Hierarchical Stochastic Neighbor Embedding
Supplemental Materials

In this document we include the extensive tests that we performed on the MNIST (60k data points, 728 dimensions), CIFAR-10 (50k data points, 1024 dimensions) and TIMIT (1M data points, 39 dimensions) datasets in order to validate the performance of HSNE. For each dataset we provide the embedding generated by tSNE, Landmark-SNE, LSP, P-LSP, LAMP and our HSNE. Furthermore we provide several *drill-down* analysis demonstrating that, thanks to an *Overview-First* and *Details-on-Demand* paradigm, HSNE outperforms tSNE on every dataset. In each analysis, the initial placement of the data points in the embedding is initialized randomly with a different seed. Finally, we provide evidence that the approximated computations used in HSNE do not reduce the quality of the embeddings. All the benchmarks are based on the *Nearest Neighbor Preservation* metric, as defined in Sec. 6 of the paper. For the TIMIT dataset we were able to analyze the complete dataset only using tSNE and HSNE. Therefore we provide a comparative analysis on a subset composed of 100k randomly selected data-points, together with an analysis on the full dataset using tSNE and HSNE (Fig. 26). This last result, in particular, shows that HSNE outperforms tSNE on big datasets both in the computation time required and in the accuracy of the results.
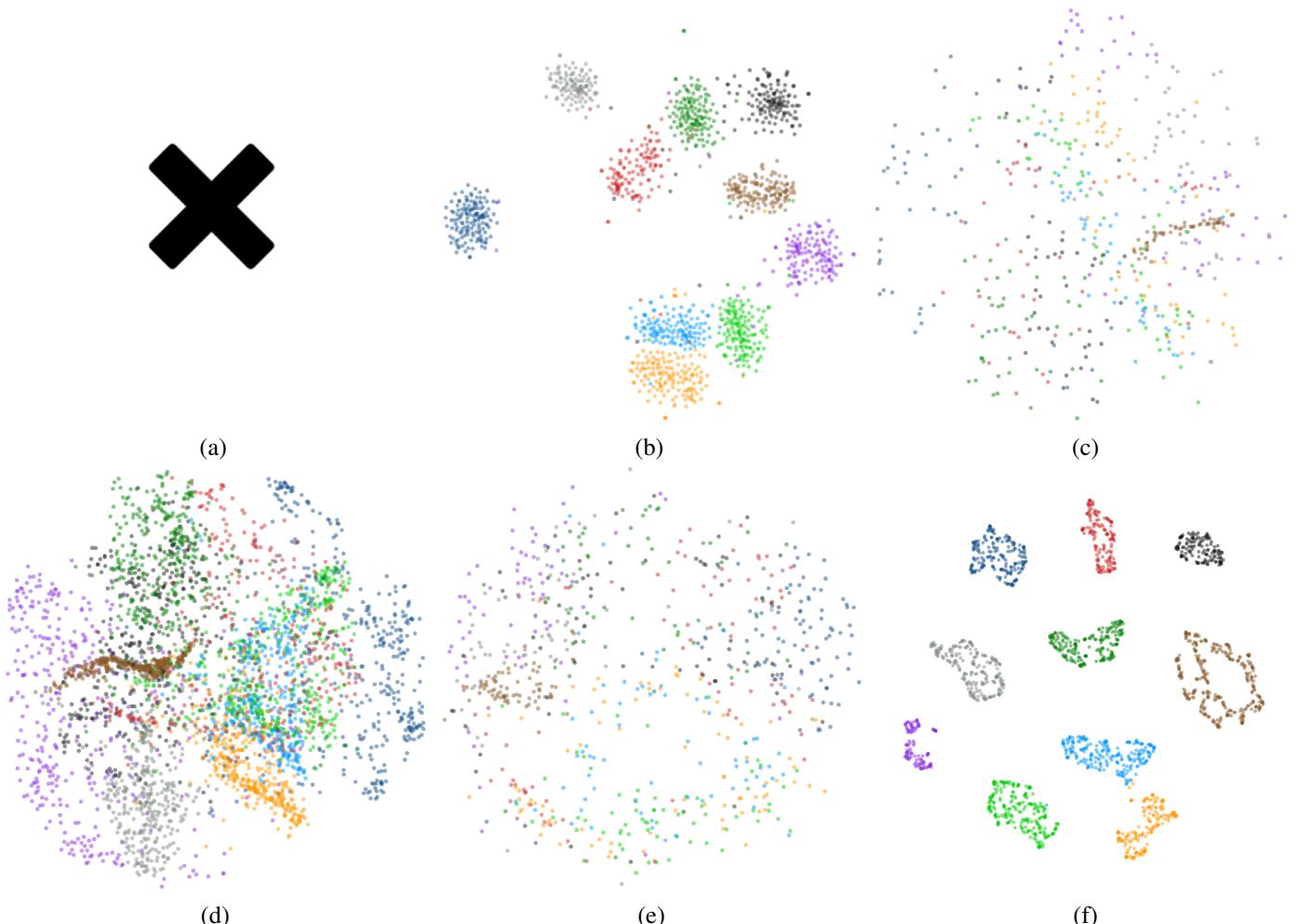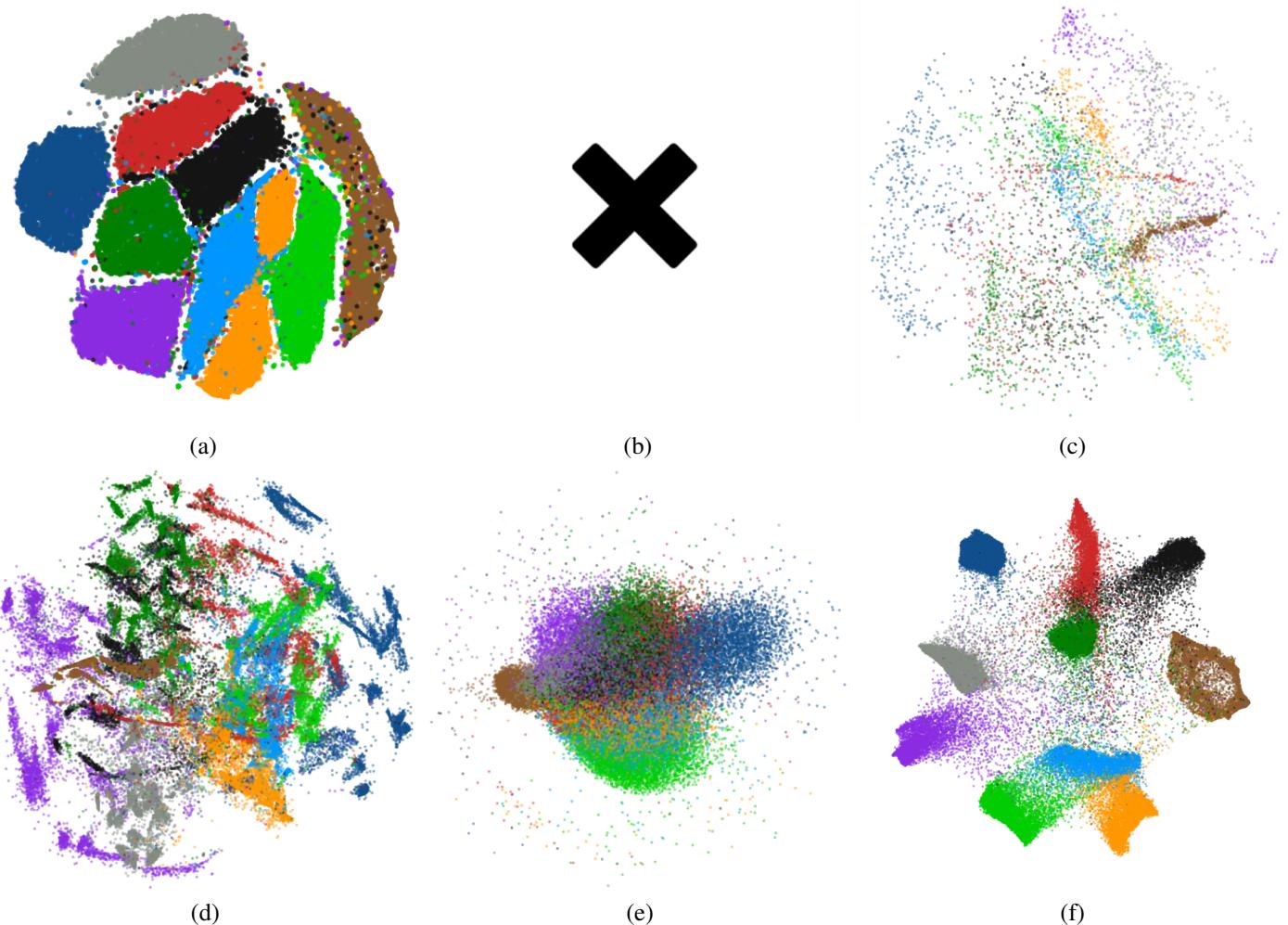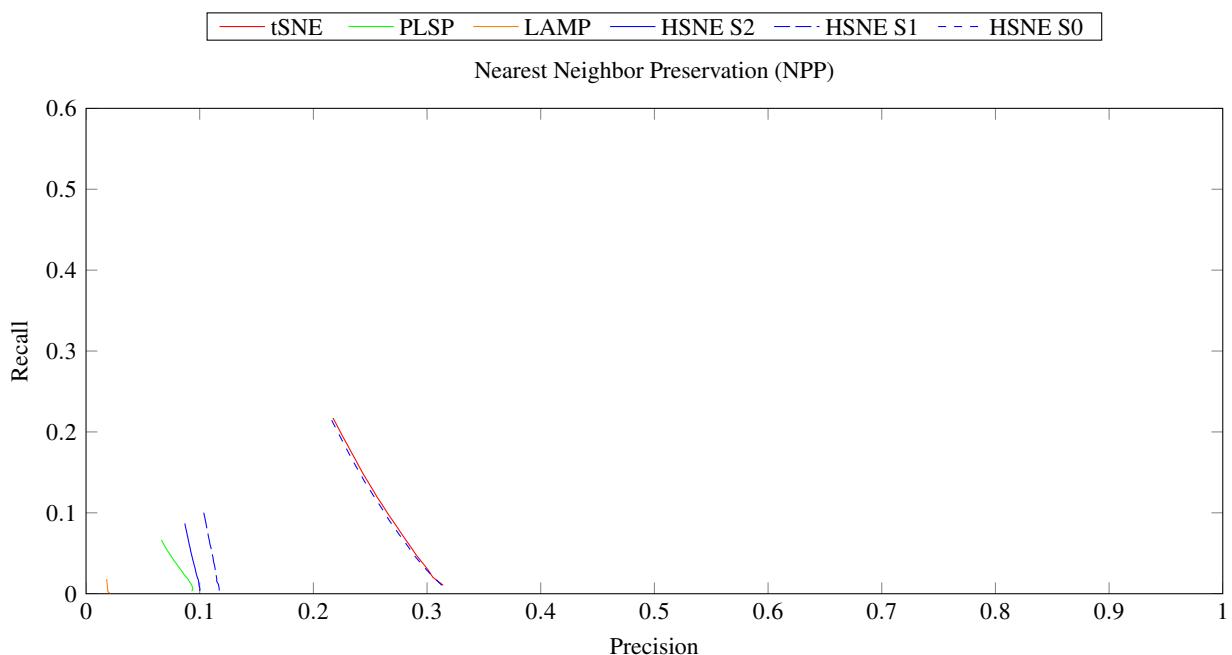

Figure 1: **Embeddings of the MNIST datasets (Landmarks)**. (a) tSNE - Time: 23'57" - #Landmarks: n/a (b) Landmark-SNE - Time: 1'13" - #Landmarks: 1431 (c) LSP with 5k data points - Time: n/a - #Landmarks: 500 (d) P-LSP - Time: 5'22" - #Landmarks: 3714 (e) LAMP - Time: 5'03" - #Landmarks: 734 (f) HSNE - Time: 1'27" - #Landmarks: 1431

Figure 2: **Embeddings of the MNIST datasets (Complete embedding)**. (a) tSNE - Time: 23'57" - #Landmarks: n/a (b) Landmark-SNE - Time: 1'13" - #Landmarks: 1431 (c) LSP with 5k data points - Time: n/a - #Landmarks: 500 (d) P-LSP - Time: 5'22" - #Landmarks: 3714 (e) LAMP - Time: 5'03" - #Landmarks: 734 (f) HSNE - Time: 1'27" - #Landmarks: 1431


Figure 3: **Precision/Recall curves** of the Nearest Neighbor Preservation for the full MNIST dataset.

(a)

(b)

(c)

(d)

