CRVUSD Explained

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1 Overview

From the CRVUSD WP

2 Analysis

The underlying idea is to create an AMM supporting concentrated liquidity whose pricing algo works differently from a standard AMM, in the following way - it depends on an external oracle price - it prices the base token (i.e. collateral) so that at the lowest price of a liquidity range the reserves are fully in quote token (e.g. USDC) and at the highest price of a liquidity range the reserves are fully in base token (e.g. ETH)

So the dynamic of the reserves is exactly the opposite of a standard AMM and it guarantees - on the lender side, the loan is always backed by enough debt token and - on the borrower side, the collateral is maintained if the price allows it

For this system to work in a fully autonomous way, it is required the arbers to adjust the reservers according to the above and this can be done by creating a pricing algo that - prices ETH below the market price when ETH goes down, this way the arbers are going to buy ETH from the LLAMMA and sell it on spot therefore removing ETH from the reserves - prices ETH above the market price when ETH goes up, this way the arbers are going to buy ETH on spot and sell it on the LLAMMA therefore adding ETH to the reserves

3 Idea

The LLAMMA is essentially a customized UniV3 AMM so it shares some aspects like - concentrated liquidity, the LLAMMA segments the price space in liquidity ranges called "bands" which are continuous to each other (according to the WP) i.e. there is no gap between them

- standard pricing algo based on preserving the usual hyperbolic bonding curve invariant resulting from the combination of real reserves and virtual reserves, since concentrated liquidity is supported

The general idea to implement such a pricing algorithm is essentially to make virtual reserves functions of an external oracle price so that the hyperbolic bonding curve can be transformed properly, so the price at a given point in time, identified by a point on the hyperbola, is associated to a different derivative as the hyperbola changes its shape as the virtual reserves also change

This can be understood using analogy with Thermal Physics as - each standard AMM pricing following an hyperbolic bonding curve is equivalent to moving along an isotherm - each external price p_o value identifies a different isotherm

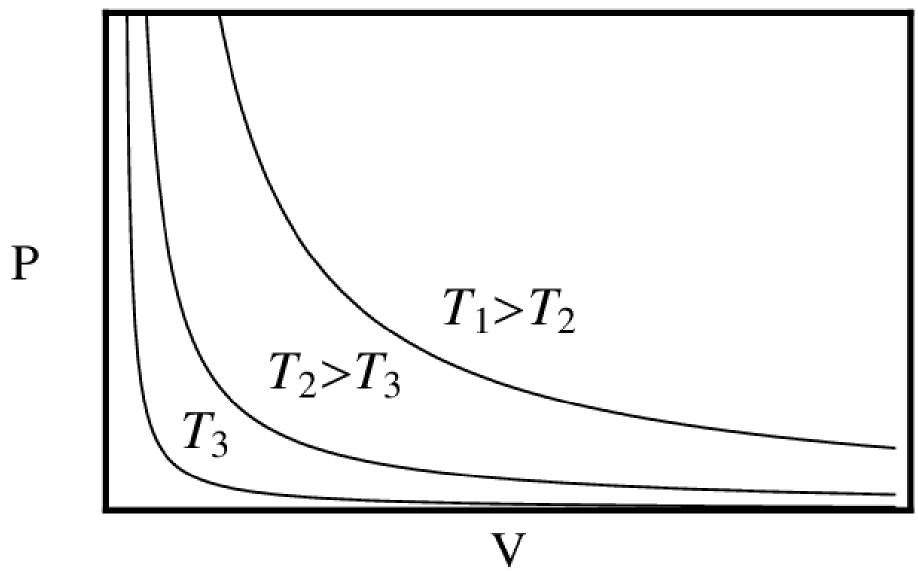

Figure 1: Alt text

So given a p_o the LLAMMA pool price p moves along the isotherm driven by a standard standard AMM pricing algo

This is what they describe in the WP here

> While oracle price p_o stays constant, the AMM works in a normal way, e.g. sells ETH when going up / buys ETH when going down. By simply substituting

