Problem set 3
SPIKE TRAINS

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1 Problem 1: *Poisson spike trains*

Neurons communicate each other through action potentials or 'spikes', which are rapid changes on membrane potential of their axons. This changes in the voltage membrane can be recorded through different means. It has been reported that spikes trains usually follows a Poisson distribution. The objective of this first exercise will be to simulate spike trains and show the distribution that they follow.

1.a

Firts, I started by creating spikes similar to what it can be seen in a Geiger counter, a window of a total of 1 second was created where 0 indicates no spike and 1 indicates the presence of a spike (each spike had 1ms duration).

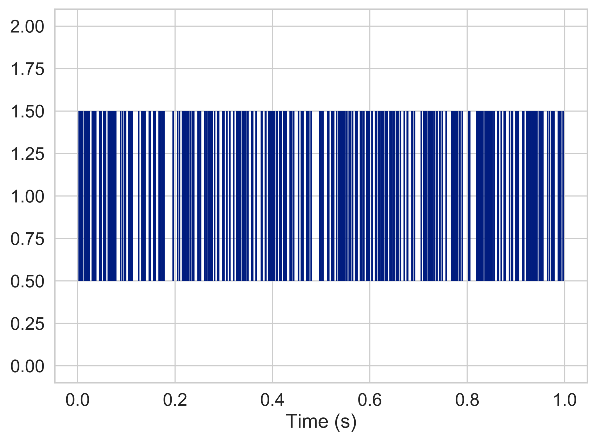

Figure 1: Ratergram of one spike train on a 1 second window, each line indicates the presence of a spike.

1.b

Now, $\Delta t = 2ms$ was used, which indicates that the presence of a spike is measured inside of that time bin. Afterwards, a 25 spikes/sec neuron was simulated:

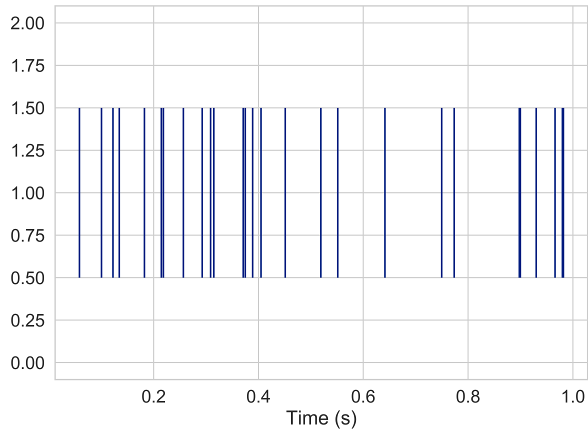

Figure 2: Ratergram of one spike train on a 1 second window, each line indicates the presence of a spike.

1.c

Here. a total of 1000 spike trains were created with an average spike rate of 25Hz. Here it can be observed a rastergram with the first 50 spike trains and the histogram of the distribution for the 1000 spike trains.

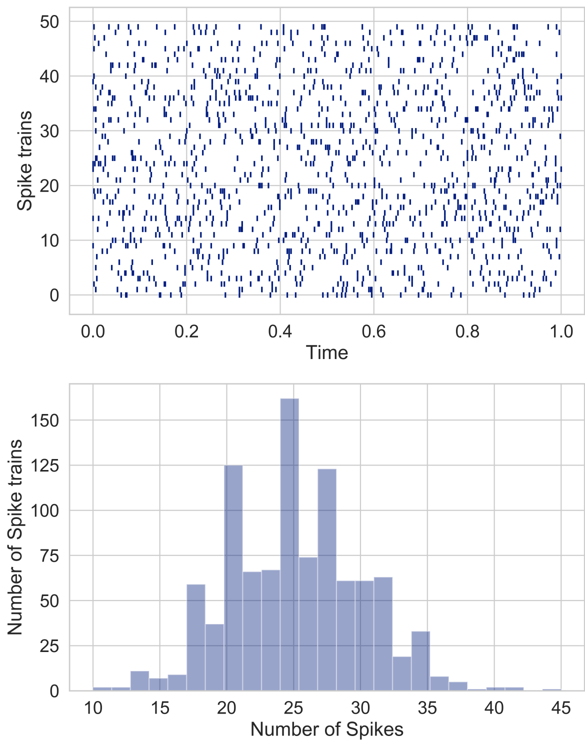

Figure 3: (Upper) Rastergram of 50 trials. (Lower) Distribution of the number of spikes per spike trains.

1.d Advanced

Finally. the distribution of the number of spikes in each spike train was plotted with a normal distribution. Also, the time intervals between each spike were plotted against an exponential curve.