| tSNE | HSNE Scale 2 | HSNE Scale 1 | HSNE Scale 0 |

(e) Nearest Neighbor Preservation (NPP)
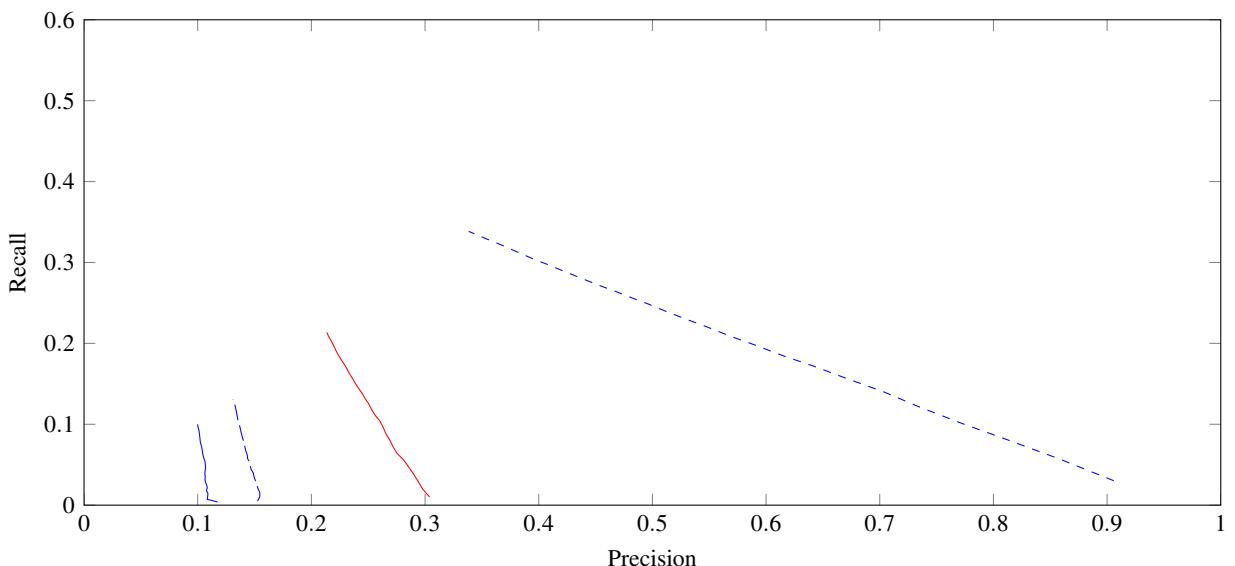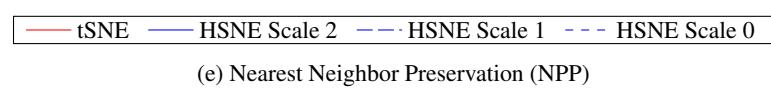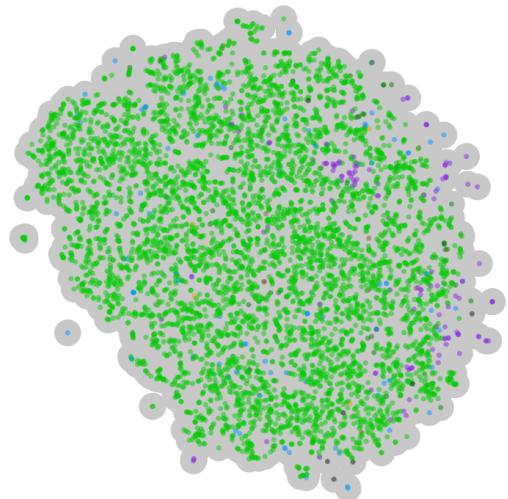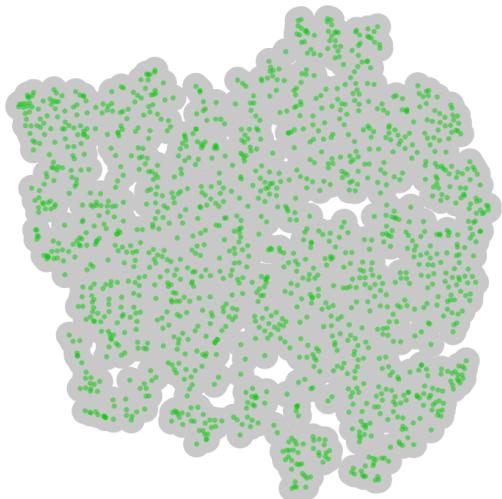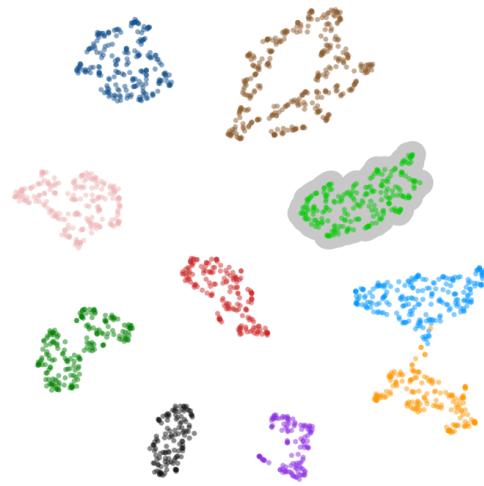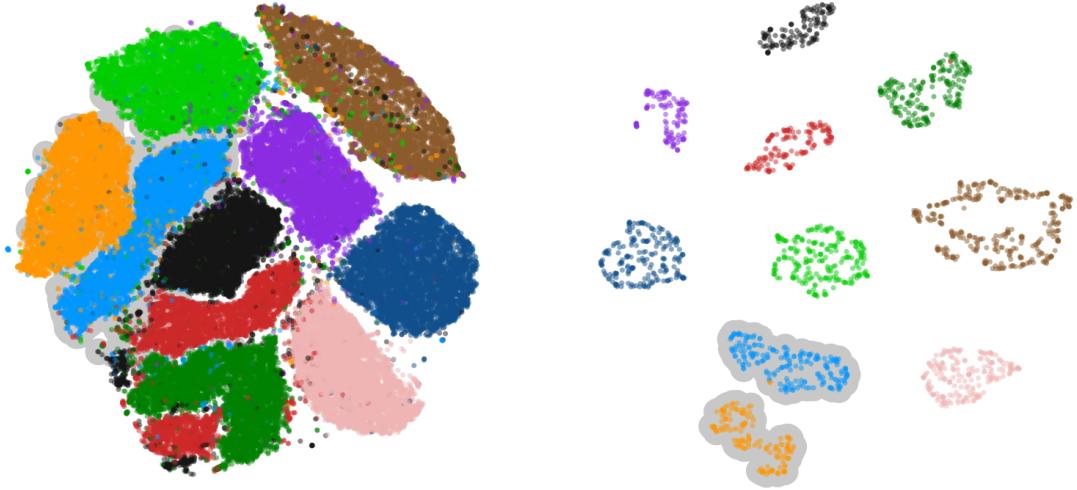
Figure 4: **Drill-down analysis (MNIST Test 0)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

3

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
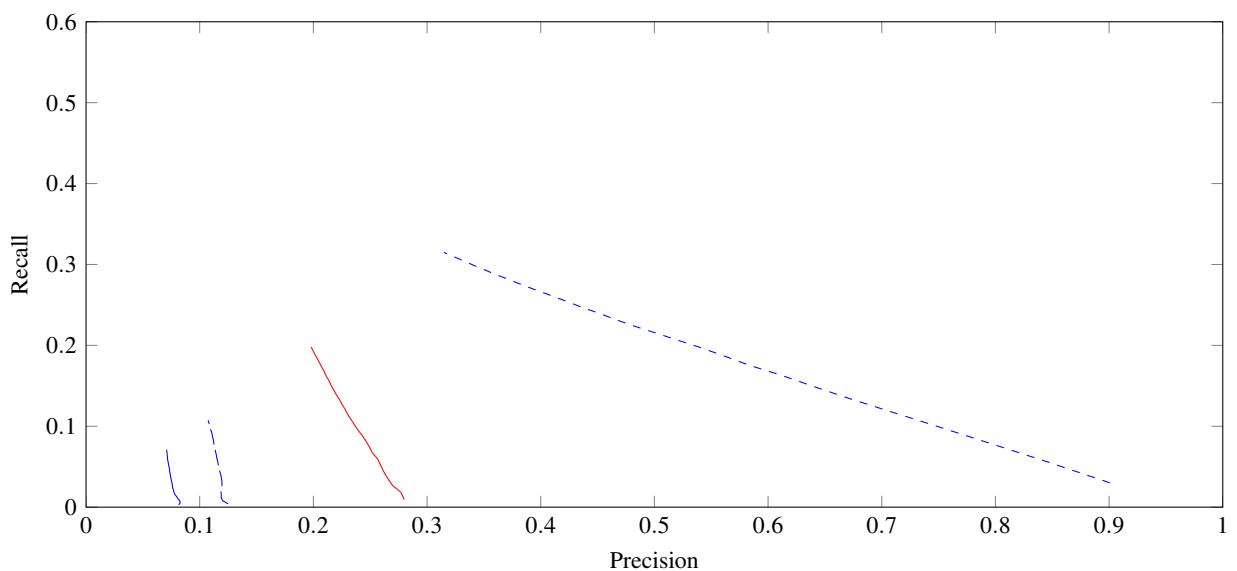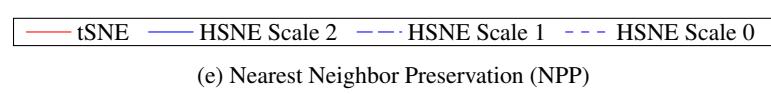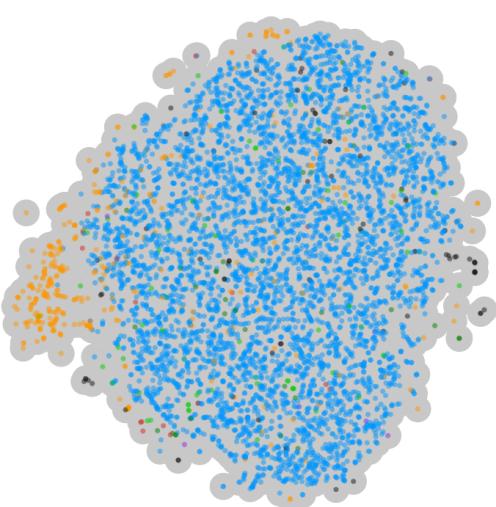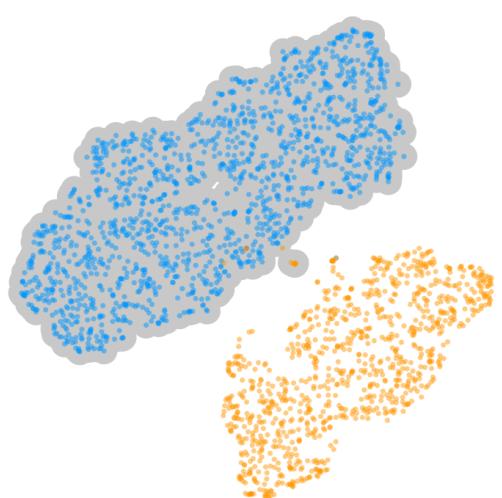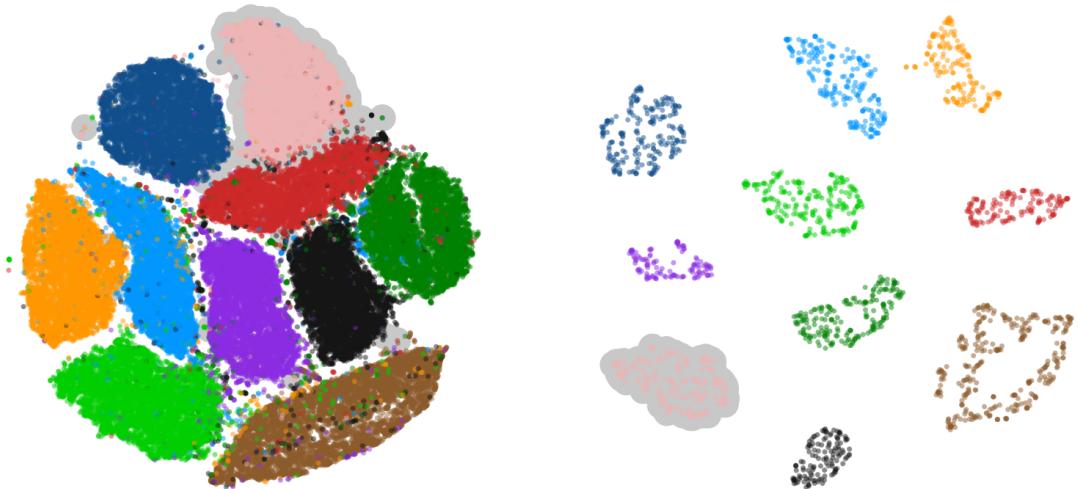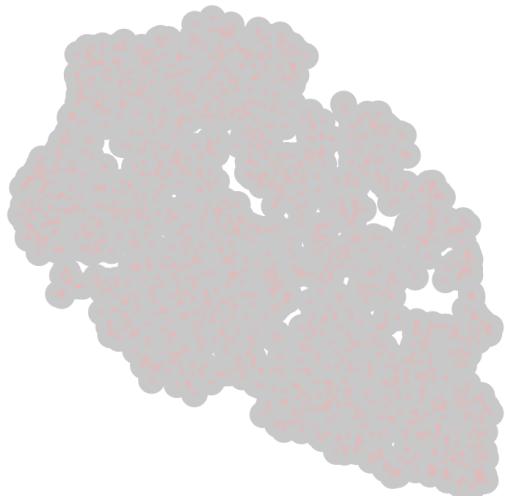
Figure 5: **Drill-down analysis (MNIST Test 1)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
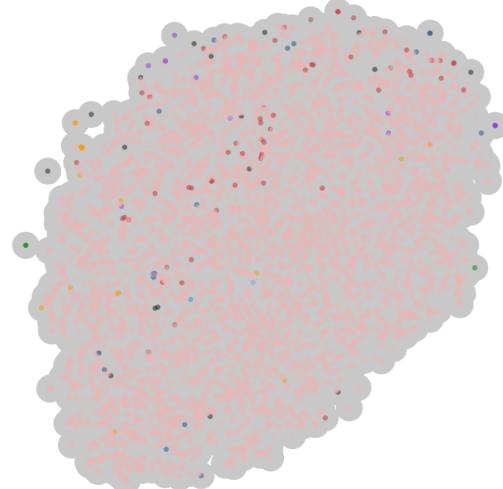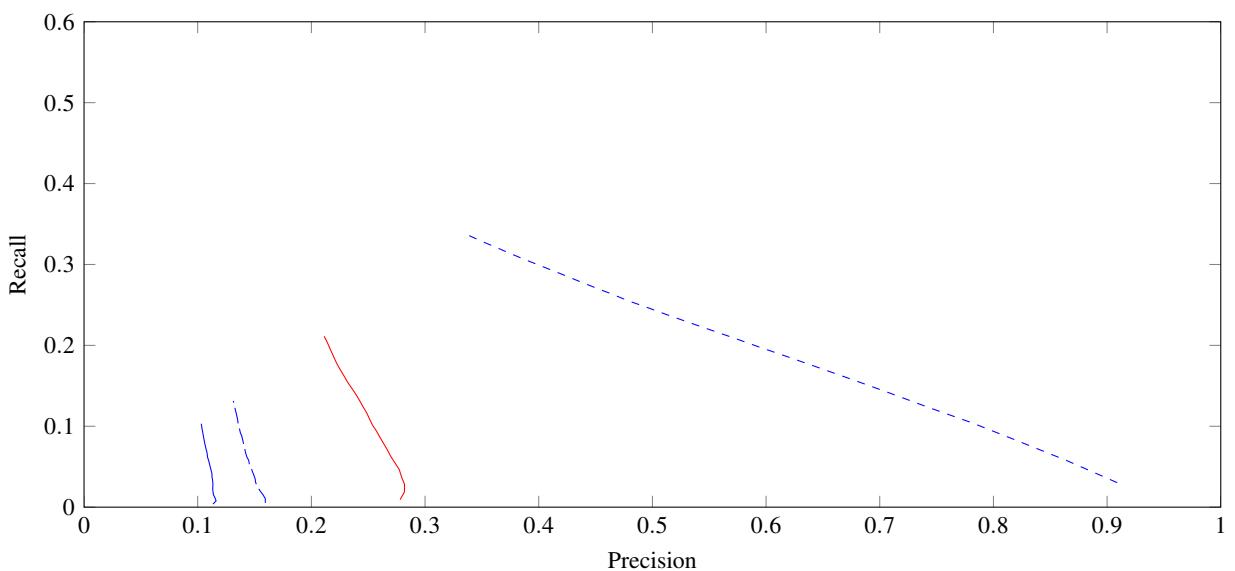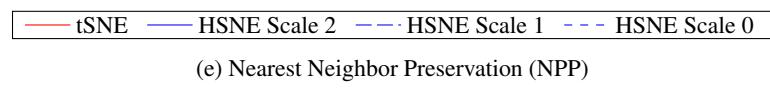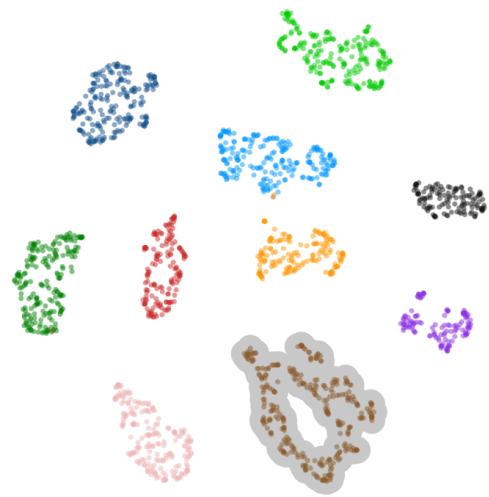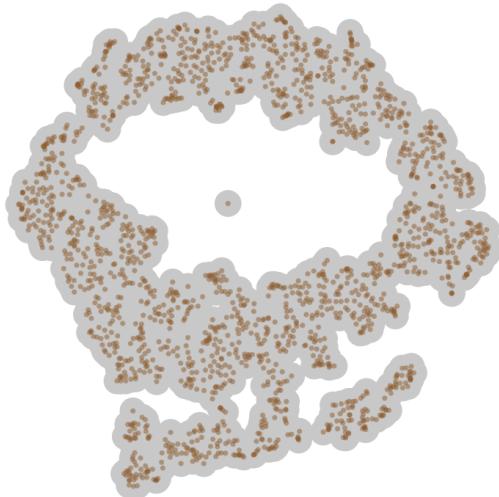
Figure 6: **Drill-down analysis (MNIST Test 2)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

