The Value of Government Debt with Domestic Arrears

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Abstract

Given the rising levels of public debt globally, a key question is whether governments will have enough fiscal space to navigate the next crisis. This paper documents the use of non-marketable financial resources by governments in order to affect the valuation of government bonds and, thus, satisfy public financing needs. The paper analyses domestic arrears, i.e. overdue payments by the government to its domestic beneficiaries, as source of non-marketable financing. A historical dataset from the UK during 1700-1900 compensates for the lack of reliable modern data on arrears. I find that arrears, whose face value averages 4.3 percent of GDP, have a risk-adjusted present value of 12.9 percent of GDP on average. Thus, arrears increase fiscal space, allowing the issuance of more marketable debt (or alternatively reducing its yield). The intuition is that arrears accumulate during bad times and redirect cash flows from suppliers to bondholders: thus, they provide risk-averse bondholders with an insurance-like cash flow that justifies higher bond valuations. The omission of arrears in the valuation of public marketable debt leads to an underestimation of its fundamental value either through mismeasurement of government expenditures (which might be inflated as compensation for suppliers), or due to the disregard of revenues extracted from suppliers via financial repression. The paper contributes to the resolution of recently raised public debt valuation puzzles and provides a new rationale for the emergence of arrears. Finally, a new dataset of outstanding arrears for EU member countries is presented, revealing size and cyclicality akin to the historical UK data.

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1 Introduction

Government debt in developed economies is high and projected to increase, raising urgent concerns about sustainability[1]. Such concerns are compounded at a time of rising interest rates and aging workforces. One overlooked factor when it comes to debt sustainability is domestic arrears. Domestic arrears are overdue payments by the government to its domestic suppliers and they can be substantial: in some OECD countries, they reach up to 9% of GDP. They provide a discretionary financing source for governments, avoiding the need for bond issuance, taxes, or expenditure cuts[2]. But this relief is also double-edged: arrears add to total debt, ultimately requiring repayment through additional future primary surpluses. In this paper, I examine arrears as both a financing tool and a hidden liability in the context of public debt sustainability.

The notion of debt sustainability is tightly linked with that of debt value, since sustainability requires that today's outstanding debt is matched by an excess of future primary surpluses over primary deficits in present value terms. By considering changes in domestic arrears as a source of financing for the government, I find that they increase the risk-adjusted valuation of marketable debt, i.e. bonds. This is an important and overlooked mechanism that allows governments to influence bond valuations by unilaterally borrowing from their suppliers. The intuition behind the mechanism is that by incurring arrears in "bad times", the government frees up financial resources ensuring the repayment of risk-averse bondholders when cash flows are highly valued. This makes bonds a more valuable asset warranting higher prices and lower yields. On the other side of this trade are government suppliers, from which the government borrows by delaying payments. Suppliers might be compensated for the liquidity service they offer. If such compensation comes in the form of surcharges on government procurement spending, it is already accounted for in the observable public spending figures. Alternatively, suppliers might be financially repressed. The failure to pay suppliers on time and to compensate them with interest is less costly to the government than a failure to repay marketable debt, which by definition would reverberate across financial markets. The possible negative repercussions of financial repression, such as firm exits, are taken into account in my analysis to the extent that they impact observable state variables, such as economic growth, that are used to assess fiscal sustainability.

My findings highlight a mechanism through which governments create fiscal space:

[1] D'Erasmo et al. (2016) provide an overview of the approaches adopted in the literature to answer the question of debt sustainability.

[2] The sovereign default literature provides compelling arguments for avoiding borrowing at excessive rates or when the debt level is too high: self-fulfilling bad equilibria emerge due to fear of future default when rates are high in the frameworks of Calvo (1988) and Lorenzoni and Werning (2019), and liquidity and rollover crises can occur if debt levels are above a certain threshold in Cole and Kehoe (2000).

by incurring arrears in "bad times", governments can issue more marketable debt at given valuations or alternatively reduce yields at given marketable debt levels. Using historical time series from the UK, where domestic arrears are defined as the unfunded debt of British Army and Navy, I find that the average size of the fiscal space gained through this mechanism is equivalent to 12.9% of GDP. The results in the paper also contribute quantitatively to the resolution of the so called *public debt valuation puzzle* raised in Chen et al. (2022) and Jiang et al. (2024). The puzzle refers to the observed market value of marketable debt exceeding the present value of primary surpluses in hegemonic countries. The puzzle disappears once domestic arrears are considered in the analysis of the historical UK debt valuations.

I build on the idea introduced by Bohn (1995) to discount future primary surpluses with some stochastic discount factor (SDF), which reflects the distribution of government revenues and spending across states of nature. The stochastic setting is advantageous compared to a deterministic one – in which we discount surpluses with the risk-free rate or the realized interest rate on government bonds – because it rules out bubble terms through which any valuation can be justified. In my framework, I consider three parties: investors that hold marketable debt in the form of bonds, government suppliers that hold non-marketable debt in the form of domestic arrears, and taxpayers. Bondholders and suppliers are responsible for pricing government debt by applying a common stochastic discount factor to the cash flows that their respective portfolios generate. The cash flow of a representative investor that holds all of government marketable debt consists of the primary surplus as it is usually defined – tax revenues minus government spending excluding interest payments – plus the amount of government financing from non-marketable sources. I refer to the resulting cash flow as the *marketable primary surplus*. Non-marketable financing denotes financing through instruments that are not usually traded on secondary markets but contribute to the total liabilities of a country. Some examples are institutional loans such as IMF loans, bilateral government loans such as the Anglo-American loan, trade credits, and domestic arrears. The public debt literature has taken either one of two approaches towards non-marketable debt. The first approach, adopted for example by Cochrane (2019) and Jiang et al. (2024), consists in disregarding non-marketable financing under the assumption that its net contribution to the total debt value is negligible. The alternative approach, taken for example by Hall and Sargent (2011), assumes that the price of non-marketable debt behaves as the price of marketable debt and, thus, the familiar primary surplus can be benchmarked against the value of total debt, marketable and non-marketable[3]. This paper demonstrates that,

[3] Hall and Sargent (2011) propose to derive the market value of non-marketable debt by multiplying its face value with the market-to-face-value ratio obtained from marketable debt.

in the presence of domestic arrears, these approaches lead to an underestimation of the fundamental value of government debt, defined as the expected present discounted value of future primary surpluses.

Domestic arrears are one of the most interesting and important component of non-marketable government financing because they can fulfil the role of shock absorbers for public finances better than other forms of non-marketable financing. Domestic arrears have the potential to generate considerable liquidity since they can be incurred on a wide base (public procurement alone averages 14% of GDP in the EU in 2023), they are a nimble instrument that can be activated just by withholding payments and, most importantly, they can be incurred unilaterally without requiring participation or trust from counterparties, which instead is required for other forms of non-marketable financing. One can easily see that other forms of non-marketable financing, such as an IMF loan, are ineffective in stabilising marketable primary surpluses and, thus, marketable debt valuations, if the creation of non-marketable liabilities is either linked with direct losses for bondholders, as was the case for the 2012 Greek debt restructuring, or if it is not likely to happen at times of financial distress, because non-marketable counterparties withdraw their support. Essentially, domestic arrears are a financing tool that offers governments greater flexibility in managing public finances and should not be neglected in the valuation of marketable debt. The next two paragraphs provide the intuition why.

One straightforward reason for considering arrears in the valuation of marketable debt is that a government can financially repress domestic suppliers by not compensating them for the liquidity service they provide through arrears. This is the case in countries with a weak rule of law or, for instance, if the government has a lot of monopsonistic power in tendering public contracts. Thus, the government extracts a revenue stream from its suppliers, which it can use for the repayment of bondholders, making holding marketable debt safer and so more valuable. The literature on debt seniority offers an additional interpretation: in the presence of arrears, marketable bonds are warranted higher valuations because they are a senior claim to future government surpluses compared to arrears and their repayment is guaranteed at the expense of government suppliers[4] [5]. A failure to pay suppliers on time has only limited impact on the government's reputation as a borrower, since arrears are generally not a liquid asset class and they are not priced or traded on secondary markets. Suppliers, naturally, bear the nega-

[4] Black and Cox (1976) and Roberts and Viscione (1984) provide theoretical and empirical evidence for the higher price of senior debt.

[5] Arrears are only one instrument through which the government pursues higher debt valuations by issuing separate claims on its revenue. The underlying mechanism, namely that the claim held by bond investors is senior to another claim which instead presents a lower default cost, can be achieved through other instruments. Hoffmann et al. (2021) provide an example for Germany.

tive consequences of late payments: in a scenario with financial repression, arrears come at a cost that should be weighed against its benefits, the same way that unexpected inflation or selective default are costly, but can be optimal in dealing with excessive public debt.

The other reason is a matter of mismeasurement. In an alternative scenario, in which government suppliers are compensated for the liquidity service they provide, arrears do not free any resources for the government, they have a present value of zero and can be disregarded from the valuation equation[6]. However, the compensation of suppliers more often than not takes the form of increased procurement costs incurred by the government, as opposed to interest payments (European Commission (2015), IMF (2019)). This is intuitive imagining that government contractors, expecting payments to be in arrears, surcharge the government to make up for it. This means that government expenditures included in the primary surplus are overstated and should be adjusted for by subtracting the servicing cost of arrears, similarly to how interest rate payments are netted out. Failing to do so leads to an underestimation of the value of public debt. Since this servicing cost is not observed, this paper infers such amount by estimating the risk-adjusted price suppliers should be paid to be indifferent between the government incurring arrears or not. Since arrears accumulate in bad times, when agents value cash flows a lot, such price is going to be large, larger than the face value of arrears.

I estimate the prices of marketable debt and arrears jointly. The starting point is recognizing that the portfolio that has as cash flow the marketable primary surplus can be modelled as a long position in a claim (i) on government revenues, and (ii) on the creation of arrears; plus a short position in a claim (iii) on government expenditures, net of marketable interest payments, and (iv) on the redemption of arrears, net of interest payments or other compensation for suppliers. This set up ensures the correct valuation of marketable debt and arrears, without the need to take a stance on the degrees of suppliers' financial repression and mismeasurement of government expenditures. This facilitates the empirical estimation and follows from the fact that, for any degree of financial repression, correcting the mismeasurement of primary surpluses by netting out arrears' surcharges from expenditures results in a reassessment of the value of marketable debt by a fixed amount. The intuition is that as revenues generated by financial repression increase, the mismeasurement in government spending decreases by the same amount, since suppliers are not being compensated, and vice versa.

As standard in asset pricing, the price of a claim on government revenues, government

[6] This does not mean that arrears do not play a role in ensuring the sustainability of public finances. On the contrary, such a scenario begs the question of why a government would choose such an expensive form of financing. One possible answer, market segmentation, is discussed in the literature review.

spending, arrears creation, and arrears redemption corresponds to the expected present discounted value of the current and future cash flows that it generates. The expected present discounted values of the these claims are derived from their price-cash flow ratios, which are estimated with the help of the Campbell and Shiller (1988) decomposition. Such decomposition allows me to write price-cash flow ratios as sums of expected cash flow changes and discount rates. A vector autoregression (VAR) is used to forecast the joint dynamics of the cash flows of taxes, spending, arrears creation, and redemption, as well as other informative state variables, while the discount rate is defined as the risk-free long-term bond yield plus a risk premium. The risk premium is anchored by the mean risk premium on a claim on output and adjusted by the comovements of the claims' cash flows with the SDF, which is given by the CAPM. Intuitively, these cash flows are safe or risky depending on how they co-move with aggregate shocks. In the long run, however, tax revenues, spending, arrears creation, and arrears redemption are cointegrated with GDP, meaning that they share a common stochastic trend and are exposed to the same long-run risk. For instance, arrears tend to accumulate during economic downturns or, historically, during wars, but they are a stationary time series as a fraction of GDP. In this context, the risk-adjusted present values of these claims ought to depend not only on average values of cash flows and discount rates, but also on covariances with aggregate risks.

The lack of reliable and comparable data on arrears hinders their understanding and their discussion within the economic literature. To the best of my knowledge, I am the first one to document how the use of domestic arrears impacts sovereign debt valuations. I do so by using Mitchell and Deane (1962)'s historical time series on British unfunded debt of Army and Navy as a measure of domestic arrears. The long time scale of the sample that covers over two centuries until WWI is ideal to study fiscal sustainability and mitigates the risk of the so called peso problem, which consists in the possibility of large fiscal adjustments with small probabilities that are anticipated by investors, but do not occur in sample when it is not sufficiently large. Prior to WWI, in the pre-Keynesian era, fiscal policy was fundamentally different than today and military spending constituted the bulk of government spending. Since the expenditures for Army and Navy often exceeded the estimates that the Parliament approved to be funded through taxes or bonds issuance, the respective military offices resorted to paying military suppliers with bills whose eventual settlement date depended on the status of public finances.

In my baseline specification, I find that domestic arrears, which have a face value that averages 4.3% of GDP in the sample period, have a present value that averages 12.9% of GDP with ranges from 5.4% up to 26.8% of GDP. This means that if we were to ignore domestic arrears in the valuation of UK marketable debt, we would underestimate the present value of marketable debt as fraction of GDP by 12.9 percentage points on average, either by disregarding resources that the government extracts from its suppliers to ensure the repayment of bondholders (if there is financial repression), or due to overstated government expenditures which include surcharges aimed at compensating government suppliers (if there is no financial repression), or, most likely, due to a combination of these two effects. The same implication holds if arrears are taken into consideration but mispriced. I find, moreover, that the present value of arrears is almost entirely driven by the covariance of changes in arrears with the SDF, while the expected value of changes in arrears is close to zero (because the UK government always repaid outstanding arrears).

By retrieving historical data, this paper provides a new motivation for the emergence of arrears, which is otherwise primarily attributed to weak public financial management (IMF (2019)). However, the use of arrears is not limited to the past and it is still relevant today for both emerging and developed countries. For instance, the European Commission reports that over 50% of invoices to the public sector were paid late in 2023 (European Commission (2023c)). In the final section of the paper, I estimate the face value of domestic arrears for all EU countries through a combination of administrative data and private survey evidence starting in 2004. Such estimates indicate that domestic arrears amount to 4.2% of GDP across EU countries in 2023, in line with the British historical average of 4.3%. In addition, arrears level fluctuations for European countries suffering from financial distress in the wake of the Great Financial Crisis display the expected counter-cyclical time patter that the historical time series has. This is a contribution to recent efforts aimed at alleviating the lack of data on arrears[7].

Discussion in Relation to the Literature The paper contributes to the current debate in economics about the valuation and the sustainability of government debt. Chen et al. (2022) and Jiang et al. (2024) have highlighted a debt valuation puzzle for the UK (prior to WWII) and the US (after WWII) consisting in the observed market value of marketable debt exceeding the present value of primary surpluses. Chen et al. (2022) find that the historical market value of UK marketable debt exceeds the present value of surpluses by 37% of GDP on average. In Jiang et al. (2024) the valuation gap for the US is 360% of GDP on average. The presence of domestic arrears alleviates this puzzle by identifying a source of unaccounted government revenues (in case of financial repression) and mismeasurement. For the same reasons, this paper contributes to the quantitative sovereign default literature which in general focuses only on marketable debt. For in-

[7]In 2020, Beers et al. (2023) started incorporating domestic arrears in the Bank of Canada and Bank of England sovereign default database for the years in which countries were part of an IMF program. Moreover, since 2023 the European Commission (2023b) has been publishing payment delays for business invoices to European governments.

stance, in the quantitative analysis of Bocola and Dovis (2019), the incidence of rollover risk in the rise of Italian bond yields between 2008-2012 is likely underestimated since the authors do not consider the maturity extension that the Italian government achieved by increasing its stock of domestic arrears, which I document in Section 8. The inclusion of arrears would reduce the amount of yield spread variation attributed to an unexplained residual shock.

The paper relates to the literature on financial repression, which, in light of the recent surge in public debt, has gained renewed attention in the academic debate[8]. The issuance of non-marketable debt is traditionally associated with financial repression, since it makes it particularly easy for the government to "tax" holders of non-marketable debt with suppressed real interest rates (Reinhart and Sbrancia (2015)). However, when non-marketable debt takes the form of domestic arrears, assessing the degree of financial repression is not as straightforward, since suppliers have the option to surcharge the government as alternative to pretending interest payments on the amounts in arrears. Evidence from the EU suggests that, even in jurisdictions with statutory interest penalties for late payments, interest on arrears is often foregone[9]. The amount that suppliers would be able to surcharge the government in turn depends on whether they expect arrears and have sufficient bargaining power. The limited evidence available in this regard indicates that the government wields some monopsony power and that its expenditure is concentrated in sectors in which prices are particularly sticky (Cox et al. (2024)).

Even in the absence of financial repression, domestic arrears can contribute to the sustainability of public finances thanks to the insurance role they play. According to the framework introduced by Jiang et al. (2021) and Gomez-Cram et al. (2024), the government faces a trade-off between insuring bondholders and insuring taxpayers against output shocks. Domestic arrears mitigate this trade-off by introducing a third category of stakeholders, domestic government suppliers. The government, then, pays a risk premium to suppliers to be able to draw on financing in the forms of arrears when the economy is hit by a large negative shock. Such explanation presumes market segmentation either between bondholders and suppliers, or between suppliers and taxpayers, depending on which group finances the risk premium. This means that the arrears cash flows are valued differently by different agents and that a Pareto improvement can be achieved by redirecting cash flows. While in the paper I assume the existence of a com-

[8]See for example the work by Jeanne (2023) and Chien et al. (2023).

[9]The EU's 2011 Late Payments Directive which mandates interest penalties of 8% above the ECB reference rate for late payments remains largely unenforced, prompting the Commission to revisit these rules in 2023 (European Commission (2023a)). Surveys across the EU confirm that, out of fear of damaging their commercial relationship with the government, firms tolerate payments delays and, for the most part, renounce interest claims on late payments despite the legislation that protects creditors (European Commission (2015)).

mon stochastic discount factor that prices all assets and that is the stochastic discount factor of a representative investors that holds both marketable debt and arrears, there are good reasons to believe that suppliers might discount cash flows differently from bondholders and taxpayers. On the one hand, the level of consumption of suppliers might be directly linked to the change in government spending. When the government ramps up spending, government suppliers are bound to benefit, even if some of the amounts due will be paid late. “Bad times” are not necessarily states of low consumption for suppliers. On the other hand, suppliers might be less financially constrained than taxpayers. Some research on public procurement shows evidence that a public contract can help firms to secure financing. In turn, this would allow suppliers to smooth consumption despite the hit caused by payments in arrears. Also in this scenario, their consumption might be higher than that of other agents.

