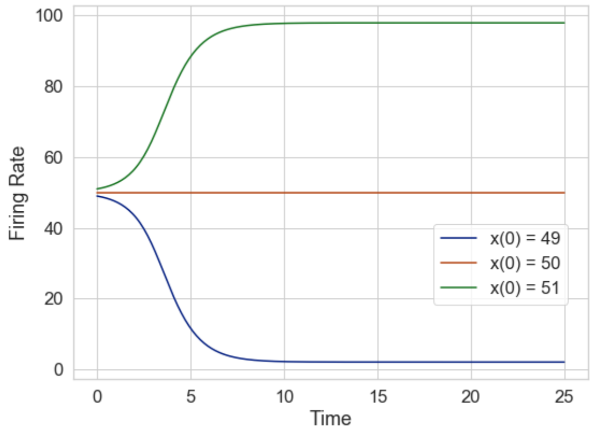

Figure 3: System firing rate simulation against time for different $x(0)$ values

It is evidenced in the plot that the firing rate increases or decreases until reaching one of the two fix points. When $x(0) = 50$ only the firing rate stays stable in this unstable fix point, for any other starting point the firing rate moves to the closes stable fix point.

1.d

Finally. to the previous equation a noise term was added giving the following equation after solving it with the Euler method:

$$x(t + \Delta t) = x(t)(1 - \Delta t) + f\big(wx(t) + I\big)\Delta t + \sigma\eta(t)\Delta t \tag{4}$$

The sane system simulated in c was redone but with this modified equation using a $\sigma = 5$ and $\sigma = 80$.

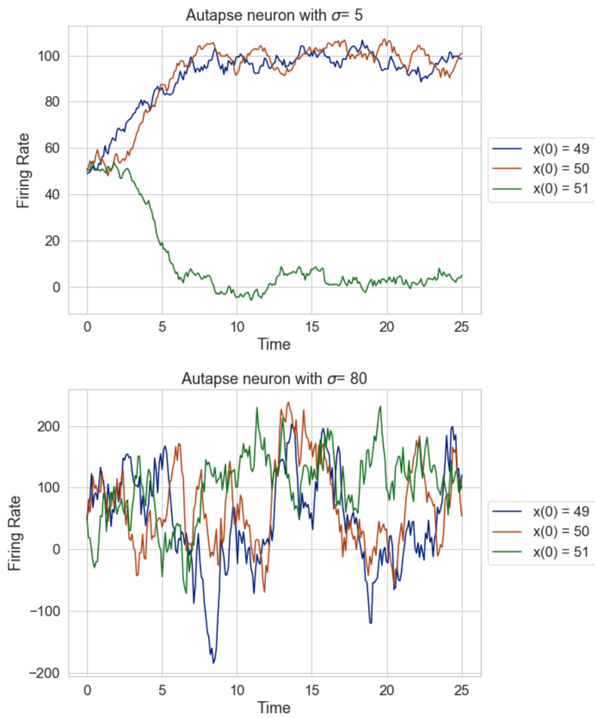

Figure 4: System firing rate simulation with a noise term against time for different $x(0)$ values.

In the upper plot where a $\sigma = 5$ was used that there is a noise effect that produced a change in comparison with Fig. 3 that now when $x(0) = 50$ it also converges to an stable fix point and doesn't remain stable to the 50 unstable fix point. In contrast, in the lower plot with a $\sigma = 80$ the system does not converge to any fix point due to such a big noise.

1.e Conclusion

It can be concluded for this problem that autapse neurons can present different fix point that the firing rate will converge to. However, if the noise is big enough the neuron won't be able to converge to any fix point.

2 Problem 2: *Circuit with mutual inhibition.*

In contrast, with the neuron modelled in the first problem neurons usually are connected to at least one neuron. In this case we will try to simulate a circuit of two neurons that inhibit each other. The firing rate of this two neurons is given by the following differential equations:

$$\dot{x}_1(t) = -x_1(t) + f(wx_2(t) + I)\dot{x}_2(t) = -x_2(t) + f(wx_1(t) + I) \tag{5}$$

The values used for the constants of this problem are:

- $w = -0.1$
- $I = 5$
- $dt = 0.1$
- $T = 25$

2.a

The nullclines of a system are obtained when $\dot{x}(t) = 0$. So the nullclines for this differential equations are:

$$x_1(t) = f(wx_2(t) + I)x_2(t) = f(wx_1(t) + I)$$

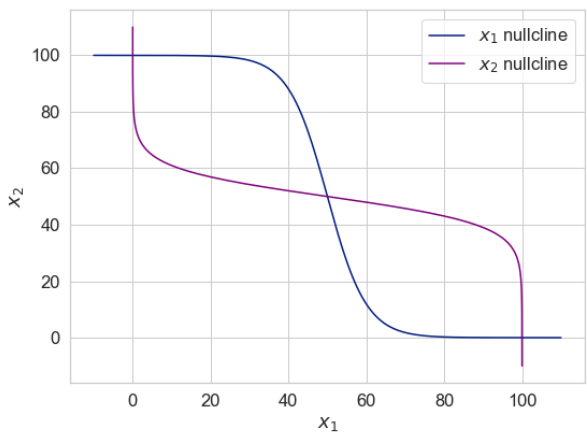

Figure 5: x_1 and x_2 nullclines plotted in the same figure

The crossing of the two nullclines indicates the fix points of this system. It can be observed that there are three different fix points.