Figure 7: **Drill-down analysis (MNIST Test 3)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.
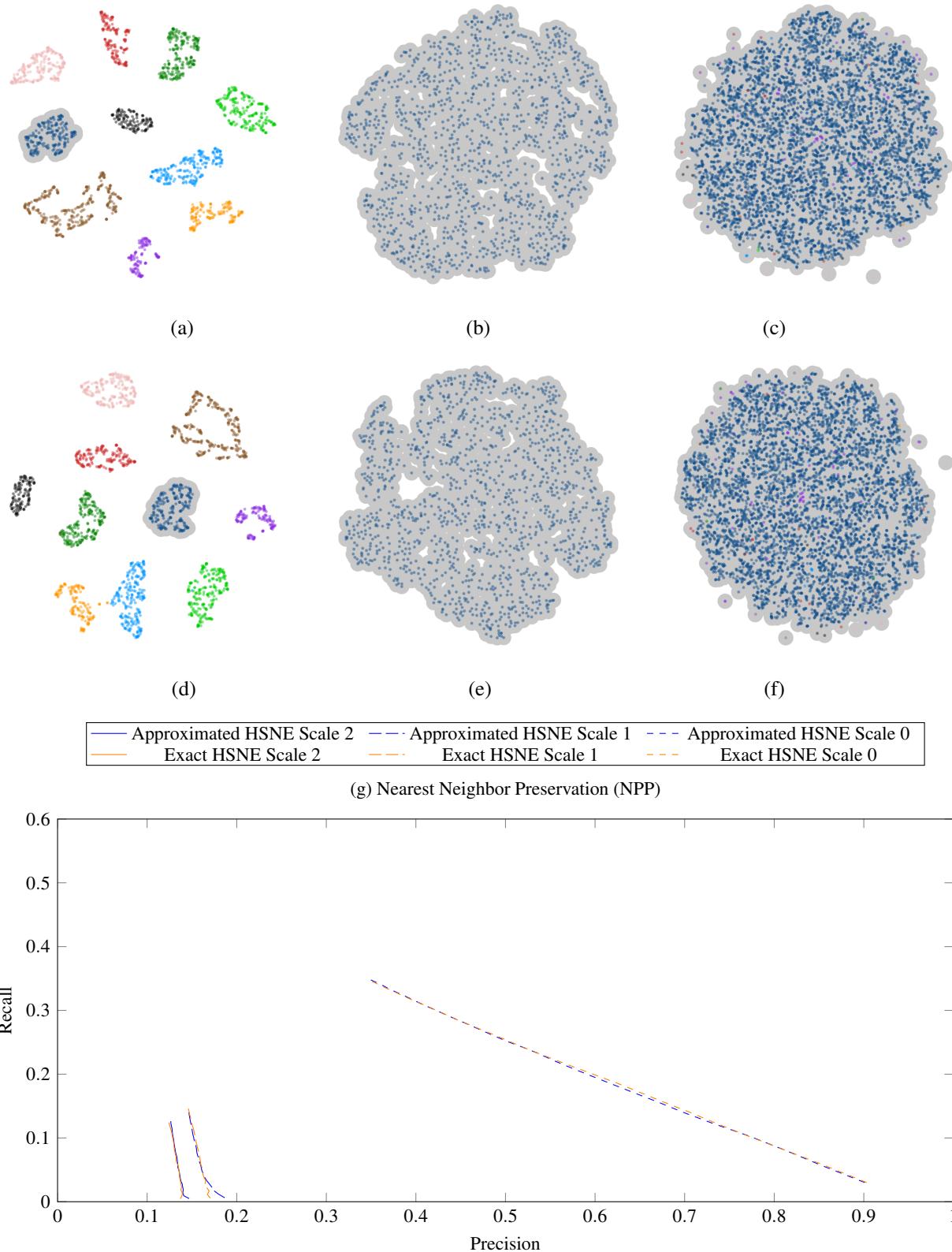
Figure 8: **Approximated vs Exact computations (MNIST Test 0)**. Selected landmarks/data-points are highlighted with a grey halo. Exact computation time: 3240". Approximated computation time: 73" (a) Approximated HSNE embedding scale 2 (b) Approximated HSNE embedding scale 1 (c) Approximated HSNE embedding scale 0 - Data points (d) Exact HSNE embedding scale 2 (e) Exact HSNE embedding scale 1 (f) Exact HSNE embedding scale 0 - Data points (g) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e)

(f)

| Approximated HSNE Scale 2 | Approximated HSNE Scale 1 | Approximated HSNE Scale 0 |
| Exact HSNE Scale 2 | Exact HSNE Scale 1 | Exact HSNE Scale 0 |

(g) Nearest Neighbor Preservation (NPP)
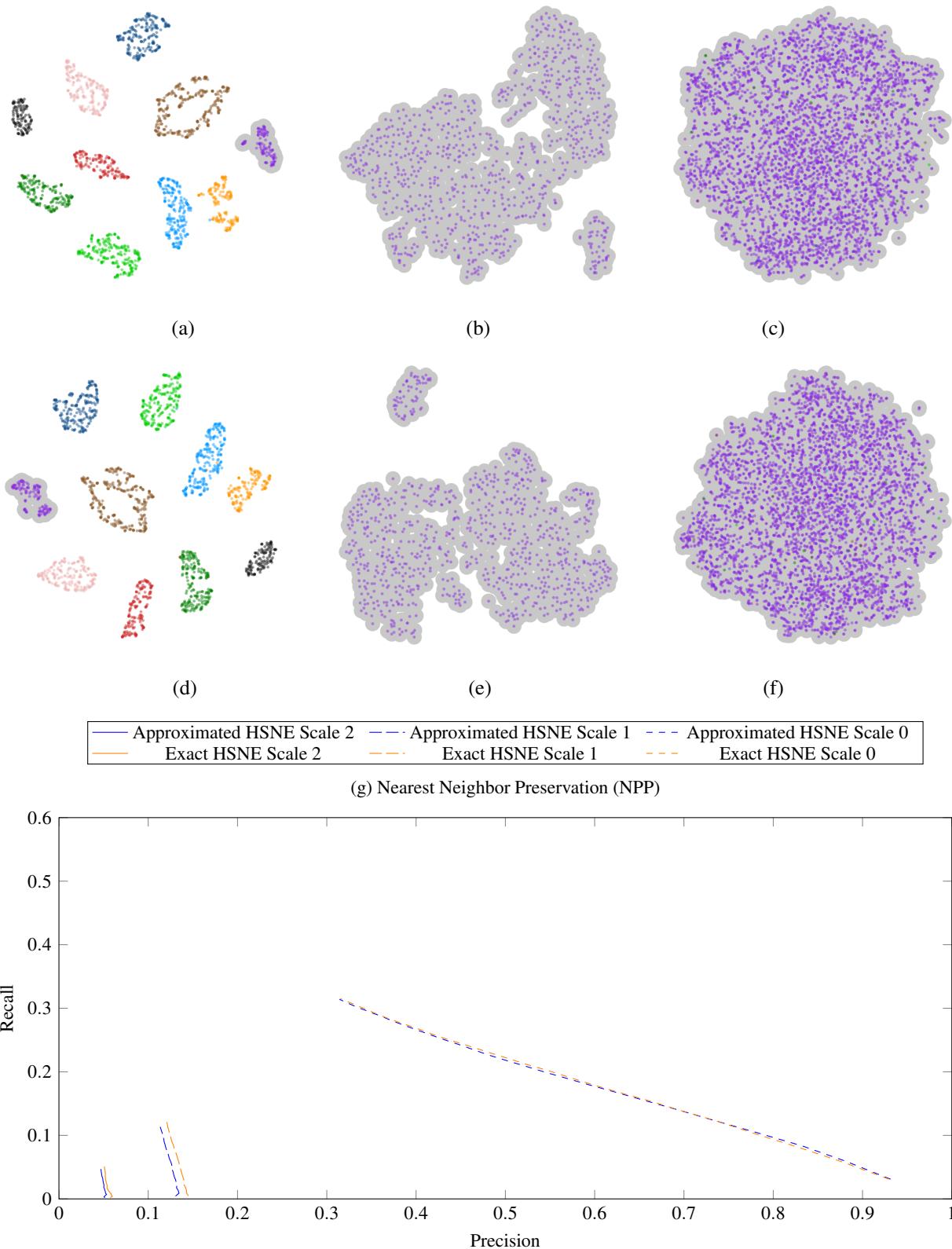
Figure 9: **Approximated vs Exact computations (MNIST Test 1)**. Selected landmarks/data-points are highlighted with a grey halo. Exact computation time: 3240". Approximated computation time: 73". (a) Approximated HSNE embedding scale 2 (b) Approximated HSNE embedding scale 1 (c) Approximated HSNE embedding scale 0 - Data points (d) Exact HSNE embedding scale 2 (e) Exact HSNE embedding scale 1 (f) Exact HSNE embedding scale 0 - Data points (g) Precision/Recall curves of the Nearest Neighbor Preservation.
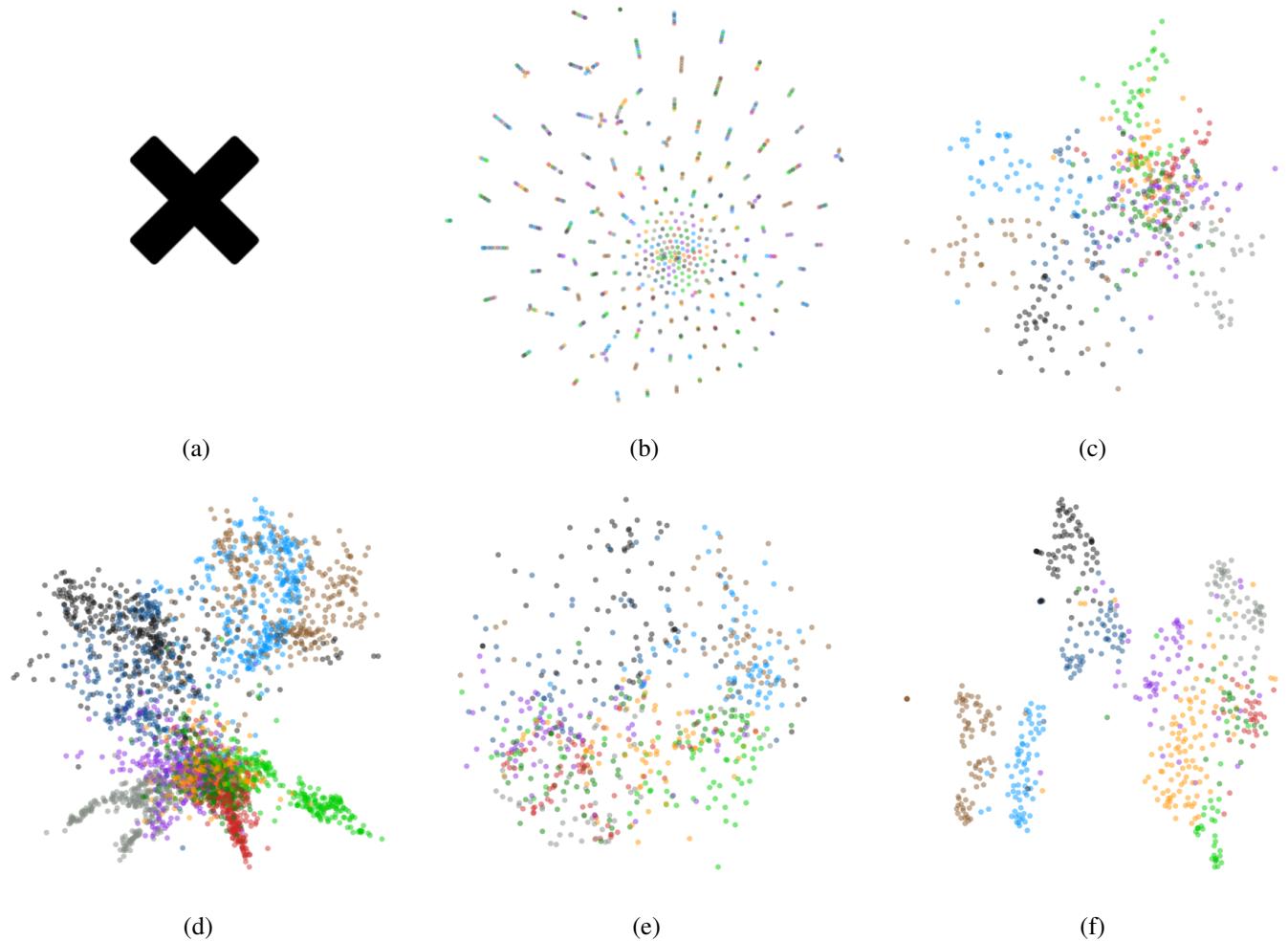
Figure 10: **Embeddings of the CIFAR-10 datasets (Landmarks)**. (a) tSNE - Time: 16'23" - #Landmarks: n/a (b) Landmark-SNE - Time: 4'23" - #Landmarks: 626 (c) LSP with 5k data points - Time: n/a - #Landmarks: 500 (d) P-LSP - Time: 5'15" - #Landmarks: 3228 (e) LAMP - Time: 4'31" - #Landmarks: 670 (f) HSNE - Time: 1'38" - #Landmarks: 626
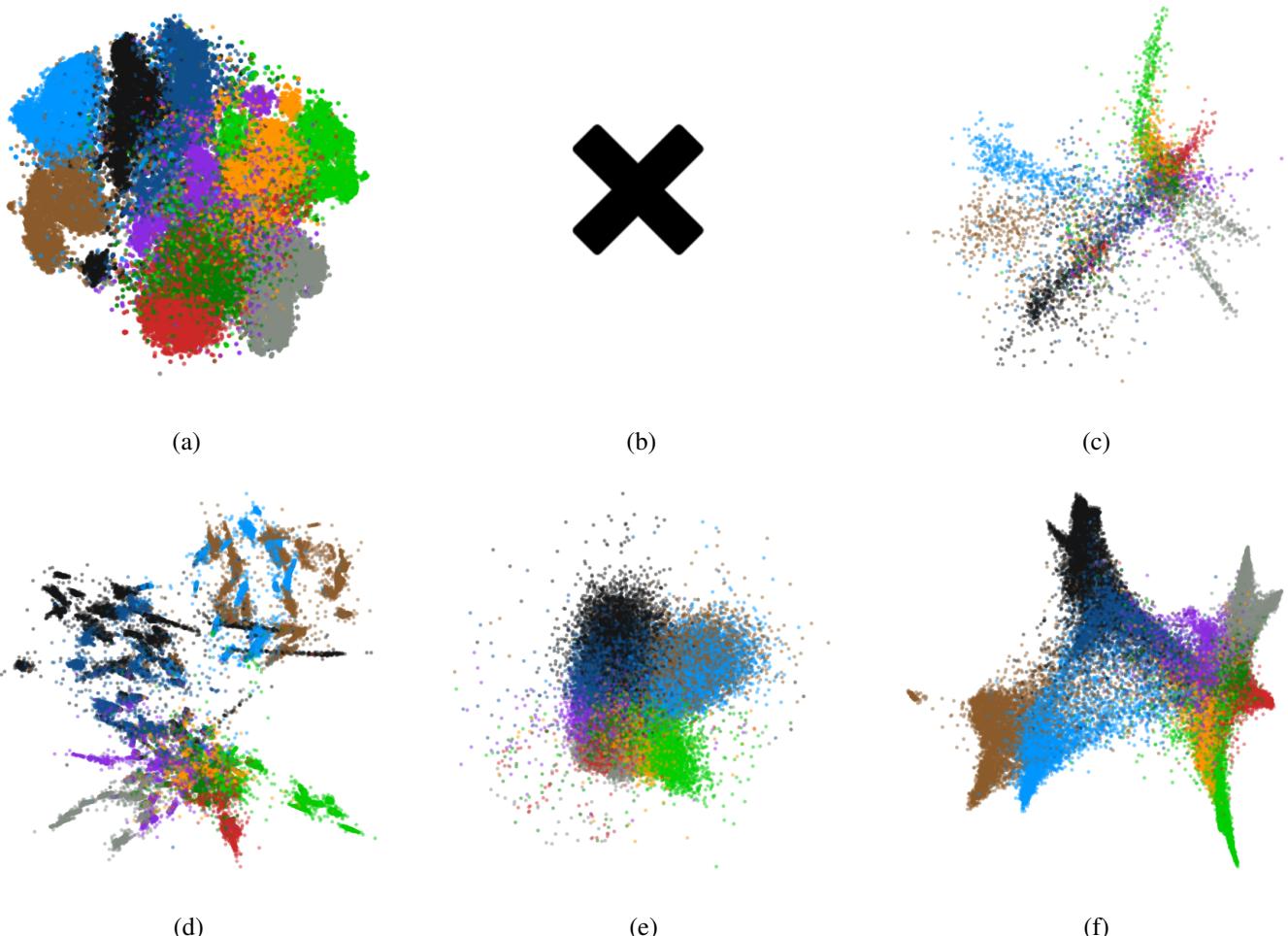
Figure 11: **Embeddings of the CIFAR-10 datasets (Complete embedding)**. (a) tSNE - Time: 16'23" - #Landmarks: n/a (b) Landmark-SNE - Time: 4'23" - #Landmarks: 626 (c) LSP with 5k data points - Time: n/a - #Landmarks: 500 (d) P-LSP - Time: 5'15" - #Landmarks: 3228 (e) LAMP - Time: 4'31" - #Landmarks: 670 (f) HSNE - Time: 1'38" - #Landmarks: 626
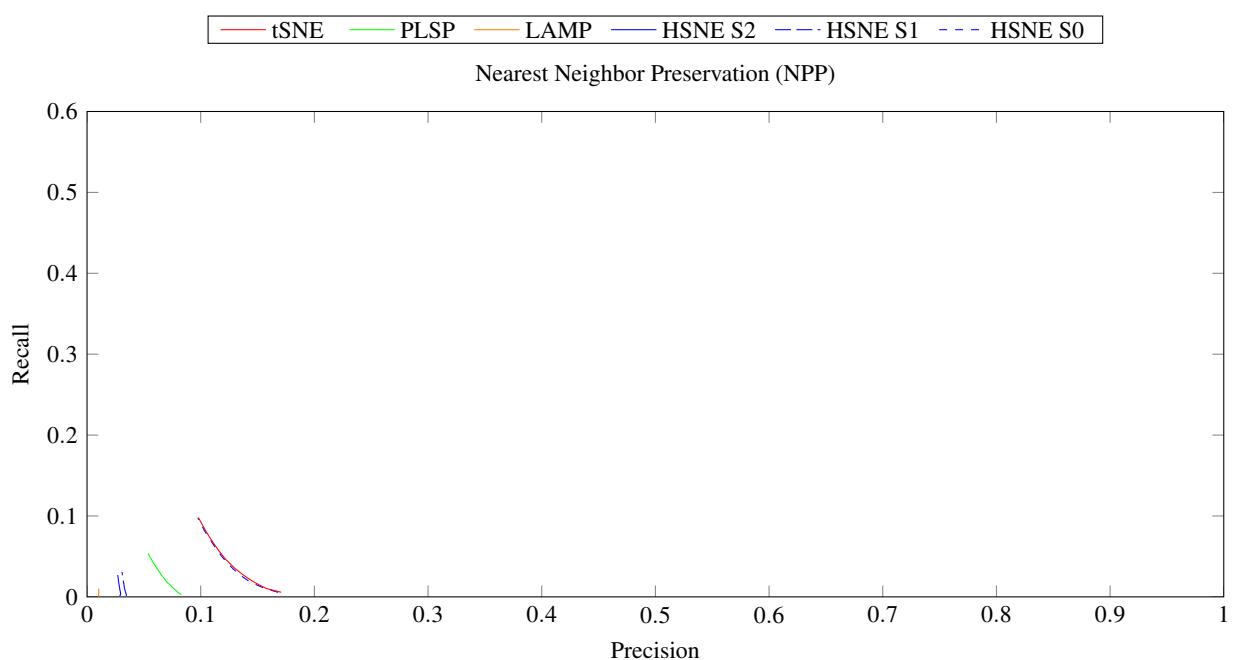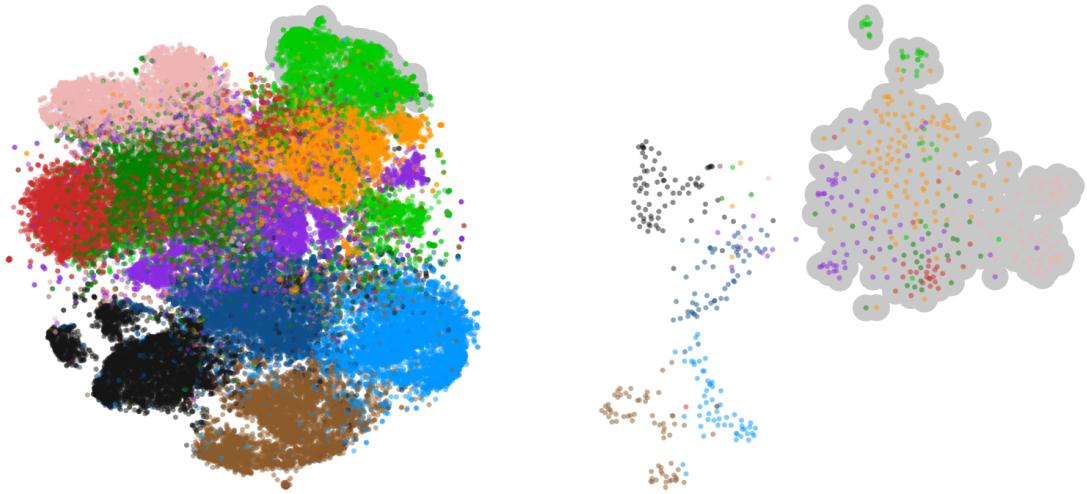