The paper complements the existing literature on payment arrears, which has primarily focused on their economic impact on governments suppliers and on how to prevent them from happening. Diamond and Schiller (1988) were among the first to frame payment arrears as a consequential fiscal issue for IMF member countries, highlighting the challenges of measuring arrears accurately. Institutional consensus attributes the emergence of payment arrears to poor budgeting and weak public financial management (Flynn and Pessoa (2014)). Quite some work has been dedicated to come up with strategies aimed at clearing arrears, such as their securitization (Ramos (1998)). In 2011 the European Commission implemented the Late Payment Directive with the objective of reducing late payments within the EU. The legislative initiative has spurred a series of papers that explore the consequences of delayed payments for suppliers, e.g. Connell (2014), Checherita-Westphal et al. (2016), Conti et al. (2021). Despite the challenges of teasing out the causal effect of late payments on firms performance, the literature finds small effects in the expected direction: payment arrears increase firm exit, lower firms profit, and slow down economic growth. Barrot and Nanda (2020) ask similar questions for the US economy and finds strong evidence of slower employment growth at firm affected by longer payment times. However, they also document substantial faster growth of non-affected firms' employment within local labor markets, with slightly negative overall effects. Another takeaway from their research is that the US government did not incur any suppliers' arrears in recent decades[10]. Along the same lines, using Spanish data, Abad et al. (2023) find that domestic arrears decrease corporate investment and cash reserves, and increase firm leverage. Conversely, there is some evidence that being a government contractor helps firms to borrow from the financial sector. Still using Span-

[10] An exception would be domestic arrears that result as a consequence of US federal government shutdowns following the reach of the debt ceiling.

ish data, di Giovanni et al. (2023) provide evidence that procurement contracts act as collateral for firms and help alleviate their financial constraint. Similarly, Duque Gabriel (2024) find that Portuguese firms can increase their credit by 0.05 Euro at lower interest rates for any additional Euro from a procurement contract. In general, a more structural approach would be helpful to assess the widespread economic ramifications of domestic arrears. The framework recently developed in the trade credit literature by Bocola and Bornstein (2023) lends itself to this task.

Finally, the paper contributes to the large economic history literature on the UK by underscoring the role of domestic arrears in financing the Crown and by quantifying their value. This is in addition to the beneficial role that scholars attribute to the issuance of debt by the British government. Ventura and Voth (2015) and Dowd and Hutchinson (2018) argue that the large returns that investors could gain by saving in government bonds accelerated the Industrial Revolution. Relatedly, Baugh (2015) underscores the advantage that the orderly management of arrears' repayments offered the British in comparison to their enemies: the trust of merchants in eventual repayment allowed arrears to accumulate in bad times and the Royal Navy to be ready for battle.

The rest of the paper is organized as follows. Section 2 presents the analytical framework. Section 3 provides some background information about the use of domestic arrears in the UK historical context. Section 4 displays the empirical strategy. Section 5 presents and discusses the results, whose robustness is assessed in Section 7. The model in Section 6 helps to rationalise the results. Section 8 explains how domestic arrears for EU countries are estimated. Section 9 concludes.

2 Analytical Framework

The one-period government budget constraint in nominal terms is:

$$\underbrace{G_t^{no\ arr} - T_t}_{\text{primary deficit}} + \underbrace{Q_{t-1}(1)}_{\substack{\text{bonds due}\\ \text{today}}} + \underbrace{A_{t-1}(1+i_t)}_{\substack{\text{arrears}\\ \text{repaid}\\ \text{today}}} = \underbrace{\sum_{h=1}^{H} \big(Q_t(h) - Q_{t-1}(h+1)\big)\, P_t(h)}_{\text{value of net new bonds issuance}} + \underbrace{A_t}_{\substack{\text{new}\\ \text{arrears}\\ \text{created}}} \tag{1}$$

where government fiscal needs on the left-hand side are equated to government financing revenues on the right-hand side. Here, $G_t^{no\ arr}$ denotes nominal government spending net of interest payment (the superscript *no arr* refers to the fact that this is the amount the government would pay for procurement if suppliers would not expect and surcharge the government for arrears since they are already compensated by interest $1 + i_t$), T_t denotes nominal government tax revenues, $Q_t(h)$ is the quantity of nominal zero-coupon bonds of maturity h outstanding in period t each promising to pay \$1 at time $t+h$, and $P_t(h)$ is today's price for a h-maturity zero-coupon bond with \$1 face value. The quantity of zero-coupon bonds of maturity $h+1$ issued at time $t-1$ evolves according to $Q_t(h) = Q_{t-1}(h+1) + \Delta Q_t(h)$, where $\Delta Q_t(h)$ is the net new issuance of h-maturity bonds at time t. Amount A_{t-1} of outstanding arrears is repaid with an interest $(1+i_t)$, while amount A_t of arrears is newly created.

By relying on the existence of a SDF and imposing a transversality condition which prevents bubbles in government debt, I can rewrite the one-period budget constraint (1) as an infinite sum that prices outstanding debt today as the expected present discounted value of tax revenues, expenditures and changes in domestic arrears:

$$\underbrace{V_t \equiv E_t \left[\sum_{j=0}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j}^{no\ arr} + \Delta \widetilde{A}_{t+j} \right) \right]}_{\text{fiscal backing}} = \underbrace{\sum_{h=0}^{H} Q_{t-1}(h+1) P_t(h)}_{\text{mkt value of mktable debt}} \tag{2}$$

where V_t is the present discounted value of all future marketable primary surpluses, and on the right-hand side is the nominal market value of the government outstanding marketable debt portfolio. The nominal multi-period stochastic discount factor $M_{t,t+j} = \prod_{k=0}^{j} M_{t+k}$ satisfies $P_t(h) = E_t \left[M_{t,t+h} \right] = E_t \left[M_{t+1} P_{t+1}(h-1) \right]$ and $P_t(0) = M_{t,t} = M_t = 1$ by no arbitrage. $\Delta \widetilde{A}_{t+j} = A_{t+j} - \widetilde{A}_{t-1+j} = A_{t+j} - A_{t-1+j}(1+i_{t+j})$ is the change in outstanding non-marketable debt, or arrears. Tildes above variables, such as in $\widetilde{A}_{t-1+j}$, indicate that such variables contain the interest earned on arrears.

I refer to the expected discounted marketable primary surpluses as the *fiscal backing* of the government, since it indicates how much marketable debt a government can issue. The same term has also been referred to as fiscal space, fiscal capacity, or the fundamental value of debt in the literature. A fiscal backing that is larger than any given nominal value of outstanding debt means that the government can issue debt at a premium to par or, equivalently, that bond yields will drop. In general, $V_t \geq 0$ provides the definition of debt sustainability under the economic approach described in Mehrotra and Sergeyev (2021); namely, it is the condition under which forward-looking investors willingly hold public debt.

By taking expectations, the valuation equation holds ex ante in both real or nominal terms taking inflation expectations into account. The valuation equations also holds if the government can default. Intuitively, the pricing kernel takes the default probability into account. Ex post, surprise inflation or a surprise default can erode the real value of the debt[11].

[11]The question of the relative contribution of actual repayments vs lower than expected returns to

Note that instead of discounting changes of arrears on the left-hand side of equation (2), one could equivalently add the market value of outstanding arrears on the right-hand side and, thus, juxtapose the market value of *all* outstanding government debt, marketable and not, with the discounted primary surpluses in their familiar form: $T_{t+j} - G^{no\ arr}_{t+j}$. However, such representation would be empirically useless because we do not observe either the market prices of non-marketable debt or $G^{no\ arr}$. The literature has taken primarily two approaches to deal with non-marketable debt. The first approach, taken e.g. by Cochrane (2019) and Jiang et al. (2024), consists in ignoring the value of non-marketable debt under the assumption that its contribution to the total debt value is negligible. Such assumption is not warranted for countries that make use of domestic arrears since – as it will be shown later – outstanding arrears are a significant fraction of GDP and their present value is even larger than their face value. The second approach, taken e.g. by Hall and Sargent (2011), assumes that the price of non-marketable debt behaves as the price of marketable debt. Such assumption is also not warranted in the presence of arrears since arrears are a different type of "security" than bonds: for one, arrears have a variable maturity which the government can lengthen unilaterally; moreover, they do not offer the same contractual safety as a bond contract since defaults on arrears do not have the same public impact as a bond default[12]; and, finally, arrears are a substitute to marketable debt financing, hence the riskier their cash flow, the less risky the bonds cash flow needs to be.

The following propositions present the cases in which government suppliers are fairly compensated, overcompensated or financially repressed by the government for the liquidity service they provide. The implications for the estimation of the fiscal backing are discussed.

Proposition 1: Zero Present Value of Arrears under Fair Pricing Under the assumptions of zero initial arrears, a transversality condition for arrears, and fair pricing of arrears, defined as $E_{t+j}(M_{t+1+j}(1 + i_{t+1+j})) = 1\ \forall j$, the expected present discounted value of arrears changes inclusive of interest payments is zero: $E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\Delta\widetilde{A}_{t+j}\right] = 0$.

The proof is given in Appendix 10.1. Intuitively, if arrears are expected to be repaid and the holders of arrears are fairly compensated for the loans they extend to the government, meaning that they are indifferent between the government delaying payments or not, then

the changes of the real value of government debt – as it is pursued for the US in Cochrane (2019) – is, however, beyond the scope of this paper, not least because the UK was under the gold standard for most of the period under consideration with negligible amounts of inflation.

[12]The government can for instance litigate a targeted default on arrears arguing substandard service by the supplier.

the government cannot improve its fiscal backing by incurring arrears. The definition of fair pricing of arrears, $E_{t+j}(M_{t+1+j}(1+i_{t+1+j})) = 1\ \forall j$, implies that if the outstanding amount of arrears is repaid with certainty every period, it should earn the risk-free rate, as a risk-free bond would. If, however, arrears repayments comove with the SDF – for instance, if they are low in bad times – then the interest that arrears earn should be the risk-free rate adjusted by a risk premium.

Corollary 1: Counterfactual without Arrears Given the result of Proposition 1, bondholders, taxpayers and suppliers are indifferent between the government incurring arrears and a counterfactual world without arrears. The presence of market frictions, such as borrowing constraints for suppliers or financial repression, that leads to unfair pricing of arrears would break this result.

Assumption 1: Arrears Interest as Surcharge Assume that arrears holders obtain interest rate $1 + i_t$ on outstanding arrears by surcharging the government a fraction ϕ_t on its procurement expenditure[13] such that $i_t A_{t-1} = \phi_t G_t^{no\ arr}$ and $G_t = G_t^{no\ arr}(1+\phi_t)$. Then, the valuation equation (2) can be rewritten as:

$$V_t = E_t \left[\sum_{j=0}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j} + \Delta A_{t+j} \right) \right] \tag{3}$$

where $G_{t+j} = G_{t+j}^{no\ arr}(1+\phi_{t+j})$ is the amount of government spending that entails the interest on arrears and $\Delta A_{t+j} = A_{t+j} - A_{t-1+j}$ is the change in the stock of arrears net of interest payments.

This follows by replacing the variables in equation (2) with their new definitions. Amounts G_{t+j} and ΔA_{t+j} are observed in the data.

Proposition 2: Mismeasurement of Primary Surpluses Assuming that Propositions 1 and Assumption 1 hold, neglecting ΔA_{t+j} in the valuation equation (3) leads to an understatement of government's fiscal backing by

$$E_t \left[\sum_{j=0}^{\infty} M_{t,t+j} \Delta A_{t+j} \right] = E_t \left[\sum_{j=0}^{\infty} M_{t,t+j} \phi_{t+j}^{*} G_{t+j}^{no\ arr} \right] \tag{4}$$

[13] There is modern and historical evidence that suppliers would surcharge the government if expecting to be paid with delay as opposed to demand an interest to be paid separately. A European Commission (2015) survey reports that 60% of sampled firms never claim compensation and/or interest in the event of late payment. Anecdotally, Wheeler (1996) reports that in 1651, victuallers for the British Navy demanded an increase of 12.5% in the amount allowed for the cost of each sailor for whom they provided food as compensation for the dramatic increase in outstanding arrears that year.

where ϕ^*_{t+j} is the surcharge that guarantees fair pricing for arrears holders as defined in Proposition 1.

The proof follows from realising that under fair pricing as in Proposition 1, the value of the cash flow of arrears changes, $E_t[\sum_{j=0}^{\infty} M_{t,t+j}\Delta A_{t+j}]$, must equal the value of surcharges, $E_t[\sum_{j=0}^{\infty} M_{t,t+j}\phi^*_{t+j}G^{no\ arr}_{t+j}]$. Thus, neglecting arrears changes ΔA_{t+j} in valuation equation (3) leads to an underestimation equal to the value of the cash flow of arrears changes. An alternative intuition is that the present value of primary surpluses is underestimated since expenditures G_{t+j} are too large due to the interest on arrears charged by suppliers.

Proposition 3: Financial Repression of Suppliers Given valuation equation (3), if the government financially represses its suppliers by withholding part of the interest payments on arrears such that $\phi_t \leq \phi^*_t\ \forall t$ with a strict inequality for at least one t, then the government's fiscal backing, V_t, increases by:

$$W_t = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\Delta A_{t+j}\right] - E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\phi_{t+j}G^{no\ arr}_{t+j}\right] > 0 \qquad (5)$$

compared to the case with fair pricing. Here, W_t is the valuation wedge introduced by financial repression.

The proof is intuitive considering that W_t is the difference between the value that the government extracts from suppliers through the timing of arrears changes minus the value of surcharges used to compensate suppliers. Note that without financial repression, $\phi_t = \phi^*_t\ \forall t$, the valuation wedge is zero, $W_t = 0$. This follows from Proposition 2. Financial repression can take the form of not paying full interests on the amounts in arrears if $0 \leq \phi_t < \phi^*_t\ \forall t$, or the government can refuse to pay even the nominal amounts of outstanding arrears which implies $\phi_t < 0\ \forall t$. To a certain degree, the financial repression discussed here could be the consequence of the government's monopsony power in pricing the goods purchased from its domestic suppliers. Suppose that the government is a monopsonist facing an upward sloping supply schedule. Then, $\phi_t = 0\ \forall t$ corresponds to the case in which $G^{*,monops}_t = G^{no\ arr}_t\ \forall t$ [14].

[14] $G^{*,monops}_t = G^{no\ arr}_t\ \forall t$ can be obtained assuming that the government is a monopsonist facing every period a supply schedule that increases exponentially with government expenditure: $P(G) = e^{G(1+\phi)} - 1$ and has elasticity equal $\frac{1}{G(1+\phi)}$. Then, the price that the government pays is $P(G^{*,monops}) = \frac{1}{1+\phi}$ and the optimal government expenditure is $G^{*,monops} = G^{no\ arr}$.

Proposition 4: Overcompensation of Suppliers The overcompensation of suppliers, defined as $E_t[\sum_{j=0}^{\infty} M_{t,t+j}\Delta A_{t+j}] < E_t[\sum_{j=0}^{\infty} M_{t,t+j}\phi_{t+j}G_{t+j}^{no\ arr}]$ or equivalently $W_t < 0$, implies that the government's fiscal backing with the inclusion of arrears is smaller than the fiscal backing derived under fair pricing.

This case is the opposite of financial repression and implies that $\phi_t > \phi_t^*$ for some t. The overcompensation of suppliers also implies that through arrears the government extracts resources from bondholder or taxpayers for suppliers, $W_t < 0$, thus reducing its fiscal backing. This further implies that suppliers would willingly use arrears as a saving instrument, since they do earn a positive return. There is no empirical support that shows that suppliers would behave this way.

Proposition 5: Estimation of Fiscal Backing Under Assumption 1 and Propositions 2, 3 and 4, and under the assumption that the degree of financial repression or overcompensation of suppliers is not infinite but is bounded below by $\underline{\phi_t} \leq 0$ and above by $\phi_t^* \leq \overline{\phi_t}$, valuation equation (3) delivers the correct fiscal backing given the presence of arrears, for any degree of financial repression or overcompensation $\underline{\phi_t} \leq \phi_t \leq \overline{\phi_t}$ $\forall t$.

The proof is given in Appendix 10.2.

In summary, this set of propositions allows me to estimate the value of marketable debt and arrears using valuation equation (3) and available data, G_t and ΔA_t, without the need to take a stance on unobservables, such as the degrees of suppliers' financial repression and surcharges on government expenditures. The assumptions required to obtain the result are that the transversality conditions on government bonds and arrears are satisfied, that financial repression or overcompensation are bounded, and that interest on arrears takes the form of surcharge. A threat to the estimation is that the government would compensate suppliers via a mix of surcharges *and* interest payments. As a result, the estimated fiscal backing would be overstated, as shown in Appendix 10.3. While it is impossible to know for certain, the anecdotal evidence we have suggests that suppliers are overwhelmingly compensated via surcharges and the magnitude of this error is likely small. Any degree of financial repression consisting in withheld interest payments also reduces this error. The empirical estimation is conducted after the presentation of the data in the next section.

