Methods in Computational Neuroscience

Problem set #3
SPIKE TRAINS

PROBLEM 1 *Poisson spike trains*

Our tour of the brain starts by looking more closely at the brain's basic signaling elements, action potentials or "spikes", and by constructing some simple tools that will allow us to deal with them.

(a) In many neurons of the brain, the omission of individual spikes appears to occur almost at random, similar to the clicks in a Geiger counter. We will therefore start by creating some artificial, random spike trains. We will quantify such a spike train as a series of 0s (no spikes) and 1s (spike). To begin, create a vector of 1000 elements so that, on average but in the most irregular manner, every fourth element in the vector is a spike. Plot the vector, by using one dot for each spike, for example, or the matplotlib function 'eventplot'.

(b) In the next step, we want to introduce time units. We will associate every 0 or 1 with a time bin of length Δt msec. Choose $\Delta t = 2$ msec and create a spike train of length 1 sec with an average rate of 25 spikes/sec. (Careful with the units!) Plot the spike train similar to above, but now use the correct time axis.

(c) Generate $N = 200$ spike trains with firing rate 25 Hz and count the total number of spikes in each of them. Plot 50 of these trials as a rastergram. Plot a histogram of the spike counts.

(d) *Advanced:* Compute the histogram of interspike intervals for the above spike trains. Verify that the histogram of spike counts follows a normal distribution (why?) and the histogram of interspike intervals an exponential distribution.

Background Literature: Gabbiani & Koch, Pages 313–314, 318–326; Dayan & Abbott, Chapter 2

PROBLEM 2 *Analysis of spike trains.*

In the next exercise, we want to do some simple analysis of real spike trains. First, download the data-file provided on the website. The data file contains the recordings of a single neuron in the primary somatosensory cortex of a monkey that was experiencing a vibratory stimulus on the fingertip. There are three variables of relevance in the file: 'f1', 'spt', and 't'. The variable 'f1' is a vector that contains the different vibration frequencies that the monkey experienced.

To load a mat file in python we need to import the loadmat function from the scipy.io package as follows: from scipy.io import loadmat then load the file with cell = loadmat('simadata.mat')

In matlab it's simpler, since we only have to load the file directly using "load".

The variable 'spt' contains the spike trains recorded. Note that this variable is a cell array — to retrieve the spike trains recorded for the i-th stimulus, you need to type 's=cell['spt'][0,i]' (Python) or 's=spt{i}' (MATLAB). Afterwards, s is a matrix of 0s and 1s as above. The variable 't' contains the respective time points.

(a) Plot all the spike trains for the first stimulus (f1=8.4 Hz) into the same graph.

(b) Plot *all* the spike trains into the same graph. (*Advanced:* Use alternating white and grey colors in the background to indicate the different stimuli.)

(c) Count the number of spikes in each trial that fall within the stimulation period (200..700 msec). For each stimulus, compute the average spike count and the standard deviation of spike counts. Plot the tuning curve of the neuron, i.e., its average firing rate (=spike count / sec) against the stimulus frequency. (*Advanced:* Additionally, add the information of the standard error of the mean (SEM) as errorbars in this plot. Remember that the standard error of the mean is defined as SEM $= \sigma/\sqrt{N}$, where N is the number of samples and σ is the standard deviation of the variable you average.)

Programming Help: *You can load the data-file using the function loadmat that you first must import from the scipy.io package. For part (c), remember the already existing MEAN. and STD functions!*

Background Literature: Gabbiani & Koch, Pages 313–314, 318–326; Dayan & Abbott, Chapter 1

PROBLEM 3 *Integrate-and-Fire neuron.*

Next, we want to investigate a simple model of how real neurons create action potentials. In a second step, we want to build a simple model of how the vibratory stimulus from Exercise (2) can be translated into a spike train.