Figure 12: **Precision/Recall curves** of the Nearest Neighbor Preservation for the full CIFAR dataset.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
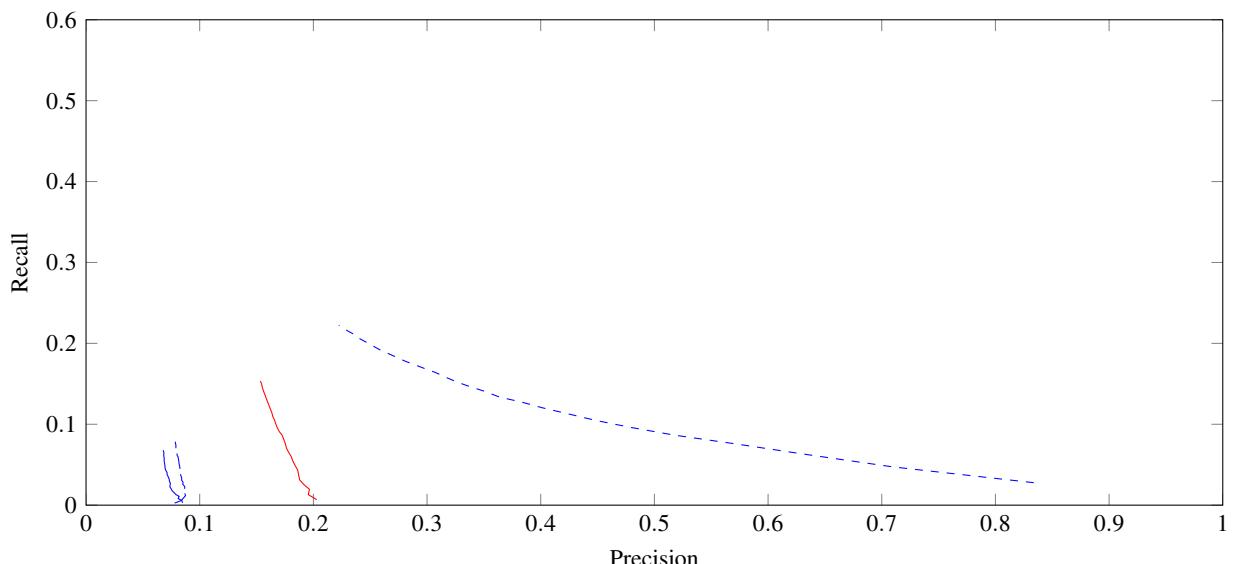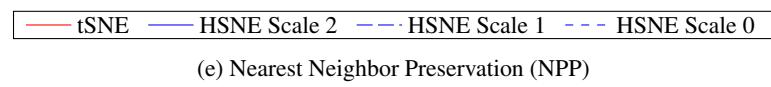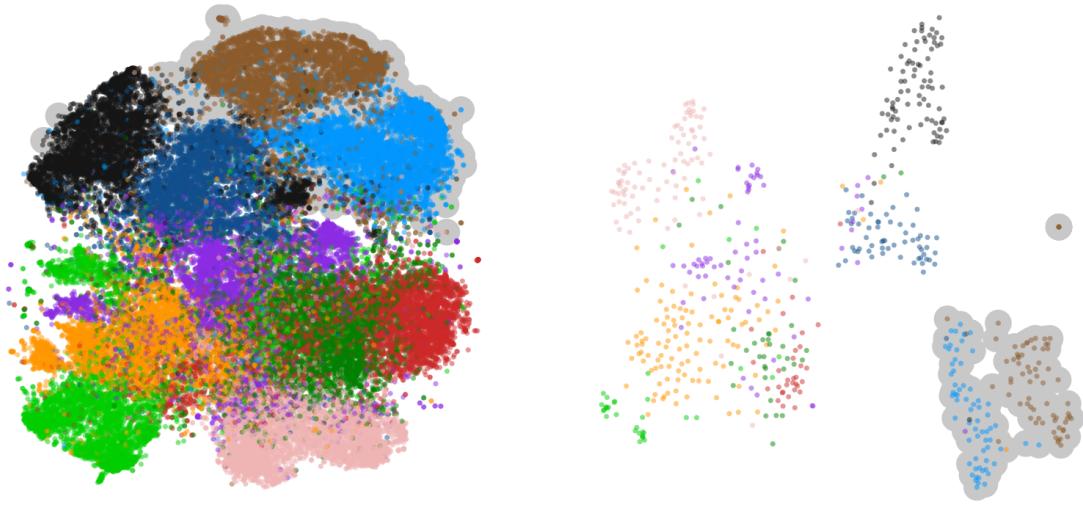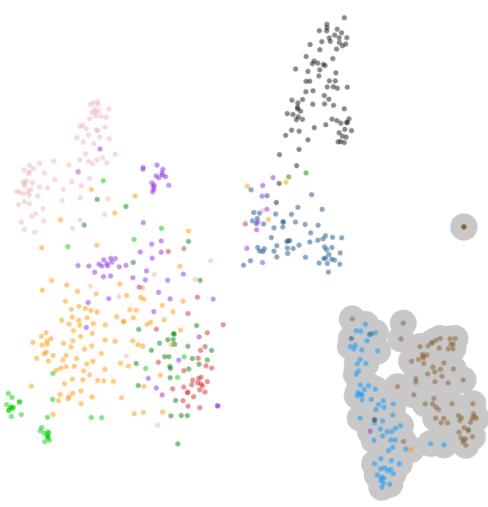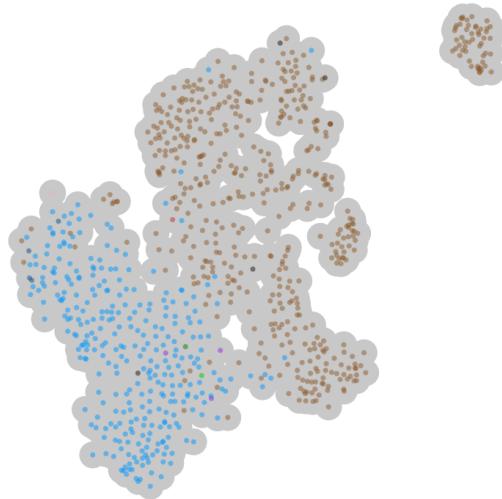
Figure 13: **Drill-down analysis (CIFAR-10 Test 0)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)