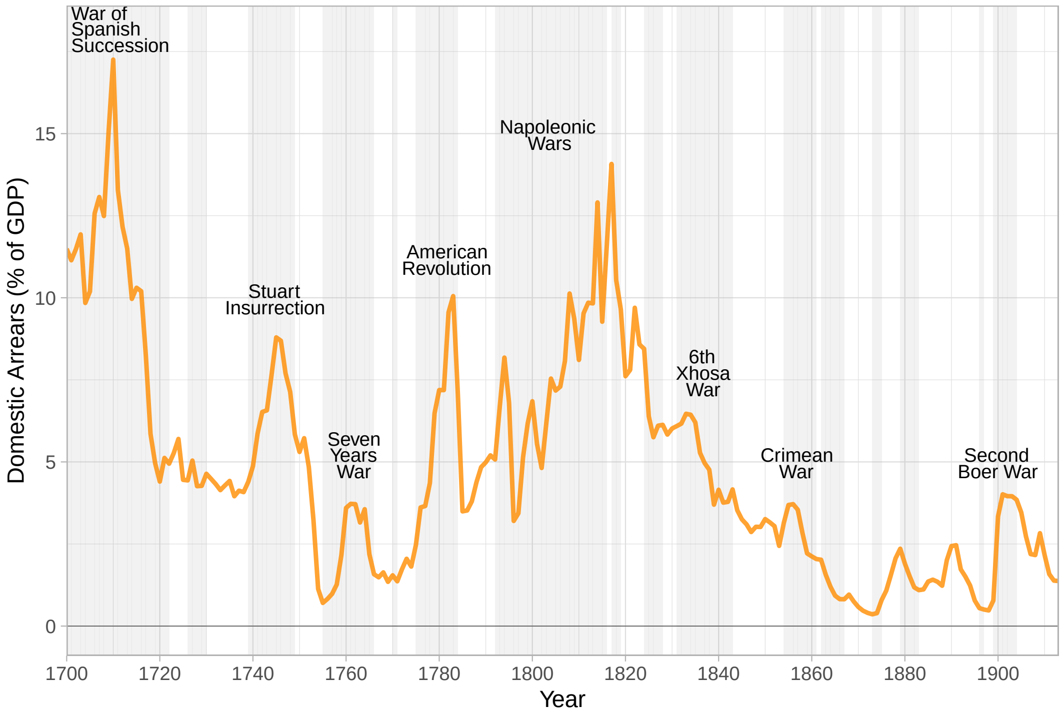
3 Background and Data

Over the past three centuries, the UK government has made ample use of domestic arrears as a source of public financing. Until the end of WWII, military spending was almost the exclusive constituent of government spending, with much smaller shares taken up by the financing of the civil service and the servicing of outstanding debts. The British Parliament had the final say in setting taxes and distributing tax revenues. However, actual expenditures for the Army and the Navy often exceeded the estimates approved by Parliament. As a consequence, the respective military offices resorted to bills to pay military suppliers. Quoting historian Baugh (2015): "the written financial instruments upon which the bulk of naval spending depended were called Navy Bills and Victualling Bills. These were issued by the Navy or Victualling Office to merchants upon their delivering goods or services. The bills served as payment but were almost never payable straightaway in cash." The actual payment of these bills ultimately depended upon the Parliament approving the necessary budget for their settlement. For this reason, debt in the form of bills was classified as *unfunded*. The trust of single merchants in the governments ability to repay was maintained by the principle that bills would be settled in the order in which they were registered, i.e. paid "in course". The course tended to lengthen in wartime when revenues did not rise nearly as fast as expenditures. It is the amount of unfunded debt issued to domestic suppliers as reported in Mitchell and Deane (1962) that I refer to as domestic arrears in my analysis. There is evidence that the bills were sometimes sold, at a discount, by merchants that needed money quickly to dealers or investors. In addition, while the bills bear no interest during the first six months, it became practice to pay interest on them if they remained unpaid for longer.

The exact time horizon of my analysis spans from 1730 – the first year of data availability on the market value of UK marketable debt – to 1907, prior to the start of WWI. While the time series by Mitchell and Deane (1962) extends until 1938, I discard the period immediately before WWI and the years after, since I do not observe the full repayment dynamic of the outsized arrears accrued in occasion of the war. Including only observations regarding the accumulation of arrears without their repayment distorts the estimation of the model parameters. Since the debt-to-GDP ratio achieves a minimum of 16.2% in 1913, years 1908-1913 are excluded for the same reason.

Figure 1 shows the stock of domestic arrears for the available time series. Arrears are on average 4.3% of GDP over the sample period. They tend to increase during wars, shaded in grey, and are repaid during peace time. Wheeler (1996) reports that the majority of outstanding arrears at any given time were overdue amounts by military offices to victuallers for the provision of food and beverage for the fleet. Notable spikes in the stock of arrears occur during of the War of the Spanish Succession (1701-1714), the Stuart Insurrection (1745-1746), the Seven Years' War (1756-1763), the American Revolution (1775-1783), the Napoleonic Wars (1793-1815), and the Second Boer War (1899-1902).

Figure 1: Domestic arrears in the UK as % of GDP
(shaded areas indicate years of war)



One limit of the arrears data is that I do not observe gross flows for arrears creation and redemption, but exclusively the stock and net flows. Gross flows, however, would be desirable in order to have two positive cash flow streams to price separately, in an analogous way to how the tax stream and the expenditure stream are each priced separately. The fact that these cash flows are positive allows me to log-linearize their price-dividend ratios in the empirical section. That would not be possible with cash flows that might occasionally turn negative. We know, moreover, that the repayment time of arrears would vary depending on the fiscal condition of the government. There are no precise information about repayment durations of domestic arrears. The accounts in Baugh (2015) and Wheeler (1996) suggests that arrears were often repaid within a year and repayment duration increased to multiple years in the wake of large spikes in government spending.

Thus, I proceed to approximate the gross flows of arrears creation and arrears redemption by assuming an underlying turnover rate of arrears of 33% of the outstanding stock. Net flows, which may be positive in the case of arrears creation or negative in case of arrears redemption, are added to the turnover to obtain the gross flows. Figure 2 shows the repayment duration of outstanding arrears given these assumptions. Since arrears were repaid in the order in which they were created, i.e. paid "in course", I can use this first-in, first-out principle to measure how many years it took to *fully* repay the arrears created in a given year. The graph shows that the repayment duration in the sample period is on average 2.1 years and as one would predict it is higher in war years and lower during peace time. By setting the turnover of arrears at 33% of the outstanding stock, I am imposing a maximum repayment duration of 3 years. Variations in the assumption about arrears turnover have minimal impact on the estimated present value of arrears as the robustness checks show. Unfortunately, there are no systematic data on the time it took the UK government to settle its procurement invoices. That data is also sparse in modern times. Note that if actual repayment times of arrears during periods of financial distress were to exceed the assumed repayment schedule which is mechanically capped at 3 years, then I would be underestimating their present value, since such repayment schedule would make a claim on arrears even more valuable.

Figure 2: Repayment duration in years for outstanding domestic arrears, estimated assuming a yearly turnover of 33% and the first-in, first-out principle (shaded areas indicate years of war)

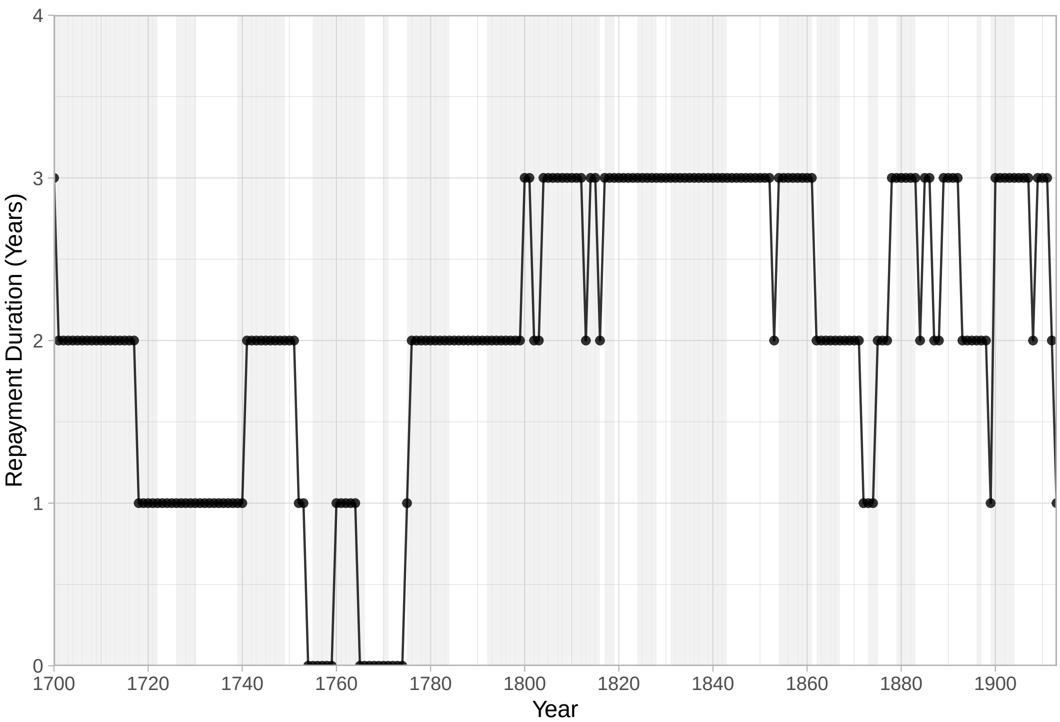
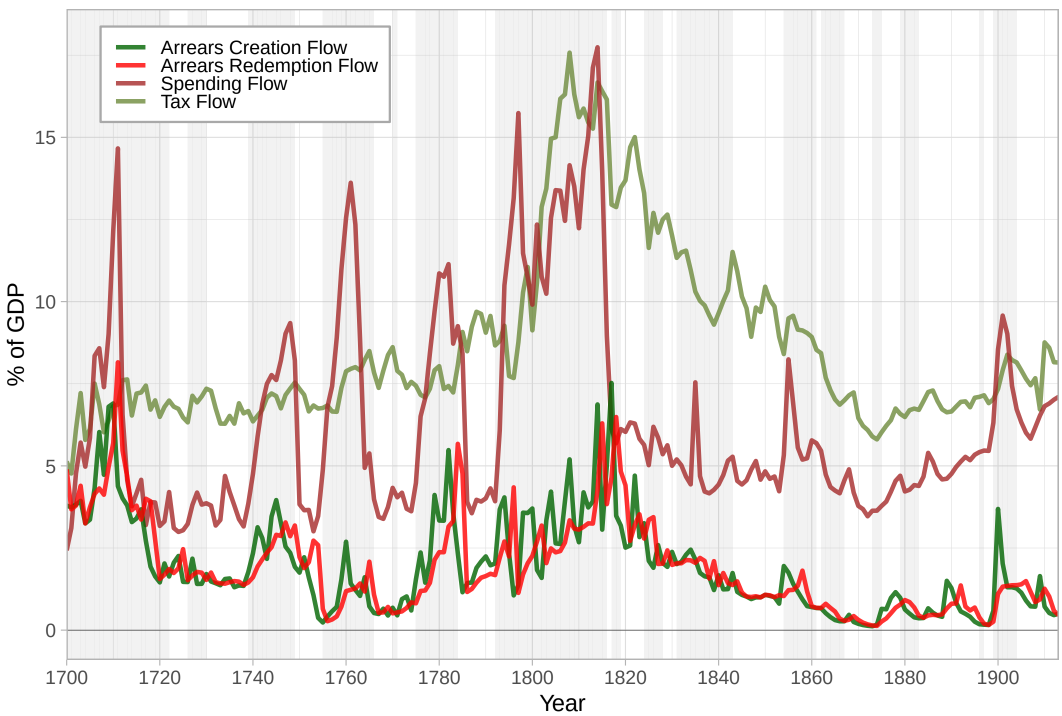


Figure 3 shows the flows of tax revenues, government spending, arrears creation and arrears redemption as percentage of GDP over time. There are a few important takeaways. Firstly, tax revenues average 9% of GDP and are much smoother than government spending. Secondly, government spending averages 6.5% of GDP and has large spikes that exceed tax revenues coinciding with war episodes. Thirdly, arrears creation averages 1.7% of GDP and it also increases in concurrence with war episodes, appearing highly correlated with government spending (correlation coefficient of 0.59). Finally, arrears redemptions also average 1.7% of GDP and closely track the creation of arrears (the contemporaneous correlation is 0.7 and the correlation with arrears creation lagged one year is 0.87) but are somewhat smoother[15]. This pattern is very much a consequence of the turnover assumption imposed in the creation of the gross flows of arrears, but also reflects the anecdotal evidence we have about arrears repayment occurring soon after they were incurred with occasional instances in which the payment period would lengthen.

Figure 3: Flows of tax revenues, spending, domestic arrears creation and redemption as % of GDP (shaded areas indicate years of war)



Concerning the degree of financial repression that government suppliers might have encountered, Wheeler (1996) suggests that government suppliers faced negligible financial repression. This is based on four observations: first, because of the large concentration of ships in a few specific seaports, victuallers were large operators whose credits with the naval administration could not be ignored without serious repercussions. Second, the

[15]Standard deviations are 2.8% for taxes, 3.2% for spending, 1.3% for arrears creation, and 1.2% for arrears redemption.

victuallers were repeat suppliers of the navy and were aware that actual payment might have been delayed. Nevertheless, they entered those procurement contracts because they would make a "tremendous amount of money". Third, anecdotal evidence such as an episode recorded in 1651 confirms that the procurement prices were inflated to indemnify the suppliers for the overdue amounts owed by the navy. Finally, there are occasions such as one reported for 1659 in which suppliers would refuse to provide services, food and material unless they received cash on delivery due to the fact that outstanding arrears had grown too large and their repayment was taking to long. Taken together, this sparse evidence suggests that domestic suppliers had a certain degree of bargaining power with the government and would try to get compensated for the liquidity they provided to the treasury. In the absence of financial repression, the presence of arrears as a financing tool for the government could be motivated by a segmentation of the debt market between bondholders and arrears holders.

With regard to the other data used in the paper, additional historical UK macroeconomic time series are taken from *A Millennium of UK Data* published by the Bank of England (Thomas and Dimsdale, 2017). The valuations of UK marketable debt are from Ellison and Scott (2020), who compute them by multiplying the actual outstanding amount of gilds of various vintages with the respective prices as reported in historical financial newspapers. The stock price-dividend ratios are taken from Golez and Koudijs (2018) until 1870 and from Jordà et al. (2019) afterwards. The years in which the UK was at war are the ones reported in Wright (1942).

4 Empirical Estimation

In this section, I explain the empirical procedure to estimate the fiscal backing.

I start by rewriting changes in arrears in the valuation equation (3) as the difference between arrears creation and arrears redemption at any given date $t + j$: $\Delta A_{t+j} = AC_{t+j} - AR_{t+j}$. The valuation equation becomes:

$$\begin{aligned} V_t &= E_t \left[\sum_{j=0}^{\infty} M_{t,t+j}(T_{t+j} - G_{t+j} + AC_{t+j} - AR_{t+j}) \right] \\ &= PV_t^{\tau} - PV_t^{g} + PV_t^{ac} - PV_t^{ar} \end{aligned} \tag{6}$$

where $PV_t^{\tau} = E\left[\sum_{j=0}^{\infty} M_{t,t+j} T_{t+j}\right]$ is the present value of current and future taxes, PV_t^{g} is the present value of government spending, PV_t^{ac} of domestic arrears creation, and PV_t^{ar} of domestic arrears redemption. Such present values represent the no-arbitrage price an investor would have to pay to acquire a claim on the cash flow of taxes, expenditures, arrears creation and arrears redemption. A forward-looking investor holding the totality of government marketable debt is long the claims on taxes and arrears creation, while at the same time being short the claims on spending and arrears redemption.

I proceed estimating the unobserved prices of the claims on taxes, spending, arrears creation and arrears redemption by modelling their price-dividend ratios. Following Campbell and Shiller (1988), I log-linearize the returns and iterate forward to obtain the log price-dividend ratio as a function of the sum of dividend changes and discount rates:

$$pd_t^c = \frac{\kappa_c}{1-\rho_c} + E_t\left[\sum_{j=0}^{\infty} \rho_c^j \left(\Delta \log Div_{t+j+1}^c - r_{t+j+1}^c\right)\right] \tag{7}$$

where $pd_t^c = \log\left(\frac{PV_t^c}{Div_t^c}\right)$ is the log price-dividend ratio with steady state value pd_{SS}^c, $\rho_c = \frac{e^{pd_{SS}^c}}{1+e^{pd_{SS}^c}}$ and $\kappa_c = \log(1+e^{pd_{SS}^c}) - \rho_c pd_{SS}^c$ are linearization constants, $\Delta \log Div_{t+j+1}^c$ are changes in log dividends, and r_{t+j+1}^c are discount rates. The dividends are the observed cash flows for each claim at each date: $Div_t^c = \{T_t, G_t, AC_t, AR_t\}$ for $c = \{\tau, g, ac, ar\}$ respectively.

The nominal expected discount rates are given by the sum of the nominal short term rate $ysht_t^{\$}$, nominal term spread $yspr_t^{\$}$ and risk premium rp_t^c for $c = \{\tau, g, ac, ar\}$:

$$E_t(r_{t+1}^c) = ysht_t^{\$} + yspr_t^{\$} + rp_t^c \tag{8}$$

In the next subsections, I describe the choice of SDF, the evolution of observed state variables such as cash flows and yields, as well as the estimation of risk premia.

Finally, given the log price-dividend ratios, the present values or prices of the claims on taxes, spending and arrears can be obtained as follows:

$$V_t = PV_t^{\tau} - PV_t^{g} + PV_t^{ac} - PV_t^{ar} = e^{pd_t^{\tau}} T_t - e^{pd_t^{g}} G_t + e^{pd_t^{ac}} AC_t - e^{pd_t^{ar}} AR_t \tag{9}$$

4.1 Stochastic Discount Factor

The SDF $M_{t,t+j}$ is the CAPM stochastic discount factor where the return on wealth is proxied by the returns on the UK stock market. On the one hand, the choice of SDF is of course constrained by the limited amount of data available historically. For instance, consumption data are not available until 1830. Moreover, bond issued prior to 1752 were illiquid and took unconventional forms by current standards[16], limiting the possibility

[16] For example, bond issuance took the form of annuities, either of a fixed duration or linked to the life of the buyer, and lotteries. For lottery loans, investor would buy tickets that entitled to various prizes in the form of bonds or annuities. Lottery loans were issued until 1776. See Velde (2018) for

to calibrate a term structure model. On the other hand, there are compelling economic arguments for the choice of the stock market return as unique factor in the SDF. For one, stock returns were tied to the returns on other securities such as marketable bonds and to some extent also non-marketable debt since the same moneyed class of aristocrats and city merchants would often hold a combination of these securities in a more or less diversified portfolios. This tie was mechanical in the first half of the sample period since the Bank of England was one of the few quoted public companies and it gained its revenues by brokering the bond issuance for the Crown. At the time of its creation in 1694, the equity that the bank raised was immediately lent to the government, meaning that the fortune of the Bank was tightly linked to the government's ability to repay its debts, and, thus, its stock price would also reflect the creditworthiness of the government. Most importantly, stock returns were closely tied to fundamental macroeconomic sources of risk. Until the 1810s, only a handful of firms were quoted on the stock market, the most important of which were the Bank of England, the British East India Company, and the South Sea Company[17]. Although technologically less advanced, the market functioned similarly to today and stock prices responded to the arrival of news in an efficient way (Koudijs (2016)), meaning that bad states of the world that would have impacted life conditions of investors, such as a war of independence in a colony or the threat of a French invasion, would have also affected the traded firms, e.g. by disrupting trade, and thus would have shown in their equity prices. Starting in the 1810s, many new equities were issued that reflected the technological advances of the time, as the Industrial Revolution gained momentum. Initially, these were mainly canals and insurances companies. Later on, banks and railroad companies became the most important issuers of new stocks.