(a) We will start by simulating the voltage across a neuron's membrane when a current $I = 1$ nA is injected. For a passive membrane, the voltage is given by the differential equation,

$$C\frac{dV(t)}{dt} = g_L\big(E_L - V(t)\big) + I \quad , \tag{1}$$

where $C = 1$nF is the membrane capacitance, $g_L = 0.1\mu$S is the conductance of the membrane ("leak" conductance), and $E_L = -70$ mV its reversal potential. This equation (and any other differential equation) can be solved numerically using the Euler method, i.e., using the approximation

$$V(t + \Delta t) = V(t) + \frac{dV(t)}{dt}\Delta t \quad . \tag{2}$$

Your task will be to implement this method for the above differential equation with initial condition $V(0) = E_L$. Choose a stepwidth of $\Delta t = 1$ ms and iterate the Euler method over 100 time steps up to time $t = 100$ ms.

(b) What happens if you change the input current I?

(c) *Advanced:* Compare the numerical solution with the exact solution to the differential equation.

(d) We will now equip the passive membrane with a very simple action-potential-generating mechanism. For that purpose, we will assume that every time when the voltage V surpasses a threshold V_{th}, the neuron fires an action potential (=spike), and the membrane voltage is reset to $V = E_L$. To implement the integrate-and-fire neuron, use the same simulation as in (a) and introduce the spiking threshold V_{th}. Use the threshold value $V_{\text{th}} = -63$mV. How many spikes do you get within the first $t = 100$ms? Change the input current and see how that changes the number of spikes!

(e) Plot the tuning curve of this neuron, i.e., the number of spikes within 100ms as a function of the input current I. At what current does the neuron start firing? Which parameters determine the *current* threshold?

(f) How could you introduce a refractory period into the model?

(g) To make the neuron more realistic, we will introduce a white noise term $\eta(t)$ in the simulation,

$$C\frac{dV(t)}{dt} = g_L\big(E_L - V(t)\big) + I + \sigma\eta(t) \quad , \tag{3}$$

where σ determines the magnitude of the noise and $\sqrt{dt}$ makes the simulation independent of the time step. (*Advanced:* why is this necessary?). Create spike trains for varying values of σ.

(h) *Advanced:* What kind of current input $I(t)$ do you need so that this integrate-and-fire neuron generates spike trains similar to the one you analyzed in Exercise (2)? Design a current input that depends on the stimulation frequency f1. Create 10 spike trains for each stimulus frequency, using the simulation from (f). How does your model compare to the data?

Background Literature: Dayan & Abbott, Chapter 5.1-5.3, 5.4 (p.11-13); Gabbiani & Koch, Pages 315-317

PROBLEM 4 *Hodgkin-Huxley model (Optional)*

So far, we have treated the action potential as a simple threshold crossing of the voltage, without further specification of how exactly it comes about. In the Hodgkin-Huxley model, the generation of the action potential itself is explained through the action of active, voltage-dependent ion channels. The membrane voltage is given by

$$C\frac{dV}{dt} = g_L\big(E_L - V\big) + \bar{g}_{\mathrm{K}}n^4\big(E_{\mathrm{K}} - V\big) + \bar{g}_{\mathrm{Na}}m^3h\big(E_{\mathrm{Na}} - V\big) + I \tag{4}$$

where the second term on the right-hand-side describes the current due to the "delayed-rectifier" K-channel and the third term the current due to the "fast" Na-channel. The parameters for this model are $C = 1\mu$F/cm^2, $g_L = 0.3$mS/cm^2, $E_L = -54.4$mV, $g_{\mathrm{K}} = 36$mS/cm^2, $E_{\mathrm{K}} = -77$mV, $g_{\mathrm{Na}} = 120$mS/cm^2, and $E_{\mathrm{Na}} = 50$mV.

The channel variables h, m, n all follow first-order kinetics, i.e. rate equations of the form

$$\frac{dx}{dt} = \alpha(V)(1-x) - \beta(V)x \tag{5}$$

and the open and closing rates, $\alpha(V)$, and $\beta(V)$ are channel-specific and voltage-dependent. You can find the equations for these rates in the textbook by Dayan & Abbott, Eq.(5.22) and Eq. (5.24).

(a) Simulate the Hodgkin-Huxley model and increase the injected current I from $I = 0$ to $I = 10$. At which value does the neuron start to spike repetitively? What is its lowest firing rate?

(b) Use your simulation to figure out how the action potential comes about. What happens at the spiking threshold?

Background Literature: Dayan & Abbott, Chapter 5.5-5.6