2.b

A simulation of this system was performed. In order to perform this simulation before the differential equation was solved:

$$x_1(t+t) = (1-t)x_1(t) + f\big(wx_2(t) + I\big)tx_2(t+t) = (1-t)x_2(t) + f\big(wx_1(t) + I\big)t$$

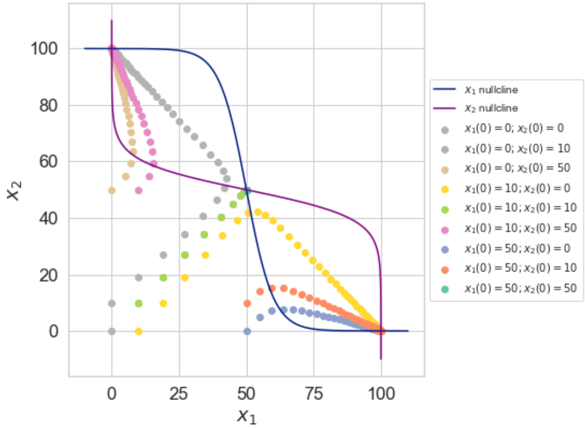

Figure 6: Simulations of the system with different initial conditions ($x(0)$)

It can be concluded from this simulations that two of the fix points are stable and one is unstable. The one in the middle is unstable as only the initial conditions in the identity line converge to this point. For all other starting points the system goest to the other two fix points.

2.c

Here, the same procedure as in **b** used but using a matrix-vector notation:

$$\dot{\mathbf{x}}(t) = -\mathbf{x}(t) + f(W\mathbf{x}(t) + \mathbf{I}) \tag{6}$$

where:

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \; W = \begin{pmatrix} 0 & w \\ w & 0 \end{pmatrix} \; I = \begin{pmatrix} I \\ I \end{pmatrix}$$

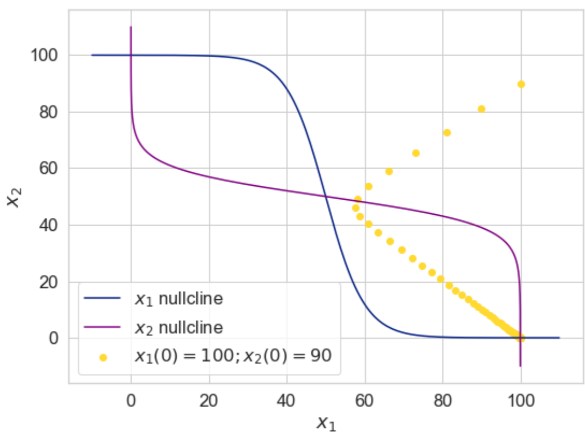

Figure 7: Simulations of the system but using matrix-vector notation.

The same results as in **b** can be observed here, which confirms that the notations does not affect the result.

2.d Advanced

Finally, the vector field of the derivatives was done.

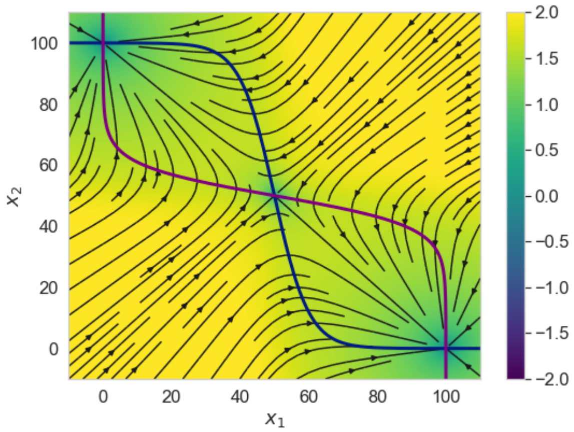

Figure 8: Streamplot of the system with the nullclines.

It can be observed that the almost the whole flow of the systems moves towards the two stable fix points.

2.e Conclusion

In this problem a more complicated system was performed in 2D. It was able to be observed more complex dynamics of fix points in comparison with the first problem.

3 Problem 3: *Optional: Hopfield model*

to be contined...