4.2 State Variables Dynamics

I assume that the representative investor in marketable government debt forms her expectations about future fundamentals using a first-order VAR model:

$$Z_t = a + \Gamma Z_{t-1} + u_t \tag{10}$$

more details.

[17]Up to 1813, stock market returns are derived from both London and Amsterdam traded companies. The most important Dutch companies were the Dutch East India Company and the Dutch West Indies Company. This is done to ensure complete data coverage for the time period. See Golez and Koudijs (2018) for full details. This circumstance does not pose a problem for my estimation since the two countries were allied and there was a substantial degree of financial interdependence between them. This implies that the SDF is an international SDF, which is all the more convenient since British marketable debt was itself a global security.

where Z_t is the vector of state variables which represents the information set of the investor, Γ is the matrix of coefficients, a is the vector of constants, and u_t are i.i.d. normally distributed innovations with homoskedastic variance Σ: $u_t \sim N(0, \Sigma)$. The state variables include macroeconomic fundamentals such as inflation, interest rates and GDP growth, financial fundamentals such as stock market dividend and price-dividend ratio, information about level and growth rate of the cash flows of interest (taxes, spending, arrears), and information about the debt-to-GDP ratio of the government. A rich set of state variables that predicts cash flows and pricing factors alleviates the concern that the model misses important shocks. The variables included in the VAR are described in Table 1. The estimated VAR coefficients are displayed in Table 6 in the Appendix. Note that the cash flows of interest enter the VAR as ratios of GDP. This ensures that the cash flows are stationary, which is equivalent to imposing a cointegration relationship between cash flows of taxes, spending, arrears creation, and redemption with GDP. The cointegration relationship is tested and, indeed, verified in the Appendix. Cointegration imposes restrictions to the matrix of coefficients Γ which ensures that cointegrated variables converge back towards equilibrium. This ensures an accurate description of fiscal dynamics where fiscal variables are allowed to react to each others' shocks.

Table 1: VAR Variables

	Variable	Name	SampleMean
1	π_t	Inflation	0.23
2	$ysht_t^{\$}$	Short-term Nominal Yield	3.75
3	$yspr_t^{\$}$	Long Minus Short-term Nominal Yield Spread	-0.23
4	x_t	Real GDP Growth	1.67
5	$\Delta \log div_t^M$	Stock Market Dividend-to-GDP Growth	0.27
6	$\log div_t^M$	Log Stock Market Dividend-to-GDP Level	3.44
7	pd_t^M	Log Stock Market Price-to-Dividend Ratio	25.29
8	$\Delta \log \tau_t$	Tax Revenue-to-GDP Growth	0.03
9	$\log \tau_t$	Log Tax Revenue-to-GDP Level	9.01
10	$\Delta \log g_t$	Spending-to-GDP Growth	0.24
11	$\log g_t$	Log Spending-to-GDP Level	6.52
12	$\Delta \log ac_t$	Arrears-Creation-to-GDP Growth	-0.37
13	$\log ac_t$	Log Arrears-Creation-to-GDP Level	1.74
14	$\Delta \log ar_t$	Arrears-Redemption-to-GDP Growth	-0.23
15	$\log ar_t$	Log Arrears-Redemption-to-GDP Level	1.68
16	$\Delta \log d_t$	Debt-to-GDP Growth	-0.59
17	$\log d_t$	Log Debt-to-GDP Level	89.66

Note: All sample means are in percent except for price-to-dividend ratio

4.3 Risk Premia

Since nominal short term rate $ysht_t^{\$}$ and nominal term spread $yspr_t^{\$}$ are part of the VAR, the risk premia are the only variables that remain to be pinned down to determine the expected discount rates. Firstly, it is useful to establish the risk premium of a claim on GDP. As standard in asset pricing models, I approximate the return on a GDP claim, which is the return on total wealth, by the unlevered return on the stock market. The equity ratio for the corporate sector in the UK in 1900 is 0.75. I assume that the same ratio is constant and also applies to the previous years[18]. Then, given the (geometric) average equity premium of 2.4% over the sample period, the GDP risk premium is: $rp^{GDP} = 0.75 * 2.4 = 1.8\%$. In my baseline specification, this risk premium is assumed constant; in one of the robustness checks it is allowed to vary over time. Secondly, the cointegration assumption between GDP and tax revenues, spending, arrears creation and arrears redemption implies that they are driven by the same stochastic trend. This means that in the long run they are exposed to the same risk. This in turn implies that in the long run the risk premia must equate: $rp_t^{GDP} = rp_t^{\tau} = rp_t^{g} = rp_t^{ac} = rp_t^{ar}$ as $t \to \infty$. In the short term, however, intuition suggests that tax revenues are more exposed to business cycle risk than spending, while the reverse is true for arrears: the cash flow from arrears creation is less risky than that of arrears redemption. This follows from the observation that spending and arrears creation display counter-cyclical behaviour. Therefore, the following holds: $rp_t^{\tau} \geq rp_t^{GDP} \geq rp_t^{g}$ and $rp_t^{ac} \leq rp_t^{GDP} \leq rp_t^{ar}$ for all t.

In the following steps, I derive the risk premium difference between the claim on taxes minus spending, and arrears creation minus redemption. Consequently, I will use it to adjust the risk premium of GDP, which anchors the risk premia in the long term.

First, I derive the price of risk. The unconditional beta representation of the stock market return, which is the factor in the SDF, is:

$$E(R_{t+1}^{M}) = \gamma + \beta^{M,M}\lambda \tag{11}$$

where $E(R_{t+1}^{M})$ is the unconditional expected return on the stock market, γ is the risk-free rate, $\beta^{M,M}$ is the risk exposure of stocks returns with itself, and λ is the price of risk. Since the exposure of stocks return with itself is 1, the price of risk equals the stock market excess return: $\lambda = E(R_{t+1}^{M}) - \gamma$, i.e. 2.4%.

Second, I estimate the difference in risk exposure between claims by regressing differ-

[18]This assumption is necessary since there are not aggregate information on the corporate sector balance sheet prior to 1900. Note that while the equity ratio certainly displays time variation, it probably moves within a narrow bandwidth. For the US since WWII, the equity ratio of the corporate sector varies between a minimum of 75% and a maximum of 84%.

ences in cumulated percentage changes of the claims' cash flows on cumulated percentage changes of the total return index of stocks, TRX_t, at different horizons. Taking arrears creation and redemption as example, I estimate:

$$\sum_{t=1}^{h} \Delta \log AC_t - \sum_{t=1}^{h} \Delta \log AR_t = c + \beta_h^{\Delta A,M} \sum_{t=1}^{h} \Delta \log TRX_t + \epsilon_t \tag{12}$$

where AC_t and AR_t are the cash flows of arrears creation and redemption respectively, c is a regression constant, and ϵ_t is the error term. The coefficient $\beta_h^{\Delta A,M}$ captures the difference in risk exposure between the claim on AC_t and AR_t. Analogously for taxes and spending, $\beta_h^{\Delta TG,M}$ measures the difference in risk exposure between the claim on T_t and G_t. To accurately measure the risk exposure of a single claim, regression 12 should employ the claim's total returns on the left-hand side. However, given that return shocks are driven by both cash flow shocks and discount rate shocks, and since we are calculating differences, the impact of discount rate shocks is effectively neutralized[19]. Consequently, we can reliably capture the underlying risk exposure despite using solely cash flows. Figure 4 displays the cash-flow betas of arrears creation and redemption, along with the cash-flow betas of taxes and spending. As one would expect, tax revenues are more risky than expenditures until their beta converge. At the same time, arrears creation is a safer claim than arrears redemption[20]. Note that the *level* of the beta displayed for each claim in Figure 4 is meaningless over a finite horizon since the returns on the claims used in the regression are based on cash flows only and do not contain discount rate shocks.

Finally, given these estimates, I obtain the maximum risk premium difference between claims as the maximum difference in the cash flows betas multiplied by the price of risk. Using arrears creation and redemption as an example:

$$\arg\max_h |rp_h^{ac} - rp_h^{ar}| = \arg\max_h |\beta_h^{\Delta A,M}|\lambda = \arg\max_h |\beta_h^{AC,M} - \beta_h^{AR,M}|\lambda \tag{13}$$

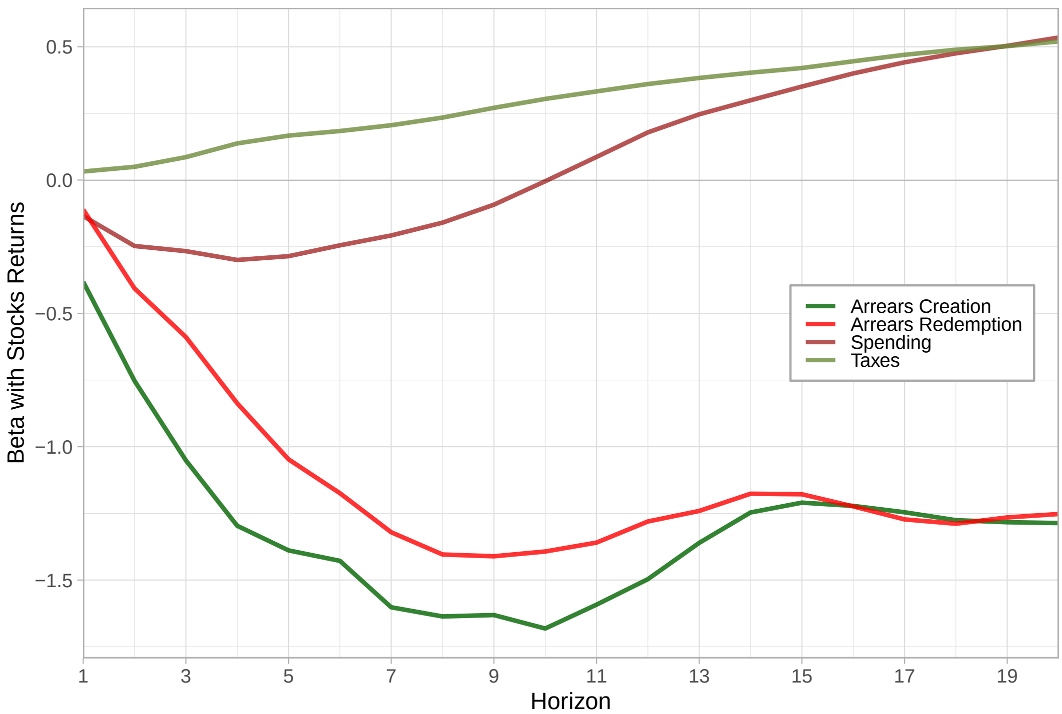
where $\beta_h^{AC,M}$ and $\beta_h^{AR,M}$ are estimated as in the beta regression 12 but with only cumulated growth of arrears creation or redemption, one at a time, as dependent variable, λ is the price of risk, and $|\cdot|$ indicates absolute values. The maximum risk premium differences amount to $\arg\max_h |rp_h^{\tau} - rp_h^{g}| = 1.0\%$ for taxes and spending, and $\arg\max_h |rp_h^{ac} - rp_h^{ar}| = 1.1\%$ for arrears creation and redemption. I take the maximum risk premium differences because together with the minimum differences, which are zero,

[19]This follows by applying the result by Campbell and Vuolteenaho (2004) to the returns of single asset i: $r_{t+1}^i - E_t r_{t+1}^i = N_{CF,t+1}^i - N_{DR,t+1}^i$, where $N_{CF,t+1}^i$ is a cash flow shock and $N_{DR,t+1}^i$ is a discount rate shock. If the same investor is holding and pricing two assets, the discount rate shocks are the same across the assets. A robustness check in Section 7 empirically confirms this results.

[20]Figure 15 in the Appendix shows the same graph for the betas with respect to real GDP growth.

as the risk premia converge in the long run, allow me to put bounds on the present values of the claims. Given $rp_t^{\tau} \geq rp_t^{GDP} \geq rp_t^{g}$ and $rp_t^{ac} \leq rp_t^{GDP} \leq rp_t^{ar}$ for all t, imposing the maximum risk premium difference yields a lower bound for the present value of a claim on taxes minus spending and an upper bound for the present value of a claim on arrears creation minus redemption. This is because the long tax claim is minimized by the large risk premium on taxes, while the short spending claim is maximized by the small risk premium on expenditures. Conversely, the long claim on arrears creation is maximized by the small risk premium, while the short claim on arrears redemption is minimized by the large risk premium. The reverse holds for the minimum risk premia difference, which is zero. It delivers an upper bound for the present value of taxes minus spending claims and a lower bound for the present value of arrears creation minus redemption claims.

Figure 4: Cash-flow beta exposure of the growth rates of taxes, spending, arrears creation, and arrears redemption with stock market returns at different horizons



In my baseline specification, I display the upper bound of the overall present value of marketable primary surpluses which means setting $rp^{\tau} = rp^{g} = rp^{GDP}$, $rp^{ac} = rp^{GDP} - (\arg\max_h |rp_h^{ac} - rp_h^{ar}|)/2$, and $rp^{ar} = rp^{GDP} + (\arg\max_h |rp_h^{ac} - rp_h^{ar}|)/2$. In the robustness Section 7, I allow the risk premia to vary over time and I show the lower bound of the present value. Also among the robustness checks, I show the validity of this approach by applying the same methodology to the post-WWII US data used in Jiang et al. (2024), recovering on average the same result as their fully fledged dynamic asset pricing model.

In conclusion, the implications of the various assumption about risk premia can be best summarized by looking at valuation equation (3) where the expectation of the products are restated as products of expectations plus covariances:

$$
\begin{aligned}
V_t = \sum_{j=0}^{\infty} [& E_t(M_{t,t+j})E_t(T_{t+j}) - E_t(M_{t,t+j})E_t(G_{t+j}) + E_t(M_{t,t+j})E_t(AC_{t+j}) - E_t(M_{t,t+j})E_t(AR_{t+j}) \\
& + \underbrace{Cov_t(M_{t,t+j}, T_{t+j})}_{<0} - \underbrace{Cov_t(M_{t,t+j}, G_{t+j})}_{>0} + \underbrace{Cov_t(M_{t,t+j}, AC_{t+j})}_{>0} - \underbrace{Cov_t(M_{t,t+j}, AR_{t+j})}_{<0}]
\end{aligned} \tag{14}
$$

The covariance terms capture the riskiness of the claims that are being priced to obtain the risk-adjusted present value of marketable primary surpluses. Without aggregate risks, there would be no covariance terms. The equation displays the covariance terms with their hypothesised signs. The assumption of cointegration of tax and spending claims with output implies $rp_t^{GDP} = rp_t^{\tau} = rp_t^{g}$, which in turn implies that $\sum_{j=0}^{\infty} Cov_t(M_{t,t+j}, T_{t+j}) = \sum_{j=0}^{\infty} Cov_t(M_{t,t+j}, G_{t+j})$. Hence, the covariance terms of taxes and spending cancel out delivering an upper bound estimation of the present value of the respective cash flows, which is based exclusively on expected values $\sum_{j=0}^{\infty}[E_t(M_{t,t+j})E_t(T_{t+j}) - E_t(M_{t,t+j})E_t(G_{t+j})]$. For arrears instead the difference in covariances is maximised. The results in the next section show how the difference in covariances is large and responsible for virtually all the present value of arrears, since the expected values of arrears creation and redemption tends to cancel each other out.

4.4 Estimation of Fiscal Backing

Given the estimation of the VAR model in equation (10), the price-dividend ratio of taxes can be obtained as follows:

$$
pd_t^{\tau} = pd_{SS}^{\tau} + \left(e_x + e_{\pi} + e_{\Delta \log \tau} - e_{ysht} - e_{yspr}\right)' \Gamma \left(I - \rho_{\tau}\Gamma\right)^{-1} Z_t \tag{15}
$$

where e_z are selector vectors that have a one in the position of the state variable z and zeros otherwise, and $pd_{SS}^{\tau}, \rho_{\tau}$, and κ_{τ} solve the system of equations:

$$
\begin{aligned}
pd_{SS}^{\tau} &= \frac{\kappa_{\tau} + (x_{SS} + \pi_{SS}) - (ysht_{SS}^{\$} + yspr_{SS}^{\$} + rp_{SS}^{\tau})}{(1 - \rho_{\tau})} \\
\kappa_{\tau} &= \log\left(1 + e^{pd_{SS}^{\tau}}\right) - \rho_{\tau} pd_{SS}^{\tau} \\
\rho_{\tau} &= \frac{e^{pd_{SS}^{\tau}}}{1 + e^{pd_{SS}^{\tau}}}
\end{aligned} \tag{16}
$$

The formulas for the price-dividend ratio of spending, arrears creation and arrears redemption are analogous. Given the price-dividend ratios, the fiscal backing can be estimated according to the formula in (9).

5 Results and Discussion

Figure 5 displays the model implied marketable debt valuation relative to GDP:

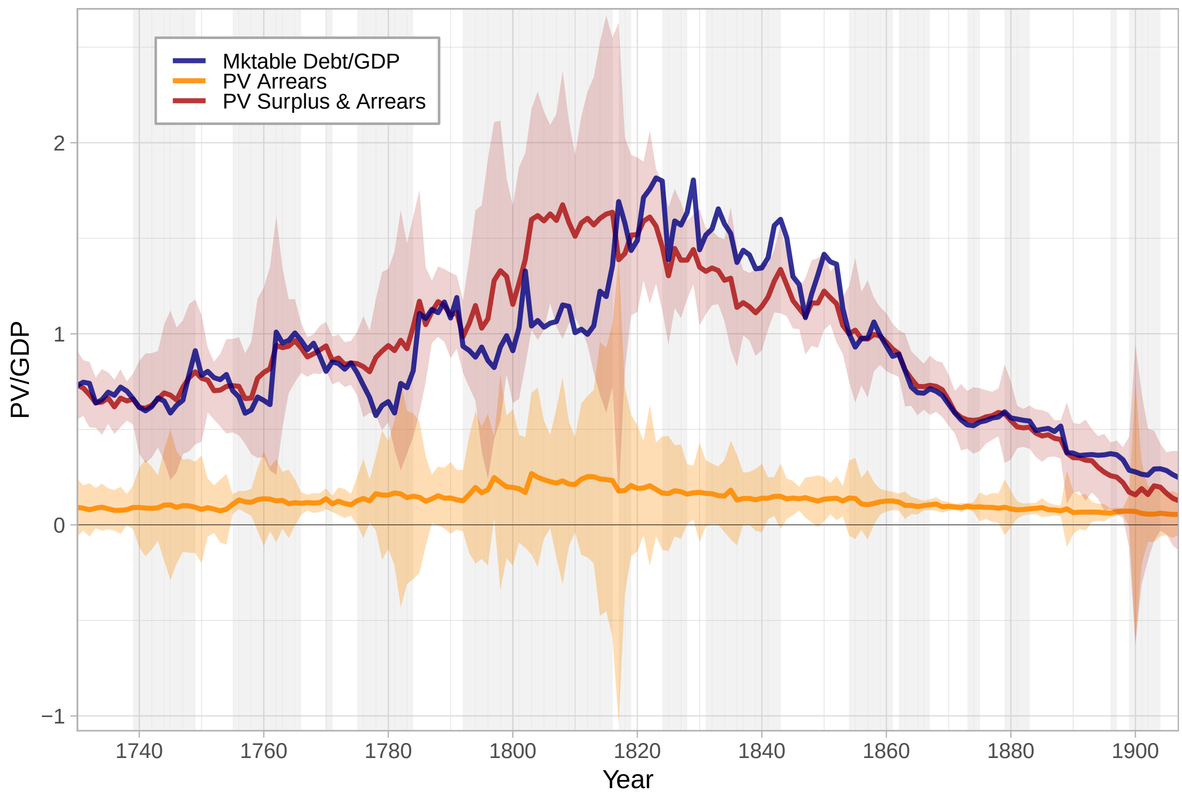
$$\frac{V_t}{Y_t} = \frac{PV_t^{\tau}}{Y_t} - \frac{PV_t^{g}}{Y_t} + \frac{PV_t^{ac}}{Y_t} - \frac{PV_t^{ar}}{Y_t} \tag{17}$$

compared with the actual market value of marketable government debt relative to GDP. The model yields a realistic debt valuation that matches level and variability of the observed market value of marketable debt. The average present value of the marketable primary surplus claim is 91.6% of GDP, while the average market value of marketable debt is 89.7% of GDP over the sample period. The correlation coefficient of the two time series is 87%. The present value of arrears $\frac{PV^{ac}-PV^{ar}}{Y_t}$ averages 12.9%.