| | tSNE | HSNE Scale 2 | HSNE Scale 1 | HSNE Scale 0 |
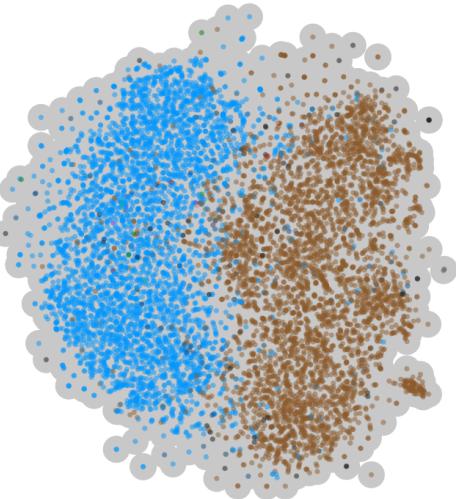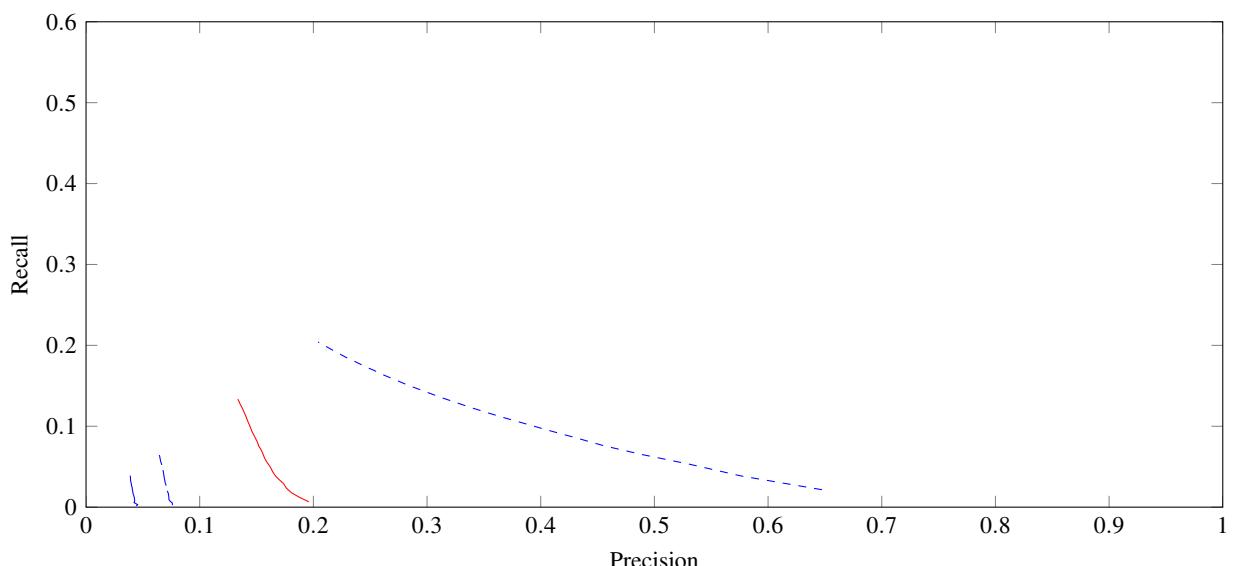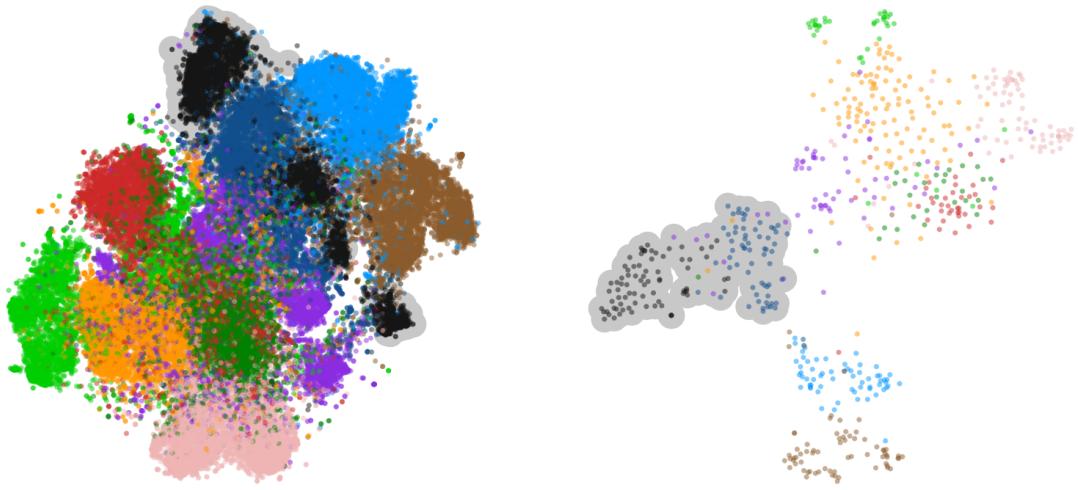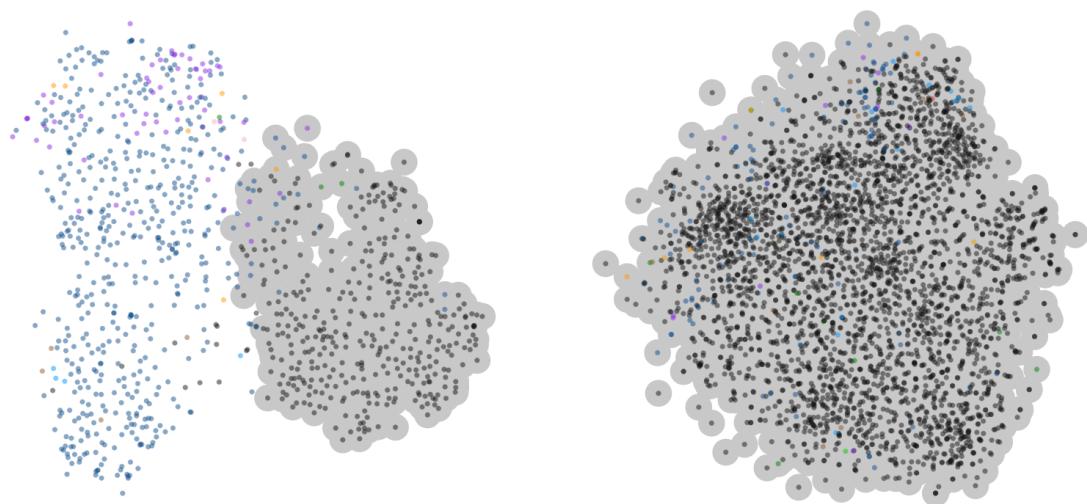
Figure 14: **Drill-down analysis (CIFAR-10 Test 1)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
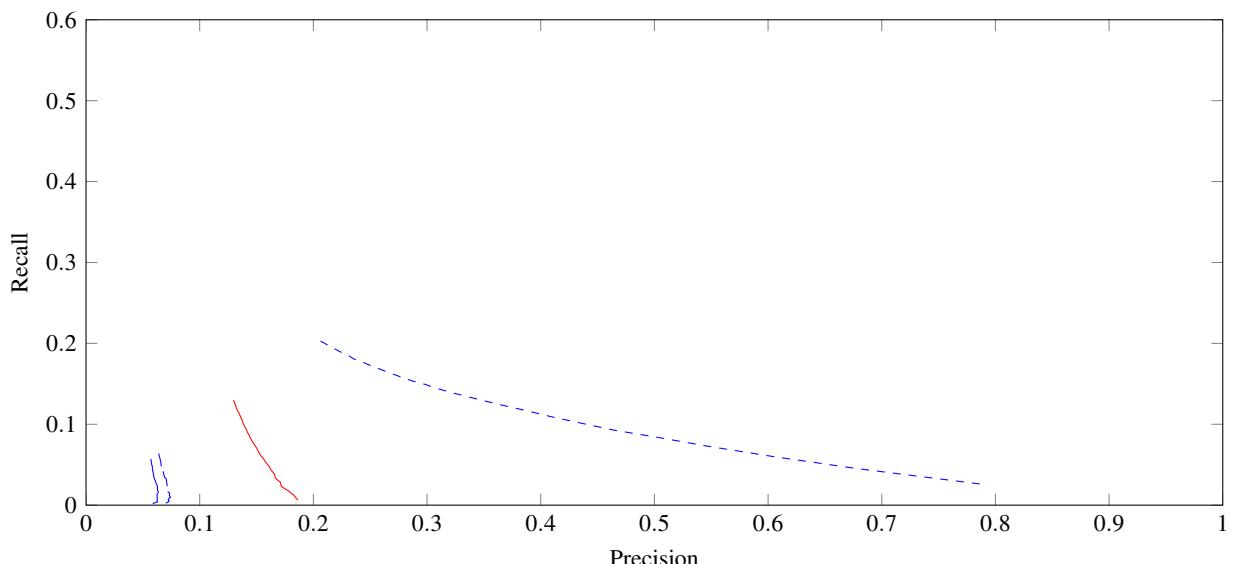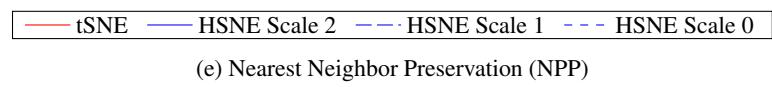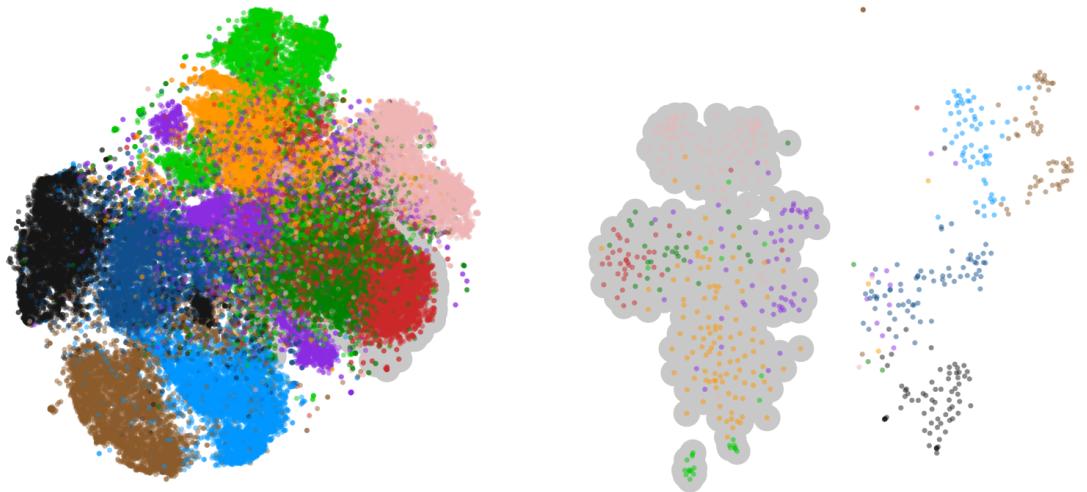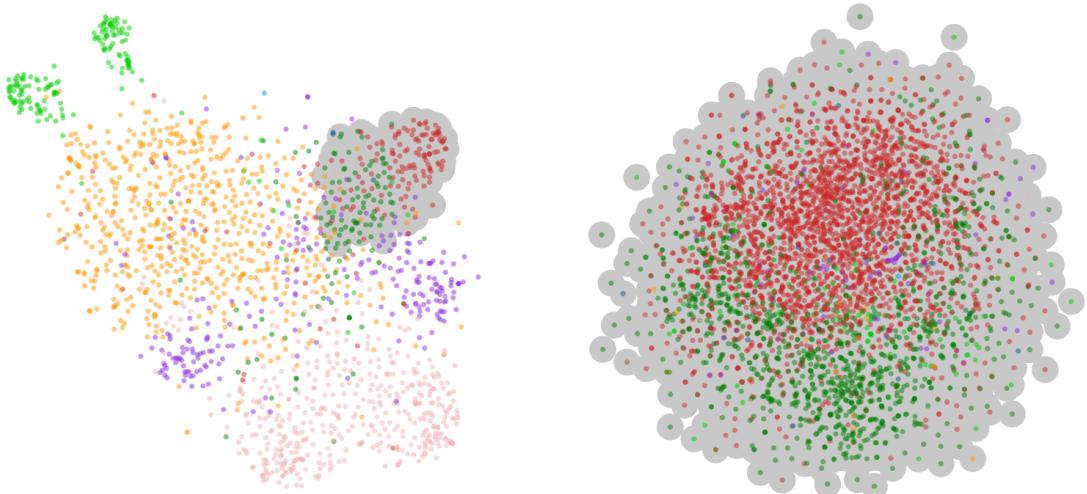
Figure 15: **Drill-down analysis (CIFAR-10 Test 2)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
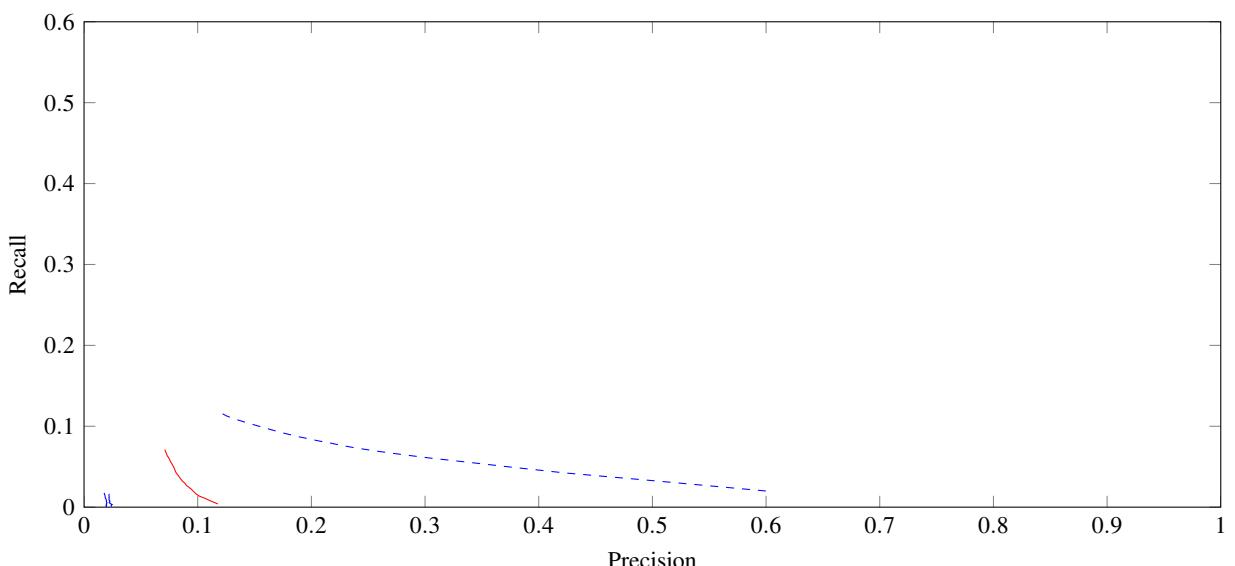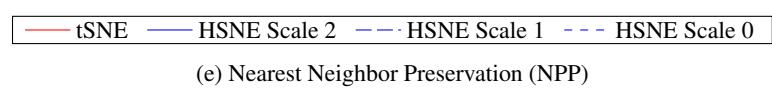
Figure 16: **Drill-down analysis (CIFAR-10 Test 3)**. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)   (b)   (c)

(d)   (e)   (f)