Figure 2: Alt text

When p_o changes, it is basically like moving an isotherm to another one like in an adiabatic transformation

and as the isotherm changes, the pool price p changes by moving on the new

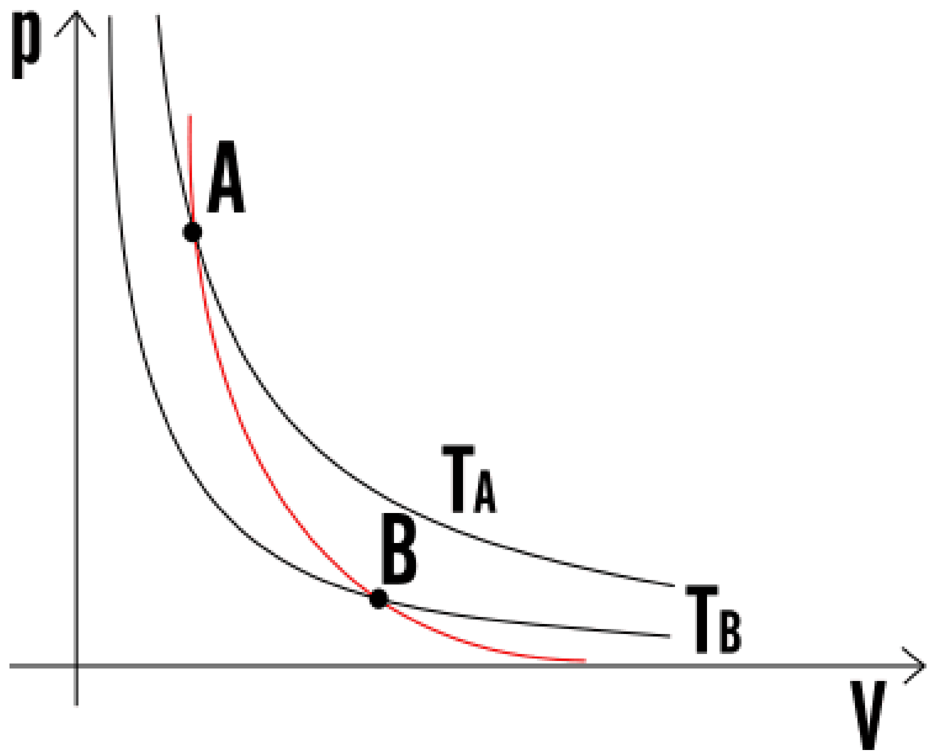

Figure 3: Alt text

instantaneous isotherm and in the (x, y) reserves space, by preserving the instantaneous invariant (polytropic process)

In the WP it is summarized here

> and ramps its "center price" from (for example) down to up, the tokens will adiabatically convert from (for example) USD to ETH while proving liquidity in both ways on the way (Fig. 3). It is somewhat similar to avoided crossing

Figure 4: Alt text

and described this way

> We start from a number of bands where, similarly to Uniswap3, hyperbolic shape of the bonding curve is preserved by adding virtual balances. Let say, the amount of USD is x, and the amount of ETH is y, therefore the "amplified" constant-product invariant would be:
>
> $$I = (x + f)(y + g). \tag{1}$$
>
> We also can denote $x' \equiv x + f$ and $y' \equiv y + g$ so that the invariant can be written as a familiar $I = x'y'$.
>
> However, f and g do not stay constant: they change with the external price oracle (and so does the invariant I, so it is only the invariant while the oracle price p_o is unchanged). At a given p_o, f and g are constant across the band.

Figure 5: Alt text

which basically means $f(p_o), g(p_o)$ determine $I(p_o)$ and each $I(p_o)$ identifies a different isotherm therefore the $I(p_o(t_1)), I(p_o(t_2))$ identifies the adiabatic process

4 Stable Situation

The price dynamic in a stable situation is like a normal Uniswap AMM i.e. like moving on an isotherm

This picture is confusing since the price are reversed: in the lower part of the description in fact we have $p_{cd} < p < p_{cu}$ while in the picture $p_{cd} > p > p_{cu}$

In the $p_{cd} < p < p_{cu}$ case then being all in ETH at p_{cu} and all in USDC at p_{cd} as described is actually the standard AMM behavior which is expected

The purpose of the oracle price in this case is just to define where the liquidity has to be concentrated, this is a strong differentiation point with respect to a Uniswap-like pool that does not need any oracle and where the liquidity has to be concentrated is decided by the LPers.

