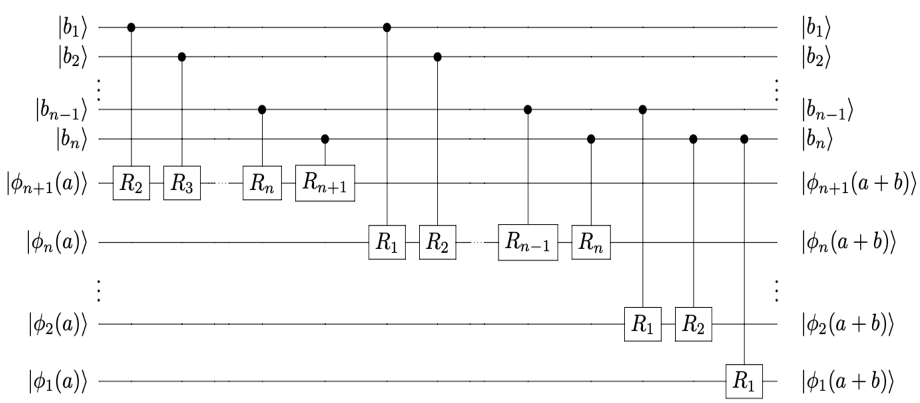
Project Description

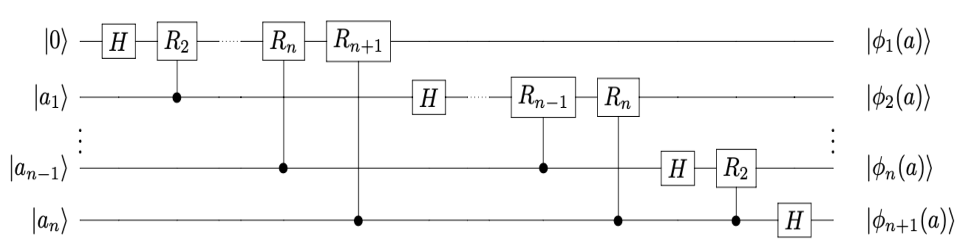
The classical function $+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ defining the addition between two natural numbers can be implemented on a quantum computer by constructing its reversible version using Toffoli gates. As we know, this realization requires additional ancillary qubits and a high number of elementary gates.

A quantum adder using no additional qubits and a minimal number of gates can be implemented by means of the QFT. The idea is as follows:

Let a and b be two integers in $\{0, 1, \dots, 2^n - 1\}$. Considering their binary representation on n bits, we can encode each of them on n qubits as $|a\rangle = |a_1\rangle \otimes \ldots \otimes |a_n\rangle$ and $|b\rangle = |b_1\rangle \otimes \ldots \otimes |b_n\rangle$. We now apply the QFT to $|a\rangle$ and use conditional phase gates controlled by each qubit $|b_j\rangle$ of $|b\rangle$ (which will only produce a change if $b_j = 1$) to obtain in the a register the QFT of $a + b$. The circuit is depicted in the figure below.



By encoding a in $n+1$ qbits as in the figure below, we can obtain the exact sum rather than the modular sum.



Your task is to

(a) verify that the circuit effectively produce $|\phi(a + b)\rangle$, where ϕ indicates the state obtained after the QFT application,

(b) implement in Qiskit the scheme for the addition for some $n \geq 2$ of your choice.