(g) Nearest Neighbor Preservation (NPP)
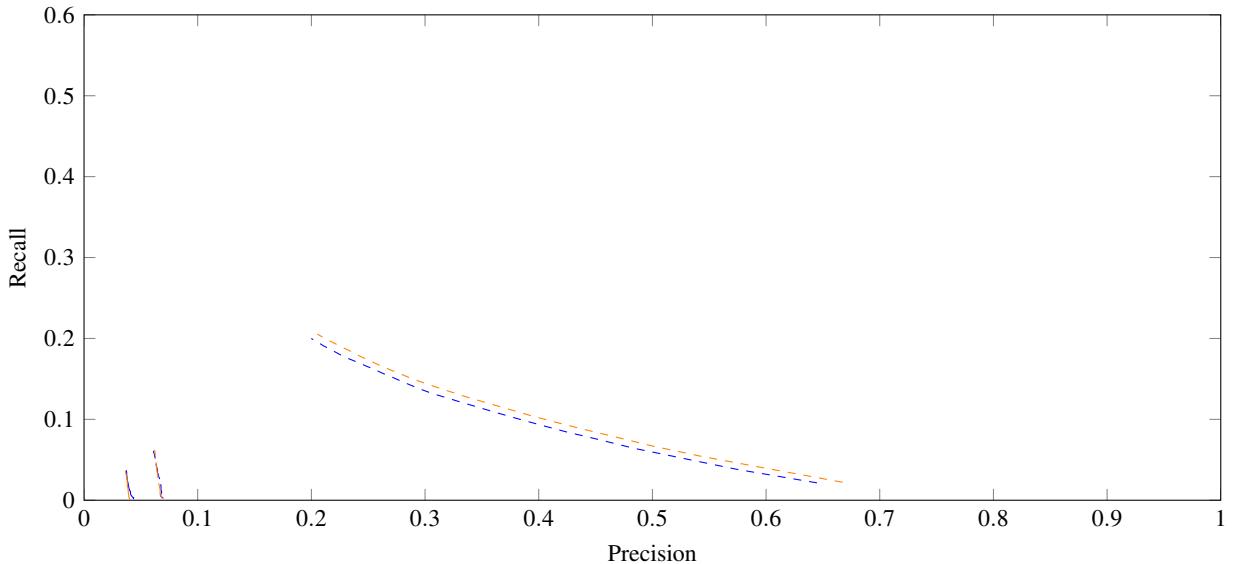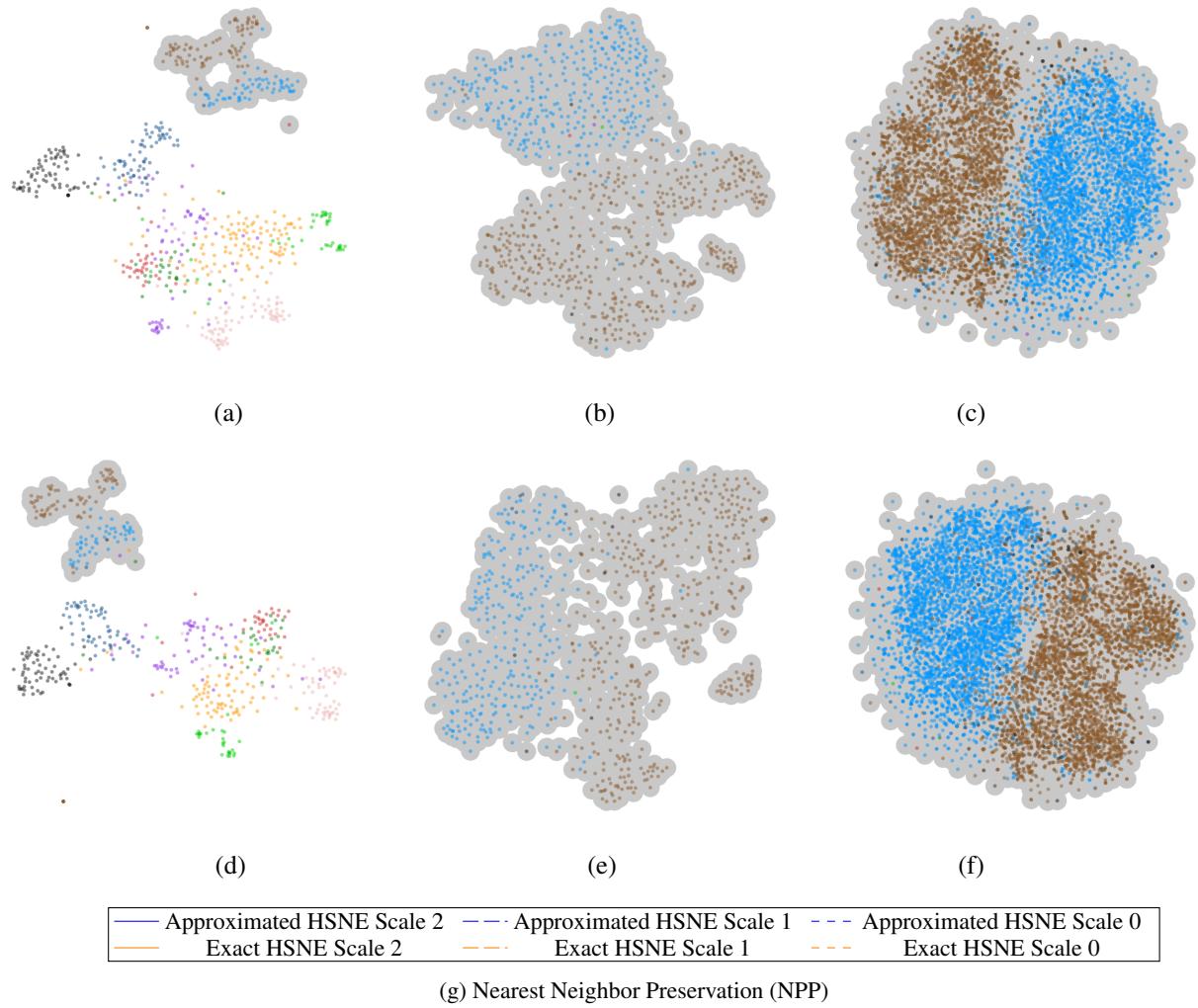
Figure 17: **Approximated vs Exact computations (CIFAR-10 Test 0)**. Selected landmarks/data-points are highlighted with a grey halo. Exact computation time: 2970". Approximated computation time: 67". (a) Approximated HSNE embedding scale 2 (b) Approximated HSNE embedding scale 1 (c) Approximated HSNE embedding scale 0 - Data points (d) Exact HSNE embedding scale 2 (e) Exact HSNE embedding scale 1 (f) Exact HSNE embedding scale 0 - Data points (g) Precision/Recall curves of the Nearest Neighbor Preservation.
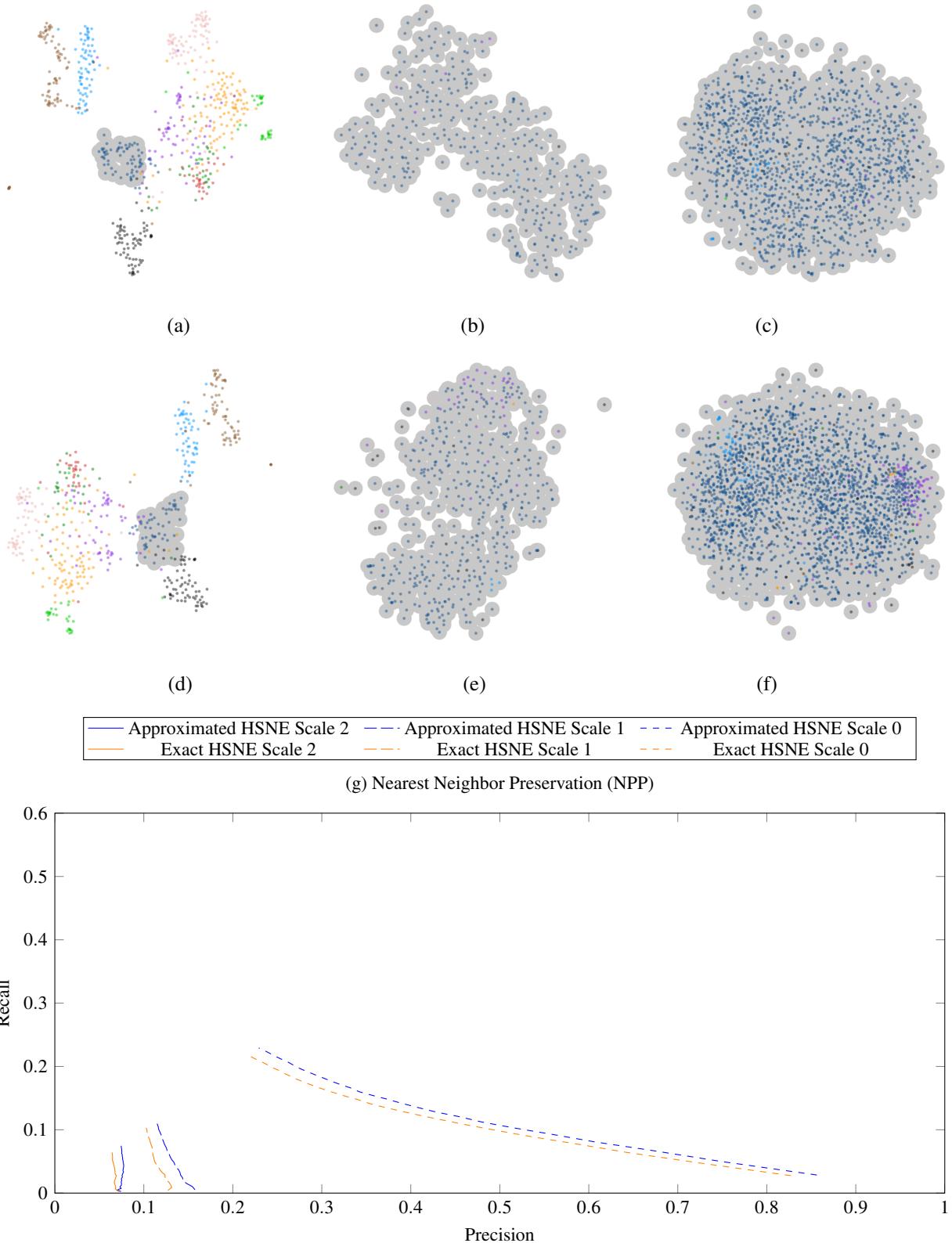
Figure 18: **Approximated vs Exact computations (CIFAR-10 Test 1)**. Selected landmarks/data-points are highlighted with a grey halo. Exact computation time: 2970". Approximated computation time: 67". (a) Approximated HSNE embedding scale 2 (b) Approximated HSNE embedding scale 1 (c) Approximated HSNE embedding scale 0 - Data points (d) Exact HSNE embedding scale 2 (e) Exact HSNE embedding scale 1 (f) Exact HSNE embedding scale 0 - Data points (g) Precision/Recall curves of the Nearest Neighbor Preservation.
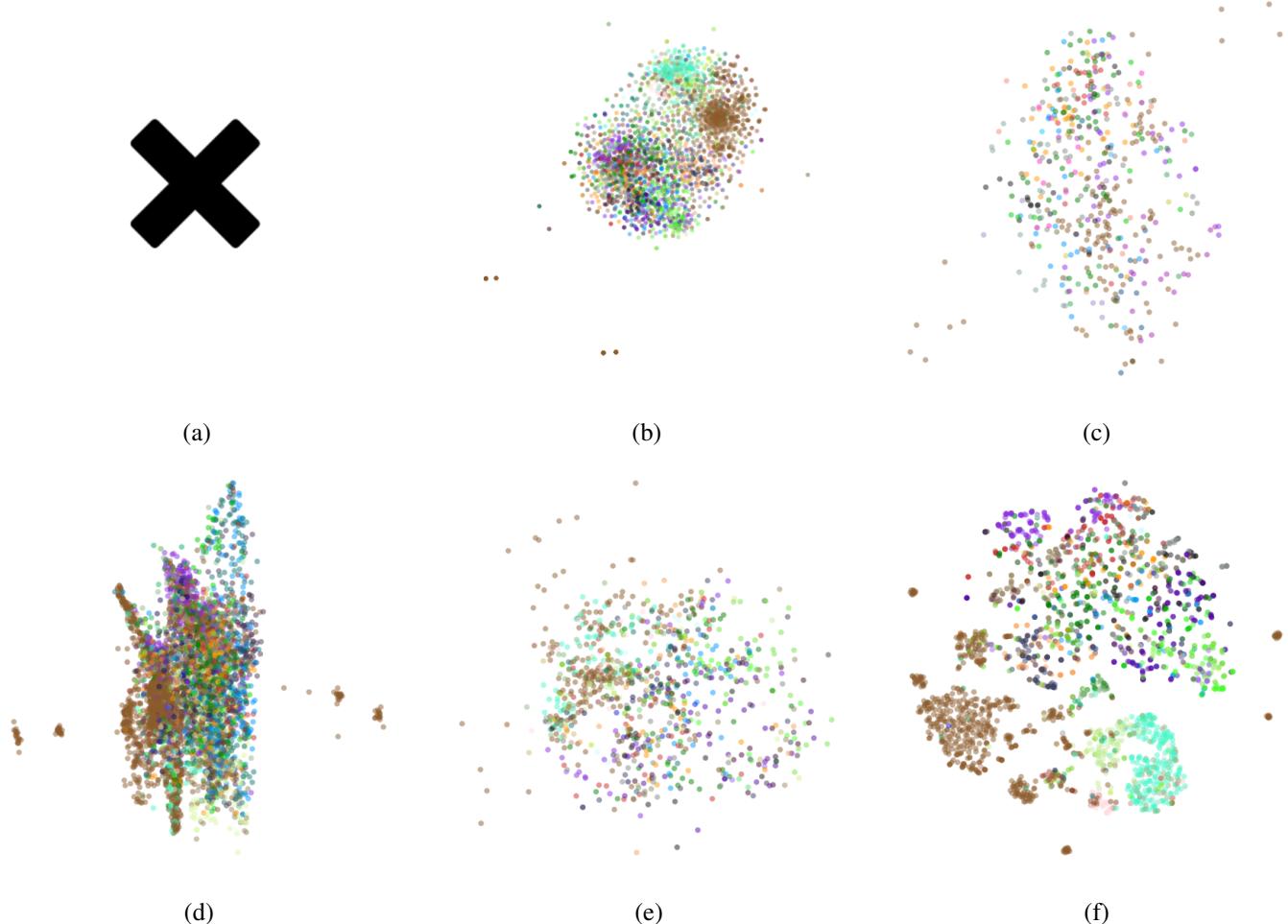
Figure 20: **Embeddings of the TIMIT datasets (Landmarks)** considering only 100k data-points. (a) tSNE - Time: 23'03" - #Landmarks: n/a (b) Landmark-SNE - Time: 3'18" - #Landmarks: 2493 (c) LSP with 5k data points - Time: n/a - #Landmarks: 500 (d) P-LSP - Time: 6'41" - #Landmarks: 5470 (e) LAMP - Time: 39" - #Landmarks: 948 (f) HSNE - Time: 3'04" - #Landmarks: 2493
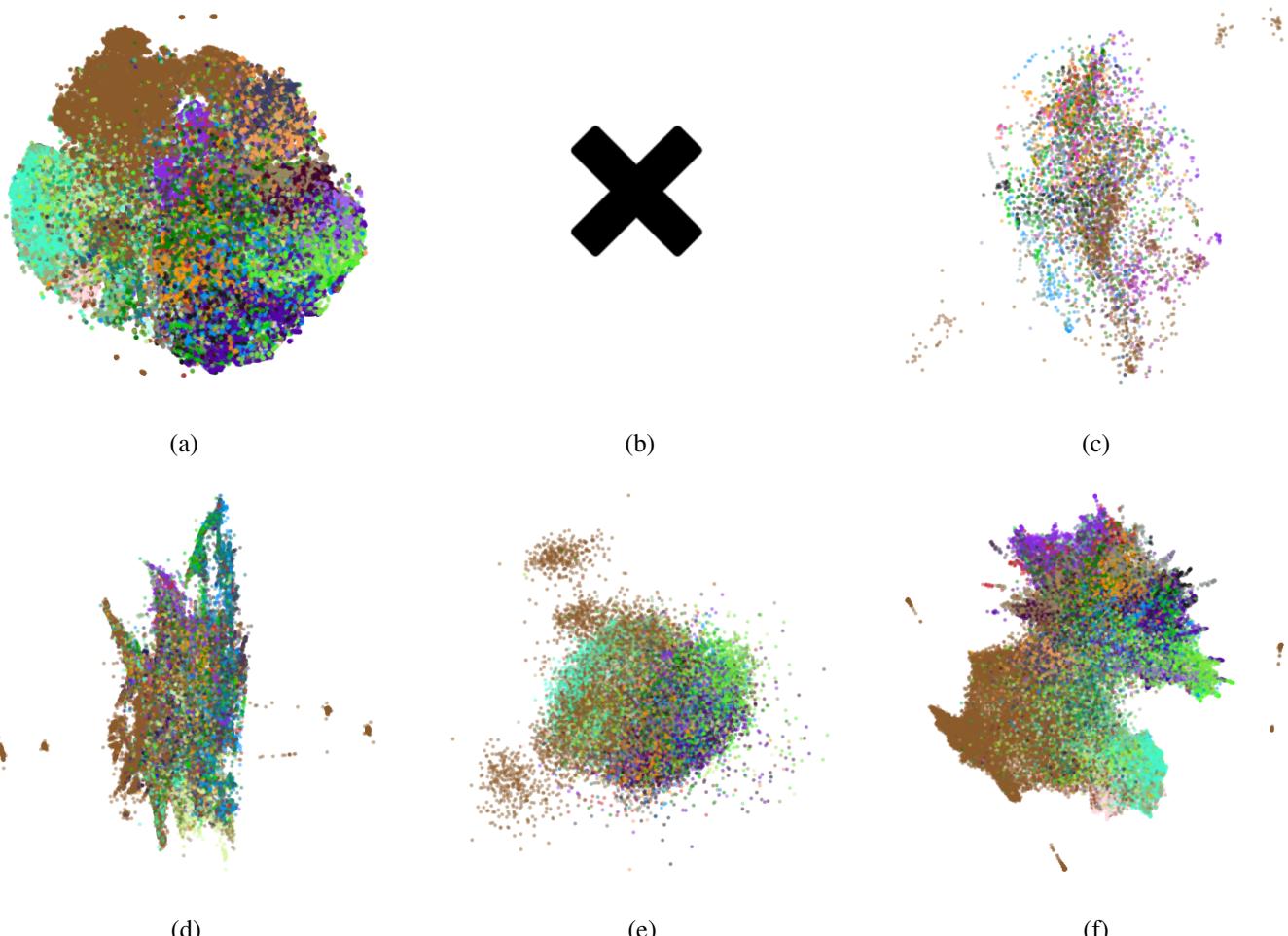
Figure 21: **Embeddings of the TIMIT datasets (Complete embedding)** considering only 100k data-points. (a) tSNE - Time: 23'03" - #Landmarks: n/a (b) Landmark-SNE - Time: 3'18" - #Landmarks: 2493 (c) LSP with 5k data points - Time: n/a - #Landmarks: 500 (d) P-LSP - Time: 6'41" - #Landmarks: 5470 (e) LAMP - Time: 39" - #Landmarks: 948 (f) HSNE - Time: 3'04" - #Landmarks: 2493
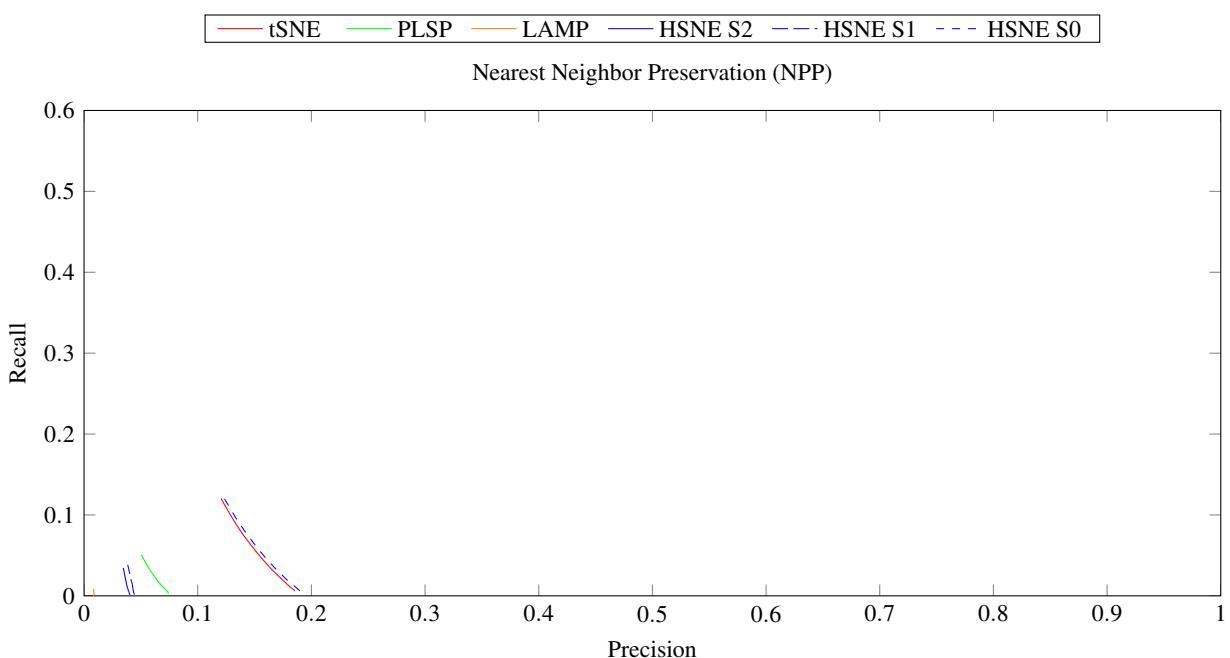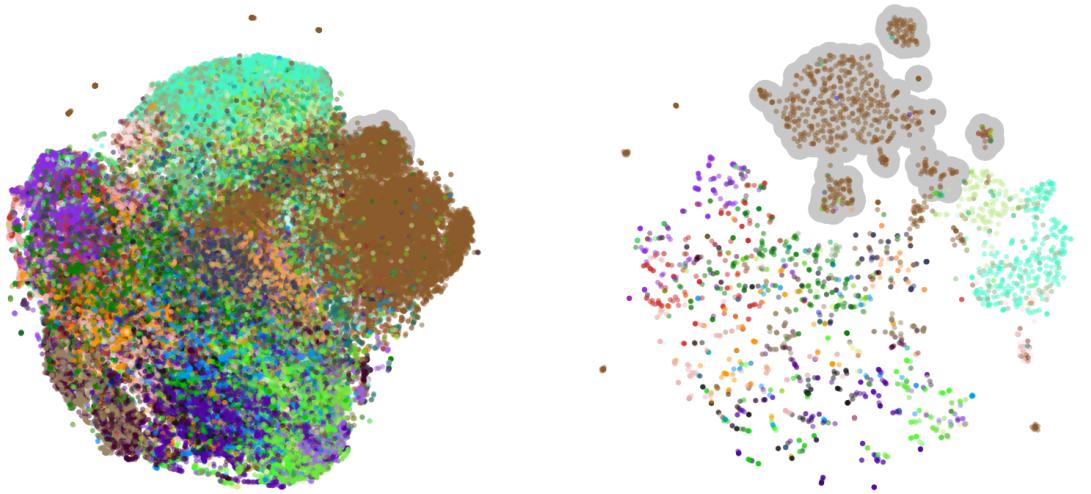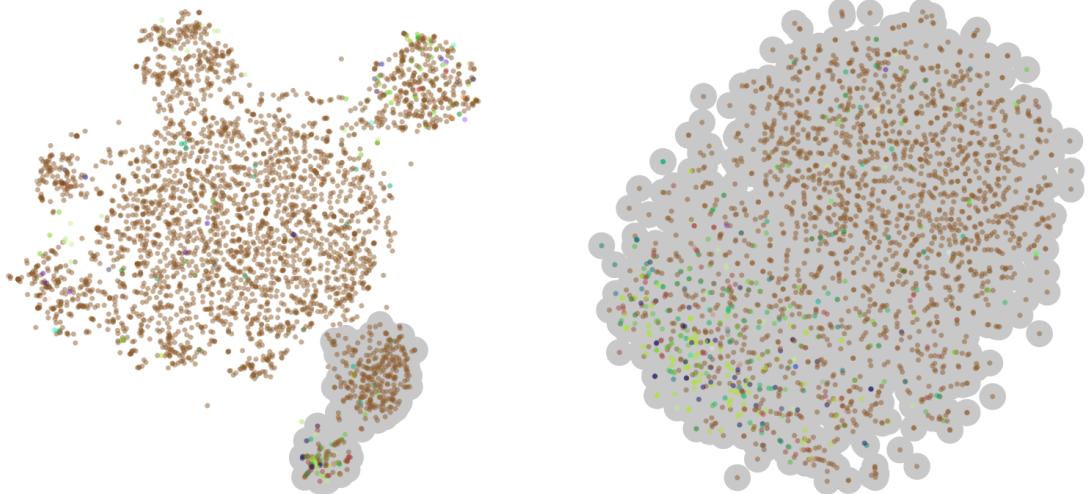