In this stable situation, the external price is assumed to remain constant and the AMM needs to ensure to keep its price pegged to the market price

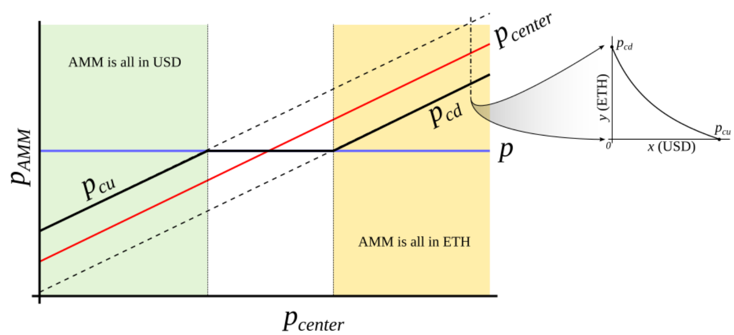

Figure 3: Behavior of an "AMM with an external price source". External price p_{center} determines a price around which liquidity is formed. AMM supports liquidity concentrated from prices p_{cd} to p_{cu}, $p_{cd} < p_{center} < p_{cu}$. When current price p is out of range between p_{cd} and p_{cu}, AMM is either fully in stablecoin (when at p_{cu}) or fully in collateral (when at p_{cd}). When $p_{cd} \leq p \leq p_{cu}$, AMM price is equal to the current price p.

Figure 6: Alt text

This is achieved by having the LLAMMA work as a standard AMM i.e. the price moves on the usual hyperbolic bonding curve (see top right of the figure) that depends on both - real local reserves - virtual reserves that depend on the band / liquidity range position in the price space

This works exactly like a UniV3 Pool

Since this is the same pricing algo used by the spot market, the LLAMMA price should not diverge from the market price otherwise this would create an arb opportunity

5 Market price moves

When the oracle price moves, what happens is the virtual reserves change which has the effect of transforming the hyperbolic bonding curve In the WP they call it "adiabatic" transformation as it resembles that

This defines a transient phase where $p_{cd} > p$ or $p > p_{cu}$ depending whether the oracle price moved up or down respectively, so the opposite as the previous situation, which effectively completely flips the reserves by the effect of the arbitrage with respect to the market price

From an AMM perspective, it is like moving the band where the liquidity is concentrated in the price space as follows - initially we can imagine we are in the $p_{cd} < p < p_{cu}$ situation, so the content of the liquidity band is a mix of ETH

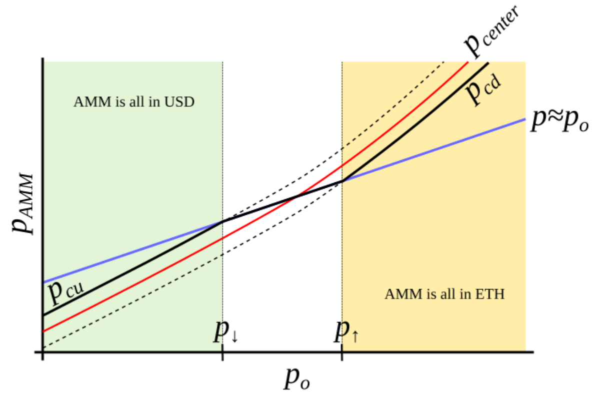

Figure 4: AMM which we search for. We seek to construct an AMM where p_{cd} and p_{cu} are such functions of p_o that when p_o grows, they grow even faster. In this case, this AMM will be all in ETH when ETH is expensive, and all in USD when ETH is cheap.

Figure 7: Alt text

and USDC - at some point price of ETH starts to go down, if the band would not move when the $p = p_{cd}$ the band was full of ETH which is the opposite of the desired behavior - instead, since $p_{cd}(p_o), p_{cu}(p_o)$ move faster than the oracle price, as we get a new value of p'_o for the oracle price, we also get a new position for the liquidity band $p_{cd}(p'_o), p_{cu}(p'_o)$ which we can call p'_{cd}, p'_{cu} to a situation where $p'_{cu} < p$ which is equivalent to having the band full of USDC, the content of the band would start to mix as soon as $p = p'_{cu}$ - eventually, since because of the arb forces at equilibrium we will have $p' \simeq p'_o$ we will have the pool price inside the new band so $p'_{cd} < p' < p'_{cu}$

From a thermodynamic perspective, this new isotherm / hyperbolic bonding curve, at the same amount of base and quote token than before the market price change, prices base token either lower or higher than the market depending if the market price moved down or up respectively For example, if the ETH price moved down, the way the LLAMMA can price it lower than the market price is by increasing its virtual amount acting on its virtual reserves, therefore the standard bonding curve will price it lower since it sees that asset as (virtually) more abundant than it is is reality

The opposite is also true

6 Acknowledgments

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