The present values plotted in Figure 5 are upper bounds derived from valuation equation (3), since they minimise risk premia for the long claims (taxes and arrears creation) and maximise risk premia for the short claims (spending and arrears redemption). The present value of the marketable primary surplus traces an upper hull for the observed market value of marketable debt in the first 100 years of observations and between 1850 and 1890. Between 1820-1850 and in the final 20 years of observations, the observed market value lies below the point estimate of the present value, but within its 99% confidence interval (not shown).

One possible explanation for the discrepancy between the point estimate and market values between 1820-1850 is that investors had some consequential information concerning debt valuations that the econometrician does not have. In particular, we observe the market value of bonds increasing rapidly after the defeat of Napoleon in 1815 and the resolution of the great uncertainty weighing down financial markets: the value of bonds that had traded below par quickly reaches par with the corresponding decline in yields. Crucially, the income tax that was introduced in 1799 to finance the Napoleonic Wars and had raised an unprecedented amount of revenue for the Crown, as Figure 3 documents, was abolished in 1816. The missing revenues are interpreted by the model as a deterioration of the fiscal backing of the country. Income taxes were however reintroduced indefinitely from 1842. Bondholders' expectations that income taxes would come back cannot be captured by the model and can explain the high market value of debt in the post Napoleonic period.

Figure 5: Upper bound of present value of marketable primary surpluses compared with the actual market value of marketable debt as % of GDP
(95% confidence intervals are displayed; standard errors are obtained from bootstrap resampling with 1,000 samples; shaded areas indicate years of war)



A similar explanation accounts for the large overshooting of the point estimate of the present value of marketable surpluses between 1800 and 1820. Exclusively over that time period (1797-1821), England suspended the gold convertibility of Bank of England banknotes. As a result, investors' inflation expectations shot up, as documented for instance in Antipa (2016), accounting for the depressed bond prices. Since inflation never materialized and Britain reintroduced gold convertibility at pre-war parity, bond prices quickly returned close to par. The model, which forms expected values based on predictions extrapolated from the data, cannot capture this particular change in expectations.

Figure 5 casts doubts on the existence of a valuation puzzle concerning the historical market value of UK debt raised in Chen et al. (2022), at least as long as the upper bound of the fundamental value of debt is considered and arrears are included. The authors finds an average gap between the true market value of debt and the model implied upper bound of the present value of surpluses of 37 percentage points of GDP over the same sample period. The reason for the different result can be attributed to the choice of risk premium and to the fact that the authors do not consider domestic arrears in their analysis. The authors chose a large fixed risk premium of 3% for the tax and spending claim (as opposed to 1.8% in this paper), which explains 65% of the 37pp gap on average. The underestimation of the primary surpluses by ignoring the funding that the government obtains via domestic arrears explains the remaining 35% of the 37pp gap on average.

Arrears The upper bound of the present value of domestic arrears as a percentage of GDP is $\frac{PV^{ac}-PV^{ar}}{Y_t} = 12.9\%$ on average over the sample period, as displayed in Figure 6. It reaches a maximum of 26.8% of GDP in 1803 and a minimum of 5.4% of GDP in 1906. The standard deviation of the time series is 5%. These numbers suggest that neglecting domestic arrears in the valuation equation leads to an underestimation of the fiscal backing by 12.9% of GDP on average. This is equivalent to increasing tax revenues by 5% – from their average value of 9% of GDP to 9.45% – all else equal. Expressed differently, under the assumption of no financial repression, the effect of neglecting domestic arrears is equivalent to using government expenditures that are inflated by 6.5% in the model estimation: average expenditures is 6.5% of GDP on average, but 0.42 percentage points of that is surcharges the government pays to suppliers in exchange for the funding it obtains via arrears[21]. The key takeaway is that the combination of a long position in

[21] These numbers are derived as follows: the mean price-dividend ratio of the tax claim or spending claim - $e^{pd^{\tau}_{SS}}$ or $e^{pd^{g}_{SS}}$ - is multiplied with the necessary change in the average tax or spending dividend to obtain a present value that is 12.9% of GDP higher.

arrears creation and a short position in arrears redemption is valuable for bondholders and shall not be neglected for the correct estimation of the fiscal backing.

Figure 6: Present value of arrears changes as fraction of GDP broken down in its components (shaded areas indicate years of war)

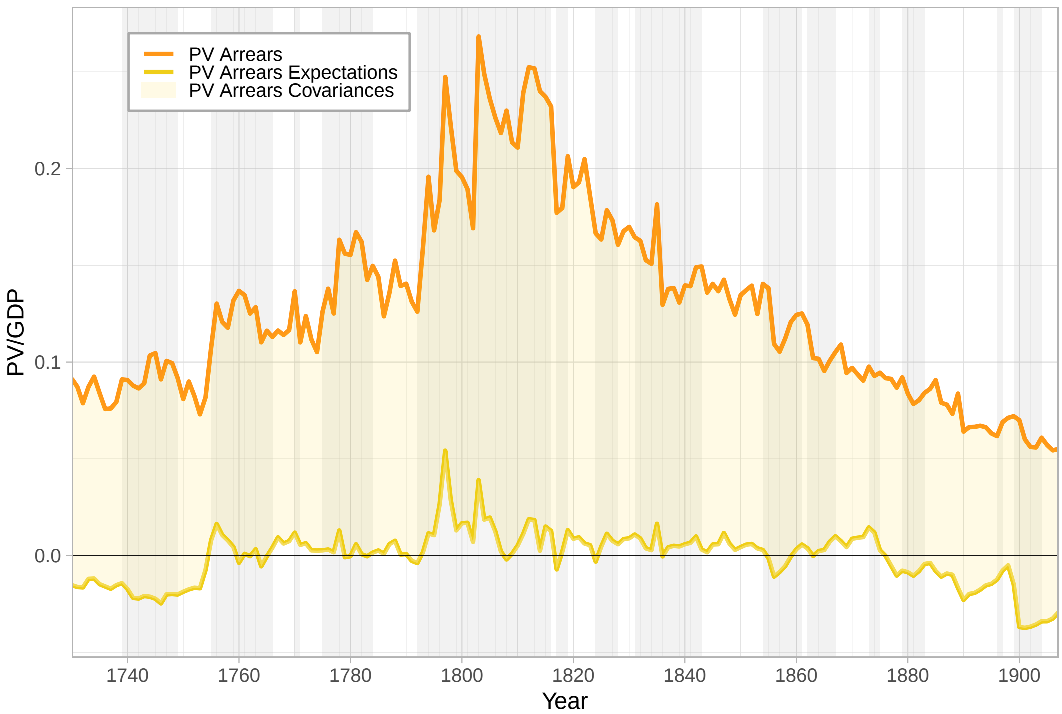


Figure 6 shows that the present value of arrears is almost entirely derived from its covariance with the SDF. The yellow line in Figure 6, which plots the present value of arrears expectations, is derived from the same empirical model described in the respective section above, but with the covariance elements for arrears creation and redemption – as in equation (14) – shut down. This is achieved by equating the risk premia of AC and AR claims to the GDP risk premium. Thus, the yellow line also corresponds to the lower bound of the present value of arrears. The reason for the negligible impact of expected values is that the model-implied expectations for arrears changes are around zero since arrears are always repaid in nominal terms in the sample period.

The present value of arrears displays an expected time pattern: it tends to increase during "bad times", when the government runs large deficits to finance war efforts. This means that ignoring arrears leads to the present value of marketable primary surpluses to be underestimated particularly during those crucial times when large debt issuance takes place. The positive covariance term of arrears changes with the SDF is responsible for their counter-cyclical increase in value. Note that this makes arrears a valuable claim from the perspective of bondholders or, equivalently, an expensive liquidity service for the government under the assumption that suppliers are fairly compensated for late payments. Recall, that under this assumption, arrears have zero present value in expectations and by themselves do not increase the fiscal backing. Nevertheless, from an ex post perspective, analogously to unexpected inflation, a lower than expected compensation of suppliers or a longer than expected repayment time leads to an net transfer of resources from suppliers to the government exactly during times when the government needs financing. From this standpoint, arrears represent an overlooked channel that can help a government navigate incomplete financial markets.

Once the possibility of financial repression against government suppliers is acknowledged, a new range of implications emerges. All else equal, incurring domestic arrears and defaulting on them increases the fiscal backing of the government. The same holds just in expectation: the belief among bond investors that when push comes to shove their claim on government revenues is senior to other claims, such as domestic arrears, warrants higher fiscal backing. Alternatively, consider a scenario in which domestic arrears need to be paid back nominally, but without interest. Then, incurring domestic arrears today and paying them back tomorrow also warrants higher fiscal backing, since tomorrow's payment is discounted by the SDF $M_{t,t+1}$. In these ways, domestic arrears can prop up the value of government debt at times when mistrust in the government ability to repay would otherwise reduce valuations. This is beneficial for the government in two ways: firstly, higher valuations allow the government to issue marketable debt at higher prices, i.e. lower yields, thus reducing the cost of servicing it and preventing the negative self-fulfilling impact that high debt servicing cost has on valuations (Calvo (1988), Lorenzoni and Werning (2019)); secondly, higher valuations help the government escape crisis zones, i.e. situations in which the government risks a liquidity crisis by not to being able to roll over its debt due to a crash in its value (Cole and Kehoe (2000)).

Figure 6 also shows that the present value of arrears peaked in the midst of the Napoleonic wars. Once the UK had established its supremacy globally following the Napoleon's defeat, wars became less frequent and less demanding to finance and arrears got steadily repaid to a level below 2.5% of GDP. This explains the steady decline in arrears' present value in the second half of the sample period.

Neglected arrears not only make a significant contribution to the level of the model implied debt valuation, but they also are responsible for some of its variability. A simple variance decomposition of the form:

$$1 = \frac{Cov(V_t, PV_t^{\tau})}{Var(V_t)} - \frac{Cov(V_t, PV_t^{g})}{Var(V_t)} + \frac{Cov(V_t, PV_t^{ac})}{Var(V_t)} - \frac{Cov(V_t, PV_t^{ar})}{Var(V_t)} \tag{18}$$

yields coefficients of 51.2% for taxes and 37.3% for spending, and 11.4% for the combined effect of arrears creation and redemption. The standard deviation of the marketable debt valuation relative to GDP is 39.6 percentage points. The results are displayed in Table 2.

Table 2: Variance decomposition of V_t

Component	Percent
$\frac{Cov(V_t, PV_t^{\tau})}{Var(V_t)}$	51.20
$-\frac{Cov(V_t, PV_t^{g})}{Var(V_t)}$	37.30
$\frac{Cov(V_t, PV_t^{ac})}{Var(V_t)}$	33.10
$-\frac{Cov(V_t, PV_t^{ar})}{Var(V_t)}$	-21.70
$\sigma(\frac{V_t}{Y_t})$	39.60

6 Theoretical Model

The following simple model helps to rationalise the results observed empirically, namely, why governments would use arrears, why arrears are cyclical and why they impact bond valuations.

Consider a stochastic two-period model of a closed economy with two agents and segmented asset markets:

- a taxpayer/bondholder[22] (bh), who has endowment Y_t and holds an asset with stochastic cash flow $-G_t(Y_t)$ at $t = 1$
- a government supplier (sup), who has endowment Y_t and holds an asset with stochastic cash flow $G_t(Y_t)$ at $t = 1$

Let $t = \{0, 1\}$. Agents have identical CRRA preferences and with the same coefficient of risk aversion:

$$U_i = \frac{C_{i,0}^{1-\sigma}}{1-\sigma} + \beta E_0 \left[\frac{C_{i,1}(Y_1)^{1-\sigma}}{1-\sigma} \right], \quad \sigma > 0$$

[22] Bondholders and taxpayers overlap in a closed economy, thus, their combined cash flow simplifies as follows: $\underbrace{-T}_{taxpayer\ flow} + \underbrace{T - G}_{bondholder\ flow} = -G$

which give rise to two distinct stochastic discount factors $M_i(Y_1) = \beta \left(\frac{C_{i,1}(Y_1)}{C_0}\right)^{-\sigma}$ due to the different consumption realisations at $t = 1$ (C_0 is the same for both agents), which are:

$$C_{bh,1}(Y_1) = Y_1 - G_1(Y_1)$$
$$C_{sup,1}(Y_1) = Y_1 + G_1(Y_1)$$

The environment is stochastic, such that $C_{i,1}(Y_1)$ and $G_1(Y_1)$ depend on a random state variable Y_1, representing aggregate output subject to permanent shocks. Specifically, and without loss of generality, I assume that government spending G_1 is countercyclical and thus positively correlated with the SDF[23]. In addition, markets are segmented meaning that bondholder and supplier cannot trade assets or insure against risk with one another. Each agent is constrained to consume only the endowment and the net cash flow associated with their role. As a result, the value of the first asset is smaller than the value of the second asset since the first one is risky, while the second one is safe, $V_0^{bh} < V_0^{sup}$, as can be seen from the following valuation equations:

$$V_0^{bh} = E_0\left[M_{bh}(Y_1) \cdot (-G_1(Y_1))\right] = E_0\left[M_{bh}(Y_1)\right] E_0\left[-G_1(Y_1)\right] - \underbrace{Cov_0\left(M_{bh}(Y_1), G_1(Y_1)\right)}_{>0} < 0$$

$$V_0^{sup} = E_0\left[M_{sup}(Y_1) \cdot G_1(Y_1)\right] = E_0\left[M_{sup}(Y_1)\right] E_0\left[G_1(Y_1)\right] + \underbrace{Cov_0\left(M_{sup}(Y_1), G_1(Y_1)\right)}_{>0} > 0$$

A social planner can change the asset cash flows through a state-contingent transfer $\Delta A_1(Y_1)$ across agents:

- The taxpayer/bondholder now receives asset cash flow $-G_1(Y_1) + \Delta A_1(Y_1)$
- The government supplier now receives asset cash flow $G_1(Y_1) - \Delta A_1(Y_1)$

The resulting consumption of each agent at $t = 1$ is:

$$C_{bh,1}(Y) = Y_1 - G_1(Y_1) + \Delta A_1(Y_1)$$
$$C_{sup,1}(Y) = Y_1 + G_1(Y_1) - \Delta A_1(Y_1)$$

The planner chooses $\Delta A_1(Y_1)$ to maximize a weighted sum of expected utilities subject to the resource constraint:

$$\max_{\Delta A_1(Y_1)} E\left[\lambda \cdot U_{bh} + (1-\lambda) \cdot U_{sup}\right] \quad \text{s.t. } C_{bh,1}(Y_1) + C_{sup,1}(Y_1) = 2Y_1$$

[23]Formally, for given shock ε, I assume $\frac{\delta Y}{\delta \varepsilon} > \frac{\delta G}{\delta \varepsilon}$. The result goes through with less stringent assumptions.

At optimum, the planner equalizes the marginal utility-weighted consumption ratio:

$$\frac{U'_{bh}\left(C_{bh,1}(Y_1)\right)}{U'_{sup}\left(C_{sup,1}(Y_1)\right)} = \frac{\lambda}{1-\lambda} \;\Rightarrow\; \left(\frac{C_{bh,1}(Y_1)}{C_{sup,1}(Y_1)}\right)^{-\sigma} = \frac{\lambda}{1-\lambda} \;\Rightarrow\; \frac{C_{bh,1}(Y_1)}{C_{sup,1}(Y_1)} = \left(\frac{1-\lambda}{\lambda}\right)^{1/\sigma}$$

Using equal weights λ, such that $\left(\frac{1-\lambda}{\lambda}\right)^{1/\sigma} = 1$, and solving for $\Delta A_1(Y_1)$, we get:

$$\begin{aligned} C_{bh,1}(Y_1) = C_{sup,1}(Y_1) \;&\Rightarrow\; Y_1 - G_1(Y_1) + \Delta A_1(Y_1) = (Y_1 + G_1(Y_1) - \Delta A_1(Y_1)) \\ &\Rightarrow\; \Delta A_1^*(Y_1) = G_1(Y_1) \end{aligned}$$

Therefore, optimal consumption is:

$$C^*_{bh,1}(Y_1) = C^*_{sup,1}(Y_1) = Y_1$$

As a result, one can see that optimal arrears creation is countercyclical (as G is) and fully smooths consumption. The value of the two assets is also equated, as shown in the following two equations, which means, in particular, that the first asset has become more valuable following the planner's intervention. Note, further, that the SDF is now unique: $M_{bh}(Y_1) = M_{sup}(Y_1) = \beta\left(\frac{C_1^*}{C_0}\right)^{-\sigma} = M(Y_1)$. Thus,

$$\begin{aligned} V_0^{*,bh} &= E_0\left[M(Y_1)\cdot(-G_1(Y_1) + \Delta A_1^*(Y_1))\right] = 0 \\ V_0^{*,sup} &= E_0\left[M(Y_1)\cdot(G_1(Y_1) - \Delta A_1^*(Y_1))\right] = 0 \end{aligned}$$

7 Robustness Checks

7.1 Alternative arrears turnover values

The estimated present value of arrears creation minus arrears redemption $\frac{PV^{ac} - PV^{ar}}{Y_t}$ does not depend crucially on the construction of the gross flows of arrears. In the baseline specification, I assume a turnover rate of arrears of 33% of the outstanding stock. The resulting present value is 12.9% of GDP. Assuming instead a turnover rate of 25%, which implies a repayment time capped at four years, yields a present value of 12.7% of GDP. Finally, assuming a turnover rate of 50%, which implies faster repayment time of maximum two years, yields a present value of 13.1% of GDP.