Figure 22: **Precision/Recall curves** of the Nearest Neighbor Preservation for the TIMIT dataset considering only 100k data-points.
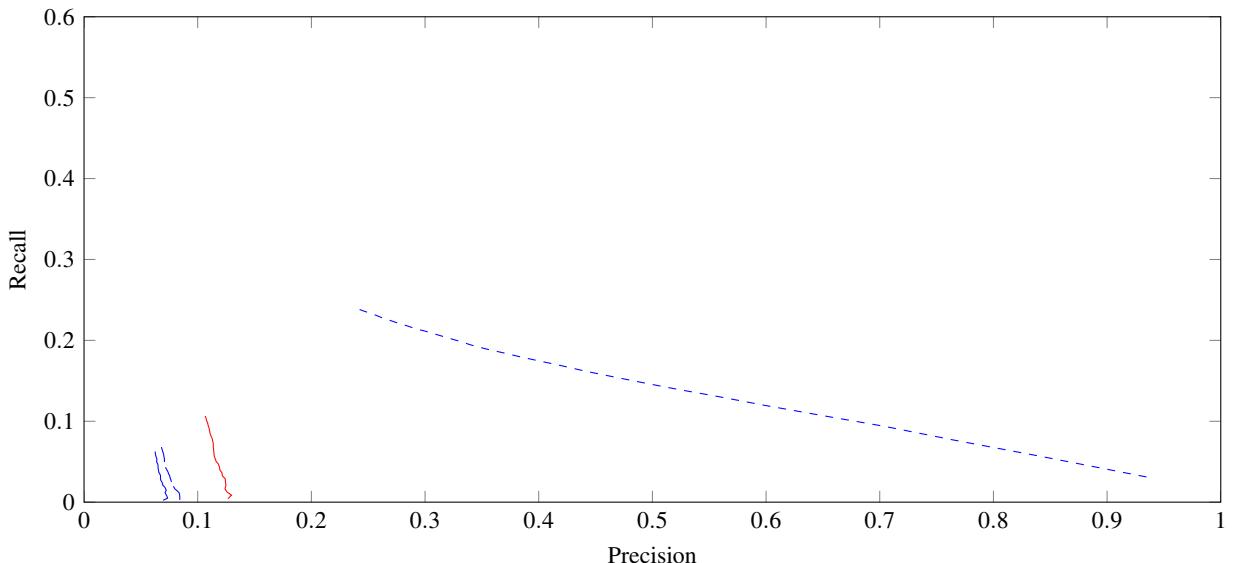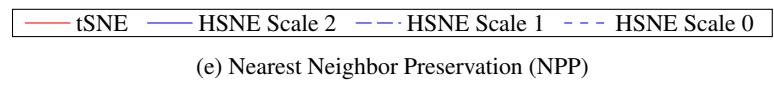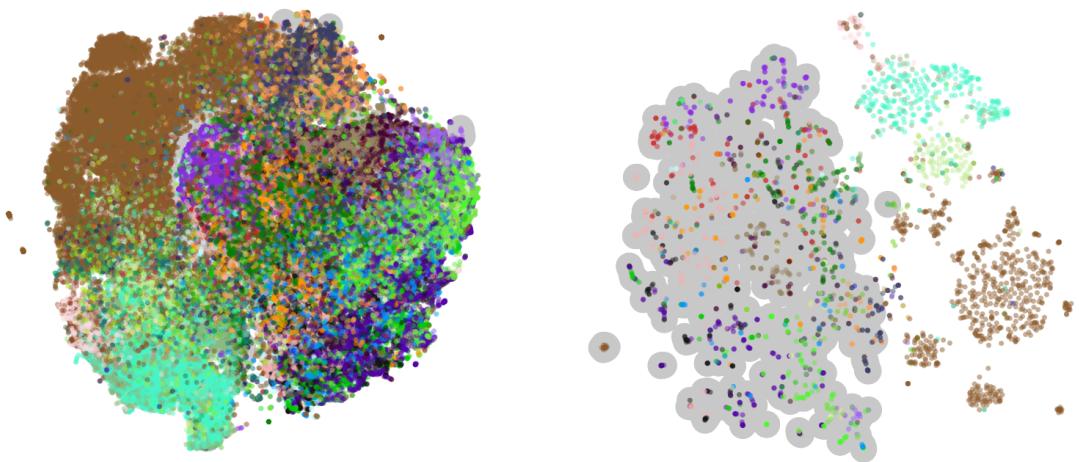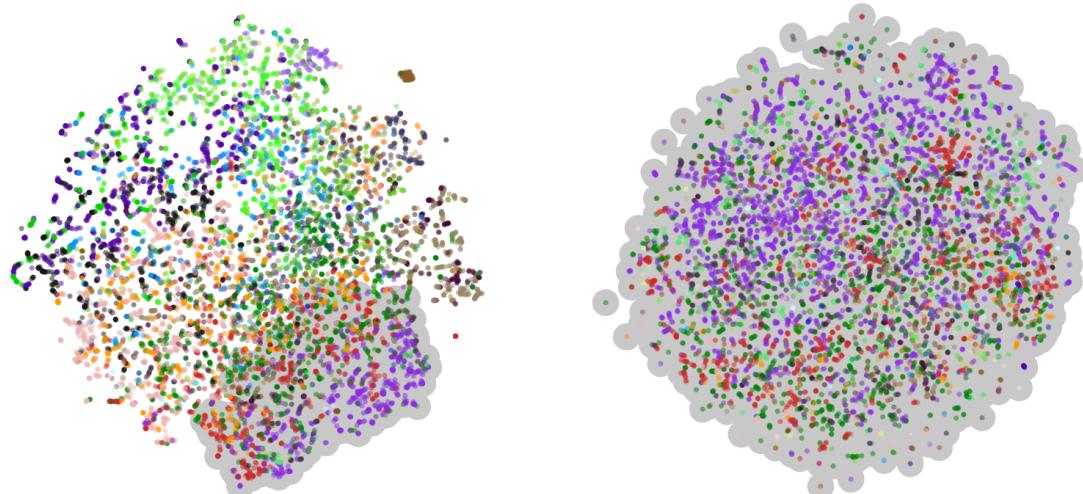
Figure 23: **Drill-down analysis (TIMIT Test 0)** considering only 100k data-points. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
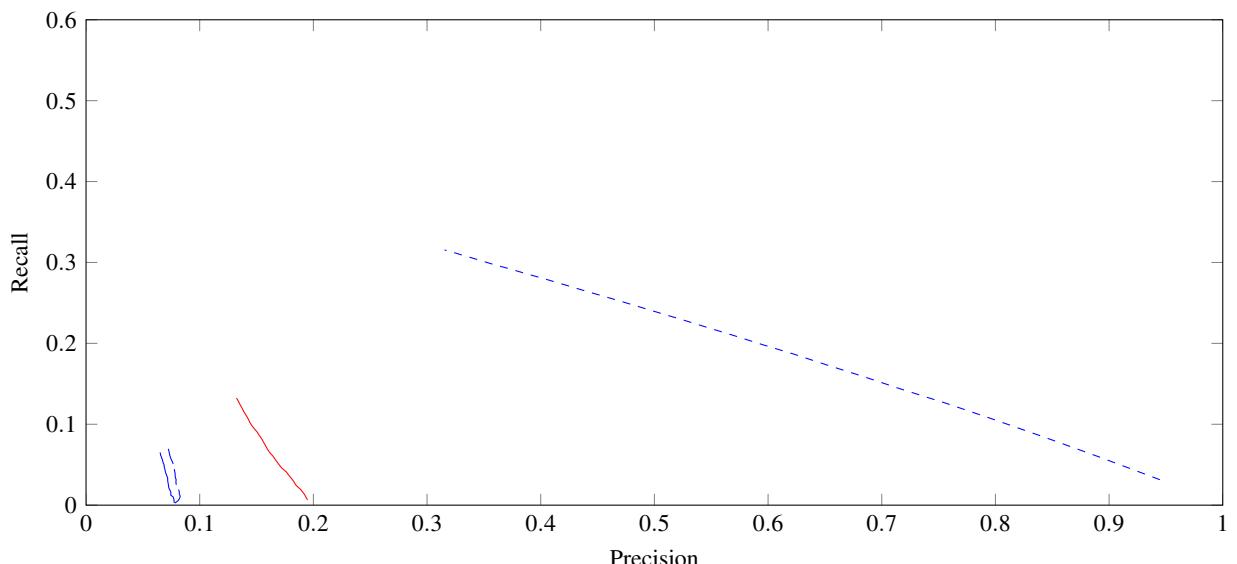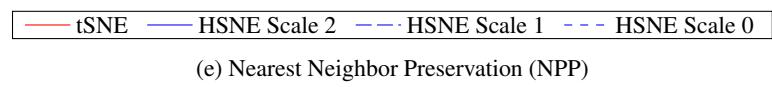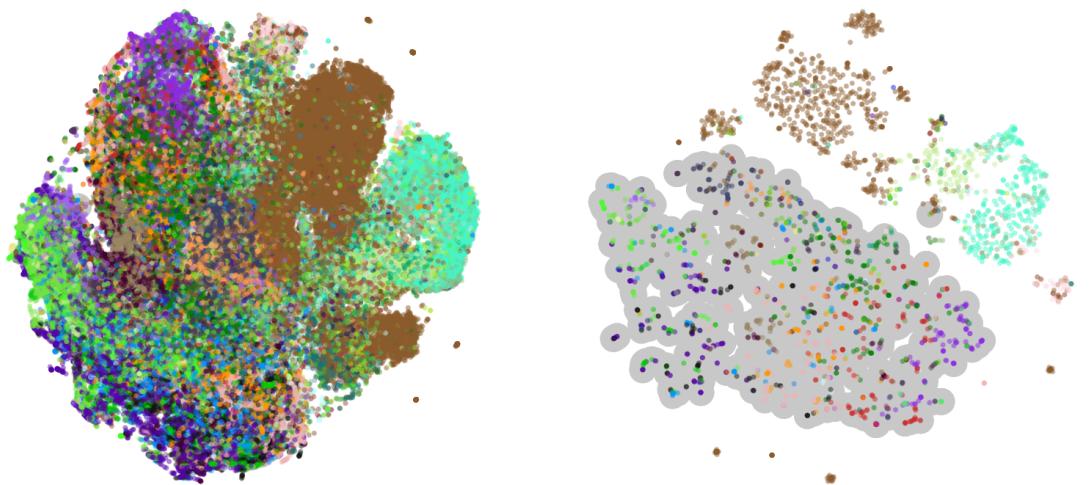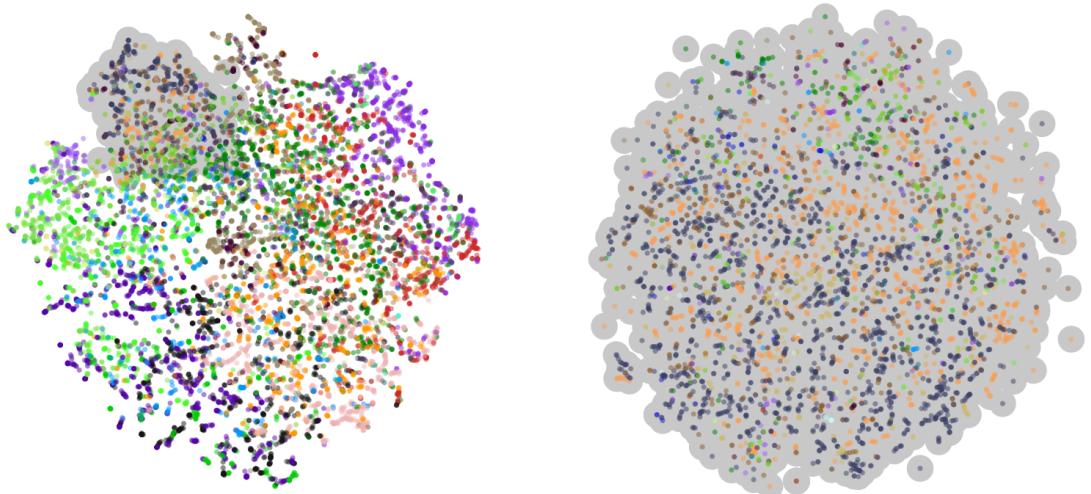
Figure 24: **Drill-down analysis (TIMIT Test 1)** considering only 100k data-points. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e) Nearest Neighbor Preservation (NPP)
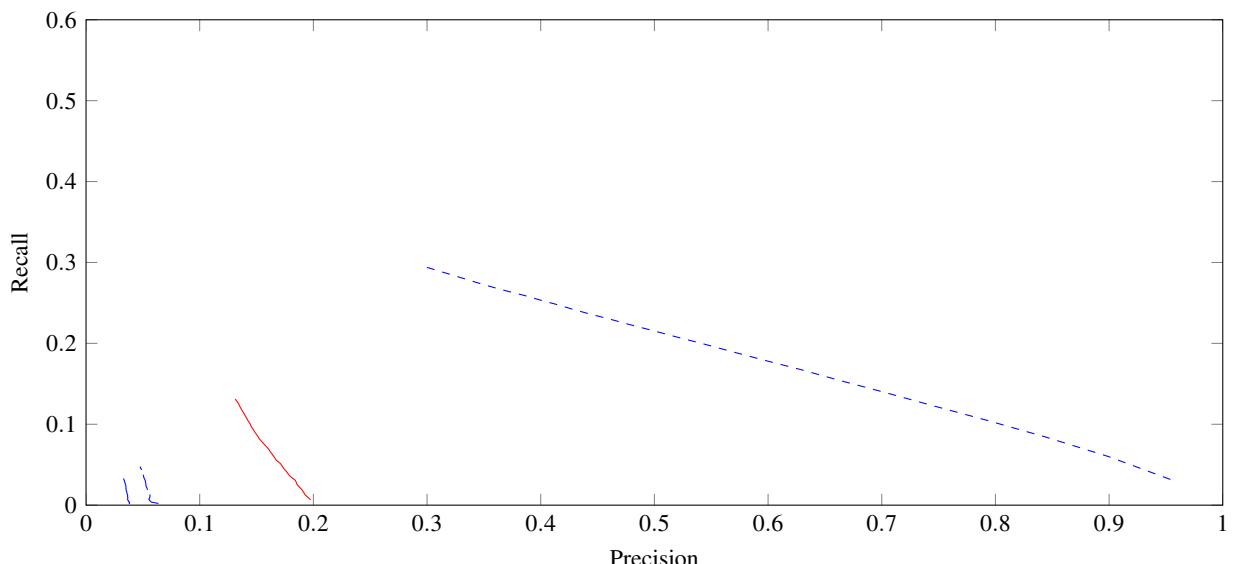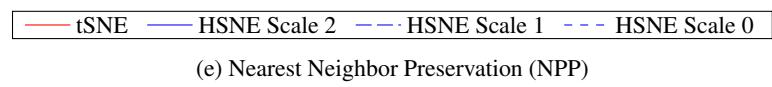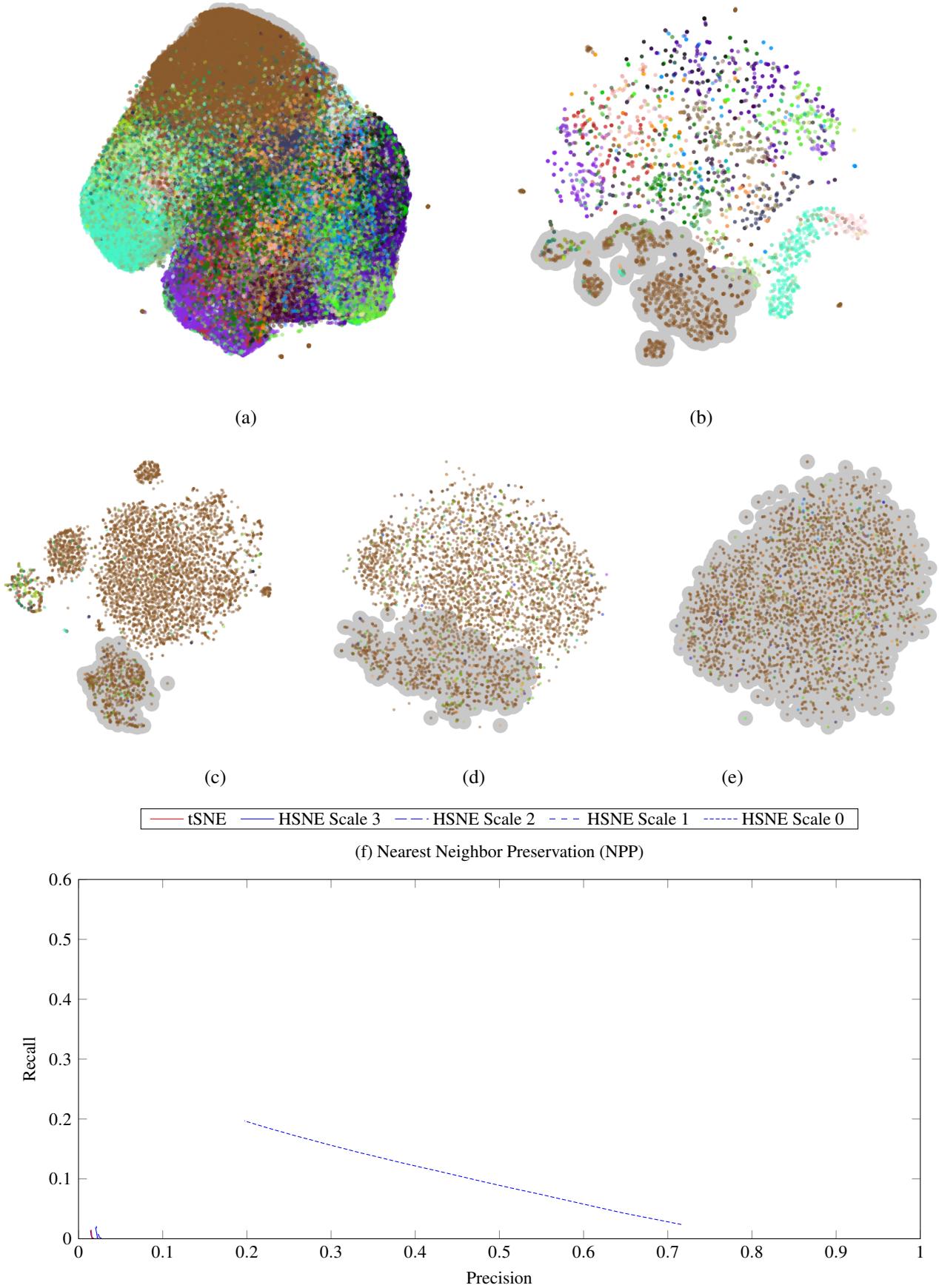
Figure 25: **Drill-down analysis (TIMIT Test 2)** considering only 100k data-points. Selected landmarks/data-points are highlighted with a grey halo. (a) tSNE Embedding (b) HSNE embedding scale 2 (c) HSNE embedding scale 1 (d) HSNE embedding scale 0 - Data points (e) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)

