



# Program Optimization – algorithmic complexity

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# Why is it important?

Good data structures, architecture, is the most crucial factor with regards to performance

## Multiple queries

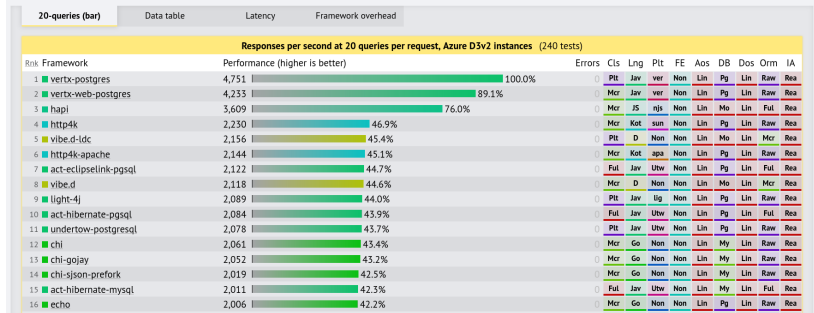


Figure 1: TechEmpower benchmark, round 17



# Big-O notation

# Examples



Which complexity corresponds to which task?

- |                                    |                        |
|------------------------------------|------------------------|
| ■ lookup in unsorted array         | ■ $O(n \cdot \log(n))$ |
| ■ lookup in a tree                 | ■ $O(1)$               |
| ■ sort                             | ■ $O(c^n)$             |
| ■ traveling salesman problem       | ■ $O(n)$               |
| ■ get $n^{th}$ element of a vector | ■ $O(\log(n))$         |

# Definition



- describes asymptotic cost of some operation at the limit (usually, infinity)
- very commonly used to describe operation costs
- ignores constant factors

# What should you know ?



- it is **very important** to know the expected cost of the important operations for several data structures,
  - cf. accidentally quadratic
- memory and computing efficiency will dominate big-O concerns for small enough  $n$  : do not underestimate compact vectors!
- there are trade-offs between space and time efficiency,
- $O(1)$  operations are usually lies, because of memory hierarchies.

# Amortized cost



**Amortized cost** : when the cost of an operation is the mean cost over multiple runs

# Amortized cost – example



```
data DQ a = DQ [a] [a]
```

```
push :: a -> DQ a -> DQ a
```

```
push a (DQ front back) = DQ front (a:back)
```

```
pop :: DQ a -> Maybe (DQ a, a)
```

```
pop dq = case dq of
```

```
    DQ [] [] -> Nothing
```

```
    DQ (a:front) back -> Just (DQ front back, a)
```

```
    DQ [] back -> pop (DQ (reverse back) [])
```

push and pop are *amortized*  $O(1)$ , can you see why?



# When does it fail?



- amortized costs on persisted structures;
- mean cost  $\neq$  worst-case cost, beware of malicious and/or degenerate input
  - example: hash tables

# Example, C++ data structures (source: Baptiste Wicht)

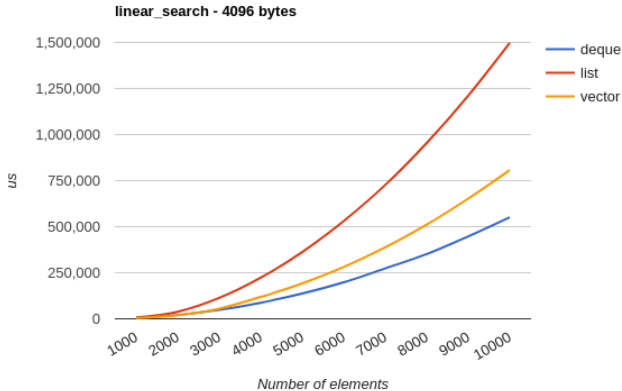


Figure 2: This is advertised as  $O(n)$



## Approximate solutions

# Why?



Some problems are too costly to tackle directly, but an approximate solution might be just good enough!

Example:

- counting problem: HyperLogLog
- set membership: bloom, cuckoo filters
- compressing a video: all major formats

# Important classes of approximation algorithm



- greedy algorithms
- local optimization
- black box optimizers, such as GA, sampling

# Greedy algorithms sometimes acceptable

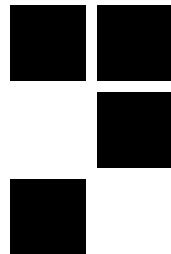


For the traveling salesman problem:

- brute force solution  $O(n!)$
- dynamic programming  $O(n^2 2^n)$
- greedy algorithm  $O(n)$ , 25% longer than optimal on average



QUESTIONS?



Thank you for your attention