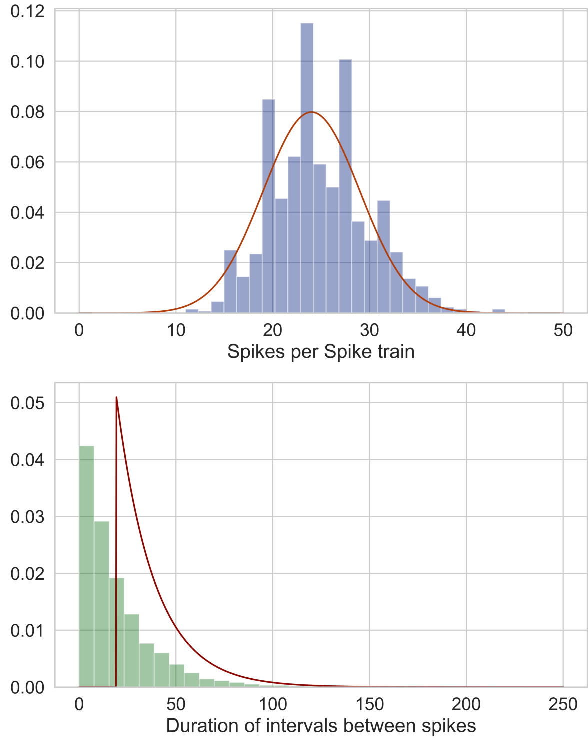

Figure 4: (Upper) Distribution of the number of spikes per spike trains in blue, normal distribution in red. (Lower) Distribution of intervals in ms between spikes in green and an exponential distribution in red.

It was observed that the distribution of spikes follows a normal distribution. Furthermore, it was seen that the distribution of intervals resembled an exponential distribution. (*Disclaimer: I used more spikes in order to have a 'better' distribution. I don't understand why the exponential distribution is moved*)

1.e Conclusion

It was observed that spike trains follow a Poisson distribution and the intervals between spikes follows an exponential distribution.

2 Problem 2: *Analysis of spike trains.*

2.a

First, the data from the first stimulus 8.4 Hz was plotted:

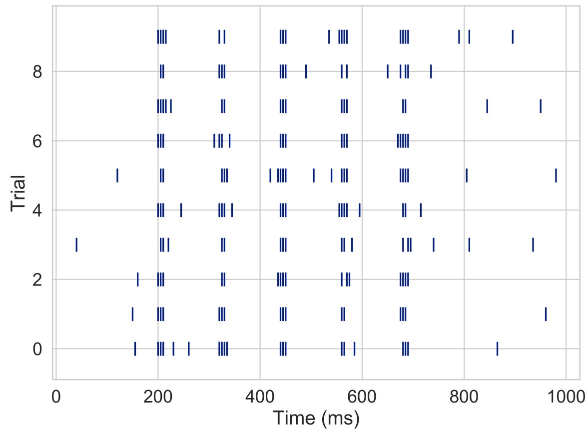

Figure 5: Ratergram of all the spike trains for the first stimulus on a 1 second window, each line indicates on the plot the presence of a spike.

It can be observed an increase in the firing rate of the neuron across all trials, this is probably due to the stimulation paradigm.

2.b

To further check the effect of the stimulation frequency paradigm all trials were plotted in the same graph.

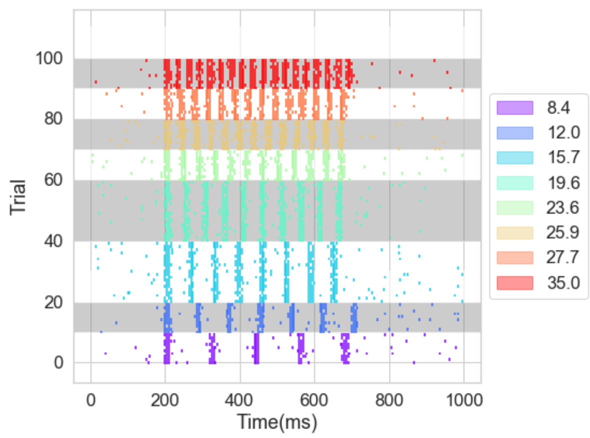

Figure 6: Ratergram of all the spike trains for the first stimulus on a 1 second window at all the stimulation frequencies

From this plot it can be drawn that firing rate changes with frequency stimulation.

2.c

Finally, to obtain a more clear idea of the effect of frequency over the firing rate of the neuron the stimulation frequency was plotted against the average firing rate.