(b)

(c)

(d)

(e)

—— tSNE   —— HSNE Scale 3   —·—· HSNE Scale 2   – – – HSNE Scale 1   ······ HSNE Scale 0

(f) Nearest Neighbor Preservation (NPP)

Figure 26: **Drill-down analysis (TIMIT complete dataset)**. Selected landmarks/data-points are highlighted with a grey halo. **Computation Time: tSNE 153' - HSNE 5'** (a) tSNE Embedding (b) HSNE embedding scale 3 (c) HSNE embedding scale 2 (d) HSNE embedding scale 1 (e) HSNE embedding scale 0 - Data points (f) Precision/Recall curves of the Nearest Neighbor Preservation.

(a)   (b)   (c)
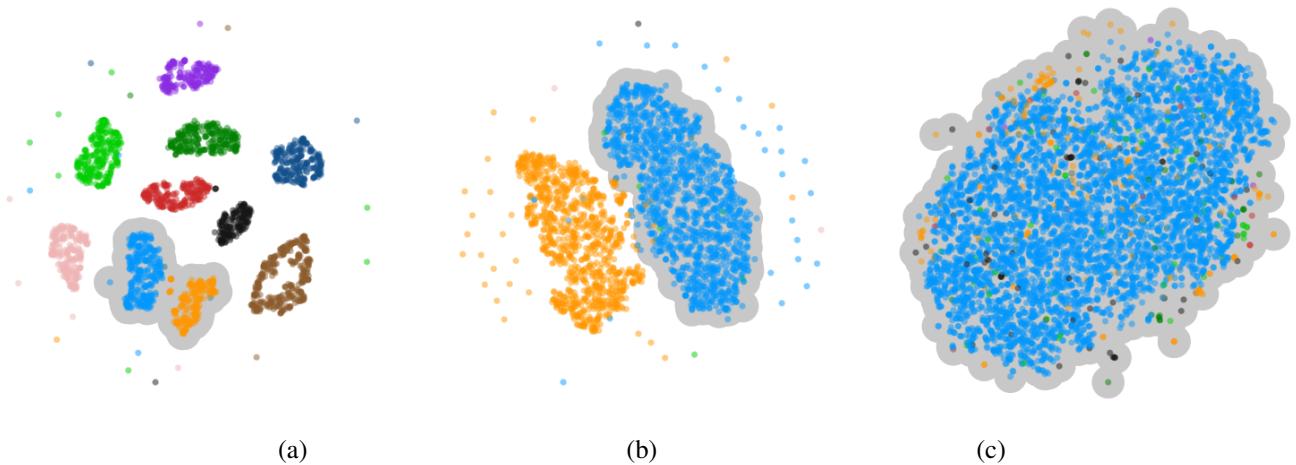
Figure 27: **HSNE embedding with random sampling (MNIST dataset)**. Outliers are selected as landmarks.

(a)   (b)   (c)
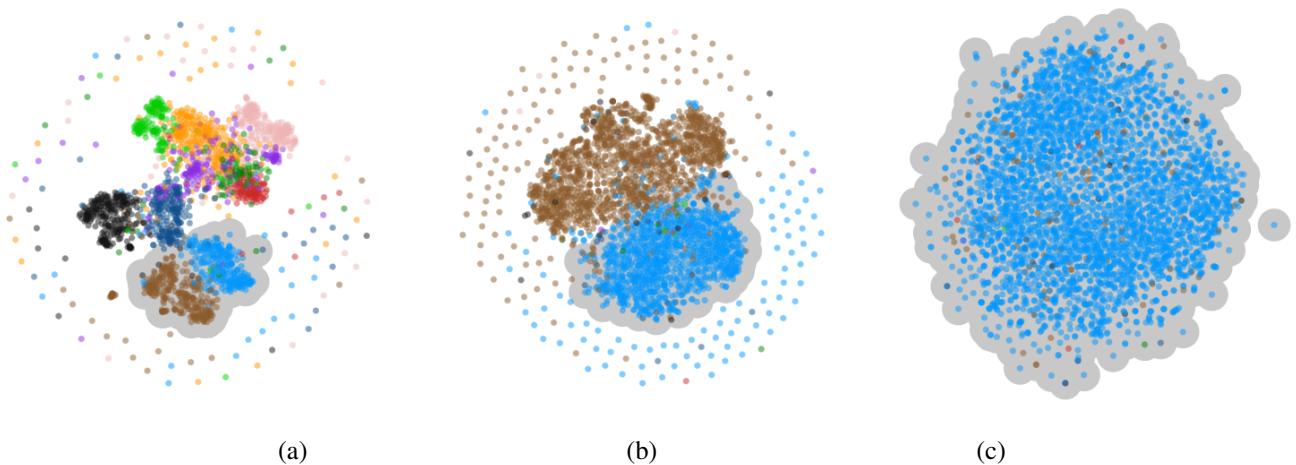
Figure 28: **HSNE embedding with random sampling (CIFAR-10 dataset)**. Outliers are selected as landmarks.


(a)   (b)   (c)
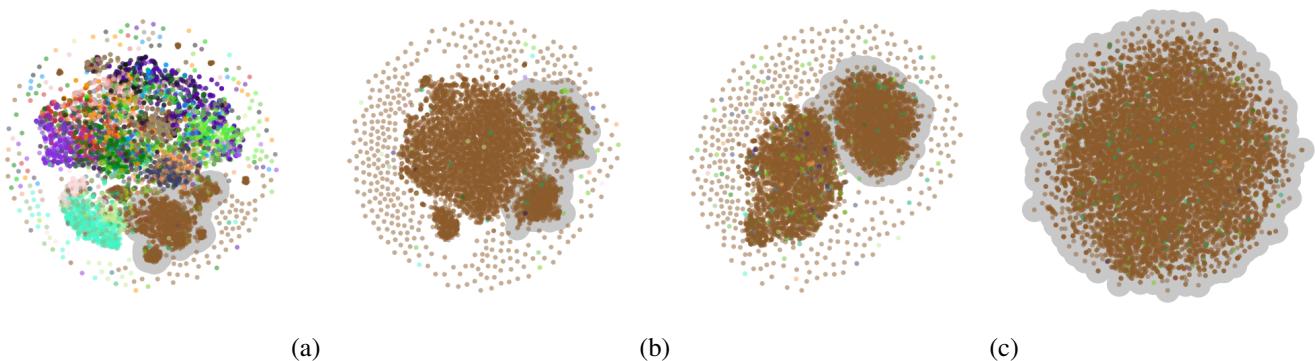
Figure 29: **HSNE embedding with random sampling (TIMIT complete dataset)**. Outliers are selected as landmarks.