7.2 Time-varying risk premia

The time-varying risk premia on a GDP claim are displayed in Figure 7. They are derived by taking the steady state value of the equity price-dividend ratio pd^M_{SS} and the equity linearization constants ρ_M and κ_M as given from the data, as opposed to finding them by solving the system of equations (16). Then, the risk premium can be computed each period from:

$$rp^M_t = rp^M_{SS} + (1-\rho_M)\left[(e_x + e_\pi + e_{\Delta \log div} - e_{ysht} - e_{yspr})' \, \Gamma \, (I - \rho_M \Gamma)^{-1} Z_t - e'_{pd} Z_t\right] \tag{19}$$

where $rp^M_{SS} = x_{SS} + \pi_{SS} - ysht^{\$}_{SS} - yspr^{\$}_{SS} - (pd^M_{SS}(1-\rho_M) - \kappa_M)$.

Figure 7: Time-varying risk premia for GDP claim
(shaded areas indicate years of war)

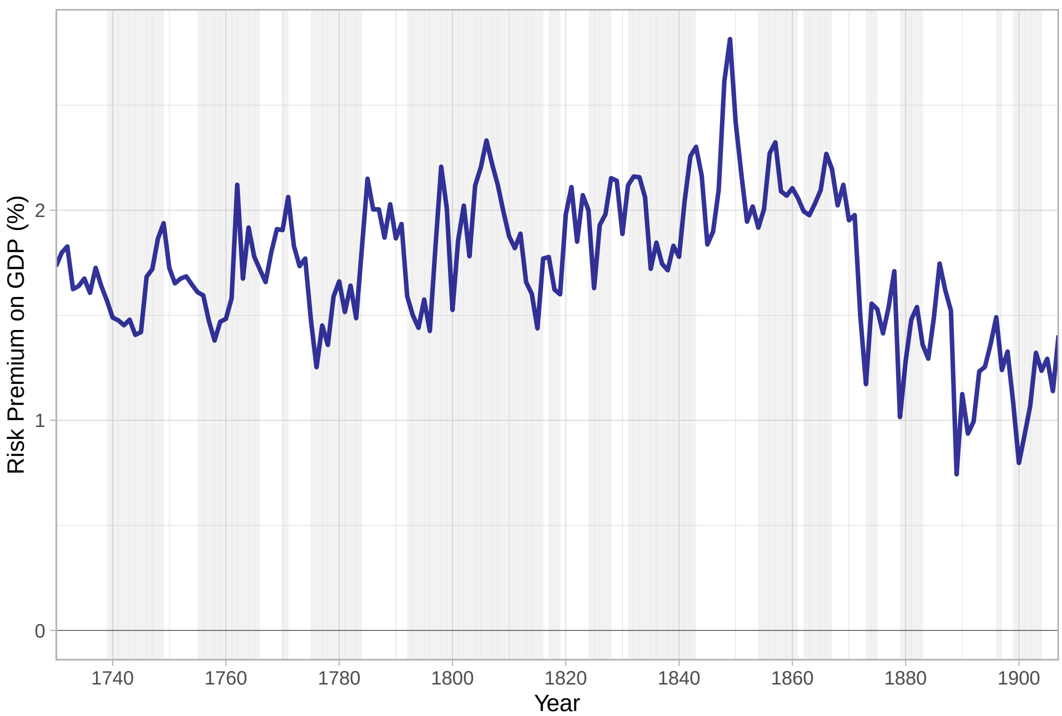
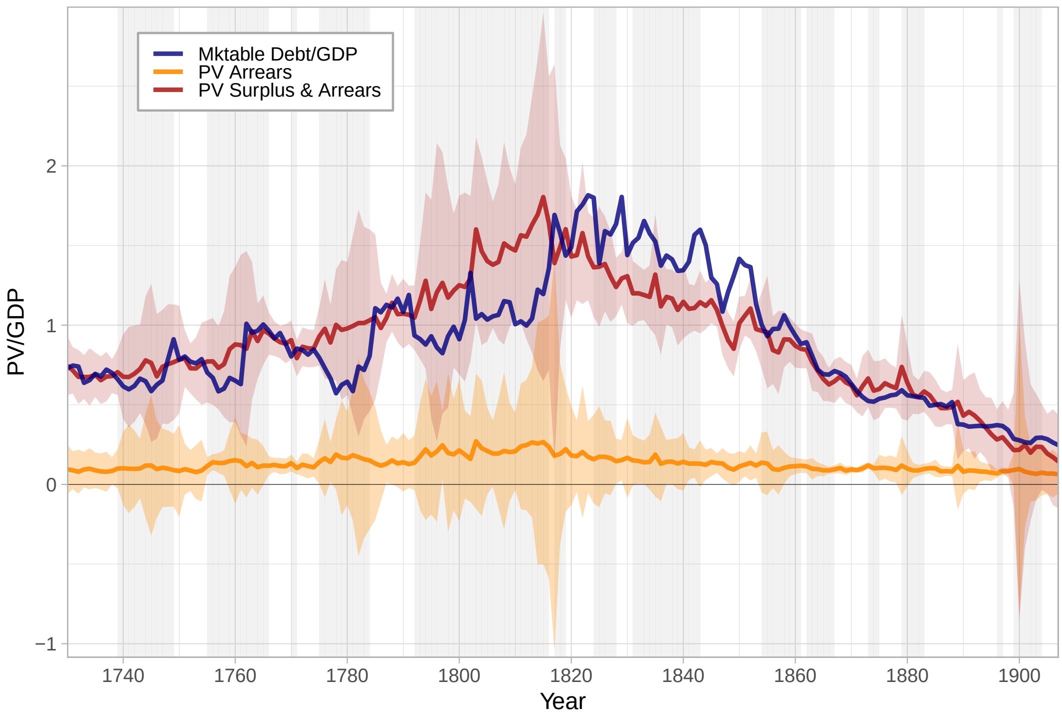


Figure 8 shows the present value of marketable debt with time-varying risk premia. The results are quite similar to the baseline results. The average present value of surpluses averages 91% and the average present value of arrears is 13%. These values are virtually identical to the baseline suggesting that the time-varying nature of risk premia is not consequential for the present value of arrears. Despite this additional degree of freedom, however, the model estimates do not improve the fit of the volatility of the observed market value of debt.

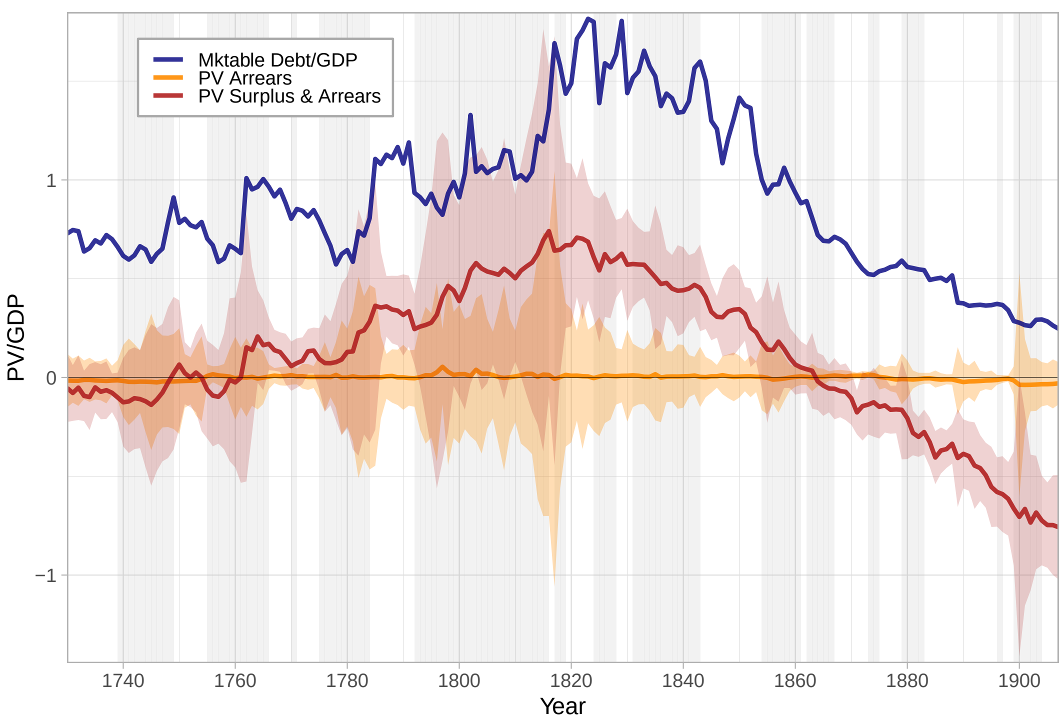
Figure 8: Upper bound of present value of marketable primary surpluses with time-varying risk premia compared with actual market value of marketable debt as % of GDP (95% confidence intervals are displayed; standard errors are obtained from bootstrap resampling with 1,000 samples; shaded areas indicate years of war)



7.3 Lower bound results

I estimate the lower bound of the present value of marketable primary surpluses by equalizing the risk premia of arrears creation and redemption and maximizing the risk premium difference between taxes and spending. It follows that the long claims of taxes and arrears creation are discounted a lot, while the short claims of spending and arrears redemption are discounted little. The results are displayed in Figure 9 and show an average present value of marketable primary surpluses of 11% of GDP, compared with the upper bound estimation of 91.6%, which is very close to the observed market prices. The present value of arrears is on average 0%, compared with the upper bound estimation of 12.9% of GDP. This result indicates that by pricing the claims on government revenues and expenditures with claim-specific risk premia, the valuation puzzle raised in Jiang et al. (2024) is reintroduced. However, this does not affect in any way the present value of arrears, which is itself derived from claim-specific risk premia. Moreover, changes in arrears should not be ignored in the valuation of marketable debt or else the valuation puzzle gets larger.

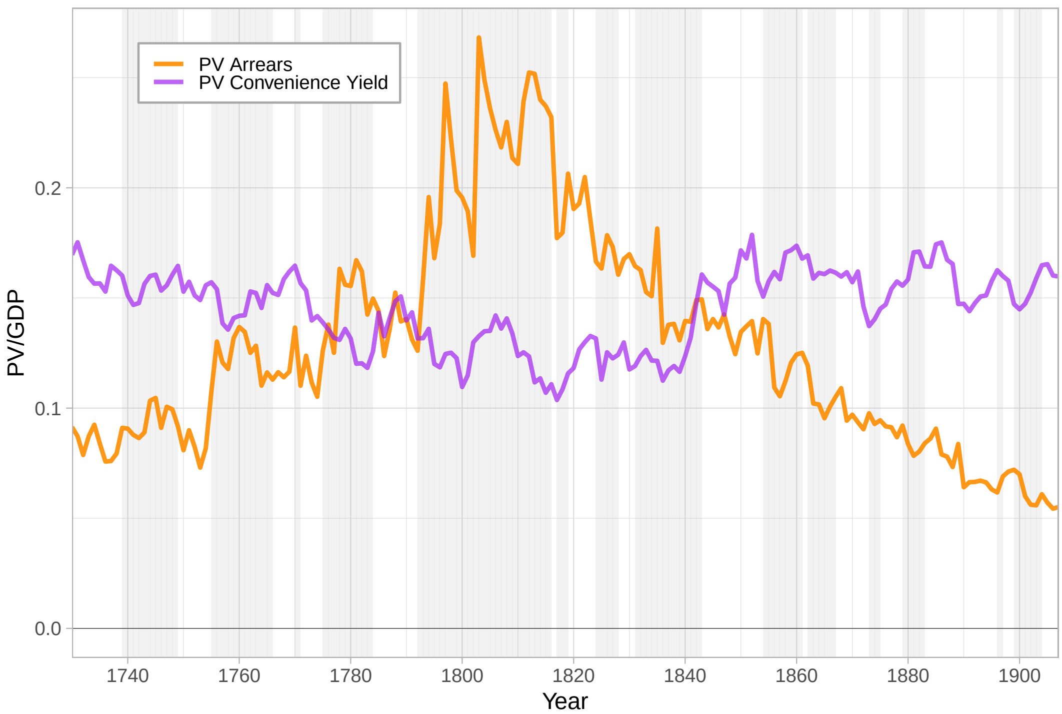
Figure 9: Lower bound of present value of marketable primary surpluses compared with actual market value of marketable debt as % of GDP
(95% confidence intervals are displayed; standard errors are obtained from bootstrap resampling with 1,000 samples; shaded areas indicate years of war)



7.4 Other sources of government revenue

In the baseline specification, I have not considered another potential source of revenue for the government: revenue from convenience yield. If bondholders are willing to pay below fundamental prices for government bonds due to non-monetary benefits, then the government can extract a stream of revenues from them. Chen et al. (2022) quantify the revenue from convenience yield to be 0.5% of GDP for the UK in the second half of the 19th century. Assuming such revenue to be constant over time, then the expected present discounted value of convenience revenue is on average 15% of GDP. This contrasts with the 12.9% upper bound present value of arrears. Figure 10 shows such comparison. As the revenues from convenience are by assumption a constant fraction of GDP, variations in their present value is driven by variation in GDP and in their discount rate. The correlation between present value of arrears and convenience yield is negative at -70%.

Figure 10: Present value of revenues from convenience yield compared with upper bound of present value of arrears changes as fraction of GDP
(shaded areas indicate years of war)

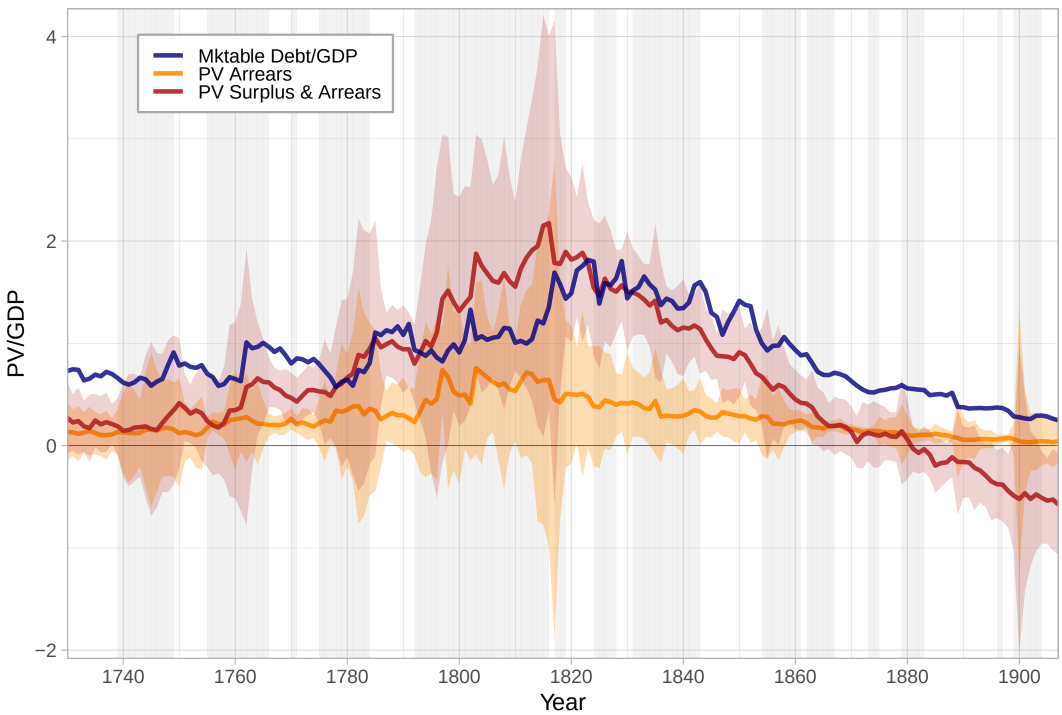


7.5 Cash flows CAPM

I estimate a standard CAPM model to obtain the price of the claims on government tax revenues, expenditures, arrears creation and arrears redemption. The model has the limitation that the loadings of each claim on the risk factor are cash flow loadings only, since the prices of such claims do not exist. However, since the final present value I aim to estimate is made of differences of present values, the same argument that justifies the estimate for the beta regressions in Section 4 also applies here. Given that return shocks are driven by both cash flow shocks and discount rate shocks, and since we are calculating differences, the impact of discount rate shocks which is common across assets is effectively neutralized.

The present values implied by the CAPM model are shown in Figure 11. The present value of marketable primary surpluses is 65% of GDP, compared to an average market value of marketable debt of 89.7%. The present value of arrears is on average 27%, compared with a baseline estimation of 12.9% of GDP. This result suggests that the choice of model can lead to different estimates of present values. The baseline specification is chosen because it better fit level and variation of the observed market price of debt. Nevertheless, even the upper bound value of arrears from the baseline appears conservative in comparison with estimates from a CAPM model.