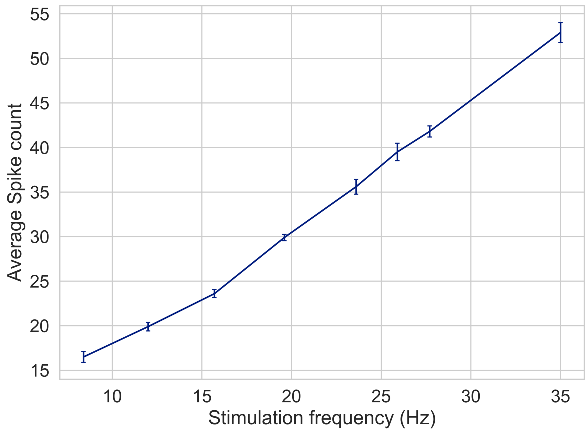

Figure 7: The average spike count for the neuron at the different stimulation frequencies

It can clearly be seen that the firing rate increases linearly with the frequency of stimulation.

2.d Conclusion

It can be concluded from this exercise that the firing rate of a neuron increases linearly with the frequency of stimulation.

3 Problem 3: *Integrate-and-Fire neuron.*

The objective of this problem will be to try model how neurons actually produce action potentials. The more simple model is called 'Leaky Integrate and Fire neuron'. For solving for this model we are going to use the passive membrane equation:

$$C\frac{dV(t)}{dt} = g_L(E_L - V(t)) + I$$

3.a

Based on these differential equations and using the following approximation

$$V(t + \Delta t) = V(t) + \frac{dV(t)}{dt}\Delta t \tag{1}$$

Now we can use the Euler method:

$$\begin{aligned}
C\frac{\Delta V}{\Delta t} &= g_L(E_L - V(t)) + I \\
\frac{\Delta V}{\Delta t} &= \frac{1}{C}(g_L(E_L - V(t)) + I) \\
V(t+\Delta t) - V(t) &= \frac{\Delta t}{C}(g_L(E_L - V(t)) + I) \\
V(t+\Delta t) - V(t) &= \frac{\Delta t g_L E_L}{C} - \frac{\Delta t g_L V(t)}{C} + \frac{\Delta t I}{C} \\
V(t+\Delta t) &= \frac{\Delta t g_L E_L}{C} - \frac{\Delta t g_L V(t)}{C} + \frac{\Delta t I}{C} + V(t) \\
V(t+\Delta t) &= \left(1 - \frac{\Delta t g_L}{C}\right) V(t) + \frac{\Delta t(g_L E_L + I)}{C}
\end{aligned}$$

Now, using the following values of the parameters the voltage for 100ms was plotted.

- $\Delta t = 1ms$
- $I = 1nA$
- $C = 1nF$
- $g_L = 0.1\mu S$
- $E_L = -70mV$
- $V(0) = E_L$

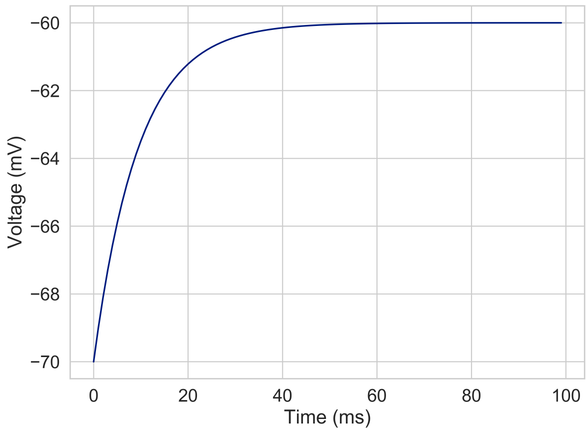

Figure 8: Voltage change of a neuron over 100ms period.

3.b

Different values of the input current I ranging from -5 to 5 were used.

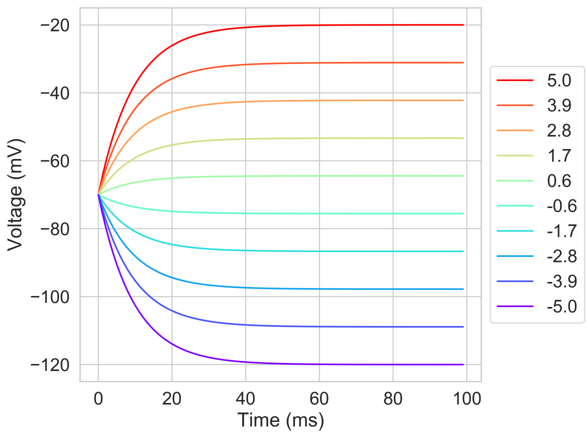

Figure 9: Voltage change of a neuron over 100ms period for different values of I.