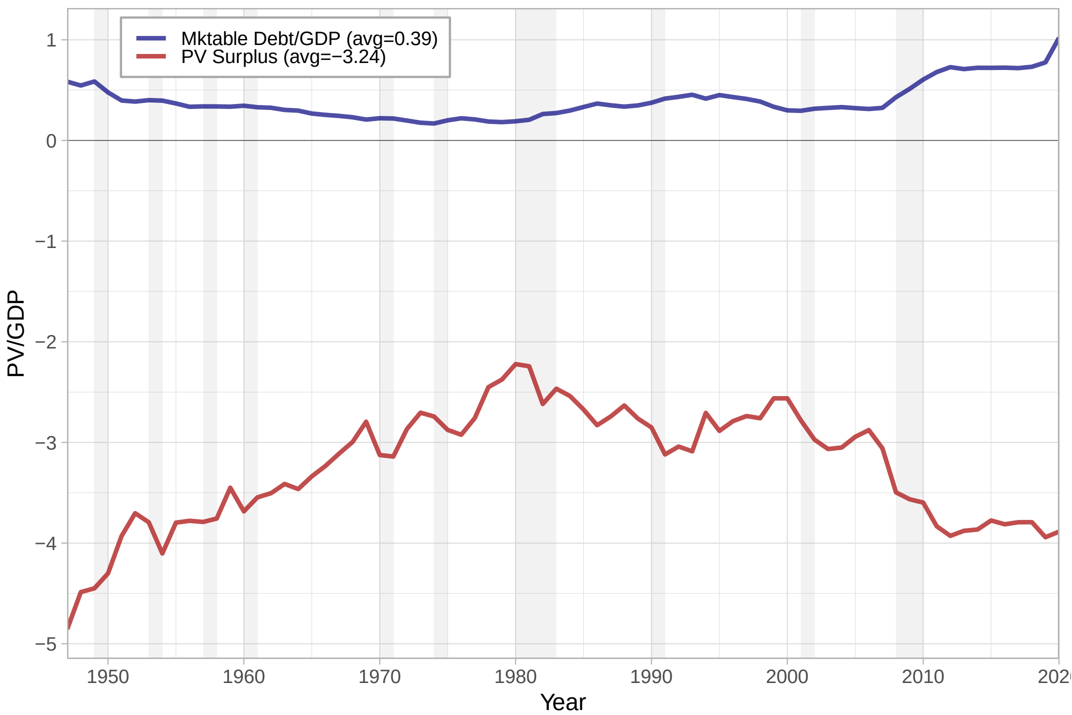
Figure 11: CAPM-implied present value of marketable primary surpluses compared with actual market value of marketable debt as % of GDP
(95% confidence intervals are displayed; standard errors are obtained from bootstrap resampling with 1,000 samples; shaded areas indicate years of war)



7.6 Validation of Empirical Methodology: Application to modern US data

The same empirical methodology from Section 4 applied to the post-WWII data used in Jiang et al. (2024) provides on average the same result as their fully fledged dynamic asset pricing model. The average lower bound of the present value of US primary surpluses is -324% of GDP compared to -320% in Jiang et al. (2024). The time series of the present value displayed in Figure 12 should be compared to Figure 9 in Jiang et al. (2024).

Figure 12: Lower bound of present value of US primary surpluses compared with the actual market value of marketable debt as % of GDP
(shaded areas indicate years of economic recession)



8 Present Day Relevance of Domestic Arrears

To estimate the incidence of domestic arrears across European countries, I combine the time series of the government *Trade credits and advances* for each country in Eurostat with the survey evidence provided by Intrum. Intrum is a Swedish debt collection company that since 2004 has been publishing the European Payment Report. In such reports, Intrum publishes for each country a payment index aimed at capturing the health of business payments. The index is constructed from a survey that Intrum conducts with a representative sample of firms in each country collecting information about payment terms, payment duration, and the difference between the two, the payment delay. Such information is collected for business-to-business transactions, as well as for business-to-government transactions and business-to-customer.

I adopt the methodology proposed by Checherita-Westphal et al. (2016) to estimate outstanding arrears. I denote $T^{dur}_{j,t}$ the average payment duration in days for business-to-government transactions for country j in year t, and $T^{term}_{j,t}$ the average payment term in days contractually agreed between government and its suppliers in country j in year t. Under the assumption that the times it takes for the government to pay an outstanding invoice can be modelled by an exponential distribution[24] with expected value equal to $T^{dur}_{j,t}$, and under the additional assumption that the size of invoices is independent of such payment duration, i.e. that the government does not systematically delay or fast-track payment of large invoices, then the cumulative distribution function of the duration of government payments is:

$$F_{j,t}(x;\lambda_{j,t}) = \begin{cases} 1 - e^{-\lambda_{j,t}x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{20}$$

where $\lambda_{j,t}$ is the rate parameter of the distribution that I can estimate with the inverse of the average payment duration, i.e. $\hat{\lambda}_{j,t} = \frac{1}{T^{dur}_{j,t}}$. Evaluating the cumulative distribution at the average payment term agreed between government and contractors, i.e. at $x = T^{term}_{j,t}$, we obtain the percentage of invoices that the government has paid within the agreed payment term. The amount of arrears is then given by:

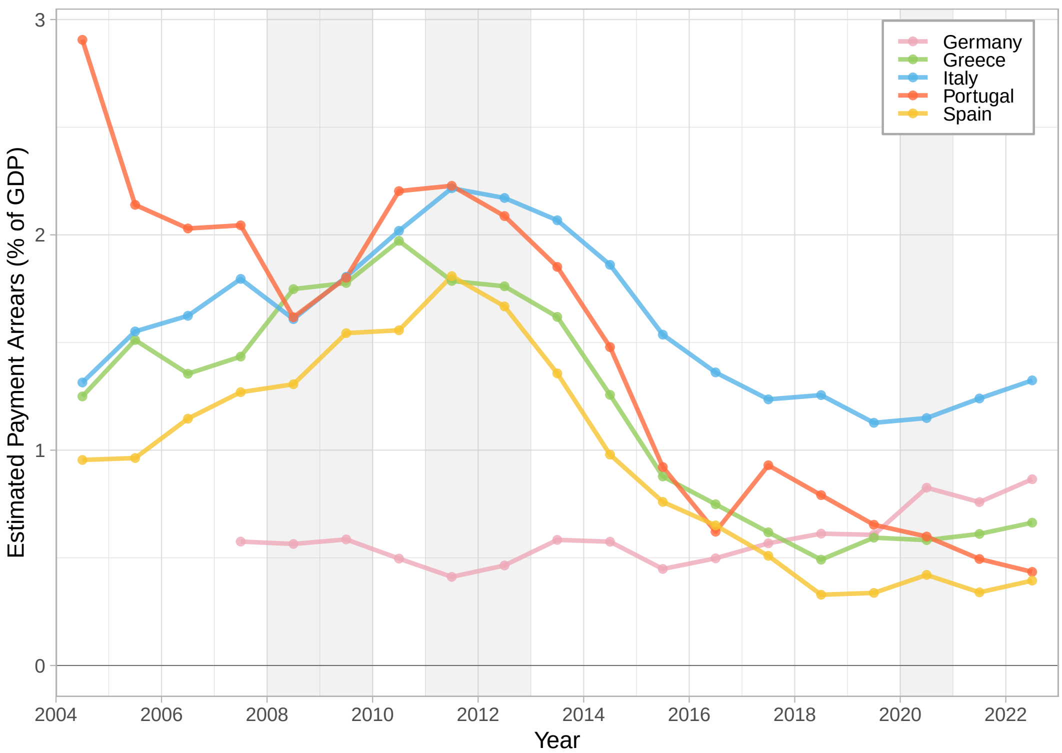
$$A_{j,t} = Trade\ credits\ and\ advances\ *\ (1 - F_{j,t}(T^{term}_{j,t}; \frac{1}{T^{dur}_{j,t}})) \tag{21}$$

Figure 13 displays the resulting time series for a subset of European countries. The figure resembles historical Figure 1 of domestic arrears in the UK in two ways. Firstly, the level of arrears is substantial: across the subset of Souther European countries displayed (Italy, Spain, Portugal and Greece) and across time it averages 1.33% of GDP. Secondly, the time variation is also similar to the historical data, with arrears creation accelerating at the onset of Great Financial Crisis, peaking at the hight of the European Debt Crisis and being repaid during non-crisis years. Given these data, I can decompose the contribution to the time variation in arrears in either increases in payment duration

[24]The exponential distribution is often used to model time-to-event data, such as waiting times, queuing times or the time until default. Here it is used to model the time until an invoice is paid.

or increase in the volume of trade credits after a simple log-linearization. Variation in payment duration contributes 20% to the variation of arrears, while variation in trade credits volume contributes 80%. Finally, it is worth nothing that some countries, like Germany, display a "natural" level of payments in arrears that does not vary over time. In conclusion, these results highlight the parallels existing between the historical and the modern use of arrears, with the resulting implications for the valuation of public debt also applying to some modern European countries.

Figure 13: Domestic arrears estimated for a subset of European countries as % of GDP (shaded areas indicate years of economic recession)



9 Conclusion

In this paper, I reconsider the government debt valuation problem in the presence of domestic arrears, which are a form of non-marketable debt consisting of overdue payments by the government to its domestic suppliers. Using UK historical data, I find that the disregard of arrears in the debt valuation equation leads to an underestimation of the risk-adjusted present value of marketable debt by 12.9% of GDP on average. This underestimation is either the consequence of the omission of revenues that the government extracts from its suppliers via financial repression or due to the mismeasurement of government expenditures which are inflated by surcharges aimed at compensating suppliers for delayed payments. Arrears play the role of shock absorbers: their present value tends to increase during bad times easing the pressure on public finances. Concretely, this means preventing rollover crises or slow-moving debt crises from happening. This motivation provides a new rationale for the existence of arrears, which institutional consensus attributes primarily to weak public financial management. Even in the absence of financial repression, the government can hope to obtain welfare gains by incurring arrears if the markets of marketable and non-marketable debt are segmented, and bondholders and arrears holders discount the future differently. The presence of domestic arrears contributes, in part, to the resolution of debt valuation puzzles raised in the literature. Moreover, to the extent that arrears represent a forced maturity extension of public debt, they help to better explain changes in yield spreads for European economies. Finally, in the paper, I address the data scarcity surrounding arrears by estimating their face value for all European countries.

10 Appendix

10.1 Proof of Proposition 1

We want to show that under fair pricing, the present value of arrears changes is zero. Start from the one-period government budget constraint in nominal terms in equation (1):

$$G_t^{no\ arr} - T_t + \underbrace{Q_{t-1}(1)}_{\text{bonds due today}} + \underbrace{A_{t-1}(1+i_t)}_{\text{arrears repaid today}} = \underbrace{\sum_{h=1}^{H} \big(Q_t(h) - Q_{t-1}(h+1)\big) P_t(h)}_{\text{net new bonds issuance}} + \underbrace{A_t}_{\text{new arrears created}}$$

Spell out the budget constraint for period t:

$$\begin{aligned}
T_t - G_t^{no\ arr} = & Q_{t-1}(1) + A_{t-1}(1+i_t) - A_t \\
& - Q_t(1)P_t(1) + Q_{t-1}(2)P_t(1) \\
& - Q_t(2)P_t(2) + Q_{t-1}(3)P_t(2) \\
& - \cdots \\
& - Q_t(H)P_t(H) + Q_{t-1}(H+1)P_t(H)
\end{aligned}$$

For $t+1$, the constraint is:

$$\begin{aligned}
T_{t+1} - G_{t+1}^{no\ arr} = & Q_t(1) + A_t(1+i_{t+1}) - A_{t+1} \\
& - Q_{t+1}(1)P_{t+1}(1) + Q_t(2)P_{t+1}(1) \\
& - Q_{t+1}(2)P_{t+1}(2) + Q_t(3)P_{t+1}(2) \\
& - \cdots \\
& - Q_{t+1}(H)P_{t+1}(H) + Q_t(H+1)P_{t+1}(H)
\end{aligned}$$

Multiply both sides by M_{t+1} and take expectations conditional on time t:

$$\begin{aligned}
E_t(M_{t+1}(T_{t+1} - G_{t+1}^{no\ arr})) = & E_t(M_{t+1})Q_t(1) + E_t(M_{t+1}A_t(1+i_{t+1})) - E_t(M_{t+1}A_{t+1}) \\
& - E_t(M_{t+1}Q_{t+1}(1)P_{t+1}(1)) + E_t(M_{t+1}Q_t(2)P_{t+1}(1)) \\
& - E_t(M_{t+1}Q_{t+1}(2)P_{t+1}(2)) + E_t(M_{t+1}Q_t(3)P_{t+1}(2)) \\
& - \cdots \\
& - E_t(M_{t+1}Q_{t+1}(H)P_{t+1}(H)) + E_t(M_{t+1}Q_t(H+1)P_{t+1}(H))
\end{aligned}$$

Use the asset pricing equations $E_t(M_{t+1}) = P_t(1)$, $E_t(M_{t+1}P_{t+1}(1)) = P_t(2)$, ... , $E_t(M_{t+1}P_{t+1}(H)) = P_t(H+1)$, as well as the definition of fair pricing $E_t(M_{t+1}(1+i_{t+1})) = 1$ to obtain:

$$\begin{aligned} E_t(M_{t+1}(T_{t+1} - G_{t+1}^{no\ arr})) =& Q_t(1)P_t(1) + A_t - E_t(M_{t+1}A_{t+1}) \\ & - E_t(M_{t+1}Q_{t+1}(1)P_{t+1}(1)) + Q_t(2)P_t(2) \\ & - E_t(M_{t+1}Q_{t+1}(2)P_{t+1}(2)) + Q_t(3)P_t(3) \\ & - \cdots \\ & - E_t(M_{t+1}Q_{t+1}(H)P_{t+1}(H)) + Q_t(H+1)P_t(H+1) \end{aligned}$$

Consider the constraint at $t+2$, multiply both sides by $M_{t+1}M_{t+2}$, take expectations conditional on time t, and use the law of iterated expectations along with $E_{t+1}(M_{t+2}) = P_{t+1}(1)$, $E_{t+1}(M_{t+2}P_{t+2}(1)) = P_{t+1}(2)$, ..., and fair pricing $E_{t+1}(M_{t+2}(1+i_{t+2})) = 1$ to obtain:

$$\begin{aligned} E_t(M_{t+1}M_{t+2}(T_{t+2} - G_{t+2}^{no\ arr})) =& E_t(M_{t+1}Q_{t+1}(1)P_{t+1}(1)) + E_t(M_{t+1}A_{t+1}) - E_t(M_{t+1}M_{t+2}A_{t+2}) \\ & - E_t(M_{t+1}M_{t+2}Q_{t+2}(1)P_{t+2}(1)) + E_t(M_{t+1}Q_{t+1}(2)P_{t+1}(2)) \\ & - E_t(M_{t+1}M_{t+2}Q_{t+2}(2)P_{t+2}(2)) + E_t(M_{t+1}Q_{t+1}(3)P_{t+1}(3)) \\ & - \cdots \\ & - E_t(M_{t+1}M_{t+2}Q_{t+2}(H)P_{t+2}(H)) + E_t(M_{t+1}Q_{t+1}(H+1)P_{t+1}(H+1)) \end{aligned}$$

Similarly, consider the constraints for $t+3$, $t+4$, ... , $t+J$. Then, add up all the constraints starting from t noting how equal terms with opposite sign cancels out leaving only:

$$\begin{aligned} E_t\left[\sum_{j=0}^{J} M_{t,t+j}\left(T_{t+j} - G_{t+j}^{no\ arr}\right)\right] =& \sum_{h=0}^{H} Q_{t-1}(h+1)P_t(h) \\ & - E_t\left[M_{t,t+J}\sum_{h=1}^{H} Q_{t+j}(h)P_{t+j}(h)\right] \\ & + A_{t-1}(1+i_t) - E_t\left[M_{t,t+J}A_{t+J}\right] \end{aligned}$$

where $M_{t,t+j} = \prod_{k=0}^{j} M_{t+k}$ is a nominal multi-period SDF.

Note that the last two terms can also be written as $A_{t-1}(1+i_t) - E_t\left[M_{t,t+J}A_{t+J}\right] = -E_t[\sum_{j=0}^{J}(M_{t,t+J}\Delta\widetilde{A}_{t+j})]$. This can be seen by grouping the arrears terms as differences instead of cancelling out the identical terms:

$$\underbrace{A_{t-1}(1+i_t)-A_t}_{-\Delta\widetilde{A}_t}+\underbrace{A_t-E_t(M_{t+1}A_{t+1})}_{-E_t[M_{t+1}\Delta\widetilde{A}_{t+1}]}+\underbrace{E_t(M_{t+1}A_{t+1})-E_t(M_{t+1}M_{t+2}A_{t+2})}_{-E_t[M_{t+1}M_{t+2}\Delta\widetilde{A}_{t+2}]}+...$$

where I used the definition of fair prices and the law of iterated expectations to write: $A_t - E_t(M_{t+1}A_{t+1}) = -E_t(M_{t+1}A_{t+1} - A_t) = -E_t(M_{t+1}A_{t+1} - A_t E_t(M_{t+1}(1+i_{t+1}))) = -E_t(M_{t+1}A_{t+1} - A_t M_{t+1}(1+i_{t+1})) = -E_t(M_{t+1}\Delta\widetilde{A}_{t+1})$ and so on for the other difference terms.

Assume that we start from zero arrears, i.e. $A_{t-1} = 0$, take the limit $J \to \infty$ and impose a transversality condition for the outstanding amount of arrears, such that they cannot grow faster than the SDF, i.e. $\lim_{J\to\infty} E_t\left[M_{t,t+J}A_{t+J}\right] = 0$. Then, we obtain the result:

$$\lim_{J\to\infty} A_{t-1}(1+i_t) - E_t\left[M_{t,t+J}A_{t+J}\right] = \lim_{J\to\infty} -E_t\left[\sum_{j=0}^{J}(M_{t,t+J}\Delta\widetilde{A}_{t+j})\right] = 0$$

Note that this results is obtained under the assumption of fair pricing of arrears, defined as $E_{t+j}(M_{t+1+j}(1+i_{t+1+j})) = 1\ \forall j$. Without such assumption, the expected present discounted value of arrears changes does not simplify and instead remains in the valuation equation as in equation (2).

10.2 Proof of Proposition 5

We want to show that valuation equation (3), delivers the correct fiscal backing given the presence of arrears for any degree of financial repression or overcompensation $\underline{\phi_t} \leq \phi_t \leq \overline{\phi_t}\ \forall t$. The most straightforward way to prove this is to start from the valuation equation for the market value of total public liabilities in the presence of arrears. Total public liabilities are composed of a marketable and a non-marketable part, whose market values are indicated by V_t and NM_t respectively. Then, under the standard assumption of no bubbles and Assumption 1 that the interest payment on arrears takes the form of surcharge to the government:

$$V_t + NM_t = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(\underbrace{T_{t+j} - G^{gross}_{t+j}}_{-\text{change in liabilities}} + \underbrace{int_{t+j} + \phi_{t+j}G^{no\ arr}_{t+j}}_{\text{interest payments}}\right)\right]$$

where in parenthesis on the right-hand side there is the cash flow destined to the holder of all public liabilities, composed of tax revenues T_{t+j}, and G^{gross}_{t+j}, which are government gross expenditures inclusive of interest payments for bondholders int_{t+j} and surcharges for suppliers $\phi_{t+j}G^{no\ arr}_{t+j}$. Gross expenditures need to be netted out by all interest payments to obtain the primary surplus. An alternative way to think about the cash flow destined to the holder of all public liabilities is as the negative change in liabilities plus the sum of all interest payments paid by the government. There is a minus in front of the change in liabilities because a surplus ($T_{t+j} > G^{gross}_{t+j}$) reduces outstanding liabilities. Note that while V_t, T_{t+j}, G^{gross}_{t+j} and int_{t+j} are observables in the data, NM_t and $\phi_{t+j}G^{no\ arr}_{t+j}$ are not. Moreover, the following definitions hold: $G_{t+j} = G^{gross}_{t+j} - int_{t+j}$ and $G^{no\ arr}_{t+j} = G^{gross}_{t+j} - int_{t+j} - \phi_{t+j}G^{no\ arr}_{t+j}$.