It can be observed from the previous plot that when the current is positive the voltage increases depolarizing the neuron and when the input current is negative the neuron is hyperpolarized, this means the membrane potential becomes more negative.

3.c Advanced

Here the exact numerical solution for the Leaky-integration firing neuron is obtained:

$$C\frac{dV(t)}{dt} = g_L(E_L - V(t)) + I$$
$$V(t) = k \cdot e^{-\frac{g_L t}{C}} + \frac{I}{g_L} + E_L$$

And considering $V(0) = E_L$ then:

$$V(t) = E_L + \frac{I}{g_L}\left(1 - e^{-\frac{g_L t}{C}}\right)$$

Finally, if we compare the result using this numerical solution against the solution obtained with the Euler method:

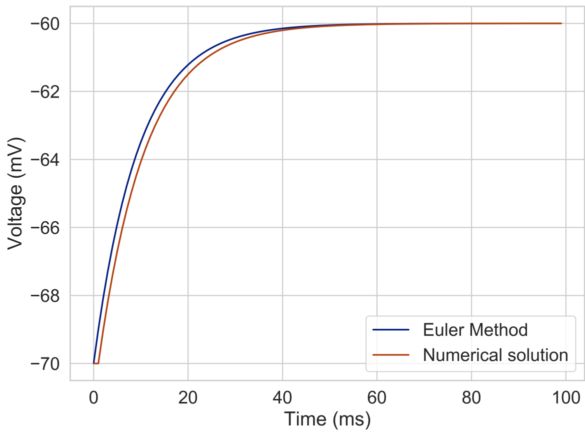

Figure 10: Voltage change of a neuron over 100ms period comparing Euler solution with exact numerical solution.

As it can be observed in the figure the difference between both solutions is really small.

3.d

In real neurons action potentials are iniciated after the membrane potential reaches a certain value called threshold. Here a threshold was added to the function that whenever is reached the voltage is reset to the membrane potential.

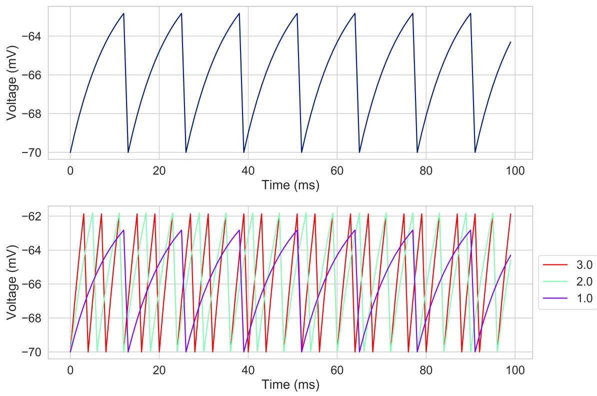

Figure 11: (upper) Voltage change of a neuron over 100ms period with a treshold of -63mV. (lower) Same as upper plot but comparing different input currents.

Using an input current of 1 it can be seen a total of 7 spikes can be seen along 1 second. However, if we increase the input current it can be seen that the amount of spikes over that period increases because the threshold is reached faster.

3.e

In order to further check the effect of input current over the amount of spikes, a rastergram and a tuning curve of spikes against input current was plotted.

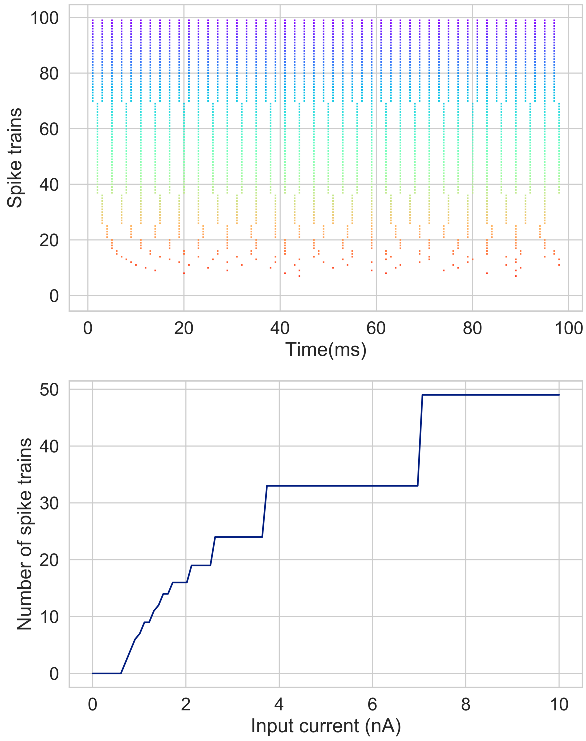

Figure 12: (upper) Rastergram of the spike trains for different input currents. (lower) Tuning curve of number of spikes for input current.