Analogously, the valuation equation for the market value of non-marketable debt is

$$NM_t = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(\underbrace{-\Delta A_{t+j}}_{-\text{change in arrears}} + \underbrace{\phi_{t+j}G^{no\ arr}_{t+j}}_{\text{interest payments}}\right)\right] = -W_t$$

where in parenthesis on the right-hand side there is the cash flow destined to the holder of arrears, composed of negative arrears changes and interest payments. W_t is the valuation wedge introduced by financial repression and defined in equation (5).

I combine these two equations to write the fiscal backing, i.e. the valuation equation for marketable liabilities, as:

$$\begin{aligned}
V_t &= E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G^{gross}_{t+j} + int_{t+j} + \phi_{t+j}G^{no\ arr}_{t+j}\right)\right] - NM_t \\
&= E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G^{gross}_{t+j} + int_{t+j} + \phi_{t+j}G^{no\ arr}_{t+j} + \Delta A_{t+j} - \phi_{t+j}G^{no\ arr}_{t+j}\right)\right] \\
&= E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G_{t+j} + \Delta A_{t+j}\right)\right]
\end{aligned}$$

where I simplified equal terms and used the definition $G_{t+j} = G^{gross}_{t+j} - int_{t+j}$. While the degree of financial repression or overcompensation affects the sign of NM_t it does not affect the valuation equation for the fiscal backing. Note that $NM_t = 0$ if $\phi_t = \phi^*_t\ \forall t$, $NM_t < 0$ if $\underline{\phi_t} \leq \phi_t < \phi^*_t\ \forall t$, and $NM_t > 0$ if $\phi^*_t < \phi_t \leq \overline{\phi_t}\ \forall t$.

10.3 Suppliers' compensation via surcharges *and* interest payments

If the government compensates suppliers for borrowing via arrears by paying an interest on the amounts in arrears in addition to inflated public procurement prices, the fiscal backing estimated from valuation equation (3) is overstated. To see this, start from the valuation equation for total public liabilities, which in this case is:

$$V_t^{act} + NM_t^{act} = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G_{t+j}^{gross} + int_{t+j} + i_{t+j}A_{t-1+j} + \phi_{t+j}G_{t+j}^{no\ arr}\right)\right]$$

where int_{t+j} is the interest payment to bondholders, $i_{t+j}A_{t-1+j}$ is the interest payment on arrears and $\phi_{t+j}G_{t+j}^{no\ arr}$ is the surcharge of government expenditures. I use the superscript *act* to indicate that those are the actual values derived under a different set of assumptions. The data reports only the overall interest rate expenditures which include int_{t+j} and $i_{t+j}A_{t-1+j}$ jointly. Then, government spending net of interest payments is $G_{t+j} = G_{t+j}^{gross} - int_{t+j} - i_{t+j}A_{t-1+j}$.

The valuation equation for non-marketable debt is now:

$$NM_t^{act} = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(-\Delta A_{t+j} + i_{t+j}A_{t-1+j} + \phi_{t+j}G_{t+j}^{no\ arr}\right)\right]$$

By combining the two equations, we get:

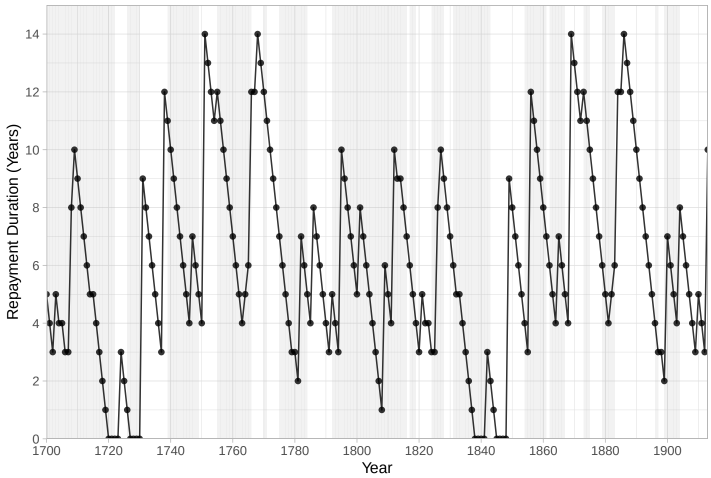
$$V_t^{act} = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G_{t+j} + \Delta A_{t+j}\right)\right] - E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(i_{t+j}A_{t-1+j}\right)\right]$$

Given $E_t[\sum_{j=0}^{\infty} M_{t,t+j}(i_{t+j}A_{t-1+j})] > 0$, meaning that the flow of interests has a positive present value, it follows that $V_t > V_t^{act}$, where V_t is derived from valuation equation (3). Historically, interests were only paid on arrears outstanding for more than six months and anecdotal evidence suggests that prices of public procurement contracts were inflated to compensate suppliers. In the modern day, a European Commission survey reports that the majority of government suppliers do not claim interest in case of late payment, despite legislation that prescribes such compensations. Therefore, $E_t[\sum_{j=0}^{\infty} M_{t,t+j}(i_{t+j}A_{t-1+j})]$ is likely small.

10.4 Repayment Duration

Assuming no turnover in the repayment of arrears and relying instead exclusively on the repayment achieved by negative net flows and the first-in, first-out principle leads to repayment durations of up to 14 years for arrears incurred in certain years. Figure 14 shows the repayment durations.

Figure 14: Repayment duration in years for outstanding domestic arrears estimated assuming gross flows equal to net flows and first in, first out principle
(shaded areas indicate years of war)



10.5 Cointegration Test

I perform a Johansen cointegration test, which estimates in VECM form the VAR:

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \mu + \varepsilon_t \quad \text{with } X_t = \begin{pmatrix} \log GDP_t \\ \log T_t \\ \log G_t \end{pmatrix} \text{and } X_t = \begin{pmatrix} \log GDP_t \\ \log AC_t \\ \log AR_t \end{pmatrix}$$

and tests for the presence of r cointegration relationships. The test statistics are reported in Table 3 and 4. In both cases, the null hypothesis of zero cointegration relationships is rejected, while the hypotheses of either 1 or 2 cannot be rejected. This means, that both taxes and spending, and arrears creation and redemption share $r = 1$ cointegration relationship with GDP. The results from a maximum eigenvalue test confirm the same finding.

Table 3: Johansen Cointegration Test for Taxes and Expenditures: Trace Statistic

	Statistic	Critical90	Critical95	Critical99
r <= 2 |	0.63	6.50	8.18	11.65
r <= 1 |	5.70	15.66	17.95	23.52
r = 0 |	38.30	28.71	31.52	37.22

In a joint cointegration test with $X_t = (\log GDP_t, \log T_t, \log G_t, \log AC_t, \log AR_t)'$, up to 3 cointegration relationships are identified. Table 5 displays the results for the trace test. The maximum eigenvalue test also rejects the hypothesis $r \leq 2$ at 1% level.

Table 4: Johansen Cointegration Test for Arrears Creation, Redemption: Trace Statistic

	Statistic	Critical90	Critical95	Critical99
r <= 2 |	0.92	6.50	8.18	11.65
r <= 1 |	15.48	15.66	17.95	23.52
r = 0 |	152.37	28.71	31.52	37.22

Table 5: Johansen Cointegration Test: Trace Statistic

	Statistic	Critical90	Critical95	Critical99
r <= 4 |	0.53	6.50	8.18	11.65
r <= 3 |	5.39	15.66	17.95	23.52
r <= 2 |	32.06	28.71	31.52	37.22
r <= 1 |	72.36	45.23	48.28	55.43
r = 0 |	218.53	66.49	70.60	78.87

10.6 VAR Estimation

The coefficients of matrix Γ from equation (10) are displayed in Table 6.

10.7 Beta with GDP growth

Figure 15 displays the cash-flow betas of taxes, spending, arrears creation and redemption with respect to real GDP growth at different horizons. The coefficients are estimated from the following regression:

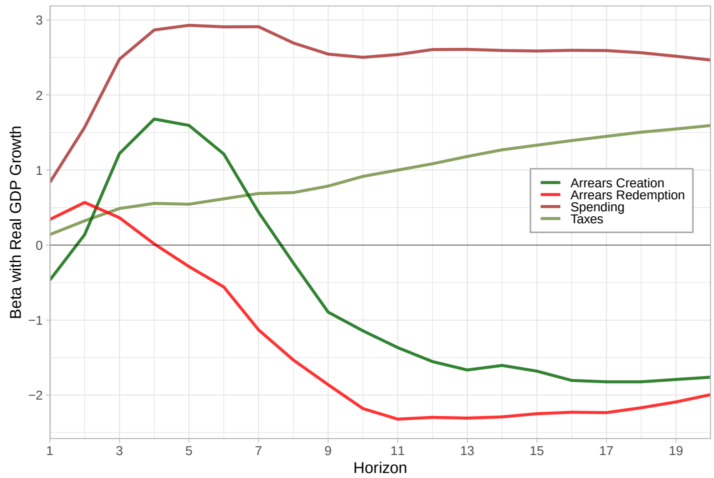
$$\sum_{t=1}^{h} \Delta \ln (CF_t) = c + \beta_h^{CF,M} \sum_{t=1}^{h} \Delta \ln (GDP_t) + \epsilon_t$$

where CF_t is either the flow of taxes, spending, arrears creation or redemption. The fact that, contrary to expectations, government spending and arrears creation appear to be pro-cyclical is testament to the fact that there was no public welfare spending in the period under consideration. Instead government spending and arrears creation were aimed at paying for war, which more often than not coincided with periods of economic growth. Nevertheless, there is no reason to presume that wars represented states of large consumption, or low marginal utility growth, for the average investor. Since historically a bad state of the world for investors in the UK would materialise when resources had to be diverted to fend off a possible French or Spanish invasion, rather than due to few quarters of negative economic growth, proxying the SDF with stock returns rather than GDP growth seems appropriate.

Table 6: VAR Estimates

	π_t	$ysht^{\$}_t$	$yspr^{\$}_t$	x_t	$\Delta \log div^M_t$	$\log div^M_t$	pd^M_t	$\Delta \log \tau_t$	$\log \tau_t$	$\Delta \log g_t$	$\log g_t$	$\Delta \log ac_t$	$\log ac_t$	$\Delta \log ar_t$	$\log ar_t$	$\Delta \log d_t$	$\log d_t$
π_{t-1}	0.13	0.06	-0.08	0.15	0.02	0.02	0.36	0.02	0.02	0.09	0.09	-0.55	-0.55	-0.49	-0.49	-0.60	-0.60
$ysht^{\$}_{t-1}$	3.29	0.88	0.03	0.51	1.76	1.76	-7.79	-2.46	-2.46	-3.05	-3.05	3.98	3.98	-11.20	-11.20	-1.97	-1.97
$yspr^{\$}_{t-1}$	3.53	0.51	0.40	0.83	0.37	0.37	-6.94	-2.71	-2.71	-1.90	-1.90	2.31	2.31	-13.63	-13.63	-2.62	-2.62
x_{t-1}	0.07	0.09	-0.10	-0.16	0.70	0.70	-0.25	0.32	0.32	-0.09	-0.09	-0.08	-0.08	0.30	0.30	-0.42	-0.42
$\Delta \log div^M_{t-1}$	-0.02	0.01	-0.01	0.03	-0.14	-0.14	0.03	0.00	0.00	-0.08	-0.08	0.42	0.42	-0.11	-0.11	-0.10	-0.10
$\log div^M_{t-1}$	0.08	0.01	-0.01	0.03	-0.03	0.97	-0.07	-0.12	-0.12	-0.18	-0.18	-0.26	-0.26	-0.29	-0.29	-0.15	-0.15
pd^M_{t-1}	0.10	0.02	-0.02	0.04	0.30	0.30	0.47	-0.14	-0.14	-0.30	-0.30	-0.11	-0.11	-0.31	-0.31	-0.30	-0.30
$\Delta \log \tau_{t-1}$	-0.14	0.01	-0.02	-0.02	0.12	0.12	0.17	0.25	0.25	-0.22	-0.22	0.51	0.51	0.01	0.01	0.19	0.19
$\log \tau_{t-1}$	0.05	0.00	-0.01	0.06	-0.07	-0.07	0.08	-0.16	0.84	-0.01	-0.01	-0.15	-0.15	-0.23	-0.23	-0.15	-0.15
$\Delta \log g_{t-1}$	0.03	0.00	-0.00	0.03	-0.03	-0.03	-0.06	-0.01	-0.01	0.28	0.28	0.00	0.00	-0.23	-0.23	-0.09	-0.09
$\log g_{t-1}$	0.02	0.01	-0.01	0.00	-0.02	-0.02	-0.06	0.02	0.02	-0.20	0.80	0.10	0.10	-0.01	-0.01	0.00	0.00
$\Delta \log ac_{t-1}$	0.02	0.00	-0.00	-0.00	-0.02	-0.02	0.01	0.02	0.02	0.03	0.03	0.06	0.06	-0.12	-0.12	-0.06	-0.06
$\log ac_{t-1}$	-0.01	-0.00	-0.00	0.00	-0.03	-0.03	0.05	-0.01	-0.01	0.02	0.02	-0.12	0.88	0.66	0.66	0.05	0.05
$\Delta \log ar_{t-1}$	-0.01	0.00	0.00	0.00	0.06	0.06	-0.05	-0.00	-0.00	0.03	0.03	-0.08	-0.08	-0.05	-0.05	-0.01	-0.01
$\log ar_{t-1}$	0.00	-0.00	0.00	-0.01	0.03	0.03	-0.04	0.02	0.02	0.00	0.00	-0.11	-0.11	-0.69	0.31	-0.02	-0.02
$\Delta \log d_{t-1}$	0.02	-0.00	-0.01	0.04	0.09	0.09	0.26	-0.07	-0.07	-0.57	-0.57	-0.48	-0.48	0.11	0.11	-0.22	-0.22
$\log d_{t-1}$	0.04	0.01	-0.01	0.02	0.05	0.05	-0.12	-0.06	-0.06	-0.20	-0.20	-0.06	-0.06	-0.16	-0.16	-0.12	0.88
const	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.01

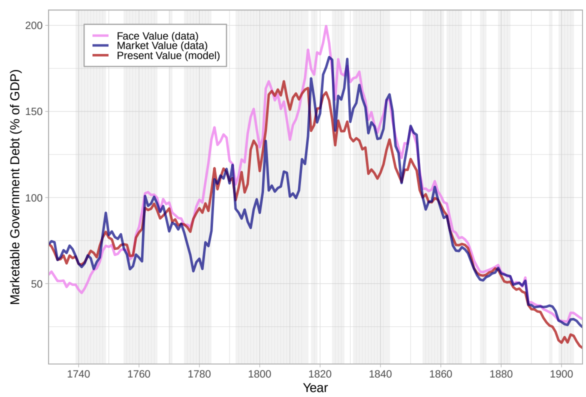
Figure 15: Cash-flow beta exposure of excess growth rates of taxes, spending, arrears creation, and arrears redemption with real GDP excess growth at different horizons



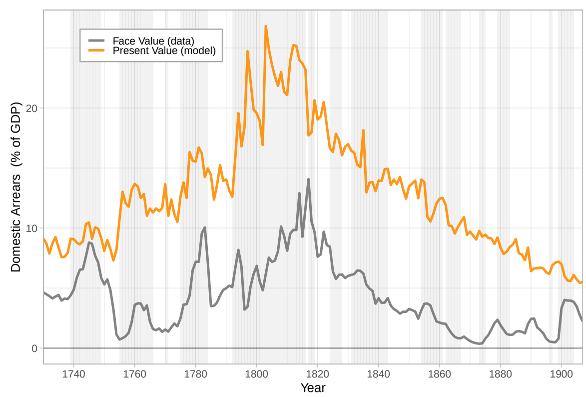
10.8 Face values

Figure 16 displays the model implied value of marketable debt and arrears in comparison with their face value. The figure underscore the importance of not attributing to arrears the same market-to-face-value ratio of marketable debt. This follows from the fact that bonds and arrears are two distinct types of security.

Figure 16: Face values of marketable debt and arrears compared with model-implied present values (shaded areas indicate years of war)



(a) Marketable government debt

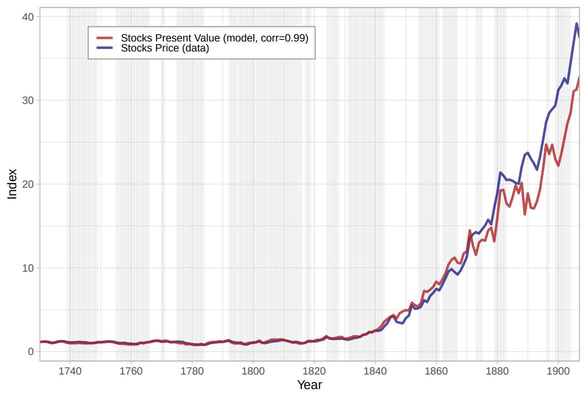


(b) Arrears

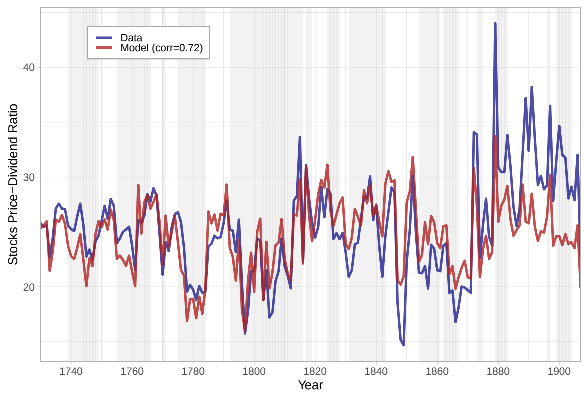
10.9 Fit of stocks prices with fixed risk premium

Under the baseline assumption of fixed risk premia, Figure 17 displays the fit of model-implied stock prices and price-dividend ratios with actual ones. The fit becomes perfect in the model variant that allows time-varying risk premia.

Figure 17: Model implied equity prices and price-dividend ratios compared with actual ones given a constant equity risk premium
(shaded areas indicate years of war)



(a) Equity prices



(b) Price-dividend ratios

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