From the previous plot it can be observed that the minimal value of the input current needed in order to generate an spike is between 0.6 and 0.7. This threshold value depends on the E_L and g_L parameters. Because if the membrane potential is lower the amount of current needed is higher, and the higher the leaky conductance parameter, the higher the loss in the membrane potential so the higher the input needed to reach threshold.

3.f

The refractory period on a neuron corresponds to the period where a neuron cannot fire after an action potential. In order to simulate this the firing an stop to the firing of the neuron was set for a number of Δt, in this case 10 was used.

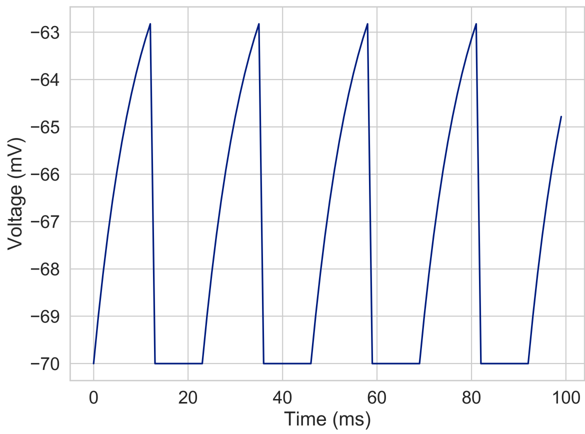

Figure 13: Voltage change of a neuron over 100ms period with a threshold of -63mV and a refractory period of 10ms

It can be observed in the plot that due to the refractory period the neuron takes 10ms after each spike to fire again.

3.g

Finally, it is important to add a noise term to the model to make it more realistic. Neuron's environments are not 'clean', thus, the dynamics of the membrane potential are not perfect neither always the same. For these reasons it is important to add noise to the model. The equation with the noise term is:

$$C\frac{dV(t)}{dt} = g_L(E_L - V(t)) + I + \sigma\eta(t)$$

Using Euler method with the same approximation used in (a):

$$
\begin{aligned}
C\frac{\Delta V}{\Delta t} &= g_L(E_L - V(t)) + I + \sigma\eta(t) \\
\frac{\Delta V}{\Delta t} &= \frac{1}{C}(g_L(E_L - V(t)) + I + \sigma\eta(t)) \\
V(t+\Delta t) - V(t) &= \frac{\Delta t}{C}(g_L(E_L - V(t)) + I + \sigma\eta(t)) \\
V(t+\Delta t) - V(t) &= \frac{\Delta t g_L E_L}{C} - \frac{\Delta t g_L V(t)}{C} + \frac{\Delta t I}{C} + \frac{\sigma\eta(t)\sqrt{\Delta t}}{C} \\
V(t+\Delta t) &= \frac{\Delta t g_L E_L}{C} - \frac{\Delta t g_L V(t)}{C} + \frac{\Delta t I}{C} + \frac{\sigma\eta(t)\sqrt{\Delta t}}{C} + V(t) \\
V(t+\Delta t) &= \left(1 - \frac{\Delta t g_L}{C}\right) V(t) + \frac{\Delta t(g_L E_L + I)}{C} + \frac{\sigma\eta(t)\sqrt{\Delta t}}{C}
\end{aligned}
$$

This was plotted using different values of sigma

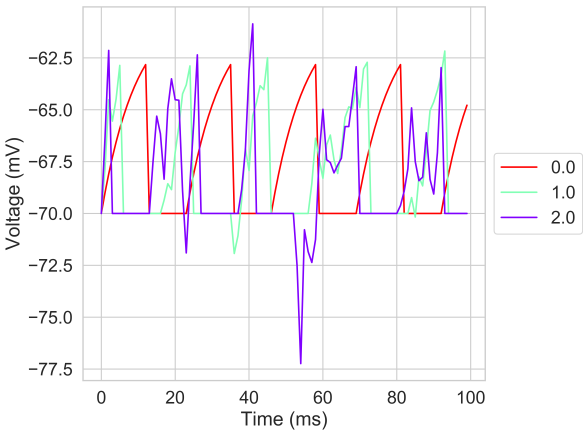

Figure 14: Voltage change of a neuron over 100ms period with a threshold of -63mV and a refractory period of 10ms and different sigma noise values

It can be observed that when $\sigma = 0$ the model is exactly the same as the original one without noise, however, with bigger σ values the noise becomes stronger.